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THEORY OF COMBUSTION NOISE

PRINCETON UNIVERSITY

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The near field study reveals (i) two different aerothermochemical roles played by steady and non-steady heat release rate with regard to the sound generation, and (ii) the conditions for the resonant oscillation and the self-sustained oscillation of the sound wave.

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THEORY OF COMBUSTION NOISE

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by

H. H. Chiu and M. Summerfield

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ABSTRACT

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NOMENCLATURE

a	=	speed of sound
A	=	nondimensional speed of sound or constant defined by Eq. (111)
A_n	=	constant in the solution of pressure fluctuation
B	=	constant defined by Eq. (111)
B_k	=	constant in the frequency factor for the k^{th} reaction
B_n	=	constant in the solution of pressure fluctuation
c	=	constant in the expression of the ratio of turbulent burning speed to that of laminar burning speed
C	=	constant defined by Eq. (111)
C_n	=	constant in the solution of pressure fluctuation
d	=	diameter of a droplet
D_n	=	constant in the solution of pressure fluctuation
E_k	=	activation energy for the k^{th} reaction
f	=	non-dimensional number density of droplet
f_{nj}	=	inhomogeneous function appears in Eq. (20)
F	=	wave function defined by Eq. (33) or constant defined by Eq. (112)
$\mathcal{F}$	=	function defined by Eq. (28)
$\tilde{g}_{nj}$	=	inhomogeneous function appearing in Eq. (19)
G	=	constant defined by Eq. (112)
h_i	=	specific enthalpy of species i
h_{nj}	=	inhomogeneous function appearing in Eq. (18)
H	=	constant defined by Eq. (112)
ΔH	=	heat of reaction
I	=	sound intensity
k	=	rate constant
K	=	function defined by Eq. (35) or kernel defined by Eq. (93)
K_{nj}	=	inhomogeneous function appearing in Eq. (17)
ℓ	=	flame thickness
L	=	linear differential operator defined by Eq. (21) or constant defined by Eq. (112), or latent heat of vaporization
m	=	mass of a droplet
$\dot{m}$	=	mass burning rate of a droplet

M = Mach number

$\mathcal{M}_i$ = chemical symbol for species i

n = number density of droplets

N = matrix defined by Eq. (91) or reference number density of droplets

p = pressure

$\dot{Q}$ = effective rate of heat release defined by Eq. (8), or by Eq. (86)

r = nondimensional radial coordinate

Re = Reynolds number

$\mathcal{R}$ = gas constant

$\mathcal{R}_{ij}$ = double correlation function

s = burning speed or specific entropy

S_{ij} = flame structural factor

$\mathcal{S}_n$ = fluctuating rate of heat release appearing in Eq. (86)

t = time

T = temperature

u = velocity component in axial direction

U = constant defined by Eq. (112)

v = velocity component in transverse direction

$\bar{v}$ = velocity vector

V = matrix defined by Eq. (91), or velocity

$\bar{w}$ = rotational velocity component

W = molecular weight

X = Mole fraction

y = mass fraction

Greek symbols

α_k = exponent determining the temperature dependence of the frequency factor for the k^{th} reaction

β = a constant defined by Eq. (56a)

γ = ratio of specific heats

Γ = constant defined by Eq. (53b)

ε = reaction progress variable

ζ = nondimensional radial coordinate defined by Eq. (23)

η = nondimensional radial coordinate defined by Eq. (22) or nondimensional diameter of droplet

θ = nondimensional temperature

λ	=	wave length
λ_n	=	constant defined by Eq. (66)
Λ	=	nondimensional parameter defined by $(\Delta H)S_b/c_pT_1U_1$
μ	=	nondimensional droplet burning rate
ν	=	kinematic viscosity
ξ	=	nondimensional radial coordinate $= r/\ell_f$
ρ	=	gas density
τ	=	nondimensional time
ϕ	=	velocity potential
Φ	=	rate of energy dissipation per unit volume of gas
χ	=	propellant response function
ψ	=	parametrix appearing in Eq. (24)
ω_i	=	rate of production of species i
Ω	=	nondimensional quantity defined by $\ln p/p_0$

1. Introduction

The mechanisms of noise generation by combustion processes are being studied in depth, since a detailed understanding of these processes is a key to providing a description of the overall noise in the combustors of various environmental, industrial, and aerospace applications. A qualitative classification of combustion noise, based on typical frequency spectra, reveals random noise, registered by the rough burning in either premixed or diffusion flames, superposed on noise at selected frequencies. The selected frequencies may be caused by physico-chemical interactions occurring between the acoustic field and the system, including the fuel injectors, flame holders, and the flame enclosure, which may have been excited by components of the random fluctuations in the flame. While the selected frequency noise can be eliminated, to some extent, by appropriate designs of the combustor system, the suppression of random noise appears to be more complicated. These two noise fields are both induced by non-steady heat release in the burning processes, and therefore, together comprise combustion noise.

Previous studies of the random noise generated by turbulent flames have been made by several investigators, including Smith and Kilham [1], Hurle, et at [2], and Giammar and Putnam [3]. Excellent correlation between the sound pressure and the emission intensity of C_2 and CH in the ethylene-air premixed flames [2] demonstrates the intrinsic nature of the turbulent driven random noise that is basically different from the sound generated at selected frequencies. Experimental studies of sound generated at selected frequency bands, provoked by fuel injectors, or air entrainment in the combustor, has been reported by Dance and Sutherland [4]. Recent experimental studies with a ducted burner conducted by the Princeton group, (Abdelhamid, et al, [5]), revealed strong unsteady combustion-cavity coupling resulting in intense noise. Indeed, the noise generation by the interaction of non-steady combustion with its enclosure constitutes a significant problem with respect to core engine noise. Basic theory on the generation and amplification of noise by combustion environmental interaction is well summarized by the books published by Gaydon and Wolfhard [6], and Markstein [7].

Examination of the far field noise patterns has established the empirical fact that the source of combustion noise is of monopole type, for both premixed and diffusion flames. This observation led to the phenomenological theory of Bragg [8] which describes the flame in terms of monopole sources. The acoustic power and thermo-acoustic efficiencies are also obtained from phenomenological reasoning, with resulting good agreement with experimental data.

Another approach to the problem was taken by Strahle [9] who adopted Lighthill's formalism for a noise field prediction. The first decisively important factor was the overall description of the turbulent flame to be used for the evaluation of the volume integral expressing the monopole sound field. Using the wrinkled flame model developed by Karlovitz [10] and others, Strahle proposed an acoustic power scaling rule similar to that of Bragg but differeing in two ways, namely, (i) appropriate pairing of the exponents appear on the gas speed and the flame speed, $U^{4-3q} S_L{}^{r+3q}$, and (ii) introduction of the scale of turbulence appropriately powered, ℓ^{2+r}, in deference to the phenomenclogical wrinkled flame model which has large scale turbulence. The exponents, r, and q, appearing in the scaling rule are functions of the Reynolds number and their numerical values are to be determined from the experimental data. While the introduction of these empirical factors is most attractive from the practical point of view, the underlying physical concept of the correlation is less clear.

The scaling rule for turbulent flames, based on the model of the distributed reaction by Summerfield et al [11] has also been used by Strahle. There is, however, reason to believe that the result thus obtained has limited applicability because of simplifying approximations made in the analysis.

In the present paper, a general formalistic approach to combustion noise is developed with the basic physical descriptions being retained at a sophisticated level, and the bifurcation in the physical as well as mathematical perceptor being postponed until the latest possible stage. The first part of the paper is concerned with (1) qualitative and quantitative descriptions of the monopole noise sources of turbulent flames, the far field noise characteristics of these sources; and (2) qualitative studies of the generation and the transmission of sound waves in the near field, i.e., in the vicinity of the flame. The principal result of the present investigation of flame noise is that an explicit far field noise expression can be formulated without specifying the structure of the flame. The indispensable feature of the analytical step is the derivation of the wave equation for the reacting gas in the most general terms. The wave equation obtained reveals the fact that the heat release rate contributes to the generation of sound and to the dispersion of the sound wave. Further, on investigating the effect of steady and non-steady heat release, it is evident that the non-steady and non-homogeneous fluctuation of the heat release in the turbulent flame produces six dominant sources contributing far field acoustic power. These six sources are represented by double correlation functions obtained from the fluctuating gas velocity, time derivative, and spatial derivatives of the fluctuating heat release rate together with the weighting factors comprised of steady state gas velocity and the spatial derivatives of heat release rate. One of these six correlation functions is identical to the acoustic power expression obtained by Strahle. It may be emphasized that the sum of these correlation functions appears explicitely in the scaling rule, and is termed the flame structural factor in view of its physical significance. This factor has been used in the scaling rules of Bragg and Strahle but the complete physical implication of the factor in reference to the flame structure has not been recognized. The present theory yields an intrinsic structural factor which may be evaluated by an experimental survey of the flame.

While the general characteristics of the far field noise can be prescribed by the correlation functions obtained from the formal asymptotic solution of the wave equation for the reacting gas, the detailed mechanisms of the near field sound generation and dispersion can also be examined within the flame and its vicinity by the solution of the wave equation.

The near field problem is of particular interest in reference to the question of detailed acoustic phenomena relative to imposing turbulence, the aerothermochemical structure of the flame, effects of the flame holder, or upstream condition, and the condition for the resonant acoustic oscillation.

The elucidation of these phenomena stems from the consideration in retrospect to the potential acoustic roles which non-steady, non-homogeneous heat release plays. Indeed, the self-contained linearized analysis could account for these intricate phenomena through simplified one-dimensional steady flame structure superposed by three dimensional turbulent fluctuations. The near field analysis has been conducted on the open flame of known flame structure, inclusive of steady and turbulent properties.

While some research has been aimed at obtaining an understanding of the noise generation by turbulent flames, little is known about the generation of noise by liquid spray combustion which is the most common mode of combustion in commercial applications. Liquid spray combustion is characterized by various complicated non-steady phenomena occurring concurrently in a two phase system. The result of these various forms of non-steadiness is the fluctuation in heat release at the frequency of the dominant mode of unsteadiness. The second part of this paper includes results of an investigation of the noise generation by liquid spray combustion. A detailed analysis reveals the non-steadiness in the combustion of droplets in a time scale of the order of magnitude of the droplet life time, generates sound. Furthermore, the turbulent fluctuation alters the burning rate and the number density of the droplets, and thus generates sound. The acoustic instability in the liquid spray combustor could yield higher rates of transfer from the reacting system to the acoustic field and thereby contributes significant sound out put. The mechanism of sound generation originated by chemical instability is of particular interest in view of the fact that the fluctuating reaction is the basic source of sound. The existing analysis is extended to consider the effect of the temperature fluctuation in the stability of a homogeneous reacting gas system. The analysis yields a basic correlation between the sound intensity and the chemical parameters, including rate constants and heats of formation of reacting species.

2 Theoretical Consideration and Mathematical Formulation

2.1 Theoretical Consideration

The analytical procedure adopted in the present study is aimed at two basic objectives of primary interest in combustion generated noise.

The first objective is the identification and quantitative description of noise emitting mechanisms from various combustion processes, including homogeneous and non-homogeneous burning, premixed and diffusion flames, open or enclosed flames. The salient feature common to these various noise emitting mechanisms and noise amplifying combustion processes is that the sound generation is attributed to the non-steady intercoupling between various aerothermochemical modes excited in the combustion zone. To be specific, the non-steady exothermic and endothermic reactions generate random noise, and steady combustion in the presence of pressure and velocity fluctuations result in noise amplification. It is evident that a detailed knowledge of the intercoupling between various modes, namely, acoustic, entropy and vortex modes in the combustion zone is essential in the identification and the quantitative description of the noise sources. For this specific reason the modal study developed by Chu and Kovásznay [12] may be extended to the burning zone characterized by rapid heat release and gas expansion.

It may be pointed out that the identification and the estimation of the sound sources is further complicated by interactions of the flame with the environment, including ducts, bluff bodies, fuel injectors, and any components capable of responding acoustically or aerodynamically with the combustion system. The subject is of great practical importance in the overall assessment of engine core noise on which active research efforts are currently underway. The analytical problems there consist of sound generation, transmission and reflection in an environment with appropriate boundary conditions. General mathematical techniques for the problem of non-reacting gases were developed in the past, however, with the exception of acoustic instability in a rocket combustion chamber, little effort has been devoted in synthesizing noise generation, amplification, and transmission in ducted combustors.

The second objective of the study is the prediction of the far field intensity of the sound generated by combustion system. The analytical schemes for the open flame and ducted flame are different in several ways.

The evaluation of the far field sound intensity of open flames is complicated by the non-isothermal zone and the non-isentropicity prevailing in the burning zone. In view of these two physical complexities, Lighthill's theory may be applied only with reservations. An alternative approach adopted in the present analysis, however, is the method of multi-intervals, frequently used in fluid mechanical problems. This method offers a self-consistent procedure of predicting pressure fluctuations in three zones; namely, the burning zone, the non-isothermal layer and the environment. Solutions in these three zones are matched by appropriate techniques for the prevailing physical conditions. The mathematical technique for the prediction of the far field noise of an open flame is illustrated in section 3.

Experimental and analytical investigation of the sound intensity from ducted combustors has been investigated by the Princeton group. The experimental correlation of the internal pressure fluctuation and the far field sound pressure level agrees well with the theoretical prediction with the correction accommodating the non-isothermal environment.

In addition to the prime objectives stipulated above, the detailed features of the sources of noise will be described in an effort to guide the analytical study. The non-steady phenomena responsible for noise generation are largely provoked by various mechanisms of fluid mechanic and acoustic origin. The scale, structure and intensities are different for different non-steady phenomena. Consequently, various sound fields may originate from the same combustion processes. For example, droplet combustion is inherently unsteady with a characteristic time equal to the order of the life-time of the droplet. The combustion of a droplet in quiescent air will therefore generate sound. In addition to this sound field generated by a cloud of droplets, turbulence with an eddy size comparable to the radius of the flame generates noise in much the same manner as an open flame. If the size of the eddies becomes of the same order of magnitude as the interparticle distance, the droplet density fluctuates. This results in local heat release fluctuations, and thereby sound generation.

The last observation suggests that relevant scaling and non-dimensionalization are essential in identifying and screening noise sources. This will be illustrated in section 4. All the sound fields produced may be adequately superposed for a final estimation of the overall sound, provided the fluctuating pressure field is substantially lower than the mean pressure. If the fluctuation becomes of a finite strength, the superposition principle ceases to apply and the identification of various sound fields becomes hopelessly complicated. Nevertheless, the perturbative scheme should shed light on the physical mechanisms of the generation and amplification of the nonlinear sound field.

It has been recognized that many reacting systems exhibit chemical instability by which the rate of the major energy releasing reaction fluctuates at characteristic frequencies. If the resulting fluctuation in the heat release is intense, the sound may be generated, either independently or cooperatively with other non-steady phenomena. The prediction of the sound intensity requires a detailed knowledge of the chemical instability, in particular, the nature of the heat release. The basic understanding of the noise generation by the chemical instability is also deemed as a pertinent object of the combustion noise theory.

2.2 Mathematical Formulation

The basic equations essential for the study of combustion noise are the conservation laws of the chemically reacting gas [13] which are summarized as follows:

$$\frac{D\rho}{Dt} + \rho\nabla\cdot\bar{V} = 0 \tag{1}$$

$$\rho\frac{D\bar{V}}{Dt} = -\nabla p + \nabla\cdot\bar{\bar{\tau}} \tag{2}$$

$$\rho T\frac{DS}{Dt} = \Phi + \nabla\cdot k\nabla T - \sum_{i=1}^{N}(\mu_i/W_i)\omega_i \tag{3}$$

$$\rho\frac{DY_i}{Dt} = \omega_i - \nabla\cdot j_i \tag{4}$$

where S is the specific entropy, ϕ is the rate of energy dissipation per unit volume, μ_i is the specific chemical potential of i th species, ω_i is the rate of production given by the phenomenological chemical kinetic expression,

$$\omega_i = W_i\sum_{k=1}^{M}(\nu''_{i,k} - \nu'_{i,k})B_k T^{\alpha_k} e^{-E_k/\mathcal{R}T}\prod_{j=1}^{N}\left(\frac{X_j p}{\mathcal{R}T}\right)^{\nu'_{j,k}} \tag{5}$$

$$\sum_i \nu''_{i,k}\mathfrak{M}_i = \sum_i \nu'_{i,k}\mathfrak{M}_i$$

where w_i is the molecular weight, $\nu'_{i,k}$ and $\nu''_{i,k}$ are the stoichiometric coefficients for species appearing as a reactant and product respectively, and E_k is the activation energy of the kth reaction. It is assumed that the gas mixture obeys the ideal gas equation of state

$$p = \rho\mathcal{R}T\sum_{i=1}^{N}(Y_i/W_i) \tag{6}$$

While the conservation laws and auxiliary equations are self-contained systems for the present study, the wave equation which describes the sound field, provides mathematical tools for a comprehensive description of the combustion noise. The derivation of the wave equation is elementary, and is given in Appendix A. The resulting equation is given in the following:

$$\frac{D^2}{Dt^2}\left[\ln\frac{p}{p_0}\right] - \frac{\partial}{\partial x_i}\left[a_f^2 \frac{\partial}{\partial x_i}\left(\ln\frac{p}{p_0}\right)\right] = \gamma \frac{\partial u_i}{\partial x_j}\frac{\partial u_j}{\partial x_i} + (\gamma-1)\frac{D}{Dt}\left[\frac{p_0}{p}\dot{Q}\right] \quad \cdots \quad (7)$$

$$\dot{Q} = \sum \omega_i h_i - \sum j_i h_i - RT\sum_{i=1}^{N}\sum_{j=1}^{N}\left(\frac{X_j D_{T,j}}{W_i D_{ij}}\right)(V_i - V_j) + \Phi_v + \frac{\partial}{\partial x_i}\left(k\frac{\partial T}{\partial x_i}\right) \cdots (8)$$

The remarkable feature of the above equation is the compact appearance of the noise generation mechanisms, including chemical reaction, heat conduction, diffusion, and viscous dissipation which are the processes common in reacting systems. (The wave equation for the two phase system is given in section 6.) The second term appearing on the right hand side of Eq. (7) represents the source of the combustion noise. Special attention may be called to the local time derivative and the spatial derivative of an effective total heat release rate. For a flame wherein a rapid rise in the rate of the reaction is accompanied by expansion, both local time derivatives and the spatial derivatives of the rate of heat release will generate a monopole type sound field. The relative strength of these two terms depends to a great extent on the basic flame structure and the turbulent flow field. This and other aspects will be discussed quantitatively in the section devoted to the open flame noise.

A qualitative picture may be drawn with regard to the difference between the diffusion flame and the premixed flame. Observation of the sound generation source reveals that the non-steady diffusion of the fuel and oxidizer may lead to fluctuation in the heat liberation and the location of the flame. Furthermore, the non-steady diffusion may contribute directly in the quadrupole-like noise field represented by the second derivation of the concentration. The order of magnitude of the noise generated, however, may be small compared with the combustion noise.

3 Noise Generation by Open Turbulent Flames

3.1 Acoustic Mode in the Near Field

The near field is the arena dominated by strong aero-thermochemical coupling between a vortex (turbulence) mode, a sound mode, and an entropy mode. The study of the near field will be focused on the sound mode with specific reference to be mde to the turbulent flame zone. This suggests that the conservation law be properly scaled by the turbulent flame thickness ℓ_f . The physical variables are non-dimensionalized by appropriate reference quantities

$$\xi_i = x_i/\ell_f\,,\quad \tau = tV_1/\ell_f\,,\quad A^2 = a_s^2/a_1^2\,,\quad M_\infty = V_1/a_1\,,\quad v_i = V_i/V_1$$

where V_1 , a_1 , are the velocity and the speed of sound at the edge of the downstream of the reacting zone. By splitting the velocity $\bar{V}$, into components consisting of basic flow velocity $\bar{U}$ and perturbative solenoidal component $\nabla\phi$, and rotational field, $\bar{W}$, i.e., $\bar{V} = \bar{U} + \Delta\phi + \bar{W}$, and by neglecting molecular transport processes the conservation laws and the wave equation become

$$\nabla^2\phi = -\frac{1}{\gamma}\frac{D\Omega}{D\tau} + \Lambda\frac{\dot{Q}}{P} \tag{9}$$

$$\frac{D\bar{W}}{Dt} = -\frac{1}{\gamma M_\infty^2}\frac{\nabla P}{\rho} - \frac{D}{D\tau}(\nabla\phi) \tag{10}$$

$$\frac{D\Omega}{D\tau} - \gamma\frac{D}{D\tau}(\ln\rho) = \gamma\Lambda\frac{\dot{Q}}{P} \tag{11}$$

$$\rho\frac{DY_i}{D\tau} = \frac{C_pT_1}{\Delta H}\Lambda\omega_i \tag{12}$$

$$\frac{D^2\Omega}{D\tau^2} - \frac{1}{M_\infty^2}\nabla\cdot(A^2\nabla\Omega) = \gamma\Lambda\frac{D}{D\tau}\left(\frac{\dot{Q}}{P}\right) \tag{13}$$

where $\Omega = \ln\left(\frac{P}{P_0}\right)$, $\Lambda = \frac{\gamma-1}{\cdot}\frac{(\Delta H)\omega_0\ell_f}{P_1\,U_1}$, ΔH is the heat of reaction and ω_0 is the rate of chemical reaction at the reference state. The non-dimensionalized parameter, Λ , represents the ratio of the heat release rate to the thermal convection, i.e., $\Lambda = (\Delta H)S_b/c_pT_1U_1$ where S_b is the burning velocity. Observation of the system of equations reveals that the density fluctuation at low Mach number, $M_\infty^2 << 1$, is of the order Λ , whereas the pressure fluctuation is of the order of $M_\infty^2\Lambda$, i.e., the compressibility effect associated with the primary density fluctuation. This suggests that the perturbative analysis on the basis of entropy mode, (Λ) and the sound mode (M_∞^2) , should render a systematic procedure for the investigation of the near field. Accordingly, the flow variables are expanded in the following two-parameter series:

$$f(r,\tau) = \sum_{j=0}\sum_{n=0} M_\infty^{2j}\Lambda^n f_{2j,n}(r,\tau) \tag{14}$$

The velocity field is expanded likewise

$$\bar{\mathcal{V}} = \bar{U} + \Lambda(\nabla\phi_{0,1} + W_{0,1}) + \Lambda^2(\nabla\phi_{0,2} + W_{0,2})$$
$$+ M_\infty^2[\Lambda(\nabla\phi_{2,1} + W_{2,1}) + \Lambda^2(\nabla\phi_{2,2} + W_{2,2}) + \cdots \quad (15)$$

By splitting the basic flow velocity into steady and non-steady parts, i.e., $\bar{U} = \bar{u} + \bar{u}'$, the total derivative becomes

$$\frac{D}{D\tau} = \frac{\bar{D}}{D\tau} + \frac{D'}{D\tau} + \Lambda\frac{D}{D\tau_{0,1}} + \Lambda^2\frac{D}{D\tau_{0,2}} + M_\infty^2\left(\Lambda\frac{D}{D\tau_{2,1}} + \Lambda^2\frac{D}{D\tau_{2,2}}\right) + \cdots \quad (16)$$

where $\frac{D}{D\tau} = \frac{\partial}{\partial\tau} + \bar{u}\cdot\nabla$, $\frac{D'}{D\tau} = \bar{u}'\cdot\nabla$, $\frac{D}{D\tau_{0,1}} = (\nabla\phi_{0,1} + \bar{W}_{0,1})\cdot\nabla$

An important physical concept pertaining to the present perturbative scheme is that the small Mach number perturbation scheme is valid in the case of compact noise sources. This is evident from the following relation $M_\infty = V_1/a_1 \simeq \ell_f/\lambda << 1$ where λ is the wave length of the sound. The noise source with $\ell_f/\lambda << 1$ is termed as an acoustically compact source. In general, turbulent open flames may be regarded as a compact noise source.

Substituting all the variables expressed in the form of Eq. (14) into the system of Eqs. (9) to (13), yeild perturbative equations for the sound mode, entropy mode, vorticity mode, and the solenoidal field.

Sound Mode	$\nabla\cdot A_0^2\nabla\Omega_{n,j} = \mathcal{K}_{n,j}$	(17)
Entropy Mode	$\frac{D\sigma_{nj}}{D\tau} = h_{nj}$	(18)
Vorticity Mode	$\frac{D\bar{W}_{nj}}{D\tau} = \bar{g}_{nj}$	(19)
Irrotational Velocity	$\nabla^2\phi_{nj} = f_{nj}$	(20)

where $\sigma_{nj} = \rho_{nj}/\rho_0$. The inhomogeneous terms K_{nj} , h_{nj} , $\bar{g}_{nj}$, and f_{nj} are given in Appendix B.

The wave equation governing the sound mode in the near field is degenerated into the eliptic system in this region. The fluctuation which drives the acoustic field acts to constrain the fluctuation of the turbulent field nearly dynamically incomprehensible ($M_\infty^2 << 1$) . The inhomogeneous term K_{nj} represents various mechanisms including true noise generation, amplification, and refraction phenomena. The first 4 terms are described in Table 1.

Table I

Compact Noise Sources and Dispersion Mechanisms

Order	Modal Interaction	Sound Sources	Amplification	Refraction	Convection
2.1	Entropy-Basic Flow	$-\gamma\left(\frac{\bar{D}\dot{Q}'}{D\tau} + \vec{u}'\cdot\nabla\dot{\bar{Q}}\right)$			
2.2	Entropy-Basic Flow	$-\gamma\left\{\left(\frac{\bar{D}}{D\tau} + \vec{u}'\cdot\nabla\right)\dot{Q}'_{0,1} + \frac{D'}{D\tau_{0,1}}\left(\dot{\bar{Q}} + \dot{Q}'\right)\right\}$			
	Entropy-Sound			$2\nabla\cdot A_0^2 a_{01}\nabla\Omega_{2,1}$	
4.1	Sound-Basic Flow				$\left(\frac{\bar{D}}{D\tau} + \vec{u}'\cdot\nabla\right)^2\Omega_{2,1}$
4.2	Entropy-Basic Flow	$-\gamma\left\{\left(\frac{\bar{D}}{D\tau} + \vec{u}'\cdot\nabla\right)\dot{Q}'_{2,1} + \frac{D'}{D\tau_{2,1}}\left(\dot{\bar{Q}} + \dot{Q}'\right)\right\}$			
	Entropy-Basic Flow -Sound		$\gamma\left\{\left(\frac{\bar{D}}{D\tau} + \vec{u}'\cdot\nabla\right)\times \Omega_{2,1}\left(\dot{\bar{Q}} + \dot{Q}'\right)\right\}$		
	Entropy-Sound			$2\nabla\cdot A_0^2 a_{01}\nabla\Omega_{4,1}$	
	Sound-Basic Flow				$\left(\frac{\bar{D}}{D\tau} + \vec{u}'\cdot\nabla\right)^2\Omega_{2,2}$

Some relevant physical aspects of the sound mode will be described categorically as follows. Firstly, the primary sound generation appearing in the $M_\infty^2\Lambda$ and $M_\infty^2\Lambda^2$ approximation is attributed to the turbulent fluctuations in the heat release rate and the gas velocity in the flame structure. However, the fluctuations may also be induced by the sound field and thus enhance the strength of the noise source. This aspect is of particular interest with reference to acoustic instability. A term represented by Entropy-Basic Flow-Sound appearing in the $M_\infty^4\Lambda^2$ approximation is the potential source responsible for the chemico-acoustic instability. Secondly, the sound amplification also plays a role as a result of entropy-acoustic intercoupling. The maximum amplification occurs at the point where the spatial derivation of the heat release reaches the highest value. The entropy and vorticity modes, within the assumption of negligible molecular transport processes, are convected along the particle path. These modes are excited primarily by heat fluctuation and to a lesser extent by sound mode. The solenoidal field, representing the rate of volume increase per unit volume of the fluid, is also primarily affected by the fluctuating heat release.

The implication of the observation being that the acoustic mode depends on other three modes within the linearized approximation, provided the pressure fluctuation remains small. This approximation ceases to predict a detailed finite strength sound field, particularly when combustion instability dominates the burning processes. The convergence of the perturbative scheme also has to be established when it is applied in the non-linear acoustic regime.

3.2 Intermediate and External Zones

3.2.1 Intermediate Non-Isothermal Zone

The sound waves generated or amplified in the flame zone are transmitted through the non-isothermal layer where the waves are refracted and reflected. In the absence of combustion, the velocity fluctuation and the non-steady heat conduction would be the primary sources of sound generation in the non-isothermal zone. The nature of the sound transmission depends on the temperature distribution in the layer and the dimension of the layer relative to the wave length. If the characteristic dimension of the layer exceeds the typical wave length, the wave emerges from the high temperature, lower density zone and propagates into the zone of low temperature, higher density. Physically, the non-isothermal zone may act as an acoustic cavity with a distributed monopole source in the inner boundary of the cavity, i.e., turbulent flame. The waves reflected from the non-isothermal layer may be incident upon the flame zone, transmitted through it and reflected back and forth inside the flame zone, (Figs. 1 and 2), where the temperature is substantially lower than the flame zone.

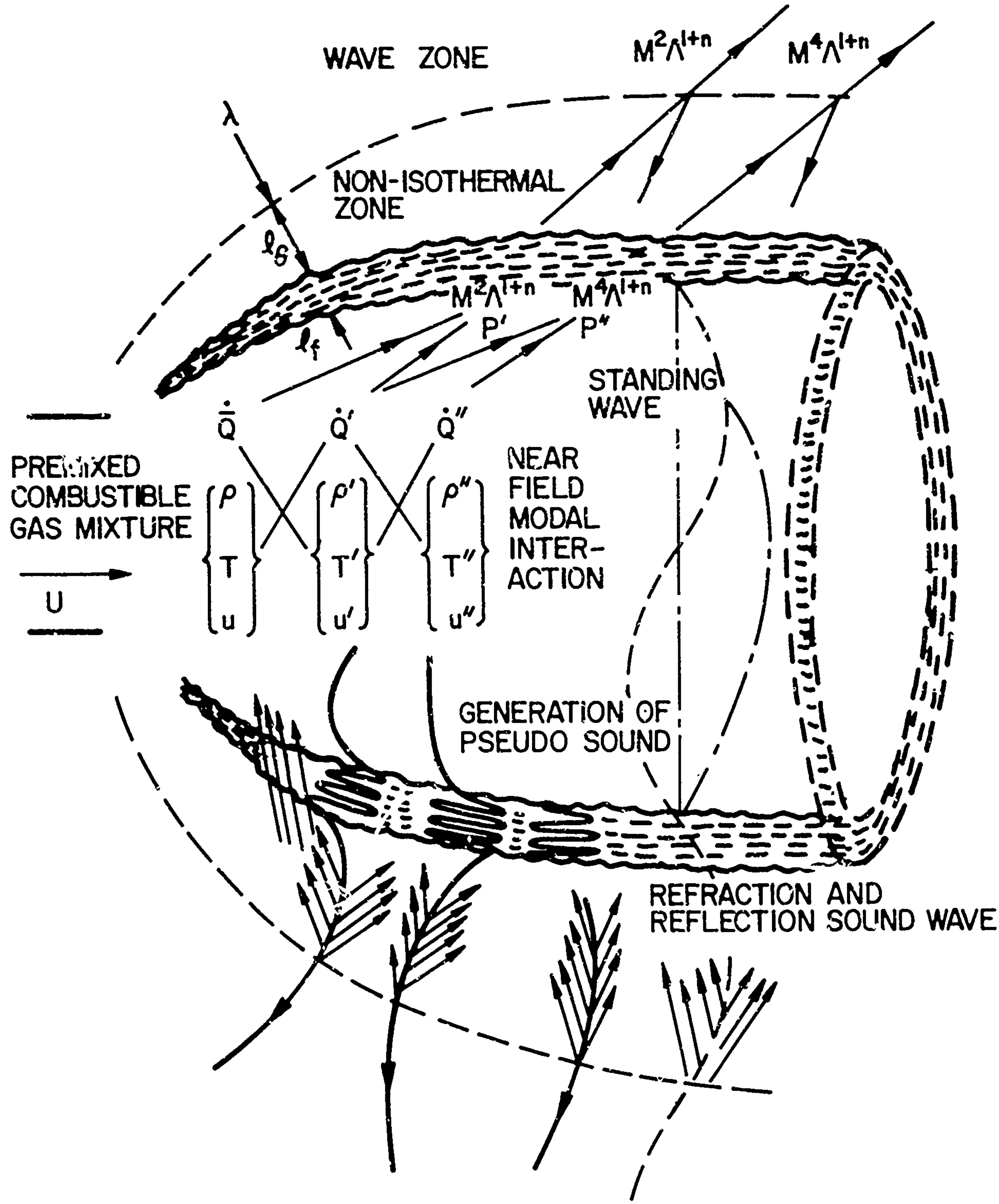


Fig. 1 Schematic diagram of sound generation by a flame: distributed reaction model.

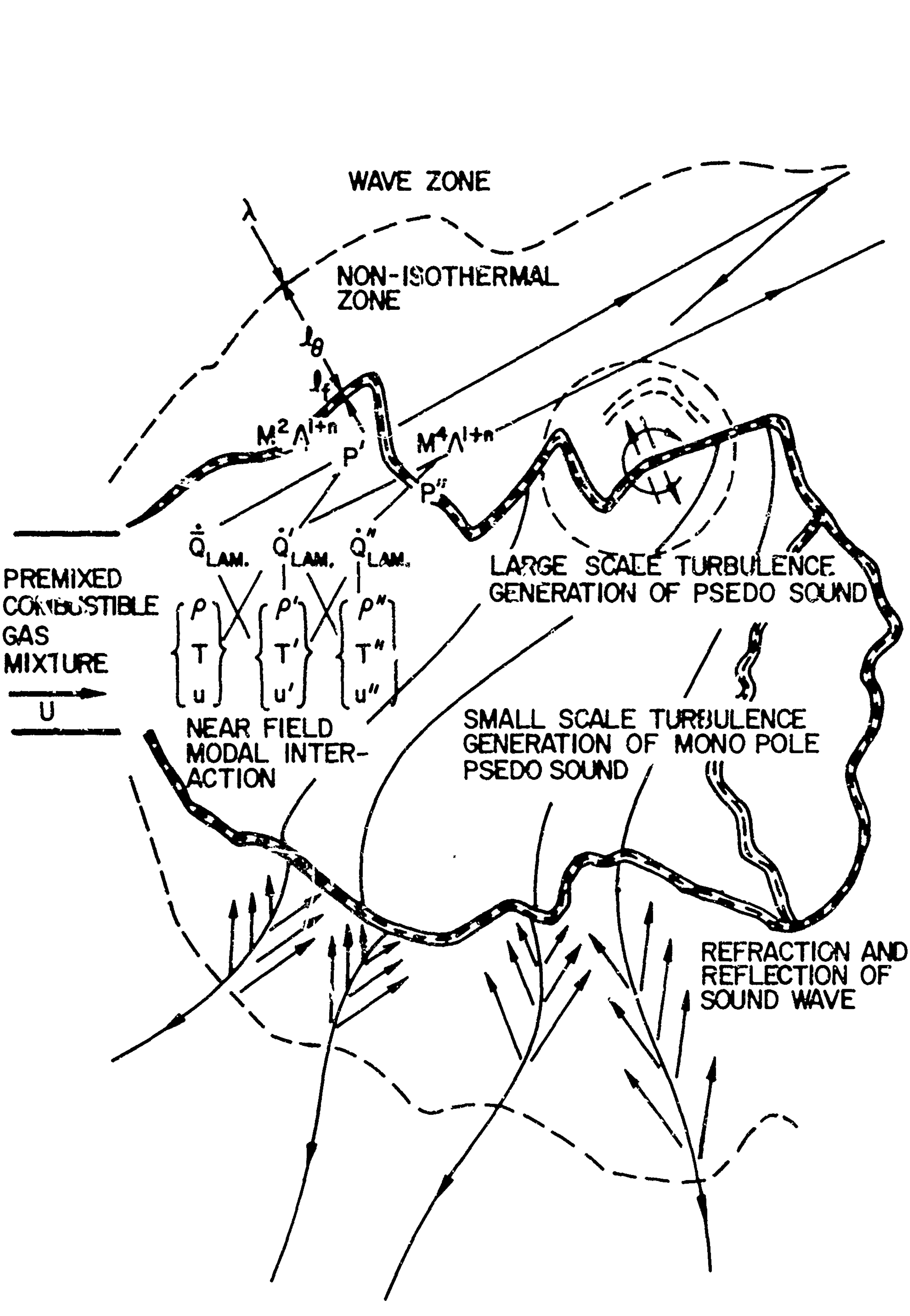


Fig. 2 Schematic diagram of sound generation by a flame: wrinkled laminar flame model.

Interaction of this sound wave with the flame may provoke flame oscillation (Fig. 3a). The overall effect of the reflected wave may not be ignored for a ducted flame with pressure sensitive exothermic reactions.

For a small size burner, the overall picture is substantially different from the previous case. Within the limit of the thin non-isothermal layer, (Fig. 3b), the overall acoustic effect may be deemed as an acoustically compact layer characterized by a rapid decrease in the gas temperature. The pressure distribution is characterized by incompressible fluctuating but non-propagating disturbances. The basic formulation for the latter problem is discussed in the next subsection.

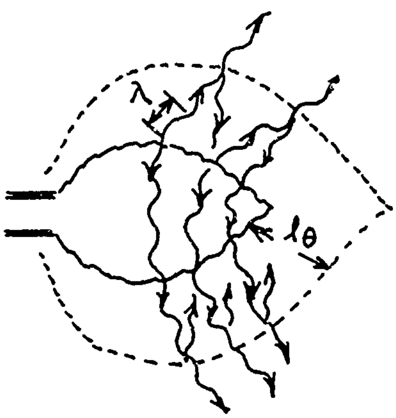


Fig. 3a Refraction and reflection of sound wave in non-isothermal acoustic cavity, $\lambda \ll \ell_\theta$.

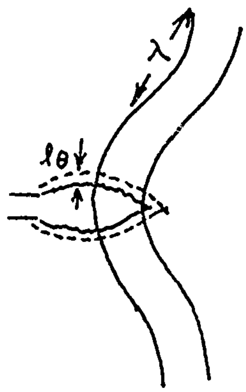


Fig. 3b Refraction and reflection of pressure fluctuation from compact non-isothermal zone $\lambda \gg \ell_\theta$.

3.2.2 External Zone

A zone extending from the outer edge of the non-isothermal layer to the geometric and acoustic far field is the free space for sound propagation. The primary source, if it exists, is the turbulent velocity fluctuation due to the jet and wake, which is neglected in this study.

While the solutions and the matching of the solutions for these three fields are discussed in the next subsection, it is relevant to point out that the overall problem of open flame combustion noise may be regarded as consisting of a thin compact noise source zone, adjoined by a compact non-isothermal layer or by a large acoustic cavity. This system is finally engulfed by an extensive wave zone. Matching of the solutions from a compact zone to a wave regime calls for the singluar perturbation technique. This approach is self-consistent and should provide a detailed aspect of the combustion noise.

3.3 Method of Matched Solution for Compact Flamelets

The equations describing the fields in the burning zone, in the intermediate non-isothermal zone, and in the external zone, will be solved concurrently and matched by appropriate physical conditions. The non-dimensionalization of the governing equations are presented in Table II, and the resulting equations within the first order approximation for pressure fluctuation in three zones are given as follows:

Inner zone:

$$L\Omega = \frac{\partial^2 \Omega}{\partial \xi_i^2} + \frac{\partial}{\partial \xi_i}(\ln A_0^2)\frac{\partial \Omega}{\partial \xi_i} = -\gamma \Lambda M_\infty^2 \frac{1}{A_0^2}\left(\frac{\bar{D}\dot{Q}'}{D\tau} + \frac{D'\dot{\bar{Q}}}{D\tau}\right) \quad (21)$$

where $\xi_i = x_i/\ell_f$, $\frac{\bar{D}}{D\tau} = \frac{\partial}{\partial\tau} + \bar{u}_i\frac{\partial}{\partial\xi_i}$, $\frac{D'}{D\tau} = (u_i' - \frac{\theta'}{\theta}\bar{u}_i)\frac{\partial}{\partial\xi_i}$, $\theta' = \frac{T'}{T_0}$

Intermediate Zone:

$$\hat{L}\hat{\Omega} = \frac{\partial^2\hat{\Omega}}{\partial\eta_i^2} + \frac{\partial}{\partial\eta_i}(\ln\hat{A}_0^2)\frac{\partial\Omega}{\partial\eta_i} = 0 \quad (22)$$

where $\eta_i = x_i/\ell_\theta$

External Zone:

$$\tilde{L}\tilde{\Omega} = \frac{\partial^2\tilde{\Omega}}{\partial\tau^2} - \frac{\partial^2\tilde{\Omega}}{\partial\zeta_i^2} = 0 \quad (23)$$

where $\zeta_i = x_i/\lambda$

The basic physical conditions for matching the solutions at zone boundaries are the continuity of pressure, and particle velocity.

3.3.1 Solution for the Inner Zone

The general solution of inhomogeneous elliptic Eq. (21) may be solved by the method of parametrix, (see for example Courant and Hilbert [14]) a method developed within the formalism of the integral equation. The general solution is expressed as

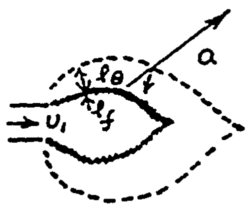
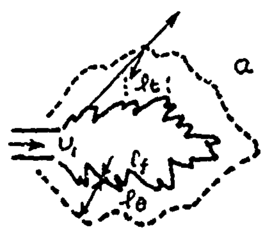
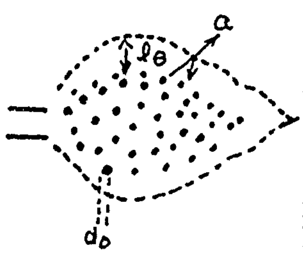
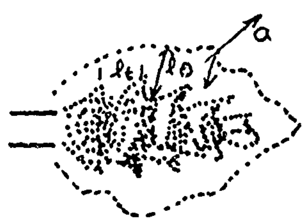
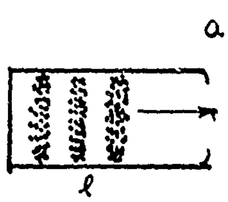
$$\Omega(\xi_i,\tau) = \Omega^{(0)}(\xi_i,t) + \iiint \psi(\xi_i:\xi_i')\mathcal{F}(\xi_i';\tau)\,dV(\xi_i') \quad (24)$$

where $\Omega^{(0)}$ is the homogeneous solution and

$\psi(\xi_i:\xi_i') = -\left[(\xi_i-\xi_i')(\xi_i-\xi_i')\right]^{-\frac{1}{2}}$ is the parametrix.

Table 2

Scaling for Turbulent Flame and Spray combustion systems

BURNING PROCESSES	TYPE OF FLAME	SCALING AND FREQUENCIES INNER ZONE	 INTERMEDIATE ZONE	 EXTERNAL ZONE	TYPE OF SOUND FIELD
OPEN PREMIXED FLAME DISTRIBU TED REAC TION MODEL		COMPACT SMALL SCALE TURB. $L=\ell_f$ $\omega=\frac{U_1}{\ell_f}$	COMPACT $L=\ell_\theta$ $\omega=\frac{U_1}{\ell_f}$	WAVE $L=\lambda$ $\omega=\frac{a}{\lambda}=\frac{U_1}{\ell_f}$	MONOPOLE
			WAVE $L=\lambda$ $\omega=\frac{a}{\lambda}=\frac{U_1}{\ell_f}$		MONOPOLE
OPEN PREMIXED FLAME WRINKLED FLAME MODEL		SMALL SCALE TURB. $L=\ell_f$ $\omega=U_1/\ell_f$	COMPACT $L=\ell_\theta$ $\omega=\frac{U_1}{\ell_f}$	WAVE $L=\lambda$ $\omega=\frac{a}{\lambda}=\frac{U_1}{\ell_f}$	MONOPOLE
		LARGE SCALE TURB. $L=\ell_t$ $\omega=U_1/\ell_t$	WAVE $L=\lambda$ $\omega=\frac{a}{\lambda}=\frac{U_1}{\ell_t}$		DIPOLE
OPEN SPRAY COMBUST ION		COMPACT $L=d_0$ $\omega=\frac{1}{t_{life}}$	COMPACT $L=\ell_\theta$ $\omega=1/t_{life}$	WAVE $L=\lambda$ $\omega=1/t_{life}$	MONOPOLE
			WAVE $L=\lambda$ $\omega=\frac{a}{\lambda}=\frac{1}{t_{life}}$		MONOPOLE
OPEN SPRAY COMBUST ION		$L=\ell_t$ $\omega=\frac{U_1}{\ell_t}$	COMPACT $L=\ell_\theta$ $\omega=U_1/\ell_t$	WAVE $L=\lambda$ $\omega=\frac{a}{\lambda}=\frac{U_1}{\ell_t}$	MONOPOLE
			WAVE $L=\lambda$ $\omega=\frac{a}{\lambda}=\frac{U_1}{\ell_t}$		MONOPOLE
DUCTED SPRAY COMBUST ION		$L=\lambda$ $\omega=\frac{a}{\ell}$ $\ell\approx\lambda$	COMPACT $L=\ell_\theta$ $\omega=a/\ell$	WAVE $L=\lambda$ $\omega=a/\ell$	MONOPOLE
			WAVE $L=\lambda$ $\omega=a/\ell$		MONOPOLE

The unknown function $\mathcal{F}(\xi_i,\tau)$ has to be chosen so that the expression for Ω given by Eq. (24) satisfies Eq. (21).

In view of the specific functional form assumed by ψ, it can be shown that

$$\nabla^2\Omega = \nabla^2\Omega^{(0)} - 4\pi\mathcal{F} \tag{25a}$$

and

$$L\Omega = L\Omega^{(0)} - 4\pi\mathcal{F} + \iiint \left[\frac{\partial}{\partial\xi_i}\ln A_0^2\right]\left[\frac{\partial}{\partial\xi_i}\psi(\xi_j;\xi_j')\right]\mathcal{F}(\xi_i',\tau)\,dV(\xi_i') \tag{25b}$$

Substituting Eq. (21) into Eq. (25b), yields the integral equation for $\mathcal{F}(\xi_i\,\tau)$.

$$\mathcal{F}(\xi_i,\tau) = \iiint K(\xi_i;\xi_i')\mathcal{F}(\xi_i',\tau)\,dV(\xi_i') - \frac{\gamma\Lambda M_\infty^2}{4\pi}\frac{1}{A_0^2}\left(\frac{\bar{D}\dot{Q}'}{D\tau} + \frac{D'\dot{\bar{Q}}}{D\tau}\right) \tag{26}$$

where $K(\xi_i;\xi_i')$ is given by

$$K(\xi_i;\xi_i') = \frac{1}{4\pi}\left[\frac{\partial}{\partial\xi_i}(\ln A_0^2)\frac{\partial}{\partial\xi_i}\psi(\xi_j;\xi_j')\right]$$

The kernel of the integral equation has a singularity, and hence, is not square integrable. However, this kernel can be reduced to a non-singular one by a sufficient number of iterations. Doubly iterated kernel $K,^{(2)}$ which is square integrable, is defined as

$$K^{(2)}(\xi_i;\xi_i') = \iiint K(\xi_i;\xi_i')K(\xi_i';\xi_i'')\,dV(\xi_i'')$$

The integral equation may be reformulated in terms of $K^{(2)}$ kernel as follows:

$$\mathcal{F}(\xi_i,\tau) = \iiint K^{(2)}(\xi_i;\xi_i')\mathcal{F}(\xi_i',\tau)\,dV(\xi_i')$$
$$- \frac{\gamma\Lambda M_\infty^2}{4\pi}\left\{\frac{1}{A_0^2}\left(\frac{\bar{D}\dot{Q}'}{D\tau} + \frac{D'\dot{\bar{Q}}}{D\tau}\right) + \iiint K(\xi_i;\xi_i')\frac{1}{A_0^2}\left[\frac{\bar{D}\dot{Q}'}{D\tau} + \frac{D'\dot{\bar{Q}}}{D\tau}\right]dV(\xi_i')\right\} \tag{27}$$

The equation is square integrable, and thus Fredholms' theorem can be applied. Since the exact solution of Eq. (27) depends on the steady state gas temperature distribution, no attempt will be made here to seek the solution for a specific case. Instead, a general series solution may be written formally in the following form

$$\mathcal{F}(\xi_i,\tau) = -\frac{\gamma\Lambda M_\infty^2}{4\pi}\left\{\frac{1}{A_0^2}\left[\frac{\bar{D}\dot{Q}'}{D\tau} + \frac{D'\dot{\bar{Q}}}{D\tau}\right] + \iiint K(\xi_i;\xi_i')\frac{1}{A_0^2}\left[\frac{\bar{D}\dot{Q}'}{D\tau} + \frac{D'\dot{\bar{Q}}}{D\tau}\right]dV(\xi_i')\right\} \tag{28}$$

provided the kernel $K(\xi_i;\xi_i')$ converges uniformly. Kernel $K(\xi_i;\xi_i')$ is the sum of the integral operators

$$\mathcal{K}(\xi_i;\xi_i') = K^{(2)}(\xi_i;\xi_i') + K^{(3)}(\xi_i;\xi_i') + K^{(4)}(\xi_i;\xi_i') + \cdots \tag{29}$$

By substituting Eq. (29) into Eq. (24), the inner solution is given by

$$\Omega(\xi_i,\tau) = \Omega^{(0)}(\xi_i,\tau) + \frac{\gamma \Lambda M_\infty^2}{4\pi}\iiint \frac{1}{|r-r'|}\left\{\frac{1}{A_0^2}\left(\frac{\bar{D}\dot{Q}'}{D\tau} + \frac{D'\dot{\bar{Q}}}{D\tau}\right)\right\} dV(\xi_i')$$
$$+ \frac{\gamma \Lambda M_\infty^2}{4\pi}\iiint \frac{\mathcal{H}(\xi_i;\xi_i',\tau)}{|r-r'|}\, dV(\xi_i') \tag{30}$$

where $|r-r'| = [(\xi_i-\xi_i')(\xi_i-\xi_i')]^{1/2}$ and $\mathcal{H}(\xi_i;\xi_i'\tau)$ is the second term which appears on the right hand side of Eq. (28). It is interesting to note that the third term appearing on the right hand side of Eq. (30) is of the order of r^{-2} at large r. In fact, in the geometric far field within the inner zone, Eq. (30) may be approximated as

$$\Omega(\xi_i,\tau) \simeq \Omega^{(0)}(\xi_i,\tau) + \frac{\gamma \Lambda M_\infty^2}{4\pi}\iiint \frac{1}{A_0^2}\left(\frac{\bar{D}\dot{Q}'}{D\tau} + \frac{D'\dot{\bar{Q}}}{D\tau}\right) dV(\xi_i') + O(r^{-2}) \tag{31}$$

The primary sound field is a monopole type as expected.

3.3.2 Solutions of the Non-Isothermal Layer and Wave Zones

As the pressure fluctuation in the intermediate zone depends explicitely on the temperature distribution, a qualitative description and assumptions with regard to the distribution of temperature will be given prior to the analytical investigation.

The temperature distribution in the vicinity of an open flame is primarily determined by convection, conduction, and turbulent mixing. While convection stretches the non-isothermal zone in an axial direction, conduction and mixing transport heat in a radial direction. Thus, the intermediate zone may be approximated as nearly spherical in shape at low convective speeds. This approximation is in accord with the unidirectional characteristic of the primary sound field. Departure from the spherical shape of this zone produces a dipole sound characteristic in the far field.

For a spherical intermediate zone, wherein the speed of sound is a function of the radial coordinate, the solution of Eq. (22) is given by

$$\hat{\Omega}(\eta,\tau) = \int^{\eta} \frac{K(\tau)\, d\eta}{\eta^2 \hat{A}_0^2(\eta)} + \hat{\Omega}^{(0)}(\eta,\tau) \tag{32}$$

Where η is the non-dimensional radial coordinate, $\eta = r/\ell_\theta$, $K(\tau)$, $\hat{\Omega}^{(0)}(\tau)$ are arbitrary functions which will be determined from the matching conditions.

The solution representing the outgoing spherical wave in the radiation zone is given by the expression

$$\tilde{\Omega}(\zeta,\tau) = \frac{F(\tau-\zeta)}{\zeta} \tag{33}$$

where ζ is the non-dimensional radial coordinate.

3.3.3 Flame Structural Correlation Functions and Far Field Noise Intensity

Two solutions, Eqs. (31) and (32), are matched at the interface of the burning zone and the non-isothermal zone. This interface is located sufficiently far downstream of the flame where the chemical reaction ceases to exist, but where the gas temperature is approximately equal to the flame temperature. It will be further assumed that the interface is located in the acoustic far field in reference to the reaction zone. Let the location of the interface be ξ, in terms of inner coordinate, and η, in terms of the intermediate coordinate, i.e.,

$$\xi_1/\ell_\theta = \eta_1/\ell_f \tag{34}$$

The physical boundary conditions are the continuity of the pressure and the particle velocity. Application of these conditions to Eqs. (31) and (32) gives the following expressions for $K_i(\tau)$ and $\hat{\Omega}^{(0)}(\xi_1\tau)$

$$K(\tau) = \frac{\gamma\Lambda}{4\pi}\frac{\ell_f}{\ell_\theta}\iiint \frac{A_b^2}{A_0^2}\left\{\frac{\bar{D}\dot{Q}'}{D\tau} + \frac{D'\dot{\bar{Q}}}{D\tau}\right\} dV(\xi_i') \tag{35}$$

$$\Omega^{(0)}(\xi_1\tau) = \hat{\Omega}^{(0)}(\eta,\tau) + \frac{\gamma\Lambda}{4\pi}\frac{\ell_f}{\ell_\theta}\left\{A_b^2\int_{\eta_1}^{\infty}\frac{d\eta}{\eta^2 A_0^2(\eta)} + \frac{1}{\eta_1}\right\}\left\{\iiint \frac{1}{A_0^2}\left(\frac{\bar{D}\dot{Q}'}{D\tau} + \frac{D'\dot{\bar{Q}}}{D\tau}\right) dV\right\} \tag{36}$$

where $A_b = A_0(\xi_1)$.

Matching Eqs. (32) and (33) is carried out by the singular perturbation technique as follows: The variables η and ζ are related so that

$$\eta = \frac{\lambda}{\ell_\theta}\zeta = \frac{\ell_f}{\ell_\theta}\frac{1}{M_\infty}\zeta \tag{37}$$

Thus, for a fixed η, $\zeta \to 0$ as $M_\infty \to 0$. The asymptotic expressions for $\hat{\Omega}(\eta,\tau)$ with $\eta \to \infty$, and $\tilde{\Omega}(\zeta,\tau)$, for $\zeta \to 0$, are given by

$$\hat{\Omega}(\eta,\tau) \simeq \frac{K(\tau)}{A_\infty^2\eta} + \hat{\Omega}^{(0)}(\eta,\tau), \qquad \eta \to \infty \tag{38}$$

and

$$\tilde{\Omega}(\zeta,\tau) \simeq \frac{\ell_\theta}{\lambda}\left\{\frac{F(\tau)}{\eta} - \dot{F}(\tau) + O(\eta)\right\}, \quad \frac{\ell_\theta}{\lambda}\eta \to 0 \tag{39}$$

The continuity conditions give the following relations

$$F(\tau) \simeq \frac{\gamma\Lambda}{4\pi}\frac{\ell_f}{\lambda}\left(\frac{A_b}{A_\infty}\right)^2\iiint\frac{1}{A_0^2}\left(\frac{\bar{D}\dot{Q}'}{D\tau}+\frac{D'\dot{\bar{Q}}}{D\tau}\right)dV(\xi_i') \tag{40}$$

$$\hat{\Omega}^{(o)}(\tau) \simeq \frac{\gamma\Lambda}{4\pi}\frac{\ell_\theta\ell_f}{\lambda^2}\left(\frac{A_b}{A_\infty}\right)^2\frac{\partial}{\partial\tau}\iiint\frac{1}{A_0^2}\left(\frac{\bar{D}\dot{Q}'}{D\tau}+\frac{D'\dot{\bar{Q}}}{D\tau}\right)dV(\xi_i') \tag{41}$$

Substituting Eq. (41) into Eq. (36) gives

$$\Omega^{(o)}(\tau) \simeq \frac{\gamma\Lambda}{4\pi}\left\{\frac{\ell_f}{\ell_\theta}\left[A_b^2\int_{\eta_1}^{\infty}\frac{d\eta}{\eta^2A_0^2}+\frac{1}{\eta_1}\right]\left[\iiint\frac{1}{A_0^2}\left(\frac{\bar{D}\dot{Q}'}{D\tau}+\frac{D'\dot{\bar{Q}}}{D\tau}\right)dV(\xi_i')\right]\right.$$

$$\left.-\frac{\ell_f\ell_\theta}{\lambda^2}\left(\frac{A_b}{A_\infty}\right)^2\frac{\partial}{\partial\tau}\iiint\frac{1}{A_0^2}\left(\frac{\bar{D}\dot{Q}'}{D\tau}+\frac{D'\dot{\bar{Q}}}{D\tau}\right)dV(\xi_i')\right\} \tag{42}$$

The far field solution, in the wave zone, is given explicitely in the following form

$$\frac{p-p_0}{p_0} \simeq \frac{\gamma\Lambda M_\infty^2}{4\pi}\left(\frac{A_b}{A_\infty}\right)^2\frac{1}{\pi}\iiint\frac{1}{A_0^2}\left[\frac{\bar{D}\dot{Q}'}{D\tau}+\left(u'-\frac{\theta'}{\bar{\theta}}\bar{u}\right)_i\frac{\partial\dot{\bar{Q}}}{\partial\xi_i}\right]dV(\xi_i') \tag{43}$$

where $\theta(\xi)=T_0(\xi)/T_0(\xi=\xi_1)$, $\theta'(\xi_i,\tau)$ is the temperature fluctuation. This expression represents the monopole sound field. Comparing the far field expressions in the radiation zone, Eq. (43), with the burning zone, Eq. (31), one notes that the former expression is similar except for the multiplicative factor of the ratio of the square of the speeds of sound, $(A_b/A_\infty)^2$, or T_b/T_∞ . The appearance of the temperature factor is consistent with the view suggested by elementary acoustics. The essential similarity preserved in far field expressions is not surprising in view of the fact that the non-isothermal layer is assumed to be acoustically compact so that the refraction and reflection of the pressure wave in the thin layer does not alter the essential emission characteristics. This suggests a model of a composite layer, consisting of a burning zone and the non-isothermal layer, which has a source strength given by the following expression

$$\left(\frac{A_b}{A_\infty}\right)^2\frac{1}{A_0^2}\left[\frac{\bar{D}\dot{Q}'}{D\tau}+\left(u'-\frac{\theta'}{\bar{\theta}}\bar{u}\right)_i\frac{\partial\dot{\bar{Q}}}{\partial\xi_i}\right] \tag{44}$$

As the thickness of the non-isothermal layer increases, the expression, Eq. (43), deviates from the actual value considerably in view of the refraction and reflection. The problem can be treated by replacing Eq. (22) with the full wave equation which can be obtained by appropriate scaling listed in Table II.

The far field noise intensity is calculated from Eq. (43), and is given by the following expression.

$$I = \frac{(\gamma-1)^2(\Delta H)^2 \omega_0^2 \ell_f^2 V_1^2}{16\pi^2 \rho_\infty a_\infty^5 r^2}(R_{11} + R_{22} + R_{33} + R_{12} + R_{13} + R_{23}) \qquad (45)$$

where the values of R_{ij} are obtained from the following double correlation

$$R_{ij} = \iint_{V(\alpha)V(\beta)} \bar{S}_{ij}(\alpha,\beta)\, dV(\alpha)\, dV(\beta) \qquad (46)$$

α and β denote the positions of two correlation elements. The integrands $S_{ij}(\alpha,\beta)$ are

$$\left.\begin{aligned}
S_{11} &= \frac{\partial \dot{Q}'(\alpha)}{\partial \tau}\frac{\partial \dot{Q}'(\beta)}{\partial \tau}, \qquad S_{22} = \bar{u}_i(\alpha)\bar{u}_j(\beta)\frac{\partial \dot{Q}'(\alpha)}{\partial \xi_i}\frac{\partial \dot{Q}'(\beta)}{\partial \xi_j} \\
S_{33} &= \left[u_i'(\alpha) - \frac{\theta'(\alpha)}{\bar{\theta}(\alpha)}\bar{u}_i(\alpha)\right]\left[u_j'(\beta) - \frac{\theta'(\beta)}{\bar{\theta}(\beta)}\bar{u}_j(\beta)\right]\frac{\partial \bar{\dot{Q}}(\alpha)}{\partial \xi_i}\frac{\partial \bar{\dot{Q}}(\beta)}{\partial \xi_j} \\
S_{12} &= 2\,\bar{u}_i(\alpha)\frac{\partial \dot{Q}'(\alpha)}{\partial \xi_i}\frac{\partial \dot{Q}'(\beta)}{\partial \tau}, \qquad S_{13} = 2\left[u_i'(\alpha) - \frac{\theta'(\alpha)}{\bar{\theta}(\alpha)}\bar{u}_i(\alpha)\right]\frac{\partial \dot{Q}'(\beta)}{\partial \tau}\frac{\partial \bar{\dot{Q}}(\beta)}{\partial \xi_i} \\
S_{23} &= 2\left[u_i'(\alpha) - \frac{\theta'(\alpha)}{\bar{\theta}(\alpha)}\bar{u}_i(\alpha)\right]\frac{\partial \dot{Q}'(\beta)}{\partial \xi_j}\bar{u}_j(\beta)\frac{\partial \bar{\dot{Q}}(\alpha)}{\partial \xi_i}
\end{aligned}\right\} \qquad (47)$$

The intensity of the far field sound is expressed by six sources represented by double correlation functions consisting of the fluctuating velocity, time derivative, and spatial derivatives of the fluctuating heat release rate, together with appropriate weighting factors comprised of steady state gas velocity and the spatial derivatives of heat release rate. In view of the physical significance attached to R_{ij} , the sum of these functions may be termed as the flame structural correlation function. The factor preceding these correlation functions is the scaling factor for the noise intensity. While the scaling factor depends on the thermochemical parameters and the upstream condition, the structural factor contains flame properties which depend on the Reynolds number, (turbulence intensity), and other aerothermo-chemical parameters. The evaluation of these structural factors, experimentally or analytically, is the central problem of the open flame noise generation. The explicit forms of structural factor based on the wrinkled flame model and the distributed reaction model are of particular interest. This is described in the following section.

4. Structural Correlation Function for Wrinkled Flame Model and Distributed Reaction Model

The contemporary theories of turbulent flames are well documented in many books, (see for example, Williams [13] and Beers [15]). The wrinkled flame model advocated by Damkohler [16], Schelkin [17], and Karlovitz [10] is based on the assumption that the flame zone is a regime in which the fuel is burned as in laminar flame. Turbulence is considered to wrinkle and perhaps to disconnect the flame; and augmentation of the burning rate results from the enlargement of the flame area due to the wrinkling of the flame surface. The distributed reaction model suggested by Summerfield, et al, [11], supports the view that the flame structure is governed by the small scale turbulence which alters the effective transport coefficient and perhaps the effective chemical reaction rate. Structural factors for these two flame models can be evaluated in terms of the turbulent burning speed S_T .

4.1 Wrinkled Flame Model

The generalized view of the wrinkled flame is that the flame is wrinkled by large scale turbulence and the flame structure remains essentially laminar, though the velocity and temperature may fluctuate within the flame, as in Fig. (4a). An element of non-steady laminar flame, Fig. (4b), may be considered as executing linear pulsation and rotation by the imposing turbulent fluctuation about a mean flame position. The overall wirnkled flame may be regarded as the collection of these small flamelets with their translational and rotational motion determined by the local turbulence. The total noise intensity is calculated by the sum of the noise produced by individual filament.

The analysis for each filament is carried on the frame of reference 0' fixed in the flame structure; (Fig. 4b) which is moving relative to a fixed point 0 . T simplify the analysis further, the rotational motion will be neglected. An elementary Galilean transformation yields the relation between the moving, $\bar{\xi}$, and the fixed, $\bar{x}$, coordinates.

$$\bar{X} = \int_0^t \dot{\bar{X}}(t)d\tau + \bar{\xi} = \bar{X}_c(t) + \bar{\xi} \qquad (48)$$

where $\dot{\bar{x}}(t)$ is the velocity of the point 0' . All the basic scalar conservation laws remain invariant under Galilean transformation, whereas the momentum equation becomes

$$\frac{D\bar{u}}{Dt} = -\frac{1}{\rho}\nabla p + \frac{1}{\rho}\nabla\cdot\bar{\bar{\tau}} + \frac{d\dot{\bar{X}}}{dt} \qquad (49)$$

The wave equation is found to be invariant under this transformation, and thus, the results obtained in previous sections remain applicable, in the new coordinate. The pressure fluctuation is the far field is given by

$$\frac{p-p_0}{p_0} \simeq \frac{\gamma \Lambda M_\infty^2}{4\pi}\left(\frac{A_b}{A_\infty}\right)^2 \iiint \frac{1}{|\bar{r}-(\bar{x}_c+\bar{\xi})|\,A_0^2}\left[\frac{\bar{D}\dot{Q}'}{D\tau}+\frac{D'\bar{\dot{Q}}}{D\tau}\right] dV(\xi_i') \qquad (50)$$

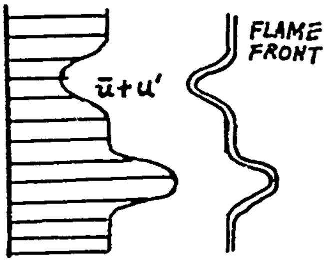


Fig. 4a
Model of Wrinkled Flame

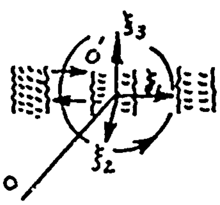


Fig. 4b
Fluctuating Flamelet

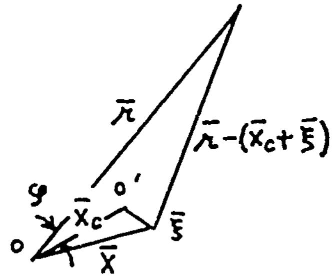


Fig. 4c
Coordinate System

In the geometrically far field $|\bar{r}| >> |\bar{x}|$, $|\bar{\xi}|$, Eq. (50) is expanded in the following form

$$\left(\frac{p-p_0}{p_0}\right)_i \simeq \frac{\gamma \Lambda M_\infty^2}{4\pi}\left(\frac{A_b}{A_\infty}\right)^2\left\{\frac{1}{r}\iiint_{V_i}\frac{1}{A_0^2}\left(\frac{\bar{D}\dot{Q}'}{D\tau}+\frac{D'\bar{\dot{Q}}}{D\tau}\right)dV_i(\xi_i')\right.$$

$$\left. + \frac{(\bar{x}_0\cdot\bar{n})_i}{r^2}\iiint\frac{1}{A_0^2}\left(\frac{\bar{D}\dot{Q}'}{D\tau}+\frac{D'\bar{\dot{Q}}}{D\tau}\right)dV_i(\xi_i')\right.$$

The first term on the right hand side represents the monopole sound field produced by the small scale turbulence in the laminar flame, whereas the second term is the dipole field generated by the fluctuation of the flamelet. The total noise produced by the wrinkled flame is

$$\frac{p-p_0}{p_0} \simeq \frac{\gamma \Lambda M_\infty^2}{4\pi}\left(\frac{A_b}{A_\infty}\right)^2\left\{\frac{1}{r}\sum_i\iiint_{V_i}\frac{1}{A_0^2}\left(\frac{\bar{D}\dot{Q}'}{D\tau}+\frac{D'\bar{\dot{Q}}}{D\tau}\right)dV_i(\xi')\right.$$

$$\left. + \frac{1}{r^2}\sum_i l_{e_i} e^{i\omega_i\tau}\cos\varphi_i\iiint_{V_i}\frac{1}{A_0^2}\left(\frac{\bar{D}\dot{Q}'}{D\tau}+\frac{D'\bar{\dot{Q}}}{D\tau}\right)dV_i(\xi_i')\right\}$$

If the flamelets are connected, and if the rate of the reaction is independent of time, despite the velocity fluctuation (relative to the point 0' fixed in the flame), the monopole field may be rewritten as follows

$$\frac{p-p_0}{p_0} \simeq \frac{\gamma \Lambda M_\infty^2}{4\pi}\left(\frac{A_b}{A_\infty}\right)^2\frac{1}{r}\iiint_V\frac{1}{\theta}\left(u_i'-\frac{\theta'}{\theta}\bar{u}_i\right)\frac{\partial\bar{\dot{Q}}}{\partial\xi_i}\,dV(\xi_i) \qquad (51)$$

It is of practical interest to deduce the sound intensity formula in terms of the turbulent burning speed S_T . Note that $\dot{Q} \sim \omega_\ell = S_L(d\varepsilon/d\xi_n)$, where ε is the reaction progress variable, ξ_n is the coordinate measured in the direction normal to the flame surface. Recalling that $S_L dA_\ell = S_T dA_T$ by virtue of the essential hypothesis of the wrinkled flame model, Eq. (51) reduces to

$$\frac{p-p_0}{p_0} \simeq \frac{\gamma \Lambda M_\infty^2}{4\pi}\left(\frac{A_b}{A_\infty}\right)^2 \frac{1}{r}\iiint\left(\frac{S_T}{S_L}\right)\frac{1}{\theta}\left(u'-\frac{\theta'}{\theta}\bar{u}\right)_i \frac{\partial^2\epsilon}{\partial\xi_i\partial\xi_n}\, dA_T\, d\xi_n \qquad (52)$$

Thus, the sound intensity is given by

$$I = \frac{(\gamma-1)(\Delta H)^2 \rho_0 S_L^2 V_1^2}{16\pi^2 a_\infty^5 r^2}\left(\frac{S_T}{S_L}\right)^2 \Gamma^2 \qquad (53a)$$

where

$$\Gamma^2 = \int_{V(\alpha)}\int_{V(\beta)} \left[\frac{1}{\theta(\alpha)}\left(u'-\frac{\theta'}{\theta}\bar{u}\right)_i\right]_\alpha \left[\frac{1}{\theta}\left(u'-\frac{\theta'}{\theta}\bar{u}\right)_j\right]_\beta \frac{\partial^2\epsilon(\alpha)}{\partial\xi_i\partial\xi_n}\frac{\partial^2\epsilon(\beta)}{\partial\xi_j\partial\xi_n}\, dV(\alpha)\, dV(\beta) \qquad (53b)$$

and $dV = dA_T d\xi_n$. Damkohler and Schelkin have derived the ratio of the effective turbulent burning speed to that of laminar flame, and this ratio is given by the following expression

$$\frac{S_T}{S_L} = \left[1 + c\left(\frac{v'}{S_L}\right)^2\right]^{\frac{1}{2}} \qquad (54)$$

For high intensity turbulence, the following expression is used

$$S_T = v' \qquad (55)$$

where v' is the turbulent velocity component normal to flame surface. The ratio of turbulent to laminar burning speed is experimentally correlated in terms of Reynolds number and physico-chemical parameters, see Table 3.

By substituting Eq. (54) into Eq. (53a) and assuming that the velocity fluctuation v' is linearly correlated with the Reynolds number, $v' = \alpha Re$, the expressions for the sound intensity becomes

$$I = \frac{(\gamma-1)(\Delta H)^2 \rho_0 S_L^2 V_1^2}{16\pi^2 a_\infty^5 r^2}\left(1 + \beta Re^2\right)\Gamma^2, \quad \beta = c\left(\frac{\alpha}{S_L}\right)^2 \qquad (56a)$$

For high intensity turbulence, the sound intensity is given, alternatively, by the following expression:

$$I = \frac{(\gamma-1)(\Delta H)^2 \rho_0 S_L^2 V_1^2}{16\pi^2 a_\infty^5 r^2}\hat{\alpha}^2 Re^2 \Gamma^2 \quad , \quad \hat{\alpha} = \frac{\alpha}{S_L} \qquad (56b)$$

Other expressions of sound intensity for different burning rate formulas are listed in Table 3, and the dependence of the intensity is illustrated in Fig. 5.

Table 3

Sound Intensity Based on Wrinkled Flame Model

Investigator	Burning velocity	$\mathcal{I} = I \times \frac{16\pi^2 a_\infty^5 h^2}{(\gamma-1)(\Delta H)^2 \rho_0 S_L^2 V_1^2 \Gamma'^2}$
Damkohler Karlovitz	$S_T/S_L = 1 + \alpha R_e$, low vel. $S_T/S_L = \frac{\alpha}{S_L} Re = \hat{\alpha} Re$, high vel.	$(1 + \alpha Re)^2$ $\hat{\alpha}^2 Re^2$
Bollinger & Williams	$S_T/S_L = 0.18 d^{0.26} Re^{0.24}$ d = burner tube diam (cm)	$0.324 d^{0.52} Re^{0.48}$
Schelkin	$S_T/S_L = (1 + \beta Re^2)^{\frac{1}{2}}$, low vel. $S_T/S_L = \frac{\alpha}{S_L} Re = \hat{\alpha} Re$, high vel.	$1 + \beta Re^2$ $\hat{\alpha}^2 Re^2$
Scurlock	$S_T/S_L = (1 + c_3 \bar{y}/\ell)^{\frac{1}{2}}$	$1 + c_3 \bar{y}/\ell$

where Γ^2 is given by Eq. (53b).

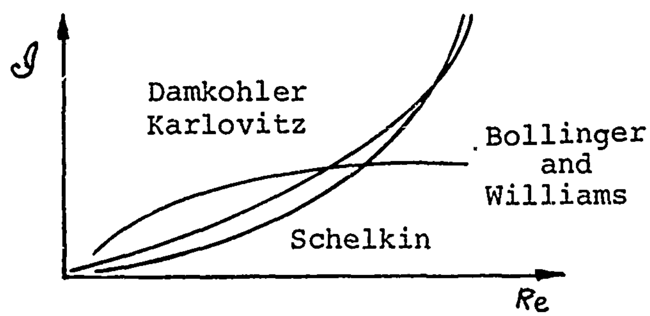


Fig. 5 Dependence of the sound intensity on Reynolds number based on wrinkled flame model.

4.2 Distributed Reaction Model

For small scale turbulence, Summerfield, et al, [11], proposed a "Similarity hypothesis" relating to the Reynolds number based on the burning speed, flame thickness, and viscosity, i.e.,

$$\frac{S_T \ell_T}{\nu_T} = \frac{S_L \ell_L}{\nu_L} = f(\phi) \tag{57}$$

where ν_T is the effective turbulent diffusivity, ℓ_T , ℓ_L are the thickness of the turbulent and the laminar flame respectively, and $f(\phi)$ is the function of the fuel-air ratio for a given fuel. Noting $\dot{Q} = S_T \frac{\partial \epsilon}{\partial \xi_n}$ the effective turbulent diffusivity is given by Prandtl mixing length theory

$$\nu_T = \nu_L \left(1 + \ell_m V_1 v' / \nu_L \right) = \nu_L \left(1 + a R_e v' \right) \qquad (58)$$

where ϵ is the reaction progress variable, ℓ_m is the effective mixing length, and v' is the turbulent veolcity component normal to the flame surface. Substituting Eqs. (57a,b) into Eq. (45) gives the following expression for the noise intensity

$$I = \frac{(\gamma-1)\Delta H^2 \rho_0 f^2(\varphi) \nu_L^2 V_1^2}{16 \pi^2 a_\infty^5 r^2 \ell_L^2} \Sigma R_{ij} \qquad (59)$$

The integrands $\bar{S}_{ij}$ appearing in the double correlation function R_{ij} are given by

$$\bar{S}_{11} = a^2 R_e^2 \left\{ \frac{\partial}{\partial \tau}\left(v' \frac{\partial \epsilon}{\partial \xi_n} \right)_\alpha \right\} \left\{ \frac{\partial}{\partial \tau}\left(v' \frac{\partial \epsilon}{\partial \xi_n} \right)_\beta \right\}$$

$$\bar{S}_{22} = a^2 R_e^2 \left\{ \bar{u}_i(\alpha) \bar{u}_j(\beta) \left[\frac{\partial}{\partial \xi_i}\left(v' \frac{\partial \epsilon}{\partial \xi_n} \right) \right]_\alpha \left[\frac{\partial}{\partial \xi_j}\left(v' \frac{\partial \epsilon}{\partial \xi_n} \right) \right]_\beta \right\}$$

$$\bar{S}_{33} = \left(u_i' - \frac{\theta'}{\theta} \bar{u}_i \right)_\alpha \left(u_j' - \frac{\theta'}{\theta} \bar{u}_j \right)_\beta \left(\frac{\partial^2 \epsilon}{\partial \xi_i \partial \xi_n} \right)_\alpha \left(\frac{\partial^2 \epsilon}{\partial \xi_j \partial \xi_n} \right)_\beta$$

$$\bar{S}_{12} = 2 a^2 R_e^2 \, \bar{u}_i(\alpha) \left[\frac{\partial}{\partial \xi_i}\left(v' \frac{\partial \epsilon}{\partial \xi_n} \right) \right]_\alpha \left[\frac{\partial}{\partial \xi}\left(v' \frac{\partial \epsilon}{\partial \xi_n} \right) \right]_\beta$$

$$\bar{S}_{13} = 2 a R_e \left(u_i' - \frac{\theta'}{\theta} \bar{u}_i \right)_\alpha \frac{\partial}{\partial \tau}\left(v' \frac{\partial \epsilon}{\partial \xi_n} \right)_\beta \frac{\partial \dot{\bar{Q}}(\beta)}{\partial \xi_i}$$

$$\bar{S}_{23} = 2 a R_e \left(u_i' - \frac{\theta'}{\theta} \bar{u}_i \right)_\alpha \frac{\partial}{\partial \xi_i}\left(v' \frac{\partial \epsilon}{\partial \xi_n} \right)_\beta \left[\bar{u}_j \frac{\partial^2 \bar{\epsilon}}{\partial \xi_j \partial \xi_n} \right]_\alpha$$

The intensity expression, Eq. (59), can be rearranged in the following form

$$I = \frac{(\gamma-1)\Delta H^2 \rho_0 f^2(\varphi) \nu_L^2 V_1^2}{16 \pi^2 a_\infty^5 r^2 \ell_L^2} \left\{ a^2 R_e^2 \left(\hat{R}_{11} + \hat{R}_{22} + \hat{R}_{12} \right) + a R_e \left(\hat{R}_{13} + \hat{R}_{23} \right) + R_{33} \right\} \qquad (60)$$

where $aRe\hat{R}_{13} = R_{13}$, $a^2Re^2\hat{R}_{11} = R_{11}$, etc. For high intensity turbulence, the source terms proportional to the square of the Reynolds number dominate over two other terms.

5. Near Field Structure

While the far field noise intensity is described by the correlation functions obtained by a matched asymptotic solution, the detailed mechanisms of the near field sound generation and dispersion are of particular interest. The pattern of pressure fluctuation in this compact zone, unlike that of the free space, is coupled to the turbulent fluctuation. The near field sound is comprised of primarily pseudo-sound field generated by the fluctuating heat release. This pseudo-sound field is subsequently joined by the true sound field in the adjacent zones. The near field analysis of the open flame of known structure, inclusive of steady and turbulent properties, is described in this section.

To facilitate the analysis, the domain of interest has been divided into three zones; non-reacting upstream region, I, reaction zone, II, and hot downstream region, III, (Fig. 6a).

The analysis will be simplified by assuming that the temperature and the velocities are uniform in the three zones, and no reaction occurs in the upstream and the downstream zones except in the reaction wave. The turbulent fluctuation is assumed to be one dimensional and the speed of sound in the upstream zone is unity, whereas the sound speed in the reaction and downstream zone is taken to be $a_2 = a_0(\theta_f/\theta_0)^{\frac{1}{2}}$.

The equations in these three zones are given in the following:

For a zone extending from upstream to the flame front $-\infty < \xi \le 0$,

$$\frac{\bar{D}^2\Omega_1}{D\tau^2} - \frac{1}{M_0^2}\frac{\partial^2\Omega_1}{\partial\xi^2} = 0 \qquad (61)$$

The linearized convected wave equation in the flame zone is, $0 < \xi < 1$,

$$\frac{\bar{D}^2\Omega_2}{D\tau^2} - \frac{1}{M_2^2}\frac{\partial^2\Omega_2}{\partial\xi^2} + \Lambda\dot{\bar{Q}}(\xi)\frac{\bar{D}\Omega_2}{D\tau} + \Lambda U_2\frac{d\dot{\bar{Q}}}{d\xi}\Omega_2$$

$$= \gamma\Lambda\left[u'\frac{d\dot{\bar{Q}}}{d\xi} + \frac{\bar{D}\dot{Q}'}{D\tau}\right] = \gamma\Lambda N(\xi,\tau) \qquad (62)$$

where $M_2 = M_0(\theta_f/\theta_0)^{\frac{1}{2}}$. The wave eqaution in the downstream of the flame $(1 < \xi < \infty)$ is

$$\frac{D^2\Omega_3}{D\tau^2} - \frac{1}{M_2^2}\frac{\partial^2\Omega_3}{\partial\xi^2} = 0 \qquad (63)$$

where the subscript, 2 , refers to the quantities in zones (II) and (III).

In the present analysis, the upstream and the downstream regions are assumed to be of semi-infinite extent, and that the sound waves generated in the flame propagate upstream and downstream without reflections. In reality, the geometrical configuration of the flame is such that the upstream region does not extend to infinity, and that the sound originate from various parts of the flame may incident on the flame front, as is illustrated in Figs. 1 and 2.

The fluctuation in the rate of heat production is given by the following expression

$$N(\xi, \tau) = \sum_n N_n(\xi)\, e^{in\tau} \tag{64}$$

The solutions in the upstream and the downstream regions are given by

$$\left.\begin{aligned} \Omega_1(\xi,\tau) &= \sum A_n\, e^{i\left(n\tau + \frac{nM_0}{1-M_0V_1}\xi + \delta_n\right)} \\ \Omega_3(\xi,\tau) &= \sum B_n\, e^{i\left(n\tau - \frac{nM_2}{1+M_2V_2}\xi + \epsilon_n\right)} \end{aligned}\right\} \tag{65}$$

The constants A_n, B_n, δ_n, and ε_n are to be determined by matching Ω_1, Ω_3 with Ω_2 at the respective boundaries.

It is physically evident that the sound waves generated in the reaction zone will propagate in the upstream and downstream directions. The waves will be partially transmitted to zones 1 and 3, and partially reflected at the boundaries. Hence, the general solution for the reaction should include the particular solution as well as two homogeneous solutions. The general solution is given by

$$\Omega_2(\xi,\tau) = \sum_n e^{in\tau}\left\{ C_n e^{\lambda_n^{(1)}\xi} + D_n e^{\lambda_n^{(2)}\xi} + \gamma\Lambda\left[\left(\frac{1}{M_2^2} - V_2^2\right)\left(\lambda_n^{(1)} - \lambda_n^{(2)}\right)\right]^{-1} \times \int_0^{\xi}\left[e^{\lambda_n^{(1)}(\xi-\xi')} - e^{\lambda_n^{(2)}(\xi-\xi')}\right] N_n(\xi')\, d\xi' \right. \tag{66}$$

where

$$\lambda_n^{(1),(2)} = \frac{V_2}{2\left(\frac{1}{M_2^2} - V_2^2\right)}\left(2in + \Lambda\dot{\bar{Q}}\right) \pm \frac{1}{2}\left\{\left(2in + \Lambda\dot{\bar{Q}}\right)^2 V_2^2 \left(\frac{1}{M_2^2} - V_2^2\right)^{-2} - 4\left(n^2 - in\Lambda\dot{\bar{Q}}\right)V_2\left(\frac{1}{M_2^2} - V_2^2\right)^{-1}\right\}^{\frac{1}{2}}$$

The boundary conditions, at $\xi = 0$ and $\xi = 1$, relate to the continuity of the amplitude and the gas velocity. Thus, at $\xi = 1$, two conditions give

$$A_n e^{i\delta_n} = C_n + D_n \tag{67}$$

and

$$iA_n\left(\frac{nM_0}{1 - M_0V_1}\right)e^{i\delta_n} = C_n\lambda_n^{(1)} + D_n\lambda_n^{(2)} \tag{68}$$

and at $\xi = 1$, continuity conditions give

$$B_n e^{-i\left(\frac{nM_2}{1+M_2V_2} - \epsilon_n\right)} = C_n e^{\lambda_n^{(1)}} + D_n e^{\lambda_n^{(2)}} + F_{P_n}(1) \qquad (69)$$

$$i B_n\left(\frac{nM_2}{1+M_2V_2}\right) e^{-i\left(\frac{nM_2}{1+M_2V_2} - \epsilon_n\right)} = C_n \lambda_n^{(1)} e^{\lambda_n^{(1)}} + D_n \lambda_n^{(2)} e^{\lambda_n^{(2)}} + \frac{dF_{P_n}(1)}{d\xi} \qquad (70)$$

where $F_{Pn}(1)$ and $\frac{dF_{Pn}(1)}{d\xi}$ are the value and the slope of the particular integral of Eq. (66), evaluated at $\xi = 1$. From Eqs. (67), (68), (69), and (70), $A_n e^{i\delta_n}$, $B_n e^{i\epsilon_n}$, C_n , and D_n are calcualted as follows:

$$A_n e^{i\delta_n} = \frac{\frac{M_2}{1+M_2V_2} F_{P_n}(1) - \frac{i}{n}\frac{dF_{P_n}(1)}{d\xi}}{\frac{M_2}{1+M_2V_2} + \frac{M_0}{1-M_0V_1}} \qquad (71)$$

$$B_n e^{i\epsilon_n} = \frac{1}{\Delta}\left\{\left(\lambda_n^{(1)} - \frac{inM_0}{1-M_0V_1}\right)e^{\lambda_n^{(1)}} - \left(\lambda_n^{(2)} - \frac{inM_0}{1-M_0V_1}\right)e^{\lambda_n^{(2)}}\right\} \times$$

$$\left\{\frac{M_2}{1+M_2V_2} F_{P_n}(1) - \frac{i}{n}\frac{dF_{P_n}(1)}{d\xi}\right\} + F_{P_n}(1) \qquad (72)$$

$$C_n = \frac{-1}{\Delta}\left\{\left(\lambda_n^{(2)} + \frac{inM_0}{1-M_0V_1}\right)\left[\frac{M_2}{1+M_2V_2} F_{P_n}(1) - \frac{i}{n}\frac{dF_{P_n}(1)}{d\xi}\right]\right\} \qquad (73)$$

$$D_n = \frac{1}{\Delta}\left\{\left(\lambda_n^{(1)} - \frac{inM_0}{1-M_0V_1}\right)\left[\frac{M_2}{1+M_2V_2} F_{P_n}(1) - \frac{i}{n}\frac{dF_{P_n}}{d\xi}(1)\right]\right\} \qquad (74)$$

where $\Delta = \left(\lambda_n^{(2)} - \lambda_n^{(1)}\right)\left[\frac{M_2}{1+M_2V_2} + \frac{M_0}{1-M_0V_1}\right]$

It is of particular interest to note that the amplitude of the pressure waves are all proportional to

$$\frac{M_2}{1+M_2V_2} F_{P_n}(1) - \frac{i}{n}\frac{dF_{P_n}}{d\xi} \qquad (75)$$

and that the contribution of the second term of this expression diminishes at higher frequencies. To examine the qualitative aspect of this noise generation term, $F_p(\xi)$, the explicit expression of F_p will be calculated when the source function $N_n(\xi)$ is given in the following Fourier series

$$N(\xi,\tau) = \sum_n N_n e^{in\tau} = \sum K_n e^{i(S_n\pi\xi + n\tau)} \qquad (76)$$

Substituting the above expression into Eq. (66), the particular integral is given by

$$F_{p_n}(\xi) = \frac{M_2^2 K_n \left(e^{iS_n\pi\xi} - 1\right)}{(1 - M_2^2 V_2^2)(iS_n\pi - \lambda_n^{(1)})(iS_n\pi - \lambda_n^{(2)})} \qquad (77)$$

It may be noted that, within the present approximation, the inhomogeneous solution approaches infinity as $iS_n\pi \rightarrow \lambda_n^{(1)}$ or $\lambda_n^{(2)}$. This resonant oscillation, however, is likely to be suppressed by the dissipative effects which are not considered in the present analysis. Some of the typical modes of outgoing pressure waves of different harmonics ($n = 1$, 2) induced by the first few modes of the turbulence $S_n = 1, 2, 3, 4$, and 5, are sketched in Fig. 6b.

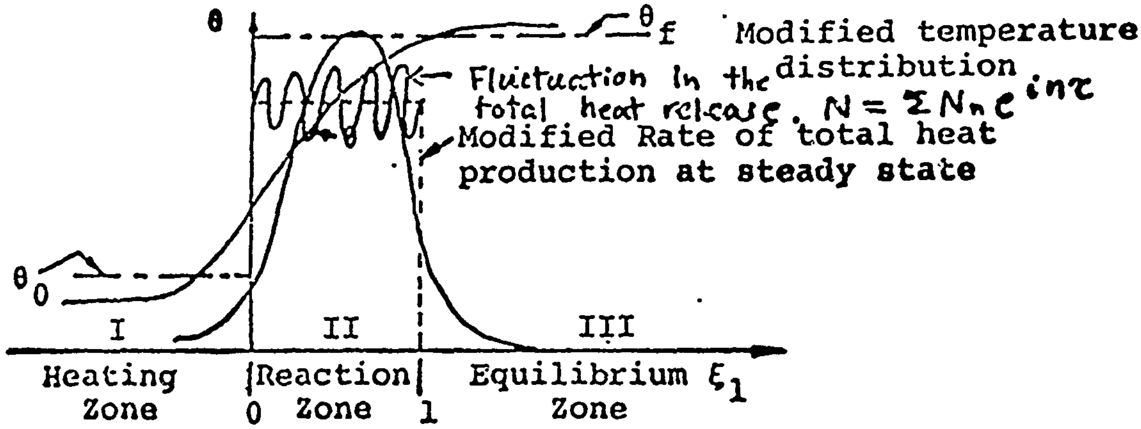


Fig. 6a. Modified Turbulent Steady State Flame Model

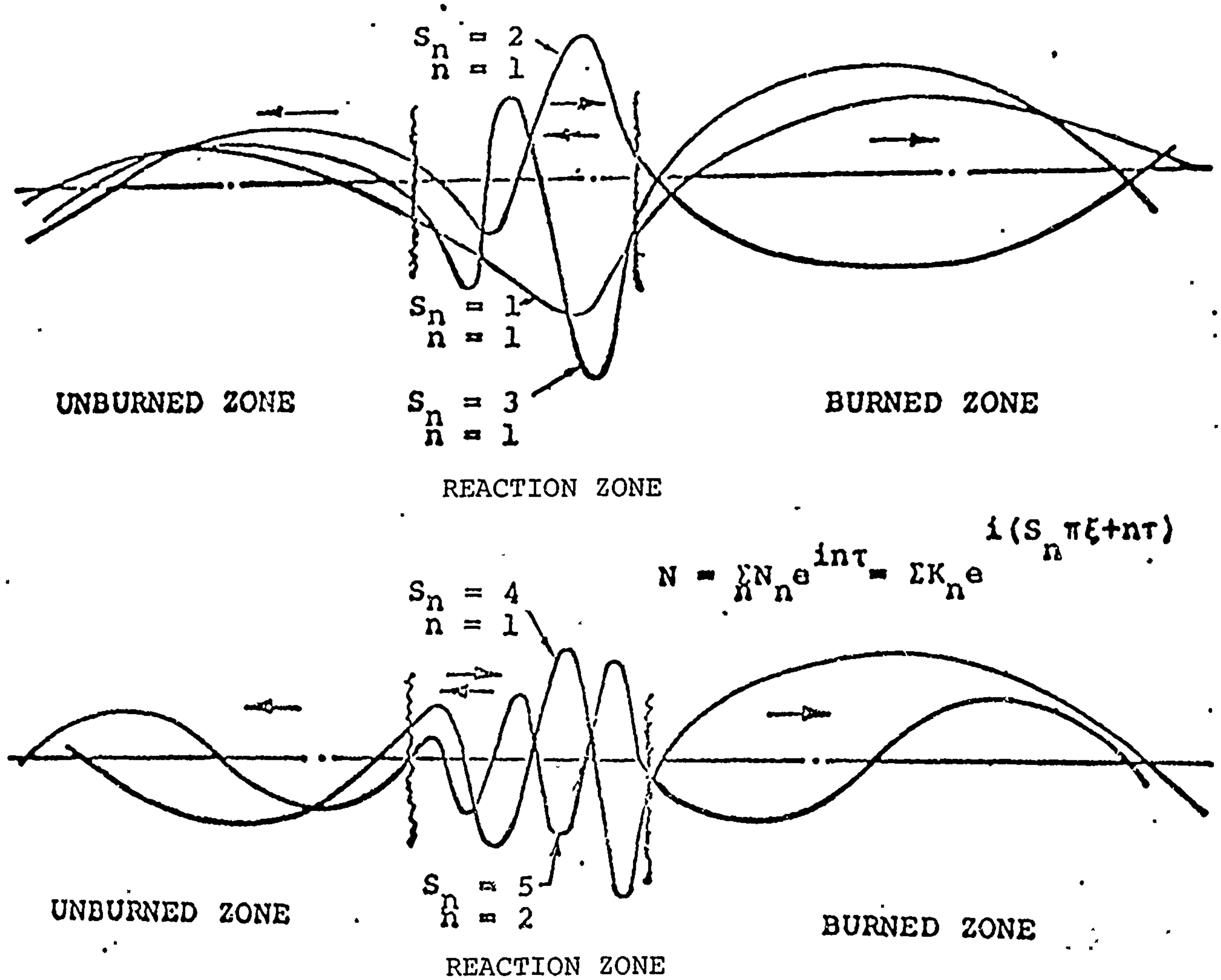


Fig. 6b. Generation and Propagation of Pressure Waves

6. Noise Generation by Spray Combustion

6.1 Intrinsic Noise and Turbulent Driven Noise

The combustion of droplets in the combustor has been recognized as complicated phenomena invoked by heterogeneous aerothermochemical systems, with mass momentum and energy exchange between two phases, in addition to the exothermic gas phase reaction. The mechanisms of the sound generation are also highly complicated in view of the many non-steady phenomena concurrently taking place in the combustion processes. The conservation laws and the derivation of the wave equation are given in Appendix C, and the wave equation is listed in the following

$$\frac{D^2\Omega}{D\tau^2} - \frac{\partial}{\partial x_i}\left[a_j^2 \frac{\partial \Omega}{\partial x_i}\right] = \gamma \frac{D}{D\tau}\left(\frac{\dot{m}}{\rho}\right) - \gamma \frac{\partial}{\partial x_i}\left(\frac{F_i}{\rho}\right) + \frac{D}{D\tau}\left\{\frac{1}{P}\left[(u_i - W_i)F_i - C\psi\right]\right\}$$
$$+ \frac{\gamma}{C_p}\frac{D}{D\tau}\left(\frac{\dot{m}}{\rho}\frac{L}{T_\ell}\right) + (\gamma-1)\frac{D}{D\tau}\left(\frac{\dot{Q}}{P}\right) + \gamma \frac{\partial u_i}{\partial x_j}\frac{\partial u_j}{\partial x_i} - \gamma \frac{\partial}{\partial x_i}\frac{1}{\rho}\frac{\partial \tau_{ij}}{\partial x_j} \qquad (78)$$

where $\dot{m}$ is the rate of the droplet vaporization per unit volume; F_i is the force per unit volume exerted on the gas phase; W_i is the droplet velocity component; T_ℓ is the temperature of the liquid; L is the latent heat of the vaporization; and ψ is the rate of the heat exchange between two phases. Two mechanisms of noise generation, namely, intrinsic and turbulent driven, and the intensities of these sound fields, will be described in the following sections.

6.2 Intrinsic Noise Generation

Spray combustion is inherently non-steady in the time scale equal to the droplet's life time, which ranges from 10^{-3} to 1 sec for 50 ∿ 200 micron sized droplets. The major physical processes responsible for the noise generation includes, vaporization accompanied by mass and heat exchange, followed by diffusion of the fuel vapor, and the non-steady combustion in the gas phase. The following simplified assumptions will be made in the analysis of the intrinsic noise generation. Firstly, the free stream flow is steady, and the droplet interactions are neglected. Secondly, droplets are all assumed to be spherical, and the combustion of the droplet starts simultaneously as the vaporization of the droplet starts. In general, time lag of the order of the gas phase diffusion, ranging 10^{-6} to 10^{-2} sec, exists between vaporization and combustion. Finally, the droplets are assumed to be in dynamic equilibrium with the gas phase, and the transport phenomena contributes an insignificant effect on the generation of intrinsic sound. The equations are non-dimensionalized as follows:

$$M_0^2 \frac{D^2\Omega}{D\tau^2} - \frac{\partial}{\partial \xi_i}\left(A^2 \frac{\partial \Omega}{\partial \xi_i}\right) = \gamma M_0^2 \Lambda_\ell \frac{D}{D\tau}\left\{\frac{f}{\rho}\left(\eta^2 \frac{d\eta}{d\tau}\right)\left[1 + \frac{L}{\Delta H}\frac{T_\ell}{T_1} + \frac{C_p T_1}{\Delta H}\right]\right\}. \qquad (79)$$

where $D/D\tau = \partial/\partial\tau + M_0 \bar{u}_i \frac{\partial}{\partial \xi_i}$ and the non-dimensional quantities are defined as $\xi_i = x_i/d_0$, $\tau = t/t_{life}$, $A^2 = a_f^2/a_1^2$, $M_0 = d_0 a_1/t_{life}$

$$u_i = U_i/(d_0/t_{life}),\ \Lambda_l = (N_0 m_l \Delta H)/\rho_1 C_p T_1\ ,\ \eta = d/d_0\ ,\ f = n/N_0$$

The reference quantity, d_0 , is the typical droplet diameter, t_{life} is the life-time of the droplet, N_0 is the typical droplet number density per unit volume, m_ℓ is the mass of a droplet, and a_1 , ρ_1 , T_1 are the speed of sound, gas density, and temperature of the hot gas in the burning zone. The effective Mach number appearing in Eq. (79) is, in general, small, and thus, the first term which appears on the left hand side of Eq. (79) is negligible. The terms which appear on the right hand side are the sound source terms, including the effects of mass ejection, vaporization, and combustion. In estimating the far field intensity, it will be further assumed that the characteristic dimension of the burning zone and the non-isothermal layer are all small compared with the wave length, and that the temperature distribution in the non-isothermal zone is nearly spherical. Following the analysis similar to what has been presented in Section 3 the far field intensity is calculated to be

$$\frac{p-p_0}{p_0} \simeq \frac{\gamma \Lambda_l M_0^2}{4\pi}\left(\frac{A_1}{A_\infty}\right)^2 \left\{ \frac{1}{r} \iiint_V f \frac{\partial}{\partial \tau}\left[\eta^2 \frac{d\eta}{d\tau}\left(1 + \frac{L T_l}{(\Delta H) T_1} + \frac{C_p T_1}{\Delta H}\right] dV \right\} \qquad (80)$$

The order of magnitude of this intensity of the monopole field can be estimated by the simplifying assumption that the numerical density of the droplet remains constant, and that the hot gas density and temperature are uniform. The change in the droplet diameter is assumed be given by the following experimental formula [13]

$$\eta^2 = 1 - \tau \qquad (81)$$

Substituting Eq. (81) into Eq. (80), the sound intensity is finally given by

$$I_{Max} = \frac{(\gamma-1)^2 (N_0 m_l \Delta H)^2}{64\pi^2 \rho_\infty a_\infty r^2}\left(\frac{B}{a_\infty t_{life}}\right)^4 \left(1 + \frac{L T_l}{(\Delta H) T_1} + \frac{C_p T_1}{\Delta H}\right) V^2 \qquad (82)$$

where V is the total volume of the combustion zone, and B is the characteristic dimension of the combustion zone. It is interesting to note that the intensity is inversely proportional to the fourth power of the life time of the droplet. This expression, however, is the maximum intensity that would be reached when all the droplets burn simultaneously in the period of the droplet life time. In practice, the burning of the droplet is random in time and in space, thus, the intrinsic noise expression should be weighed by the appropriate statistical factor g(r). The final expression, Eq. (82), will be accordingly modified.

6.3 Turbulent Driven Noise in Spray Combustion

Turbulent fluctuations in the spray combustion zone produce complicated non-steady combustion processes. It is physically evident that the turbulence, with its eddy sizes smaller than typical inter-droplet distances, will affect the number density of the droplets, and more importantly, the burning rate. The generation of noise by turbulent fluctuations can be treated by the method applied in the preceding subsection, except that scaling and non-dimensionalization need to be modified accordingly, as shown in Table 7.

The physical dimension is the size of the turbulent eddy, ℓ_e, and the reference time scale is, ℓ_e/u_1. The wave equation, Eq. (78), is now deduced into the following form:

$$M_0^2 \frac{D^2\Omega}{D\tau^2} - \frac{\partial}{\partial \xi_i}\left(A^2 \frac{\partial \Omega}{\partial \xi_i}\right) = \gamma M_0^2 \tilde{\Lambda}_\ell \frac{D}{D\tau}\left\{\frac{f}{\rho}\left(\frac{\dot{m}}{\dot{m}_0}\right)\left[1 + \frac{L\,T_0}{(\Delta H)T_\ell} + \frac{C_p T_1}{\Delta H}\right]\right\} \quad (83)$$

where $M_0^2\tilde{\Lambda}_\ell = \frac{N_0 \dot{m}_0}{\rho_1}\left(\frac{\Delta H}{c_p T_1}\right)\left(\frac{u}{\ell_e}\right)\left(\frac{\ell_e}{a_1}\right)^2$, $\dot{m}_0$ is the reference burning rate of the droplet. Rewriting the right hand side of Eq. (80) in terms of steady and fluctuating quantities, the far field expression for the pressure fluctuation becomes

$$\frac{p-p_0}{p_0} \simeq \frac{\gamma \tilde{\Lambda}_\ell M_0^2}{4\pi}\left(\frac{A_1}{A_\infty}\right)^2 \frac{1}{r} \iiint \left\{ \frac{\bar{f}}{\bar{\rho}}\left[1 + \frac{L}{\Delta H}\frac{\bar{\theta}_1}{\theta_\ell} + \frac{C_p T_1 \bar{\theta}_1}{\Delta H}\right]\left[\frac{\bar{D}\mu'}{D\tau} + \frac{D'\bar{\mu}}{D\tau}\right]\right.$$

$$+ \left[1 + \frac{L}{(\Delta H)}\frac{\bar{\theta}_1}{\theta_\ell} + \frac{C_p T_1 \theta_1}{\Delta H}\right]\left[\frac{1}{\bar{\rho}}\left(\frac{\bar{D}f'}{D\tau} + \frac{D'\bar{f}}{D\tau}\right) - \frac{\bar{f}}{\bar{\rho}^2}\left(\frac{\bar{D}\rho'}{D\tau} + \frac{D'\bar{\rho}}{D\tau}\right)\right]$$

$$\left. + \frac{\bar{f}}{\bar{\rho}}\bar{\mu}\left[\frac{L\,\theta_1}{(\Delta H)\theta_\ell} + \frac{C_p T_1}{\Delta H}\right]\left(\frac{\bar{D}\theta'}{D\tau} + \frac{D'\bar{\theta}}{D\tau}\right)\right\} dV(\xi_i) \quad (84)$$

The fluctuations in the velocity, the density, and the temperature of the gas, as well as the numerical density and the burning rate, affect the strength of noise sources. Those quantities are either to be determined experimentally or analytically. If the characteristic time of the turbulence is larger than the mass diffusion and the heat transfer between the gas and the droplets, the quasi-steady expression of the mass burning rate may be used. For example, the burning rate of a droplet with convective effect is given by the following expression [13]

$$\dot{m}_0 \bar{\mu} = (1 + 0.27 Re^{\frac{1}{2}} Sc^{\frac{1}{3}}) \frac{2\pi K d_0}{C_p} \ln\left\{1 + \frac{C_p T_1}{L}(\bar{\theta}_1 - \bar{\theta}_\ell) + \frac{(\Delta H) Y_{0,\infty}}{W_0 \nu_0 L}\right\}$$

and the fluctuation in the burning rate is given by

$$\dot{m}_0 \mu' = (1 + 0.27 Re^{\frac{1}{2}} Sc^{\frac{1}{3}}) \frac{2\pi K d_0}{C_p} \frac{C_p T_1 \theta_1'/L - (\Delta H) Y_{0,\infty}/W_0 \nu_0 L}{1 + \frac{T_1}{L} C_p(\bar{\theta}_1 - \bar{\theta}_L) + (\Delta H) Y_{0\infty}/L\,W_0 \nu_0}$$

At higher frequencies, the quasi-steady approximation is inapplicable, and therefore, the explicit non-steady burning rate expression should be used in estimating the high frequency noise characteristics.

6.4 Acoustic Amplification in Combustion Zone

In all the problems treated in previous sections, the sources are assumed to be compact. In large combustors, however, the size of the combustion zone may exceed the prevailing acoustic wave length. The heat release may generate pressure fluctuations, as well as amplify the sound waves passing through the combustion zone. The detailed picture of the generation and the amplification can be studied for the case of large spray combustors.

Consider a one-dimensional spray combustor of total length L_C, in which a combustion zone of liquid droplets extends form $x = 0$ to L. The chamber is closed at $x = 0$, and open to the atmosphere at $x = L_C$. Equation (78) is non-dimensionalized by the following quantities $\xi_i = x_i/L$

$\tau = t a_1/L$, $A^2 = a^2/a_1^2$, $M_1 = V_1/a_1$

$$\frac{D^2\Omega}{D\tau^2} - \frac{\partial}{\partial \xi_i}\left(A^2 \frac{\partial \Omega}{\partial \xi_i}\right) + \gamma\epsilon\left(\dot{Q}\frac{D\Omega}{D\tau} + \frac{D\dot{Q}}{D\tau}\Omega\right) = \gamma\epsilon\frac{D\dot{Q}}{D\tau} \qquad (85)$$

where $\Omega \equiv (p - p_0)/p_0$, $\epsilon = (N_0 m_\ell \Delta H)/c_p T_1$, $\frac{D}{D\tau} = \frac{\partial}{\partial \tau} + M_1 \bar{u}_i \frac{\partial}{\partial \xi_i}$.

The effects of mass addition and the heat of vaporization are assumed to be negligible compared with that of the heat release due to the reaction which is assumed by the following expression

$$\dot{Q}(\xi,\tau) = \bar{\dot{Q}} + \Sigma \dot{Q}_n'(\xi,t) = \bar{\dot{Q}} + \Sigma \mathcal{S}_n(\xi) e^{in\tau} \qquad (86)$$

The steady state heat release is constant, and the fluctuating component $S_n(\xi)$ is assumed to be known. The non-steady burning rate of the liquid propellant is often expressed in terms of the product of the response function and the pressure fluctuation, as for example,

$$\dot{Q}_n'(\xi,t) = \mathcal{R}_n(\xi,t)\Omega_n(\xi,t) \qquad (87)$$

It can be shown that the analysis of the generation and the amplification, based on Eqs. (86) and (87), are essentially similar within the linearized approximation. In order to retain the generality in the present analysis, the heat release rate given by expression (86), is adopted. The solution for the pressure fluctuation is assumed in the following form:

$$\Omega(\xi,\tau) = \Sigma \phi_n(\xi) e^{in\tau} \qquad (88)$$

Substituting Eq. (88) into Eq. (85), and neglecting the terms containing M_1, the following equation governing ϕ_n, yields

$$\phi_n'' + \lambda_n^2 \phi_n = an \sum_{j+\ell=n} V_{j\ell}\phi_\ell + an\mathcal{S}_n \qquad (89)$$

where

$$\lambda_n^2 = (n^2 - in\gamma\epsilon\bar{\dot{Q}})/A_1^2, \quad a = i\gamma\epsilon, \quad nV_{j\ell} = (j+\ell)\mathcal{S}_j = n\mathcal{S}_j$$

The distributed burning zone extends between $x = 0$ and $x = h$. Equation (89) may be rewritten in the matrix form

$$\phi'' + \lambda^2 \phi = a N V \phi + a N \mathcal{S} \quad (90)$$

where $\phi, \lambda, N, V, \mathcal{S}$ are given by

$$\left.\begin{aligned} &\phi = \begin{bmatrix} \varphi_1 \\ \varphi_2 \\ \vdots \\ \varphi_N \end{bmatrix} \quad N = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 2 & & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & n \end{bmatrix} \quad \mathcal{S} = \begin{bmatrix} \mathcal{S}_1 & 0 & \cdots & 0 \\ 0 & \mathcal{S}_2 & & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & & \mathcal{S}_n \end{bmatrix} \\ &\lambda = \begin{bmatrix} 1 - a\dot{\bar{Q}} & 0 & \cdots & 0 \\ 0 & 2^2 - 2a\dot{\bar{Q}} & & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & & n^2 - na\dot{\bar{Q}} \end{bmatrix} \quad V = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ \mathcal{S}_1 & 0 & & \cdot \\ \mathcal{S}_2 & \mathcal{S}_1 & & \cdot \\ \vdots & \vdots & & \vdots \\ \mathcal{S}_n & \mathcal{S}_{n-1} & & \mathcal{S}_1 \end{bmatrix} \end{aligned}\right\} \quad (91)$$

The general solution of Eq. (90) consists of two homogeneous solutions $\phi_{1,2}$, and an inhomogeneous solution ψ, which can be calculated from the following integral equation

$$\psi(\xi) = a\int_0^{\xi} N K(\xi-\xi')\mathcal{S}(\xi')\,d\xi' + a\int_0^{\xi} N K(\xi-\xi')V(\xi')\psi(\xi')\,d\xi' \quad (92)$$

and the kernel $K(\xi-\xi')$ is given by

$$K(\xi-\xi') = \frac{i}{2}\lambda^{-1}\left\{\exp[-i\lambda(\xi-\xi')] - \exp[i\lambda(\xi-\xi')]\right\} \quad (93)$$

where λ^{-1} is the inverse of matrix λ.
The matrix integral equation, Eq. (92), is Volterra's type and can be solved if the inhomogeneous term is absolutely integrable. The solution of Eq. (92) is approximated as follows

$$\psi(\xi) = f(\xi) + a\int_0^{\xi} N K(\xi-\xi')V(\xi')f(\xi')\,d\xi'$$
$$+ a^2\int_0^{\xi} N K(\xi-\xi')V(\xi')\int^{\xi'} N K(\xi'-\xi'')f(\xi'')\,d\xi'' + \cdots \quad (94)$$

where

$$f(\xi) = a\int N K(\xi-\xi')\mathcal{S}(\xi')\,d\xi' \quad (95)$$

Note that the convergence of the series solution, Eq. (94), is assumed to be a given parameter a. Two homogeneous solutions, ϕ_1 and ϕ_2 are given respectively by the following expressions

$$\phi_1^{(1)}(\xi) = \lambda^{-1}\sin\lambda\xi + \int_0^{\xi}\lambda^{-1}\sin\lambda(\xi-\xi')V(\xi')\phi^{(1)}(\xi')\,d\xi' \quad (96)$$

$$\phi^{(2)}(\xi) = \cos\lambda\xi - \cos\lambda\xi\int_0^{\xi}\lambda^{-1}\sin\lambda\xi' V(\xi')\phi^{(2)}(\xi')\,d\xi' \quad (97)$$
$$- \lambda^{-1}\sin\lambda\xi\int_{\xi}^{1}\cos\lambda\xi V(\xi')\phi^{(2)}(\xi')\,d\xi'$$

These two solutions satisfy the following canonical boundary conditions:

$$\phi^{(1)}(\xi=0) = 0 \qquad \phi^{(1)\prime}(\xi=0) = 1 \tag{98}$$

$$\phi^{(2)}(\xi=0) = 1 \qquad \phi^{(2)\prime}(\xi=0) = 0 \tag{99}$$

The general solution is obtained by approximate linear combinations of the inhomogeneous solution, Eq. (94), and the two homogeneous solutions, Eqs. (96) and (97).

In the non-reacting zone, the pressure fluctuation is governed by the homogeneous wave equation, i.e.,

$$\frac{\partial^2\hat{\Omega}}{\partial\tau^2} - A_1^2\frac{\partial^2\hat{\Omega}}{\partial\xi^2} = 0 \qquad \frac{L_c}{L} = h \geq \xi \geq 1 \tag{100}$$

The solution for Ω is given by

$$\hat{\Omega}(\xi,\tau) = [(\sin\mu_n\xi)A + (\cos\mu_n\xi)B]e^{in\tau} \tag{101}$$

where A and B are constant vectors to be determined from the boundary conditions. The solutions in the reacting zone and the hot non-reacting zone are to be joined at the intersection of the two zones. The boundary conditions are given as follows:

$$\xi = 0 \qquad u' = 0 \tag{102}$$

$$\xi = 1 \qquad \Omega = \hat{\Omega}, \qquad u' = \hat{u}' \tag{103}$$

$$\xi = h \qquad (\partial u'/\partial\xi) = 0 \tag{104}$$

Since the particle velocity vanishes at $\xi = 0$, the solution in the reacting zone is given by

$$\left.\begin{aligned}\Omega(\xi,\tau) = \Big\{&\cos\lambda\xi - \cos\lambda\xi\int_0^\xi \lambda^{-1}(\sin\lambda\xi')V(\xi')(\cos\lambda\xi')d\xi' \\ &-\lambda^{-1}\sin\lambda\xi\int_\xi^1(\cos\lambda\xi')V(\xi')\cos\lambda\xi' d\xi'\Big\}C + f(\xi) \\ &+ a\int_0^\xi NK(\xi-\xi')V(\xi')f(\xi')d\xi' + \\ &+ a^2\int_0^\xi NK(\xi-\xi')V(\xi')\int^{\xi'} NK(\xi'-\xi'')V(\xi'')f(\xi'')d\xi'' + \cdots\end{aligned}\right\} \tag{105}$$

where C is an arbitrary vector, to be determined from the boundary conditions at $\xi = 1$. The continuity of the pressure at $\xi = 1$ is given by

$$\left.\begin{aligned}&\Big[\cos\lambda - \cos\lambda\int_0^1 \lambda^{-1}\sin\lambda'\xi \cos\lambda\xi' V(\xi')d\xi'\Big]C + f(1) \\ &+ a\int_0^1 NK(1-\xi')V(\xi')f(\xi')d\xi' + \\ &+ a^2\int_0^1 NK(1-\xi')V(\xi')\int_0 NK(\xi'-\xi'')V(\xi'')f(\xi'')d\xi'' = (\sin\mu)A + (\cos\mu)B\end{aligned}\right\} \tag{106}$$

The continuity of the velocity is

$$\left\{-\lambda \sin\lambda + \lambda \sin\lambda \int_0^1 \lambda^{-1} V(\xi') \sin\lambda\xi' \cos\lambda\xi' d\xi'\right\} C$$
$$+ a\int_0^1 N\left[\frac{\partial K(\xi-\xi')}{\partial \xi}\right]_{\xi=1} V(\xi') f(\xi') d\xi' + \left(\frac{df}{d\xi}\right)_{\xi=1}$$
$$+ a^2\int_0^1 N\left[\frac{\partial K(\xi-\xi')}{\partial \xi}\right]_{\xi=1} V(\xi')\int_0^{\xi'} N K(\xi'-\xi'') V(\xi'') f(\xi'') d\xi'' = (\mu\cos\mu)A - (\mu\sin\mu)B \tag{107}$$

At $x = h$, $\partial u'/\partial\xi = 0$ thus

$$(\sin\mu h)A + (\cos\mu h)B = 0 \tag{108}$$

If $\mu h \neq m\pi$, or $(m+\frac{1}{2})\pi$, then neither $\sin\mu h$ nor $\cos\mu h$ is singluar. Thus,

$$B = -(\cos\mu h)^{-1}(\sin\mu h)A \tag{109}$$

From Eqs.(106),(107), and (109) the vectors are calculated from the following matrix equations:

$$\left.\begin{aligned} FC + H &= [\sin\mu - (\cos\mu)(\cos\mu h)^{-1} \sin\mu h] A \\ GC + L &= [\mu\cos\mu - \mu(\sin\mu)(\cos\mu h)^{-1}\sin\mu h] A \end{aligned}\right\} \tag{110}$$

Solving for A and C, there yield

$$\left.\begin{aligned} A &= U^{-1}H + [U^{-1}F][WU^{-1}F - G]^{-1}[L - WU^{-1}H] \\ B &= -(\cos\mu h)^{-1}(\sin\mu h)A \\ C &= [WU^{-1}F - G]^{-1}[L - WU^{-1}H] \end{aligned}\right\} \tag{111}$$

where

$$\left.\begin{aligned} F &= \cos\lambda - \cos\lambda\int_0^1 \lambda^{-1}\sin\lambda\xi'\cos\lambda\xi' V(\xi') d\xi' \\ G &= -\lambda\sin\lambda + \lambda\sin\lambda\int_0^1 \lambda^{-1}\sin\lambda\xi'\cos\lambda\xi' V(\xi') d\xi' \\ H &= a\int_0^1 N K(1-\xi') V(\xi') f(\xi') d\xi' + f(\xi=1) \\ L &= a\int_0^1 N\left[\frac{\partial K}{\partial\xi}\right]_{\xi=1} V(\xi') f(\xi') d\xi' + \left(\frac{df}{d\xi}\right)_{\xi=1} \\ U &= \sin\mu - (\cos\mu)(\cos\mu h)^{-1}(\sin\mu h) \\ W &= \mu\cos\mu - \mu(\sin\mu)(\cos\mu h)^{-1}(\sin\mu h) \end{aligned}\right\} \tag{112}$$

The noise generation by intense acoustic instability is of particular importance in view of the fact that the amplitude of the wave is proportional to $(N m_0 \Delta H)/C_p T_1 \rho_1$. This is M_1^{-2} times greater than the sound generation by any turbulent driven, compact noise source. For a large combustor with a pressure sensitive reaction, therefore, acoustic instability should be adequately controlled.

The mechanisms of the generation of the higher harmonics and the modulation of the sound wave, as illustrated in the analysis, are of special interest. Since the velocity and pressure fluctuations at the exit plane of the duct affect the nature of the monopole-like sound field for a ducted burner, it is of practical importance to design an optimal duct minimizing these results. Note that in an open flame, the acoustic instability is not likely to occur, and hence, the noise essentially dominates lower frequency regions.

7. Noise Generation by Chemical Instability

It has been demonstrated, in previous sections, that the monopole-like sound field generated by a turbulent flame is primarily attributed to the small scale fluctuations in the rate of heat release, velocity, and temperature. The estimation of fluctuation in the rate of chemical reaction is, therefore, essential to the determination of the source strength. The small scale in the turbulence may enter the flame from the upstream region, or be generated within the reaction zone.

The stability of the flame structure has been the object of intense research with regard to turbulent flame theory. However, the question of chemical stability has not been investigated to a satisfactory level due to the fact that such an analysis is exceedingly complicated, and that the chemical kinetics involved are not well understood. In the analysis of chemical stability, (see for example Bradley [18] Glansdorff, and Prigogine [19]), the question has been focused on the chemical stability associated with the response of the fluctuation of species concentration, whereas the effect of the temperature is ignored. The inclusion of the temperature variation is essential in estimating the non-steady heat release.

Following the reaction scheme considered by Frank-Kamenetiky, [20]

$$\begin{aligned} A + M &\rightarrow B + 2M \\ M + N &\rightarrow B + 2N \\ A + N &\rightarrow B \end{aligned} \tag{113}$$

where A represents reactants, B final products, and M, N intermediates. For a homogeneous medium, the rate of reaction of species M and N are given by

$$\rho \frac{dY_M}{dt} = k_1(T) Y_A Y_M - k_2(T) Y_M Y_N \tag{114a}$$

$$\rho \frac{dY_N}{dt} = k_2(T) Y_M Y_N - k_3(T) Y_A Y_N \tag{114b}$$

Note that k_1, k_2, and k_3 are rate constants which depend on activation energy, and temperature among other things. The linearized equation describing a small departure of the concentrations of M and N from the steady state including the temperature variation are given in the following non-dimensional form,

$$\frac{dY_M'}{d\tau} = -K_3 \bar{Y}_A Y_N' + a\,\theta' \tag{115a}$$

$$\frac{dY_N'}{d\tau} = K_1 \bar{Y}_A Y_M' + b\,\theta' \tag{115b}$$

$$\frac{d\theta'}{d\tau} = \frac{K_1 h_2}{\alpha} \bar{Y}_A Y_M' - \frac{K_3 h_1}{\alpha} \bar{Y}_A Y_N' + \frac{\beta}{\alpha}\theta' \tag{115c}$$

Where $K_i = k_i/K_0$, $\theta' = T'/T_0$, $\tau = K_0 t/\rho_0$

$$a = \left(\frac{\partial K_1}{\partial \theta} - \frac{K_1}{K_2}\frac{\partial K_2}{\partial \theta}\right)\bar{Y}_A \bar{Y}_M \, , \quad b = \left(\frac{K_3}{K_2}\frac{\partial K_2}{\partial \theta} - \frac{\partial K_3}{\partial \theta}\right)\bar{Y}_A \bar{Y}_N$$

$\alpha = c_p T_0/H$, $\beta = (ah_1 + bh_2)/K_0 H$, K_0 is the numerical value of, say, k_1 at reference temperature. H is the overall heat of reaction, h_1 and h_2 are non-dimensional heat of formation of intermediates M and N respectively. The frequencies of the system are given by the following characteristic equations,

$$\omega^3 - \gamma\omega^2 + [K_1 K_3 \bar{Y}_A^2 + (bK_3 h_1 - aK_1 h_2)\bar{Y}_A/\alpha]\,\omega = 0 \qquad (116)$$

where $\gamma = \beta/\alpha$

It can be shown that when a and b are set to zero, the characteristic equation gives two pure imaginary roots

$$\omega = \pm i\,(K_1 K_3)^{\frac{1}{2}}\,\bar{Y}_A$$

Thus, the concentrations of M and N fluctuate at the frequency determined by the reaction rate K_1, K_3, and the steady state concentration of the reactant.

In general, three characteristic frequencies are found to be

$$\omega_{1,2} = \frac{\gamma}{2} \pm i\,[K_1 K_3 \bar{Y}_A^2 + (bK_3 h_1 - aK_1 h_2)\bar{Y}_A/\alpha - \frac{\gamma^2}{4}]^{\frac{1}{2}} \qquad (117)$$

$$\omega_3 = 0$$

Thus, according to the linearized theory, if a and b are sufficiently small, such that the quantity in the bracket remains positive, the system is unstable if $\gamma>0$, or

$$\left(\frac{\partial K_1}{\partial\theta} - \frac{K_1}{K_2}\frac{\partial K_2}{\partial\theta}\right)h_1 + \left(\frac{K_3}{K_2}\frac{\partial K_2}{\partial\theta} - \frac{\partial K_3}{\partial\theta}\right)h_2 > 0 \qquad (118)$$

The Condition stipulated by Eqn.(118) suggests that the stability of the reaction depends, to a great extent, on the derivative of the rate of formation of the intermediates M and N. This stability limit of this reaction scheme is a function of gas temperature, among other things.

The instability, or oscillatory behaviour indicated by the reaction schemes typical of the combustion process, together with the upstream turbulence, require further investigation, in particular, when the oscillation becomes self-sustained (19).

The source of noise produced by the chemical instability induced by a reaction scheme suggested by Eqn.(113) can be evaluated from Eqn.(7) as follows,

$$(\gamma-1)\frac{D\bar{\dot{Q}}'}{D\tau} = (\gamma-1)\frac{D}{D\tau}\Big\{k_1 H_2 \bar{Y}_A \dot{Y}_M' - k_3 H_1 \bar{Y}_A \dot{Y}_N' + \Big[\left(\frac{\partial k_1}{\partial T} - \frac{k_1}{k_2}\frac{\partial k_2}{\partial T}\right)\times$$

$$\bar{Y}_A \bar{Y}_M H_1 + \left(\frac{k_3}{k_2}\frac{\partial k_2}{\partial T} - \frac{\partial k_3}{\partial T}\right)\bar{Y}_A \bar{Y}_N H_2\Big]\dot{T}'\Big\} \qquad (119)$$

The order of the magnitude of the frequency is inversely proportional to the reaction line which is ρ_0/κ_0 . The first and second terms appearing in the curly bracket are the rate of heat release associated with the fluctuations of the intermediates M and N, whereas the third term represents the effect of the temperature fluctuation. Essentially the same procedure of calculations for other homogeneous reactions may be used.

8. Conclusion

A theoretical description of noise generation and amplification by combustion systems has been developed on the basis of theory of reacting gas dynamics and the theory of sound generation. The principal result of the open flame noise generation formulation is that the far field intensity of the sound generated by acoustically compact open flames can be predicted by the flame structural factor consisting of six double correlation functions. The explicit forms of the structural factors are obtained for two flame models; wrinkled flame model and distributed reaction model. The experimental determination of the structural factor of a flame appears to be of particular interest in the prediction of the noise intensity. The analysis also disclosed that the small scale turbulence in the reaction zone is the primary source of the monopole sound filled whereas the large scale turbulence contributes dipole emission. The burning zone is dominated, primarily by psudo sound, which acts to constrain the field to remain dynamically incompressible. The rate of thermo-acoustic energy conversion is determined by the eddy size and the gas velocity among other factors. Consequently a given combustible gas burning with intense turbulence of small eddie size is likely to be an intense acoustic emitter.

Noise generation in liquid spray combustion process is attributed to various non-steady phenomena including turbulent fluctuations, intrinsic non-steady burning characteristic of droplets, acoustic instability and other types of instability. The excitation of the specific sound mode is largely determined by the interaction of the burning zone and its environment. For example, liquid spray combusion in a ducted burner could be a potential noise emitter when the acoustic instability, or resonance phenomena trigger intense fluctuations in the heat release. The investigation of such non-steady burning processes in a duct is essential to the overall assesment of core engine noise.

Noise may be generated independently by chemical instability or cooperatively by various instability phenomena. The effect of the chemical instability on noise generation is, to some extent, obscured by the uncertainty in the chemical kinetics of the major energy yielding reactions. Further research is necessary to assess the over all noise contribution by chemical instability.

References

1. Smith, T.J.B. and Kilham, J.S., Combusion and Flame 9, 426. (1965).

2. Hurle, I.R., Price, R.B., Sugden, T.M., & Thomas, A., Proc. Roy. Soc., A303, 409. (1969).

3. Giammar R.D. and Putnam. Jo. of Engineering for Power 157. (1957).

4. Dance, E.W.G. and Sutherland, A., "Noise Research in the Field of Domestic Gas Utilization," 10th Int. Gas Conference, Hamburg. (1967).

5. Abdelhamid, A.N., Harrje, D.T., Plett, E.G., and Summerfield, M., AIAA 11th Aerospace Science Meeting, AIAA Paper No. 73-189. (1973).

6. Gaydon, A.G. and Wolfhard, H.D., Flames, 3rd Ed., London; Chapman and Hall. pp. 161-174 (1970).

7. Markstein, G.H., "Non-Steady Flame Propagation," N.Y.; Macmillan. pp. 183-286 (1964).

8. Bragg, S.L. J. Inst. Fuel, 36, 12. (1963).

9. Strahle, W.C., J. Fluid Mech., 49, 399. (1971).

10. Karlovitz, B., J. Chem. Phys., 19, 541. (1951).

11. Summerfield, M., Reiter S.H., Kebely, V., and Mascolo, R.W., Jet Propulsion 25, 377. (1955).

12. Chu, B.T. and Kovasznay, L.S.G., J. Fluid Mech., 3, 494, (1958).

13. Williams, F.A., Combusion Theory, Addison-Wesley Publishing Co., Reading, Mass. Chaps. 1 and 3.(1965).

14. Courant, R. and Hilbert, D. Method of Mathematical Physics, Vol. 2. Interscience Publishers Inc ., N.Y. pp. 362-367.(1962).

15. Beer, J.M. and Chiger, N.A. Combustion Aerodynamics John Wiley & Sons, Inc., N.Y. pp.49-84 (1972).

16. Damkohler, G., Z. Electrochem, 46, pp. 601-626 (1940), English translation, NACA Tech. Memo no. 1112 (1947).

17. Shelkin, K.I., Zhur. Tekhn. Fiz (U.S.S.R.) 13, pp. 520-530 (1943): English translation, NACA Tech. Memo no. 1110 (1947).

18. Bradley, J.N., Flame and Combustion Phenomena, Methuen & Co., London. pp. 119-125. (1969).

19. Glansdorff, P., Prigogine, I., Thermodynamic Theory of Structure, Stability and Fluctuations. Wiley-Interscience, N.Y. pp. 222-271. (1971).

20. Frank-Kamenetsky, D.A., C.R. Acad. Sci. U.S.S.R., 25, 671 (1939); J. Phys. Chem., Moscow, 14, 30 (1940).

21. Sirignana, W.A., Liquid Propellant Rocket Combustion Instability, Ed. by D.T. Harrje and F.H. Reardon, NASA SP-194, pp.167-175 (1973).

Appendix A

Convected Wave Equation of a Reacting Gas

The convected wave equation of the reacting gas is derived from the conservation laws as follows. The overall continuity equation for a reacting gas is

$$\frac{D\rho}{Dt} + \rho \frac{\partial u_i}{\partial x_i} = 0 \qquad (A-1)$$

where $\frac{D}{Dt} = \frac{\partial}{\partial t} + u_j \frac{\partial}{\partial x_j}$

and $\rho = \rho(p, S, Y_i)$ where p is the pressure, S is the entropy and Y_i is the concentration of species "i" . The momentum equation for the reacting gas is given by

$$\frac{Du_i}{Dt} = -\frac{1}{\rho}\frac{\partial p}{\partial x_i} + \frac{1}{\rho}\frac{\partial \tau_{ij}}{\partial x_j} \qquad (A-2)$$

where τ_{ij} is the viscous stress tensor.

To obtain the convected wave equation for the reacting gases, the divergence of the momentum equation and substantial derivative of the continuity equation is taken and combined. That is,

$$\frac{\partial}{\partial x_i}\left[\frac{Du_i}{Dt}\right] = -\frac{\partial}{\partial x_i}\left(\frac{1}{\rho}\frac{\partial p}{\partial x_i} - \frac{1}{\rho}\frac{\partial \tau_{ij}}{\partial x_j}\right) \qquad (A-3)$$

and expanding the density derivative in terms of its functional variables, the continuity equation becomes after some reduction

$$\frac{D}{Dt}\left(\frac{\partial u_i}{\partial x_i}\right) = -\frac{1}{\gamma}\frac{D^2}{Dt^2}\left(\ln p/p_o\right) - \frac{D}{Dt}\left\{\frac{1}{\rho}\left(\frac{\partial \rho}{\partial S}\right)_{p,Y_j}\left[\frac{DS}{Dt} - \sum_{j\neq i}\left(\frac{\partial S}{\partial Y_i}\right)_{\rho,p,Y_j}\frac{DY_i}{Dt}\right]\right. \qquad (A-4)$$

By subtracting equation (A-3) from (A-4) and after applying some rearrangement, the convected wave equation for a reacting gas is obtained in the form

$$\frac{D^2}{Dt^2}\left(\ln\frac{p}{p_o}\right) - \frac{\partial}{\partial x_i}\left[\frac{\gamma p}{\rho}\frac{\partial}{\partial x_i}\left(\ln\frac{p}{p_o}\right)\right] = \gamma\frac{\partial u_i}{\partial x_j}\frac{\partial u_i}{\partial x_j} - \gamma\frac{\partial}{\partial x_i}\left(\frac{1}{\rho}\frac{\partial \tau_{ij}}{\partial x_j}\right)$$

$$-\frac{1}{\gamma}\frac{D}{Dt}\left\{\frac{1}{\rho}\left(\frac{\partial \rho}{\partial S}\right)_{p,Y_i}\left[\frac{DS}{Dt} - \Sigma\left(\frac{\partial S}{\partial Y_i}\right)_{p,\rho,Y_j,\,j\neq i}\left(\frac{DY_i}{Dt}\right)\right]\right. \qquad (A-5)$$

For a reacting gas, Eq. (A-5) is reduced to the following form

$$\frac{D^2}{Dt^2}\left(\ln\frac{p}{p_o}\right) - \frac{\partial}{\partial x_i}\left[a_j^2\frac{\partial}{\partial x_i}\ln\left(\frac{p}{p_o}\right)\right] = \gamma\frac{\partial u_i}{\partial x_j}\frac{\partial u_j}{\partial x_i} + (\gamma-1)\frac{D}{Dt}\left(\frac{p_o}{p}\dot{Q}\right) \qquad (A-6)$$

where

$$\dot{Q} = \Sigma w_i h_i - \Sigma j_i h_i - RT\Sigma\Sigma\frac{X_j D_{T,j}}{W_j D_{ij}}(V_i - V_j) + \frac{\partial}{\partial x_i}\left(k\frac{\partial T}{\partial x_i}\right) + \phi_v$$

V_1 is the diffusion velocity of species i, $D_{\tau,i}$ is the thermal diffusion coefficient for species i, and ϕ_v is the viscous dissipation.

Appendix B

Perturbation Equations for Acoustic Mode, Entropy Mode, and Vortex Mode in the Burning Zone

The perturbation equations for the burning zone are obtained by substituting the assumed two-parameter solutions of the flow variables, (14), into Eqs. (9), (10), (11), (12), and (13). By collecting the terms associated with the same power of M_∞ and Λ , they yield systematic perturbation equations.

Acoustic Mode:

$$\nabla \cdot A_0^2 \nabla \Omega_{n,j} = \mathcal{K}_{n,j}$$

$$\mathcal{K}_{2,1} = -\gamma \left(\frac{\bar{D}\dot{Q}'}{D\tau} + \bar{u}' \cdot \nabla \dot{\bar{Q}} \right)$$

$$\mathcal{K}_{2,2} = -\gamma \left\{ \left(\frac{\bar{D}}{D\tau} + \bar{u}' \cdot \nabla \right) \dot{Q}'_{0,1} + \frac{D'}{D\tau_{0,1}} (\dot{\bar{Q}} + \dot{Q}') \right\} + 2 \nabla \cdot (A_0^2 a_{01} \nabla \Omega_{21})$$

$$\mathcal{K}_{4,1} = \left(\frac{\bar{D}}{D\tau} + \bar{u}' \cdot \nabla \right)^2 \Omega_{2,1}$$

$$\mathcal{K}_{4,2} = -\gamma \left\{ \left(\frac{\bar{D}}{D\tau} + \bar{u}' \cdot \nabla \right) \dot{Q}'_{2,1} + \frac{D'}{D\tau_{2,1}} (\dot{\bar{Q}} + \dot{Q}') \right\} + \gamma \left\{ \left(\frac{\bar{D}}{D\tau} + \bar{u} \cdot \nabla \right) \Omega_{2,1} (\dot{\bar{Q}} + \dot{Q}') \right\}$$

Entropy Mode:

$$\left(\frac{\bar{D}}{D\tau} + \bar{u}' \cdot \nabla \right) \sigma_{n,j} = h_{n,j}$$

$$h_{0,1} = -\dot{Q}' - \frac{D'}{D\tau_{0,1}} (\ln \rho_{0,0})$$

$$h_{0,2} = -\dot{Q}_{0,1} - \frac{D'}{D\tau_{0,2}} (\ln \rho_{0,0}) - \frac{D'\sigma_{0,1}}{D\tau_{0,1}} - \left(\frac{\bar{D}}{D\tau} + \bar{u}' \cdot \nabla \right) \frac{1}{2} \sigma_{0,1}^2$$

$$h_{2,1} = \frac{1}{\gamma} \left(\frac{\bar{D}}{D\tau} + \bar{u}' \cdot \nabla \right) \Omega_{2,1} - \frac{D'}{D\tau_{2,1}} (\ln \rho_{0,0})$$

$$h_{2,2} = \frac{1}{\gamma} \left(\frac{\bar{D}}{D\tau} + \bar{u}' \cdot \nabla \right) \Omega_{2,2} + \frac{1}{\gamma} \frac{D' \Omega_{2,1}}{D\tau_{0,1}} - \left\{ \dot{Q}'_{2,1} - \Omega_{2,1} (\dot{\bar{Q}} + \dot{Q}') \right\}$$
$$- \frac{D'}{D\tau_{2,2}} (\ln \rho_{0,0}) - \frac{D'\sigma_{0,1}}{D\tau_{2,1}} - \frac{D'\sigma_{2,1}}{D\tau_{0,1}} - \left(\frac{\bar{D}}{D\tau} + \bar{u}' \cdot \nabla \right) \sigma_{0,1} \sigma_{2,1}.$$

Rotational Mode:

$$\left(\frac{\bar{D}}{D\tau} + \bar{u}' \cdot \nabla \right) \bar{w}_{n,j} = \bar{g}_{n,j}$$

$$g_{0,1} = -\frac{1}{\gamma \rho_{0,0}} \nabla \Omega_{2,1} - \left(\frac{\bar{D}}{D\tau} + \bar{u}' \cdot \nabla\right) \nabla \phi_{0,1}$$

$$g_{0,2} = -\frac{1}{\gamma \rho_{0,0}} \left(\nabla \Omega_{2,2} - \sigma_{0,1} \nabla \Omega_{2,1} \right) - \left(\frac{D}{D\tau} + \bar{u}' \cdot \nabla\right) \nabla \phi_{0,2} - \frac{D'}{D\tau_{0,1}} (\nabla \phi_{0,1} + W_{0,1})$$

$$g_{2,1} = -\frac{1}{\gamma \rho_{0,0}} \nabla \Omega_{4,1} - \left(\frac{\bar{D}}{D\tau} + \bar{u}' \cdot \nabla\right) \nabla \phi_{2,1}$$

$$g_{2,2} = -\frac{1}{\gamma \rho_{0,0}} \left(\nabla \Omega_{4,2} + \nabla \frac{1}{2} \Omega_{2,1}^{2} - \sigma_{0,1} \nabla \Omega_{4,1} - \sigma_{2,1} \nabla \Omega_{2,1} \right)$$
$$- \left(\frac{\bar{D}}{D\tau} + \bar{u}' \cdot \nabla\right) \nabla \phi_{2,2} - \frac{D'}{D\tau_{2,1}} \left(\nabla \phi_{0,1} + \bar{W}_{0,1} \right) - \frac{D'}{D\tau_{0,1}} \bar{W}_{2,1}$$

Dilation: Field

$$\nabla^2 \phi_{n,j} = f_{n,j}$$

$$f_{0,1} = \dot{Q}' \quad , \qquad f_{2,1} = -\frac{1}{\gamma}\left(\frac{\bar{D}}{D\tau} + \bar{u}' \cdot \nabla\right) \Omega_{2,1}$$

$$f_{0,2} = \dot{Q}'_{0,1} \quad , \qquad f_{2,2} = -\frac{1}{\gamma}\left(\frac{D\Omega_{2,2}}{D\tau} + \frac{D'\Omega_{2,1}}{D\tau_{0,1}}\right) - \left(\dot{\bar{Q}} + \dot{Q}'\right) \Omega_{2,1} + \dot{Q}'_{2,1}$$

Appendix C

Wave Equation for Liquid Fuel Spray

For a two-phase system, the conservation laws given by Eqs. (1) - (4), need be supplemented by the conservation laws for a condensed phase and a set of equations describing the rate of exchange of mass, momentum, and energy between two phases, see for example Sirignano [21]. A straight-forward algebraic manipulation shows that three additional terms enter into the right hand side of Eq. (A-5). These three terms represent the effects of mass addition, $\gamma \frac{D}{Dt}\left(\frac{\dot{m}}{\rho}\right)$ distributed body force, $-\gamma \frac{\partial}{\partial x_i}(F_i/\rho)$ and the entropy generation $\rho\left(\frac{DS}{Dt}\right)_{T.P.}$ due to non-equilibrium phenomena, and exchange of thermal energy between two phases. The rate of entropy $\rho\left(\frac{DS}{Dt}\right)_{T.P.}$ is given by the following expression

$$\rho\left(\frac{DS}{Dt}\right)_{T.P.} = \rho_\ell \frac{\gamma R}{a^2}\left[(u_i - W_i)\frac{D_\ell u_\ell}{Dt} - C_\ell \frac{D_\ell T_\ell}{Dt}\right] + \frac{\dot{m} L}{\rho T_\ell} \qquad (C\text{-}1)$$

Addition of (C-1) in the right hand side of (A-5), which contains the entropy generation due to gas phase reaction, gives the wave equation for liquid fuel spray combustion.