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MECHANICAL INSTABILITY GROUND DYNAMICS PROGRAM

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MECHANICAL INSTABILITY GROUND DYNAMICS PROGRAM

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By

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Prepared by

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Bloomfield, Connecticut

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ABSTRACT

A state-of-the-art survey has been performed to determine if any work has been performed which associates soil dynamics with mechanical instability in helicopters. None was found. No data pertaining to the range of near surface soil dynamic properties was found. A method of determining the mechanical stability of a helicopter in contact with the soil has been developed. A parametric study has been performed to examine the UH2 helicopter under conditions of a range of soil properties. It was found that there are ranges of parameters which will cause mechanical instability. It is recommended that a program be initiated to obtain quantitative near surface dynamic characteristics of representative soils through field testing.

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FOREWORD

The work presented in this report was performed by Kaman Aerospace Corporation under Contract N00019-73-C-0460, for the Naval Air Systems Command, Department of the Navy. The program was under the technical direction of Mr. R. E. Malatino of the Naval Air Systems Command. This work was performed by the author under the direction of Messrs R. Jones and A. Berman of Kaman Aerospace Corporation.

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INTRODUCTION

Mechanical instability or ground resonance is a phenomenon associated with helicopters that can lead to the total destruction of the aircraft. Therefore, the helicopter must be designed to be free from mechanical instability in any phase of its operation whether it be airborne or on the ground. Ground resonance is a well understood problem and for operation from prepared sites, the present methods of analysis can be used to insure that the vehicle is designed to be stable. Design problems arise when a helicopter is required to operate from unprepared sites.

In a normal mechanical stability analysis it is assumed that the aircraft is resting on an infinitely rigid base. This assumption is valid for a prepared site since the stiffness of the surface is much greater than the stiffness of the landing gear. When the aircraft is operating from an unprepared site, the assumption may not be valid. The problem arises not from the ground resonance analysis, which can take soil properties into account, but from the fact that very little is known about soil properties as applied to ground resonance. A considerable amount of data is available pertaining to the soil dynamics involved in the landing of fixed wing aircraft but virtually no research has been done in the rotary wing field. The concerns relating to the landing of fixed wing aircraft are much different than those relating to helicopters.

OBJECTIVES

The objectives of this study were to determine if any work has been done which applies directly to mechanical instability in helicopters due to soil dynamics and to perform a mechanical instability analysis which included soil dynamics. The work was divided into the following phases:

1. State-of-the-art survey
2. Soil Dynamic Model
3. Mechanical Instability Analysis

The state-of-the-art survey had three aspects. The first was to determine if any work pertaining to the effect of soil dynamics on mechanical instability in helicopters has been done. The second required that existing soil modeling techniques be examined to determine if they are compatible with the present methods of mechanical instability analysis. The third aspect consisted of searching the appropriate literature in an effort to find values for various soil properties which were needed to perform the mechanical instability analysis.

The second phase required that a soil dynamic model be developed which could be used as part of a mechanical instability analysis. The two requirements on the model were that it must represent the soil adequately and that it must be compatible with the present methods of mechanical instability analysis.

The third phase of the study was the performance of a mechanical instability analysis which included soil dynamics, to determine what range of soil property values are associated with mechanical instability.

STATE-OF-THE-ART SURVEY

A state-of-the-art survey was performed to achieve the following:

1. Determine if any work concerning mechanical instability associated with soil dynamics has been reported.

2. Determine the state of the art of soil modeling and its applicability to the present state of the art of mechanical instability analysis.

3. Determine what soil property data has been collected and how it was collected.

Four bibliographies were obtained. These bibliographies contained citations on well over 1000 books, papers, and reports pertaining to soil dynamics. Three of these bibliographies were compiled specially for this study, the fourth (Ref. 24) is a government report. From these bibliographies, 52 papers and reports and two books were selected as being worthy of further investigation. Of the 52 papers and reports, 21 (Refs. 1-21) were reviewed in some detail.

There was no literature found which made any association between soil dynamics and mechanical instability in helicopters.

Most of the reports which were obtained could be generally classified as falling into one of three categories: (a) wave propagation through soil; (b) tire-soil interaction; (c) soil foundation dynamics. The works which dealt with wave propagation were scanned for soil property data which could be useful in the mechanical stability analysis. The modeling techniques used for wave propagation were much different than the type of models which were needed for this study, and unfortunately, the data used for wave propagation models was not the type which was required for the mechanical instability analysis. At first glance, the tire-soil interaction reports seemed to contain exactly the type of information that was sought. However, further investigation revealed that these reports dealt with traction and non-elastic deformation and the soil models developed did not model the properties of interest. Therefore, these reports were not used for this in-depth study.

The last type of report, soil-foundation dynamics, was found to contain models most similar to the type of models which were required to perform the mechanical instability analysis. Elastic theory is used almost universally for soil-foundation models for several reasons. The theory of elasticity has a strong theoretical background, and therefore, provides a solid basis from which to start (Ref. 5). The methods of modeling which incorporate linear elastic theory lend themselves to direct methods of solution (Refs. 22-25). Most soils exhibit linear properties over small changes in strain.

The two methods of employing elastic theory most widely cited in the literature are the elastic half-space theory and the lumped parameter method. The elastic half-space theory treats the soil as a semi-infinite solid with a rigid plate resting on the surface. Figure 1 is a schematic of a plate resting on a layered elastic half space. The elastic half-space theory is particularly useful for investigation of the response of a body which is remote from the source of vibration, but is in contact with the soil. This type of problem can be solved using a finite element analysis. Elastic half-space theory is also useful when analyzing layered soils, the properties of which are known individually but not collectively.

The lumped parameter method is based on the elastic half-space theory but rather than treating the soil as a continuum, all the properties are lumped into a single spring, mass, and damper system for each degree of freedom (Ref. 22). Figure 2 is a schematic representation of a lumped parameter model. The lumped parameter method is particularly useful because of its simplicity. Numerical solutions are easy to obtain using a linear lumped parameter model. It must be noted that several simplifying assumptions are made in the transition from the elastic half-space theory to the lumped parameter method. It is assumed that the soil is isotropic and homogeneous. A model using lumped parameters is not necessarily linear but it has been demonstrated that linear models yield very good results over small changes in strain, and in some cases, can even be used for very large changes.

Before examining the values of different soil properties, it is appropriate to briefly discuss the phenomenon under study. As previously mentioned, modern helicopter design precludes mechanical instability when the aircraft is on a rigid surface but not necessarily when the aircraft is operating from an unprepared site.

The primary forces in a helicopter which lead to mechanical instability are caused by coupled in-plane hub and blade motions. The geometry of all helicopters is such that the forces at the landing gear are in all three reference planes. Therefore, values for soil properties must be for the near surface in both the vertical and horizontal directions. The near surface is the most difficult to classify since it is made up of both soil and vegetation and extreme variations in properties can be found over short distances. In all of the literature reviewed, the values of soil properties were for prepared samples or prepared sites. The samples were prepared so that they would be homogeneous. The sites were prepared by removing the near surface to obtain a uniform surface. Unfortunately, data obtained by either one of these methods is not representative of the near surface except in those rare cases where the soil is homogeneous over the first several feet of depth and is not covered by vegetation.

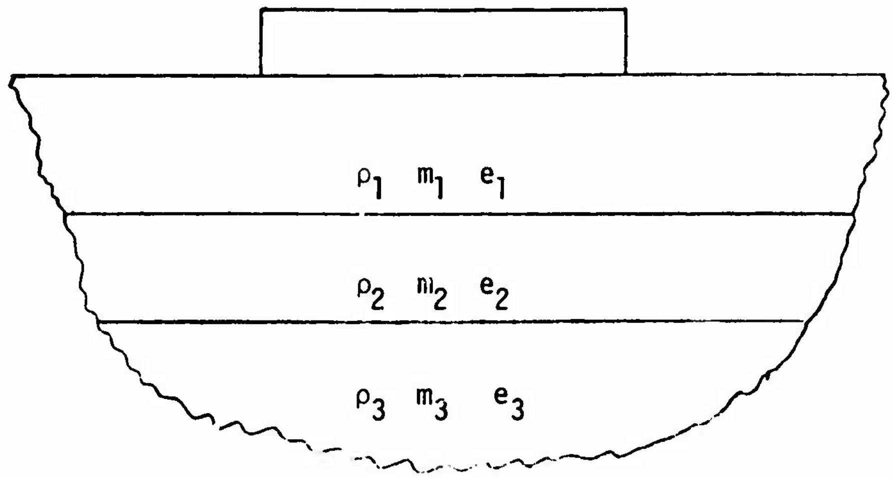

Figure 1. Two-Dimensional Layered Elastic Half Space

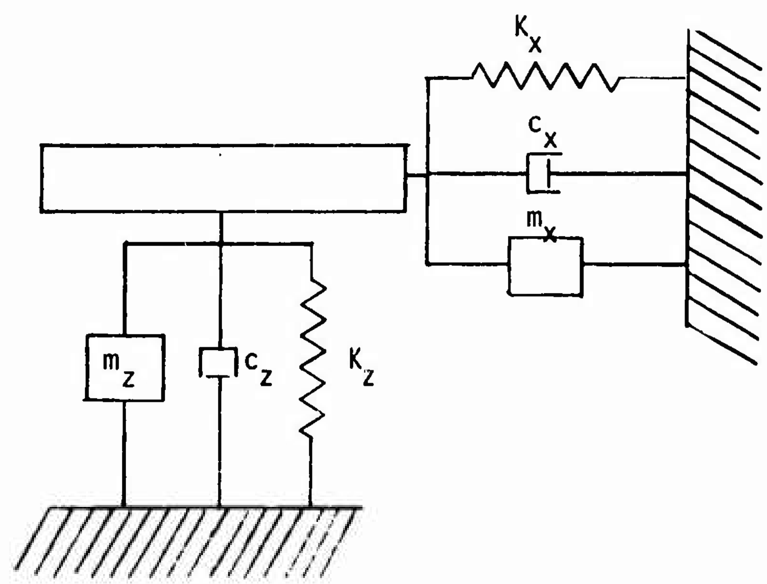

Figure 2. Two-Dimensional Lumped Parameter Model

The U. S. Army Corps of Engineers Waterway Experiment Station in Vicksburg, Mississippi was visited to determine the type of soil testing currently being done and the types of dynamic soil modeling most germane to the situation. Discussions were held with the Chief of Geodynamics, Mr. R. Ballard, and Dr. F. McClean. Both Ballard and McClean confirmed the results of our literature search, i.e., there is virtually no data for soil properties of the near surface and, in general, the data which does exist is for vertical, rotational and rocking motions, not for horizontal. They also stated that they felt that a linear lumped parameter model would be valid for the near surface if data were available.

SOIL MODEL

In order to perform a mechanical stability analysis, a soil model had to be developed which met two criteria: valid representation of the soil over small deflections, and compatibility with existing methods of ground resonance analysis.

The ground resonance analysis requires the impedance (force per unit response) of the supporting structure. The most direct method of modeling the soil would be to use measured impedance data directly, but since no data could be found, a linear lumped parameter model was used to calculate the soil impedance.

The second, and most difficult, aspect of the soil model was determining the range of values that should be used for each of the parameters in the model. As previously mentioned, there is a serious lack of data about the near surface. Even if some data were available, it would very likely be biased because the bulk of soil data is taken from locations where a structure is intended to be built and any site with a low spring rate would be rejected on sight, or feel, before any quantitative tests were made.

The damping, spring rate, and effective mass of the soil are not measured directly. The parameters which are frequently reported are density, ρ, Poissons ratio, ν, and shear modulus, G. These values can be used to compute the values of the spring and damper rate for a lumped parameter model using the following equations (Ref. 22):

$$K_z = \frac{4Gr_o}{(1 - \nu)} \quad (1)$$

$$C_z = \frac{3.4r_o^2}{(1 - \nu)} \sqrt{\rho G} \quad (2)$$

$$K_x = \frac{32(1 - \nu)Gr_o}{(7 - 8\nu)} \quad (3)$$

$$C_x = \frac{18.4(1 - \nu)}{(7 - 8\nu)} r_o^2 \sqrt{\rho G} \quad (4)$$

where:

C_x = the horizontal damping rate of the soil

C_z = the vertical damping rate of the soil

G = the shear modulus of the soil

K_x = the horizontal spring rate of the soil

K_z = the vertical spring rate of the soil

r_o = the radius of the base

ρ = the mass density of the soil

ν = Poissons ratio of the soil

These equations were derived for a rigid circular base on an elastic half-space with uniform properties throughout. In the elastic half-space theory, the only mass used is the mass of the footing: therefore, the effective mass of the soil is considered zero.

As with most sciences, the phenomenon which is easiest to classify and predict is studied most diligently. There is a wealth of data pertaining to various types of sand. It was felt that none of the data found was representative of the near surface, but two soil types which were investigated are used here as samples.

The first type, referred to as type A in this report, is made up of clay and silt which was located in the Granite Creek Desert, two miles southwest of Gerlach, Nevada (Ref. 10). The average in-place density is 45 lb/ft^3. The Poissons ratio and shear modulus are .47 and 1370 lb/in^2, respectively. Using these values in Equations (1) - (4) yields the following results:

K_z = 297,781 lb/ft

C_z = 135 lb-sec/ft

K_x = 206,535 lb/ft

C_x = 63 lb-sec/ft

The second type of soil, which is referred to as type B in this report, is a fine sand (Ref. 10). Data was taken at a site in the Nebraska Sand Hills about 50 miles southeast of O'Neil, Nebraska. The density of the sand is 98 lb/ft^3, the Poissons ratio is .31 and the measured shear modulus is 9800 lb/in^2. Again, applying the data to Equations (1) - (4) yield the following results.

K_z = 1,636,173 lb/ft

C_z = 405 lb-sec/ft

K_x = 1,378,729 lb/ft

C_x = 231 lb-sec/ft

The data for both types A and B are for embedded footings so the properties of the near surface are not reflected in the data, but this data was the best found.

MECHANICAL STABILITY ANALYSIS

In order to establish the range of soil parameter values which are associated with mechanical instability, an analysis was made which included soil properties. Rather than go directly into the effects of soil parameters, it would be well to review the mechanical instability problem as it is normally approached and then consider the effects of various soil properties. The UH-2 helicopter was used for this study, but the methods used apply equally as well to other helicopters with three or more blades.

The UH-2 helicopter has a fully articulated four-bladed rotor system. The fully articulated rotor system has potentially greater susceptibility to mechanical instability than other rotor systems in use today. Mechanical instability is a coupling between in-plane hub natural frequencies and blade natural frequencies, and because in-plane hub natural frequencies are associated with the rigid body modes of the helicopter on its landing gear, the stiffness of the landing gear in both the vertical and lateral directions are of primary importance.

The vertical and lateral spring rates of the landing gear and tire produce two modes of coupled roll and lateral translations of the helicopter. The range of instability associated with the low coupled mode usually occurs below the operating range, and is well damped due to shock absorber strut motion. However, the range of mechanical instability associated with the high mode of coupled motion is usually not well damped, because it involves lateral deflection of the gear and tire, and, therefore, little damping is obtained from the struts.

To insure freedom from mechanical instability in the operating range, either the lateral stiffness of the gear and tire should be soft enough so that the range of instability occurs well below the operating range, or the lateral stiffness of the gear should be stiff enough so that the range of mechanical instability is above the operating range. In the design of the UH-2 the latter, high stiffness, method was chosen (Ref. 25).

Because the normal approach assumes that the tires are on an infinitely rigid surface, the addition of soil properties will reduce the stiffness in both the vertical and lateral directions. The effect of reducing the stiffness will be that the natural frequencies will be lowered. The obvious problem associated with reducing the stiffness is that the second natural frequency could be lowered to such an extent that it drops within the operating range. The first natural frequency will be even further below the operating range than it previously was. The amount of damping required to permit safe passage through the natural frequency during start-up and shut-down must be reevaluated.

The means chosen to perform a numerical analysis was to modify a ground resonance computer program to include the previously discussed soil model.

The computer program consists of two parts. The first part calculates the hub mobilities which are required by the stability analysis. The second part is the stability analysis which is described in Appendix I

The first section of the program is capable of handling a helicopter with up to ten restraining points. Each restraining point can consist of up to four sets of springs, masses, and dampers in series. Each set can consist of up to three springs, three masses and three dampers, one for each of the three directions.

For the purposes of this study, three restraining points were used, one for the tail wheel and two for the main landing gear. The fuselage attachment is represented by a spring, the shock absorber strut by a spring and damper, the tire by a spring, and the soil by a spring damper and mass. Figure 3 is a schematic of the representation of one main landing gear. The other main gear and the tail gear are similarly represented. Table 1 contains the values of springs and dampers used to represent the UH-2 helicopt r. This data is converted to lateral and fore and aft hub mobilities which are used in the stability section.

The printed output from the program consists of the two neutrally stable frequencies for each rotor rpm, the blade damping required to make the system neutrally stable and the hub impedance in each direction. The program was run by varying the rpm from 2 to 310 so that the full operating range plus run-up was considered. The stability of the configuration under analysis is determined by comparing the value of required blade damping for stability to the actual value of blade damping. If the required damping exceeds the actual value, the configuration is unstable.

A parametric study was carried out to determine the values of soil parameters which have an impact on mechanical instability. The soil model consists of springs, dampers, and masses. A series of runs were made to map the entire region of instability due to soil spring rate, damping and mass in any combination.

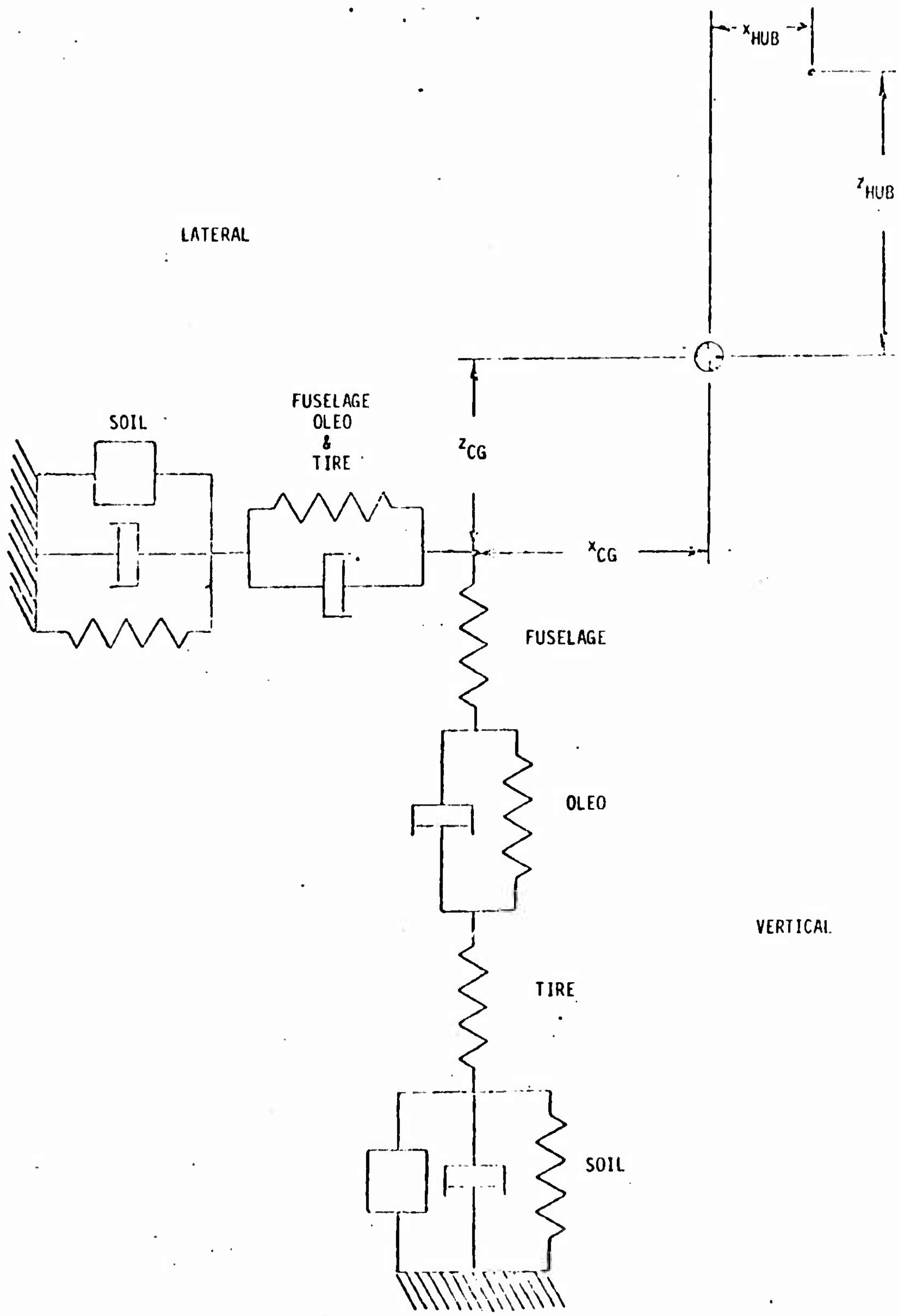

Figure 3. Schematic of Right Ma'n Gear - Front View

TABLE 1. UH-2 PROPERTIES

Inertial Properties

Mass	=	402.98	Slugs
I_x	=	7167.00	$Slug\text{-}Ft^2$
I_y	=	20900.00	$Slug\text{-}Ft^2$
I_z	=	20000.00	$Slug\text{-}Ft^2$

Dynamic Properties

	Left Main		Right Main		Tail Wheel	
	K Lb/Ft	C Lb-Sec/Ft	K Lb/Ft	C Lb-Sec/Ft	K Lb/Ft	C Lb-Sec/Ft
Vertical-Structure	96000	0	96000	0	26040	0
Vertical-Oleo	10200	900	10200	900	2040	350
Vertical-Tire	144000	0	144000	0	35700	0
Lateral	24986	0	24986	0	2846	0
Fore & Aft	30000	0	30000	0	0	0

Locations

	x(ft)	y(ft)	z(ft)
CG	0	0	0
Left Main	-5.5	2.98	-6.66
Right Main	5.5	2.98	-6.66
Tail Wheel	0	-18.94	-6.66
Hub	0	0	6.583

Blade Properties

Blade Mass - 8.95 Slugs
Static Moment About Lag Hinge - 76.25 Slug-ft
Moment of Inertia About Lag Hinge - 1148.2 $Slug\text{-}ft^2$
Distance From Lag Hinge to CG - 8.52 ft
Distance From Lag Hinge to CR - .6875 ft
Static Lag Frequency Due to Damper Spring Rate - 43.073 Rad/Sec
Number of Blades - 4

MECHANICAL INSTABILITY ANALYSIS RESULTS

The mechanical instability analysis parametric study was made to determine the range of possible soil property values associated with mechanical instability. The range of soil property values used for the study was selected to insure that the entire range over which instability occurs was covered. These values were not obtained from measured soil properties, because realistic data was not found in the literature search. Therefore, the overlap, if any, of soil property values which are associated with mechanical instability and those values which represent soil properties found in tactical situations cannot be determined at this time.

The properties of the UH-2 which were used for this study are presented in Table 1.

The results of the parametric study are presented in two different ways. The first set of curves, Figures 4 to 6, defines the regions of instability. These figures are plots of the soil spring rate versus soil damping required for neutral stability of the aircraft. The curves are shown for two different effective soil masses. The area under each curve is a region of instability.

The second type of plot, Figures 7 to 18, shows the amount of blade damping required for neutral stability versus soil spring rate. These curves are shown for several values of soil spring rate and soil damping. The value of $\lambda\beta$ for the UH-2 is 2.58, therefore, any place where the curve exceeds 2.58 is an unstable condition.

The curves on the plot are labeled K_x, K_z and $K_xK_yK_z$. These labels refer to the direction of the soil spring constant. For instance, a curve labeled K_x has a spring rate in only the x direction, the y and z directions have soil spring rates which are infinitely stiff.

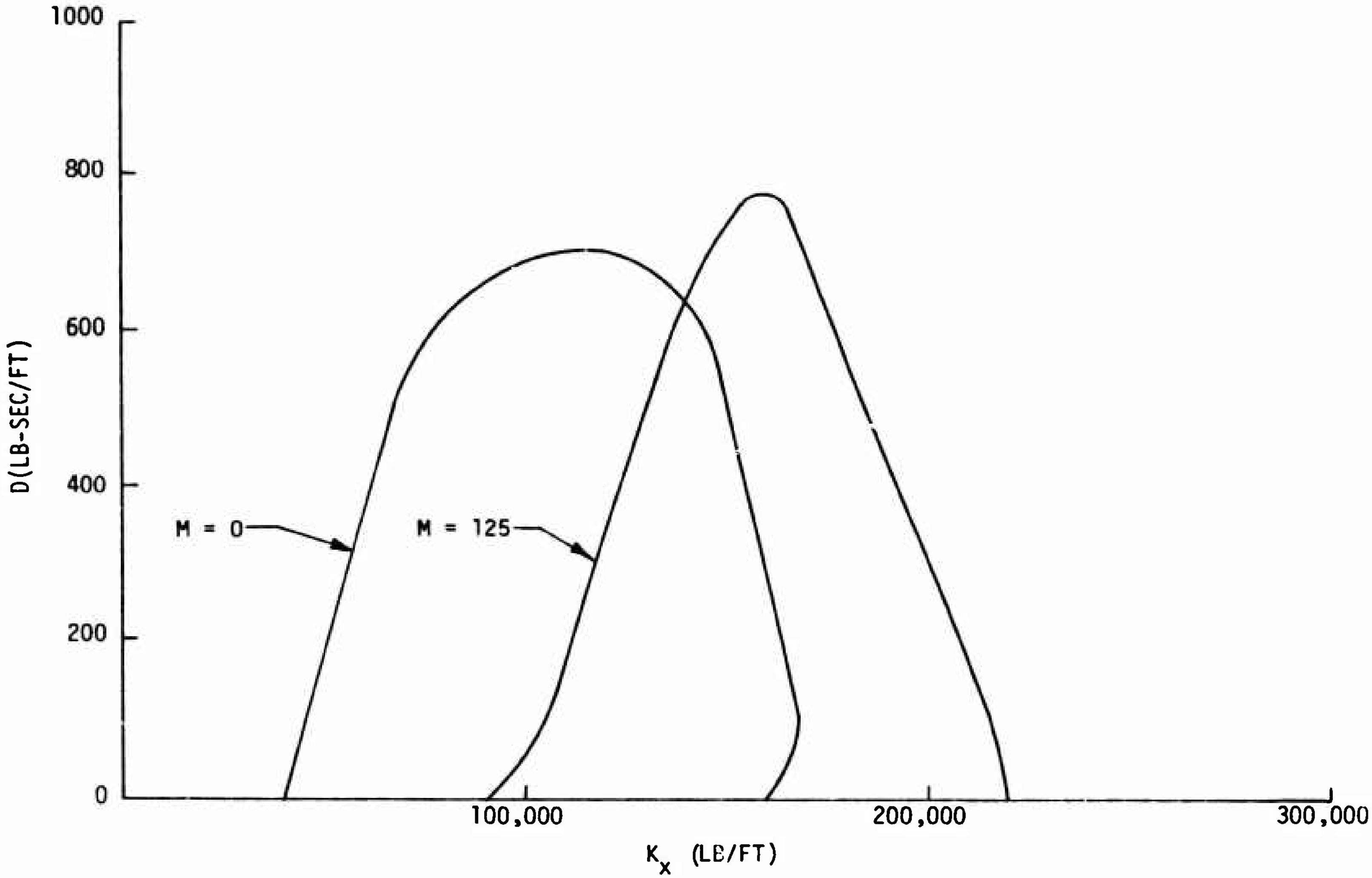

Figure 4. Soil Damping Required for Neutral Stability
(Soil Spring Rate in x Direction Only)

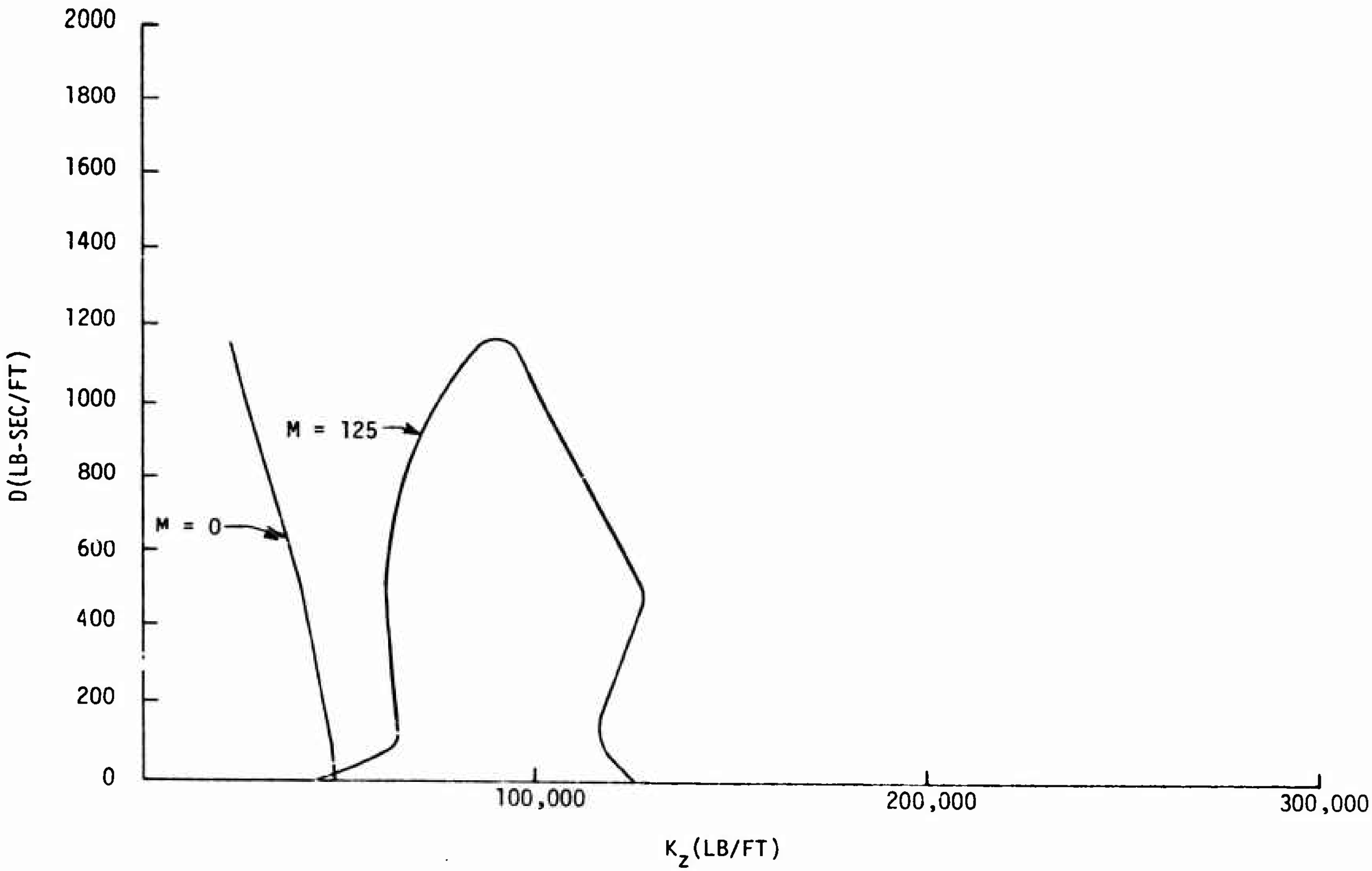

Figure 5. Soil Damping Required for Neutral Stability (Soil Spring Rate in z Direction Only)

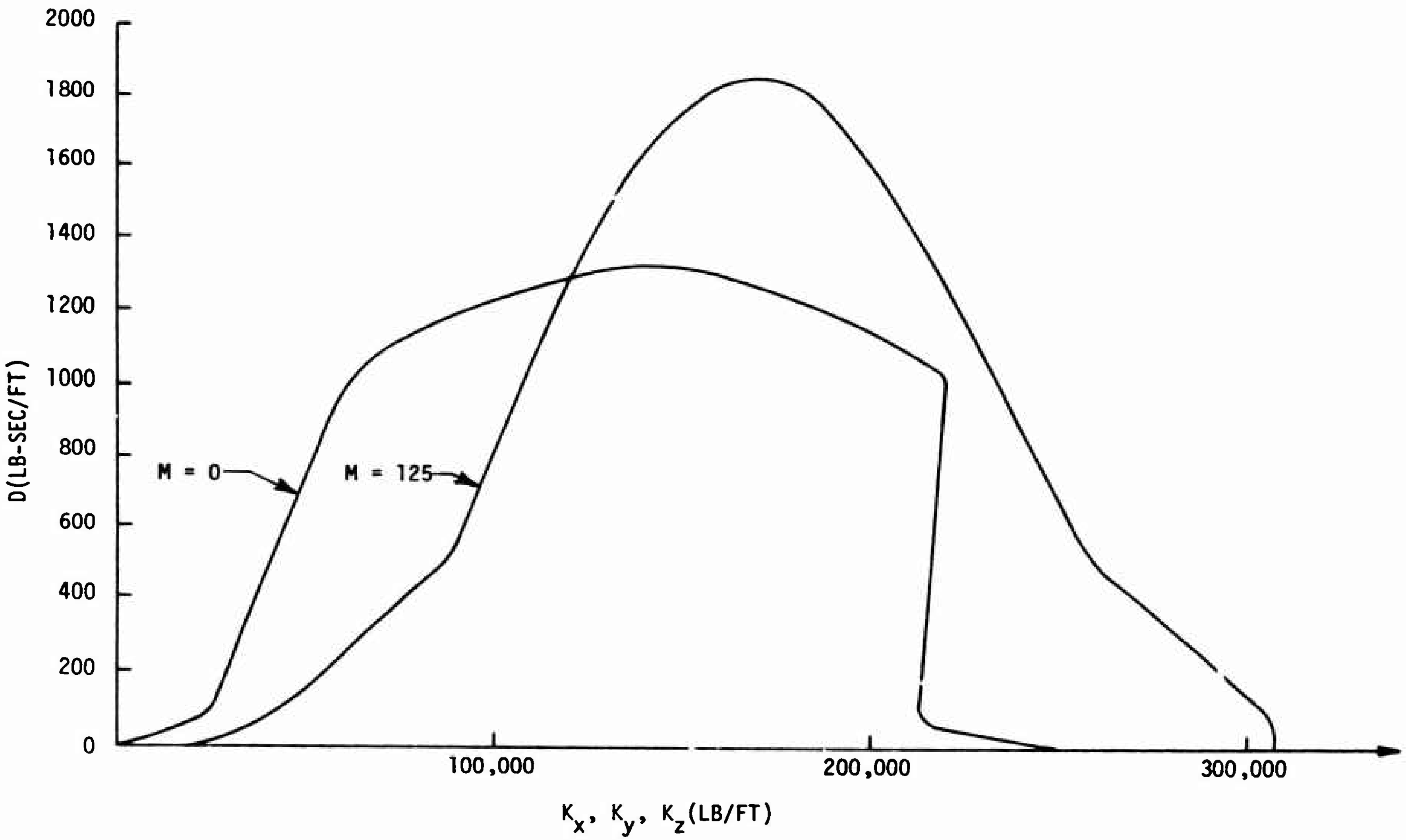

Figure 6. Soil Damping Required for Neutral Stability
(Soil Spring Rate in x, y, & z Direction)

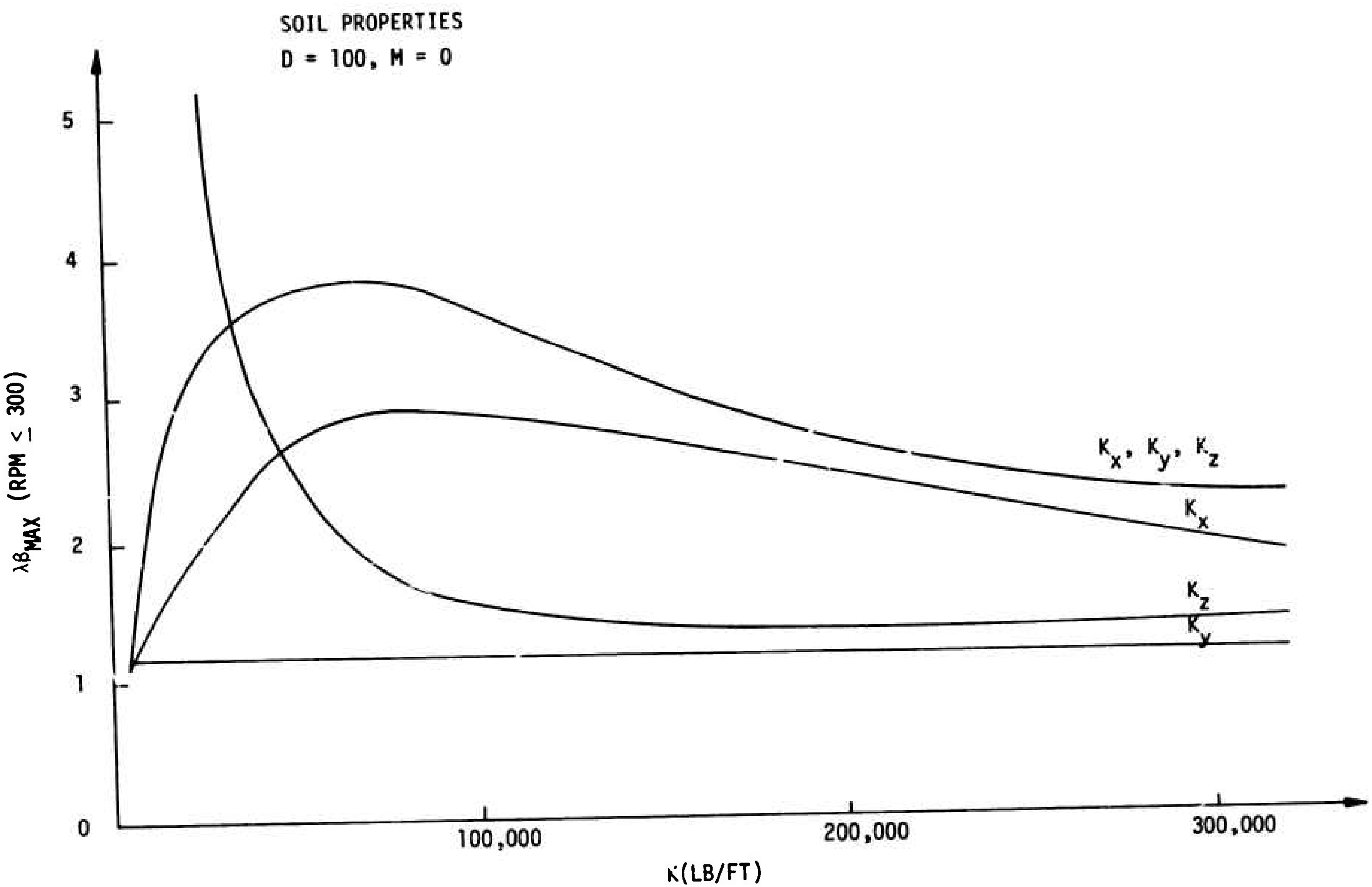

Figure 7. Blade Damping Required for Neutral Stability Vs Soil Spring Rate

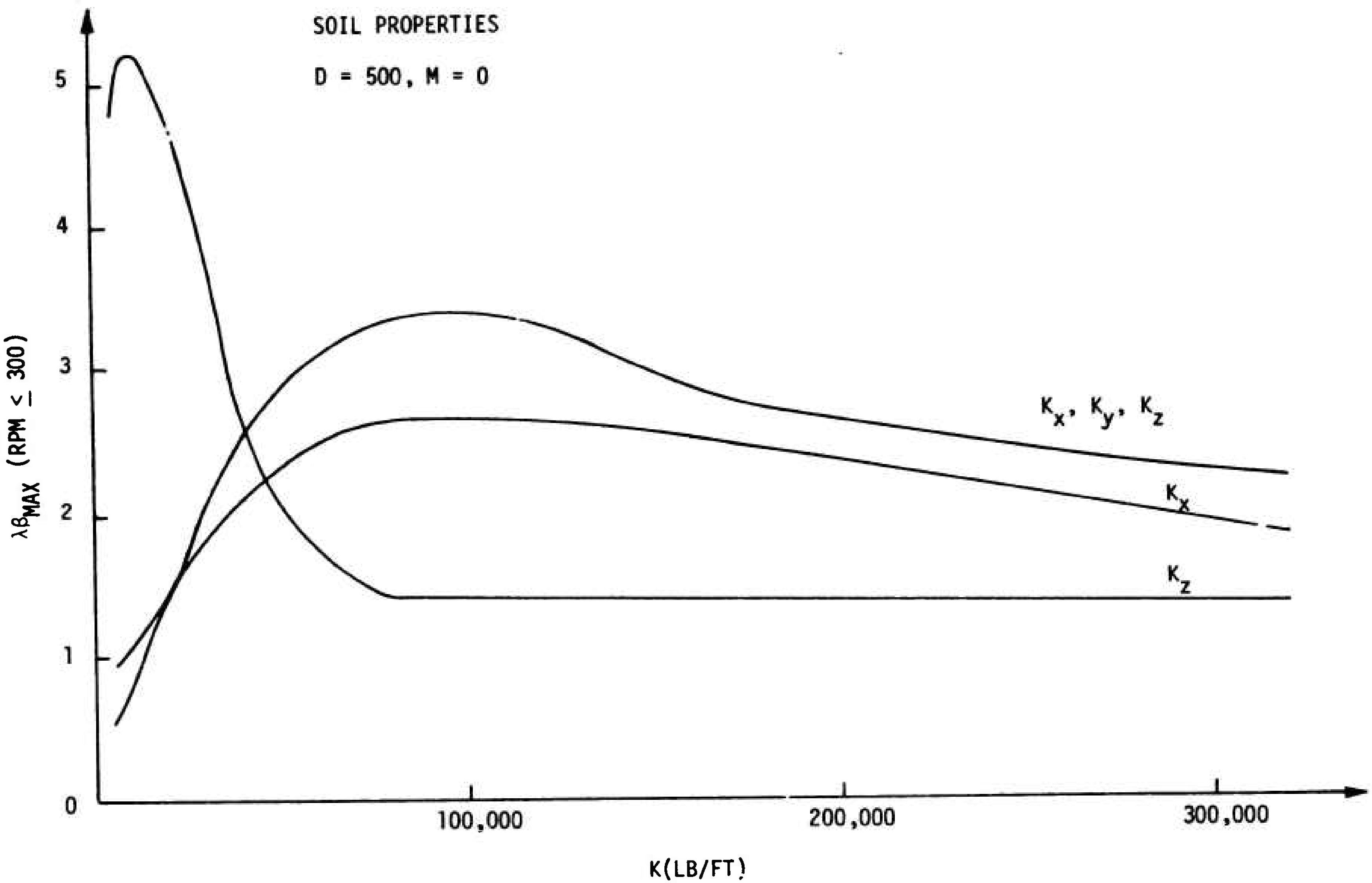

Figure 8. Blade Damping Required for Neutral Stability Vs Soil Spring Rate

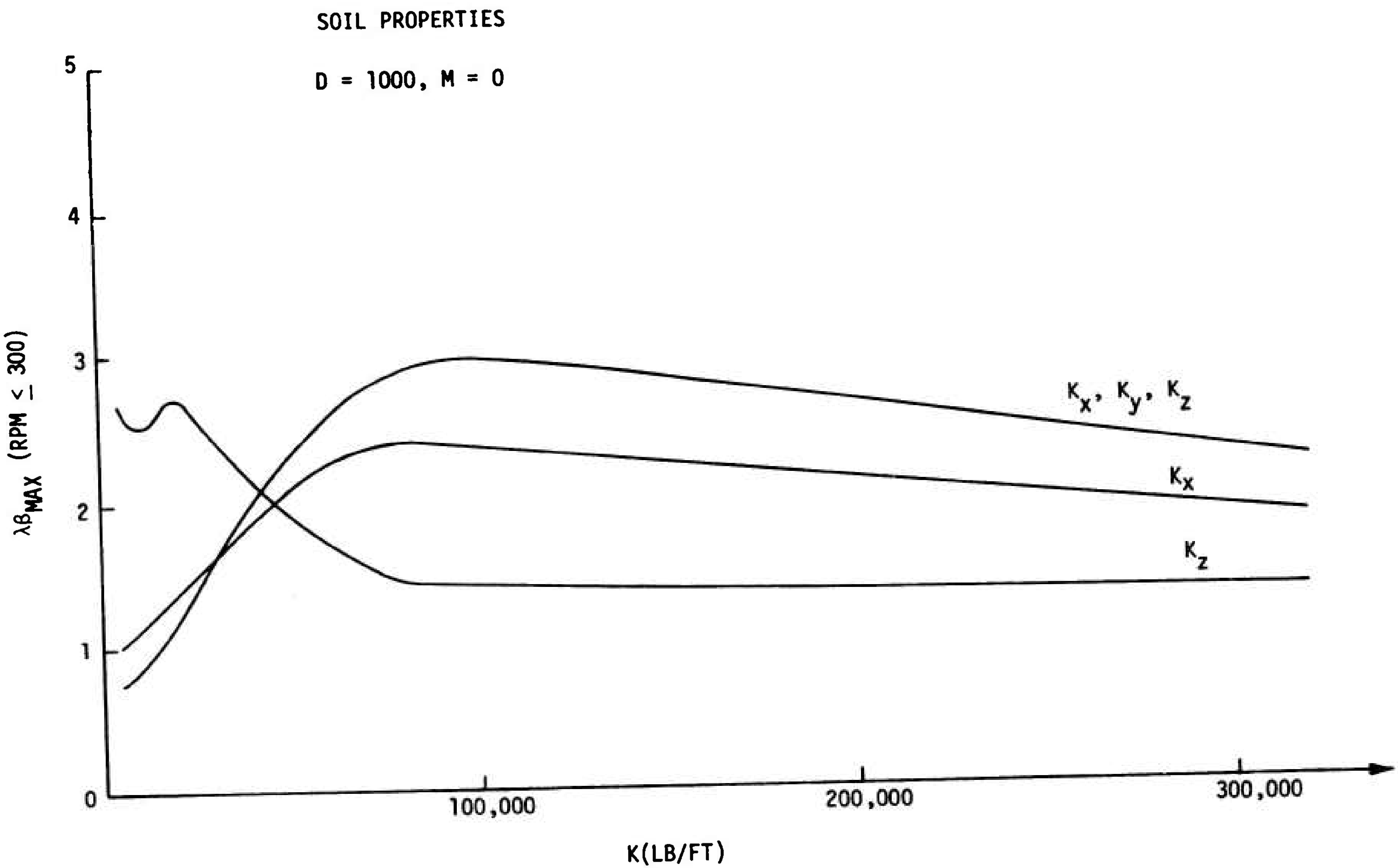

Figure 9. Blade Damping Required for Neutral Stability Vs Soil Spring Rate

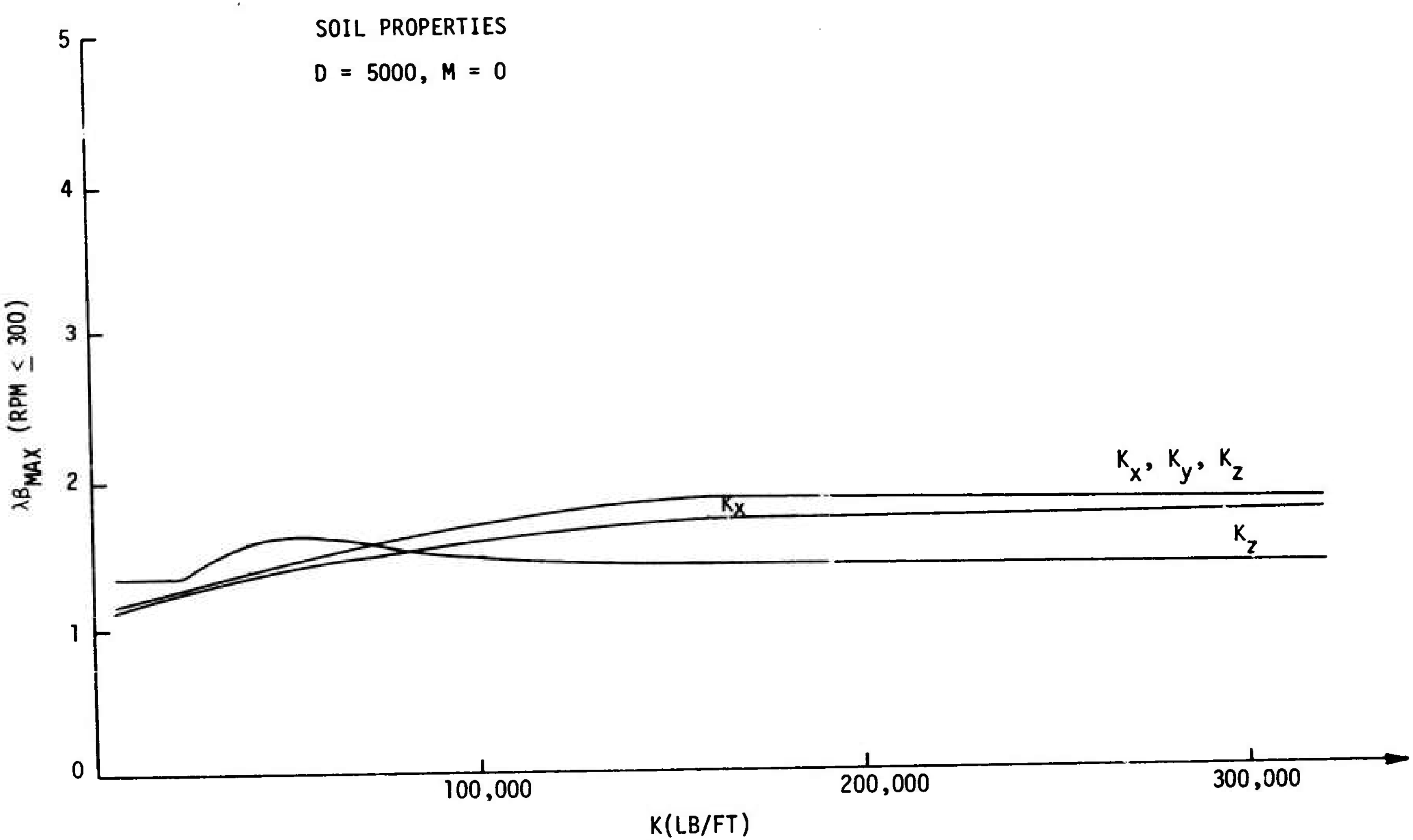

Figure 10. Blade Damping Required for Neutral Stability Vs Soil Spring Rate

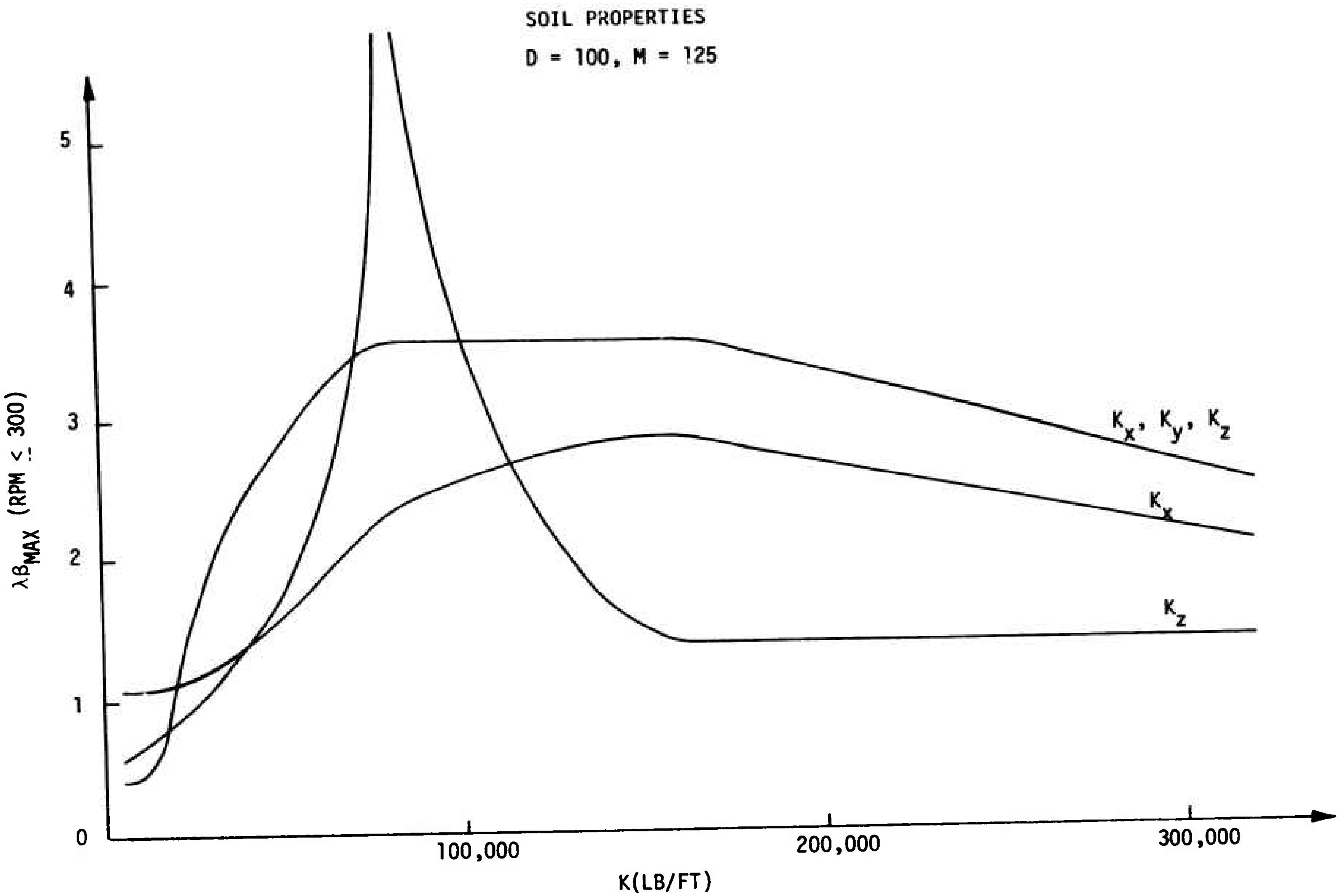

Figure 11. Blade Damping Required for Neutral Stability Vs Soil Spring Rate

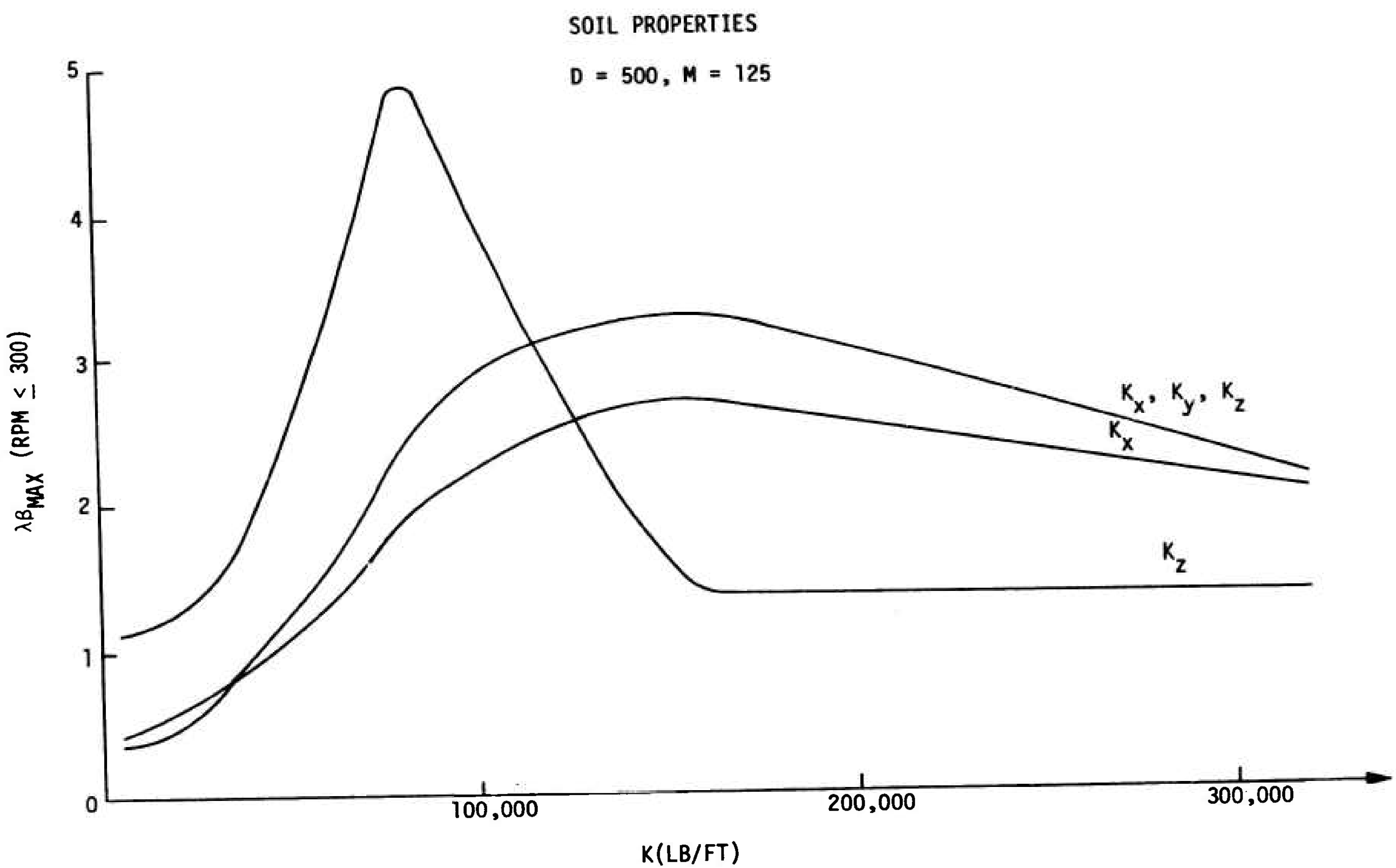

Figure 12. Blade Damping Required for Neutral Stability Vs Soil Spring Rate

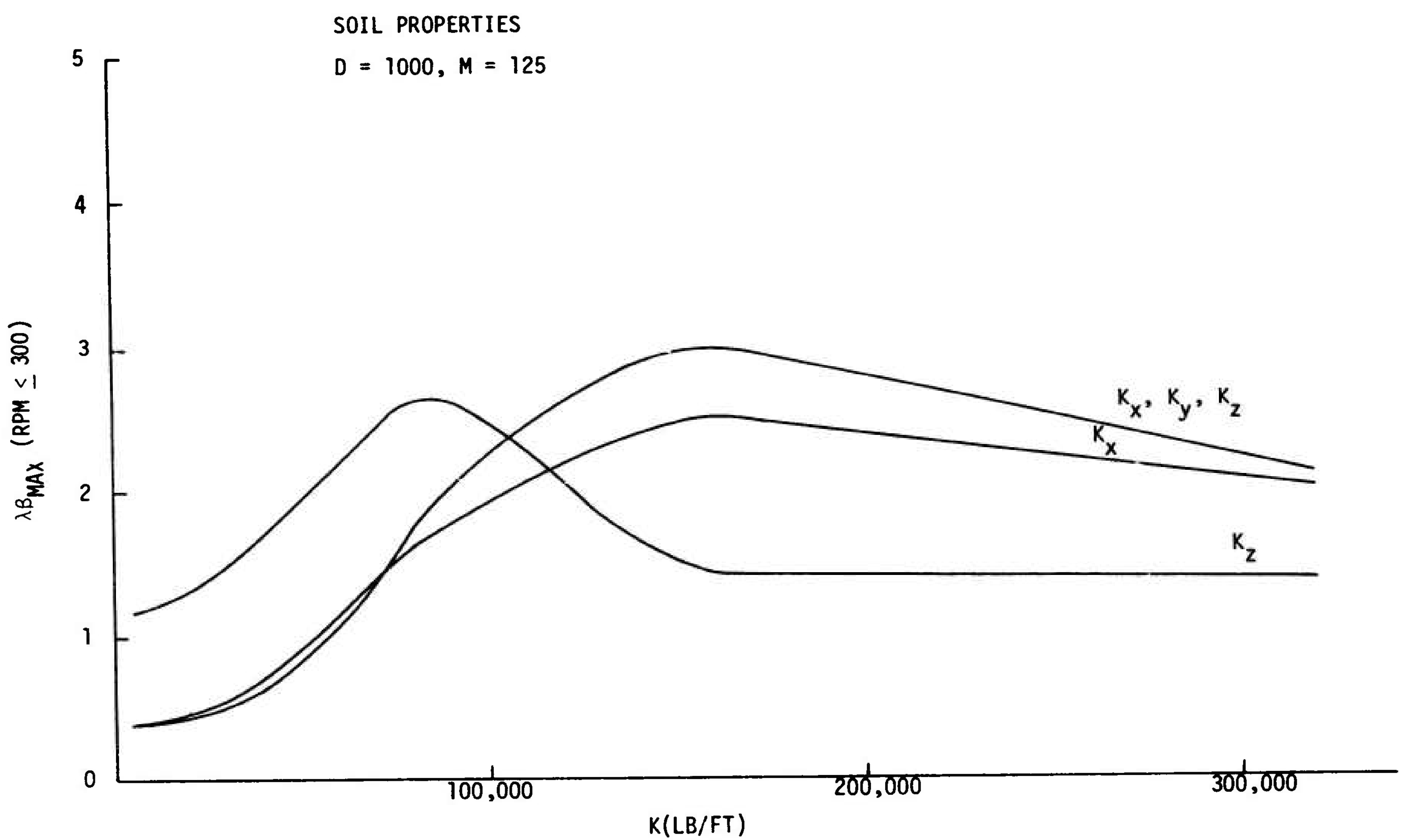

Figure 13. Blade Damping Required for Neutral Stability Vs Soil Spring Rate

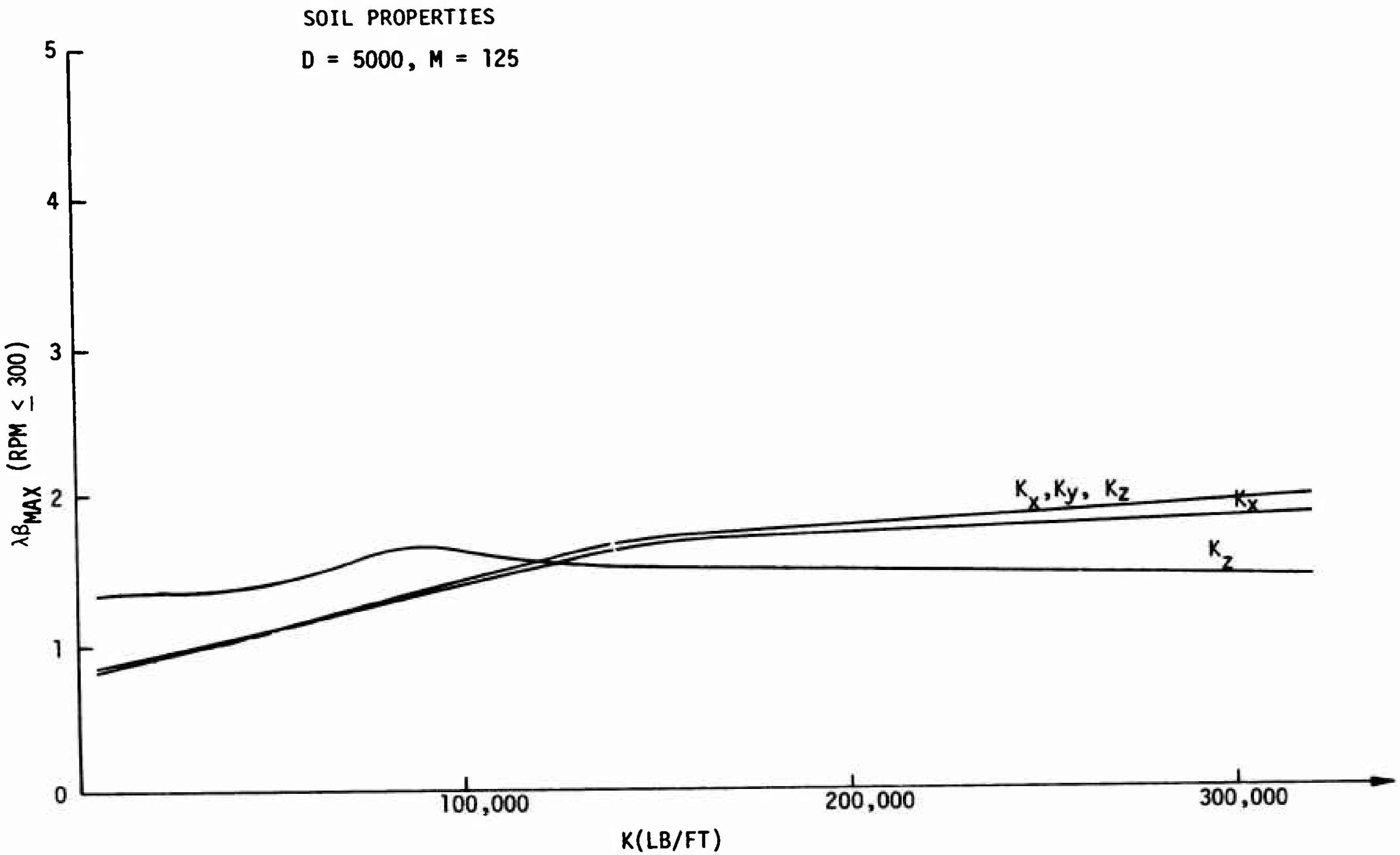

Figure 14. Blade Damping Required for Neutral Stability Vs Soil Spring Rate

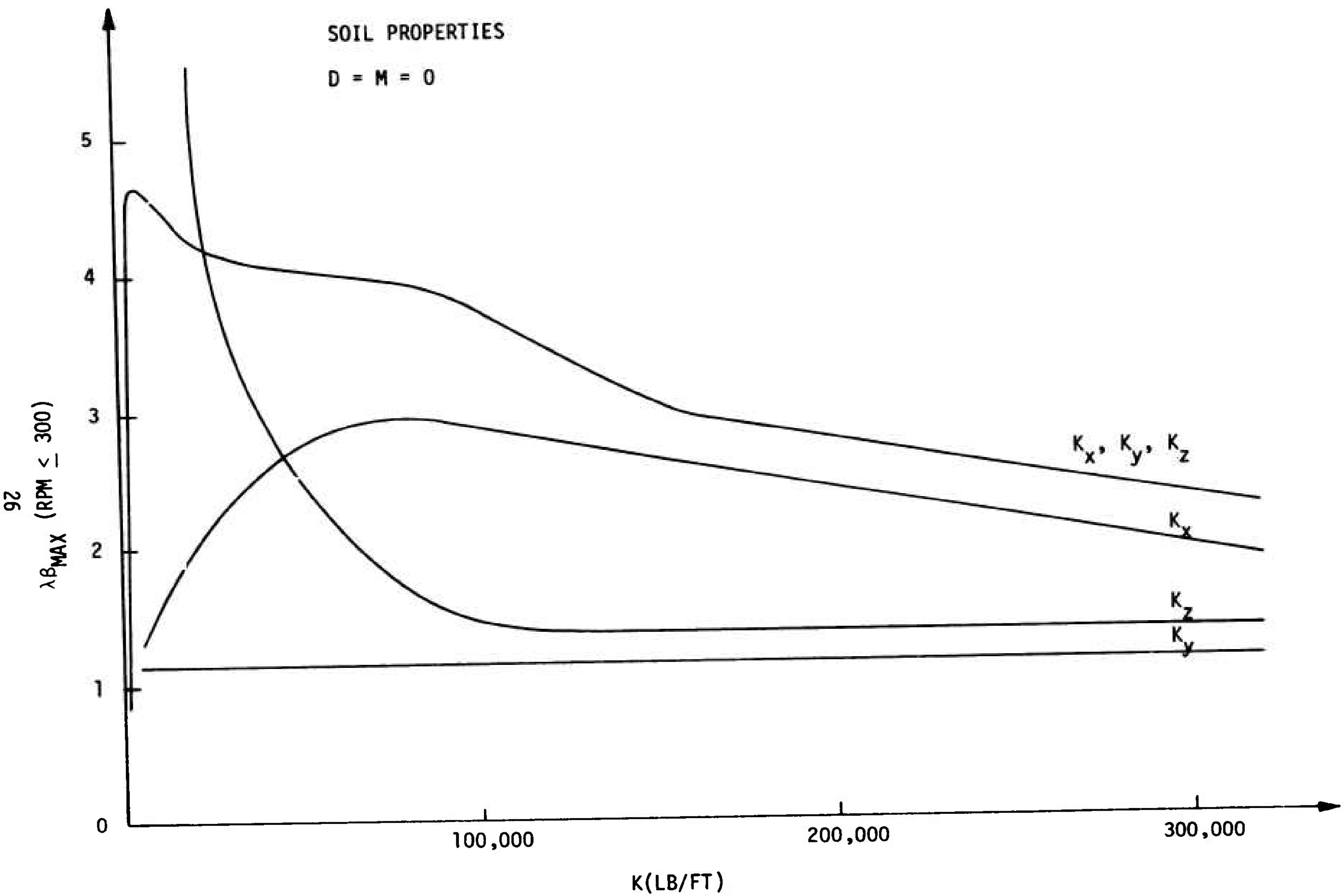

Figure 15. Blade Damping Required for Neutral Stability Vs Soil Spring Rate

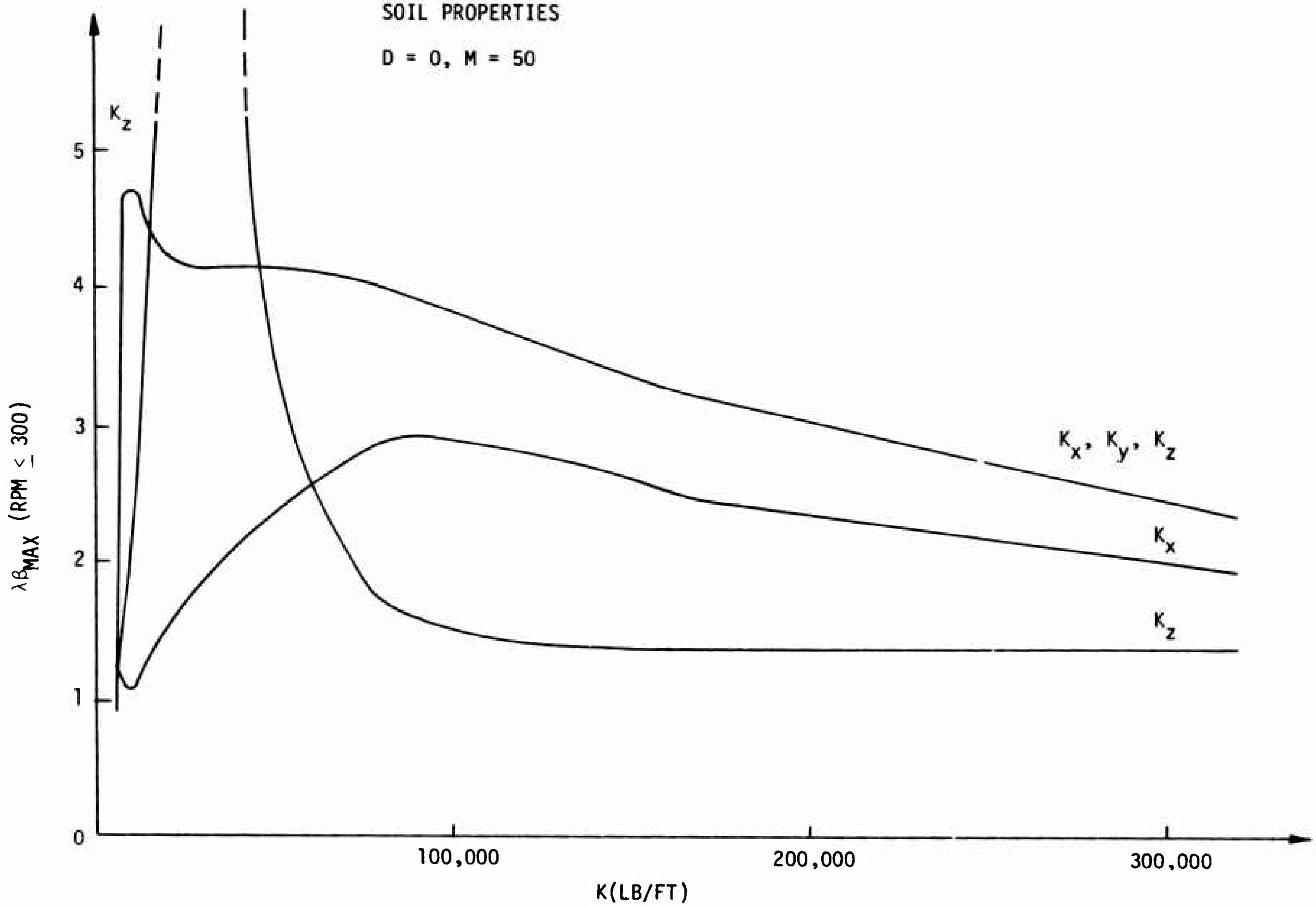

Figure 16. Blade Damping Required for Neutral Stability Vs Soil Spring Rate

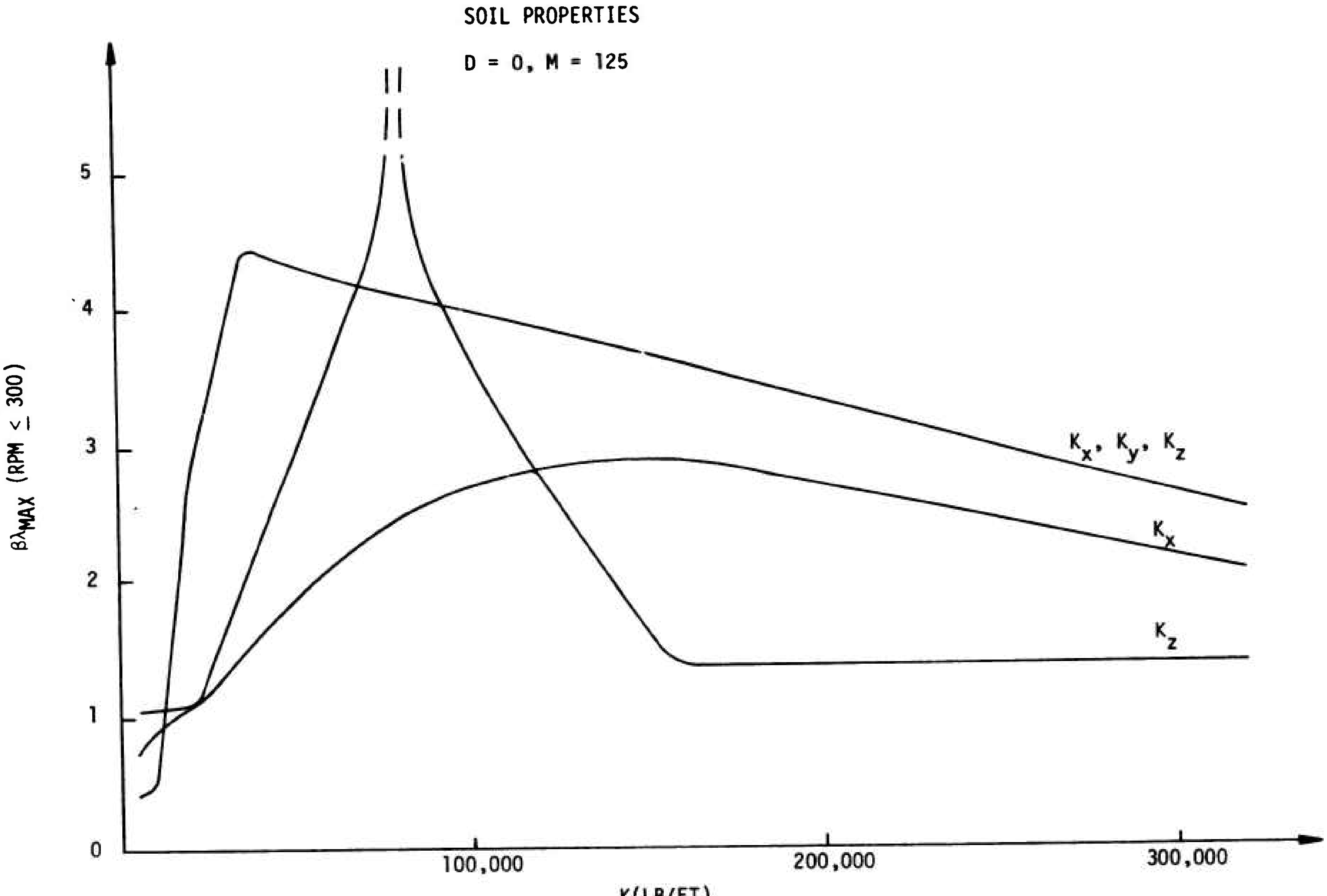

Figure 17. Blade Damping Required for Neutral Stability Vs Soil Spring Rate

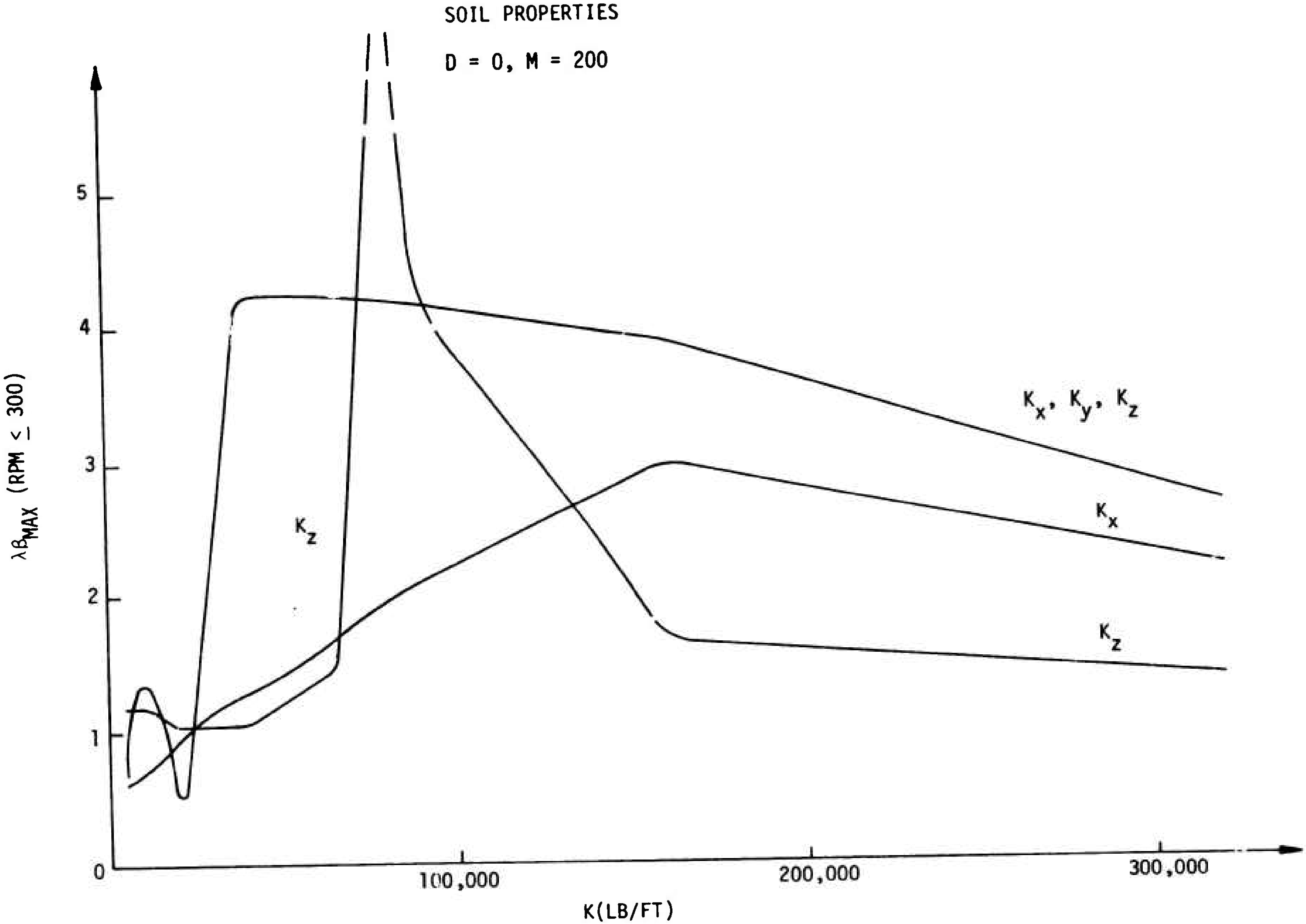

Figure 18. Blade Damping Required for Neutral Stability Vs Soil Spring Rate

CONCLUSIONS

Based on the state-of-the-art survey and the mechanical instability analyses, the following conclusions can be made.

1. No work pertaining to mechanical instability in helicopters associated with soil dynamics has been reported.

2. A linear lumped parameter soil model is adequate to represent the soil for a mechanical instability analysis.

3. No data representing the near surface in a manner which was applicable to a dynamic model was found.

4. There is a range of spring-rate, damping rate, and effective mass of the soil which can cause mechanical instability.

5. No determination could be made as to whether or not the spring, damping and mass values which can cause mechanical instability are representative of soil conditions which are found in tactical situations.

6. Measured soil impedance data can be used to perform a mechanical instability analysis for a helicopter in contact with the soil.

RECOMMENDATIONS

The mechanical properties of prepared landing sites are not well defined but the daily operation of helicopters from such sites is proof enough that they provide stable areas for helicopter operations. There is serious question as to the stability of aircraft landing on some unprepared sites. The vertical properties of soil have been investigated in some depth and empirical relationships exist to determine what the lateral properties are. Unfortunately, most of the data reported is for soil as a purely inorganic material. The near surface on which a helicopter would hand at an unprepared site contains both organic and inorganic material. There is virtually no data for this type of soil. Therefore, it is recommended that a program be initiated to measure the effects of organic materials on near surface soil properties.

It is further recommended that the program contain the following elements:

1. Development of a method of obtaining and analyzing soil data under both laboratory and field conditions.

2. Gather data from two sites under two conditions:

   a. With the soil in its natural undisturbed state.

   b. With the near surface removed.

3. Analyze the data to determine the effects of the organic materials in the near surface.

4. Make recommendations based on the conclusions drawn from 3.

It is estimated that this effort will require 2800 man-hours and material costs of $1500.00.

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APPENDIX I

MECHANICAL INSTABILITY OF ROTORS ON GENERALIZED SUPPORTS

MECHANICAL INSTABILITY OF ROTORS ON GENERALIZED SUPPORTS

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The "ground resonance" type of mechanical instability is generalized to include a large class of supporting structures for the rotor hub. Coleman's equations for three or more blades are used. The fuselage, however, is represented by the hub mobility in each of two perpendicular directions. This mobility will vary with frequency and may be either computed or measured independently of the rotor dynamics. The resulting nonlinear equations are readily solved at each frequency of vibration for the rotor speed and blade damping required for neutral stability.

The generality and resulting simplicity of the approach makes the method quite suitable for the analysis of such diverse situations as a helicopter with brakes off or a wing mounted rotor-prop in flight in various stages of transition.

NOTATION

a	Radial position of lag hinge
A_I	$= (A_{XI} + A_{YI})/2$
A_R	$= 1 - \bar{\Lambda}_2 + (A_{XR} + A_{YR})/2$
A_{XR}, A_{XI}, A_{YR}, A_{YI}	Real and imaginary parts of $\Lambda_4 \omega_f^2 P_X$, $\Lambda_4 \omega_f^2 P_Y$
b	Distance from lag hinge to center of mass of blade
B_β	Blade lag damping rate
F_X, F_Y	Forces acting on hub
F_{Xo}, F_{Yo}	Coefficients in $F_X = F_{Xo} e^{-i\omega_f t}$, etc
I	Blade moment of inertia about hinge $= m_b b^2 (1 + r^2/b^2)$
K_β	Centering spring rate of blade
m_b	Mass of blade
n	Number of blades
P_X, P_Y	Displacement mobility of hub in x, y directions
r	Radius of gyration of blade about center of mass
x, y	Coordinates of hub in fixed system
x_o, y_o	Complex amplitudes of x, y, $x = x_o e^{-i\omega_f t}$, etc
z_f	Complex coordinate of hub, $= x + iy$
Δ_I	$= \frac{1}{2} (A_{XI} - A_{YI})$
Δ_R	$= \frac{1}{2} (A_{XR} - A_{YR})$
ζ_1	Complex coordinate of motion of center of mass of rotor $= \zeta_X + i\zeta_Y$
ζ_X, ζ_Y	Coordinates of motion of center of mass of rotor

ζ_{Xo}, ζ_{Yo}	Complex amplitudes of ζ_X, ζ_Y, $\zeta_X = \zeta_{Xo}e^{-i\omega_f t}$, etc
λ_β	$= B_\beta/I$
$\bar{\lambda}_\beta$	$= \lambda_\beta/\omega_f$
Λ_1	$= m_b ab/I$
Λ_2	$= K_\beta/I$
$\bar{\Lambda}_2$	$= \Lambda_2/\omega_f^2$
Λ_4	$= nm_b/2(1 + r^2/b^2)$
ω	Rotor angular velocity
$\bar{\omega}$	ω/ω_f
ω_f	Angular whirling velocity measured in fixed coordinate system

In 1943, the analysis of the self-excited mechanical instability of hinged rotor blades commonly called ground resonance was published by Coleman. This work was republished in a NACA Technical Note in 1957[1]. The original analysis was a significant technological breakthrough and provided helicopter designers with an understanding of, and a means of avoiding, this potentially destructive condition.

Since 1943 the theory was interpreted and simplified[2,3], expanded in scope[4,5], charted[6] and tested[7]. Some of the work made the theory easier to use and some increased the applicability but in no case has there been any significant change in the basic theory.

The analysis presented in this paper also claims no improvement in the theory. The interaction of the in-plane motion of the blades and the response of the hub remains as the cause of the instability. The original blade equations given by Coleman are used but the representation of the hub dynamics is made very general and at the same time very simple. The resulting equations are applicable to almost any structure whose response can be calculated or measured. The only limitation is that at each frequency the response must be proportional to the applied force.

It is the generality of application and ease of solution which is claimed as being novel, but the basics of the original theory are unchanged.

DERIVATION OF EQUATIONS

The equations from Coleman coupling the motion of the rotor center of gravity and the motion of the hub are presented for reference (Equation (24) of Reference 1)

$$(m_f + nm_b)\ddot{z}_f + B\dot{z}_f + B_a(\dot{z}_f - i\omega z_f) + Kz_f + \Delta m\ddot{\bar{z}}_f + \Delta B\dot{\bar{z}}_f + \Delta K\bar{z}_f + nm_b\ddot{\zeta}_1 = 0 \quad (1)$$

$$nm_b\ddot{z}_f + 2nm_b\left[\left(1 + \frac{r^2}{b^2}\right)(\ddot{\zeta}_1 - 2i\omega\dot{\zeta}_1 - \omega^2\zeta_1) + \frac{B_\beta}{m_b b^2}(\dot{\zeta}_1 - i\omega\zeta_1) + \omega^2\frac{a}{b}\zeta_1 + \frac{K_\beta}{m_b b^2}\zeta_1\right] = 0 \quad (2)$$

where z_f and ζ_1 are the deflection of the hub and the center of gravity of the blades respectively in complex fixed coordinates. The first equation describes the motion of the hub. Notice that the mass of the blades are included. The last term of this equation is the only one containing ζ_1 and can be thought of as the negative of the force acting on the hub due to the motion of the center of gravity of the blades. Because this hub has only a single degree of freedom in each direction, the equation shall be replaced by a more general representation. The second equation describes the motion of the center of gravity of the rotor of three or more blades. The first term represents the coupling with the hub motion. This equation is quite adequate and shall be retained as presented.

The variables in these equations are complex. It will be convenient to write the blade equation in real coordinates using $z_f = x + iy$ and $\zeta_1 = \zeta_X + i\zeta_Y$ where x, y, ζ_X, ζ_Y represent the motion of the hub and center of gravity in the fixed system. Making these substitutions and separating the real and imaginary parts of Equation (2):

$$\frac{1}{2(1 + r^2/b^2)} \ddot{x} + \ddot{\zeta}_X + \lambda_\beta \dot{\zeta}_X - [\omega^2(1 - \Lambda_1) - \Lambda_2]\zeta_X + \omega[2\dot{\zeta}_Y + \lambda_\beta \zeta_Y] = 0$$

$$\frac{1}{2(1 + r^2/b^2)} \ddot{y} + \ddot{\zeta}_Y + \lambda_\beta \dot{\zeta}_Y - [\omega^2(1 - \Lambda_1) - \Lambda_2]\zeta_Y - \omega[2\dot{\zeta}_X + \lambda_\beta \zeta_X] = 0 \qquad (3)$$

The forces applied to the hub are

$$F_X = -nm_b\ddot{\zeta}_X$$

$$F_Y = -nm_b\ddot{\zeta}_Y \tag{4}$$

For the condition of neutral stability, each of the variables may be written in the following forms

$$x = x_o e^{-i\omega_f t}$$

$$\zeta_X = \zeta_{Xo} e^{-i\omega_f t} \tag{5}$$

$$F_X = F_{Xo} e^{-i\omega_f t}$$

and similarly for y, ζ_Y, F_Y where x_o, ζ_{Xo}, F_{Xo}, etc. are complex, representing relative phases, and ω_f is real, being the frequency of the motion in fixed coordinates. Equations (3) and (4) now become:

$$\frac{x_o}{2(1 + r^2/b^2)} + \{1 + i\overline{\lambda}_\beta + [\overline{\omega}^2(1 - \Lambda_1) - \overline{\Lambda}_2]\}\zeta_{Xo} + \overline{\omega}[2i - \overline{\lambda}_\beta]\zeta_{Y_o} = 0 \tag{6}$$

$$\frac{y_o}{2(1 + r^2/b^2)} + \{1 + i\overline{\lambda}_\beta + [\overline{\omega}^2(1 - \Lambda_1) - \overline{\Lambda}_2]\}\zeta_{Yo} - \overline{\omega}[2i - \overline{\lambda}_\beta]\zeta_{X_o} = 0 \tag{7}$$

$$F_{Xo} = nm_b\omega_f^2\zeta_{Xo}$$

$$F_{Yo} = nm_b\omega_f^2\zeta_{Yo} \tag{8}$$

where the bars indicate division by ω_f (or ω_f^2 in the case of Λ_2).

At this point, the analysis will deviate from previous approaches by describing the hub dynamics in a rather general way. It is assumed that the response of the hub to sinusoidal forcing at any frequency can be expressed as

$$x_o = P_X(\omega_f)F_{Xo}$$
$$y_o = P_Y(\omega_f)F_{Yo} \qquad (9)$$

where P_X, P_Y (the displacement mobilities) are complex and are functions of the forcing frequency, ω_f. This assumption allows the possibility of spring and damping rates which are functions of frequency as would be found in the usual series arrangement of tire spring rate - gear damper and spring - fuselage spring rate. There is also no limit on the number of normal modes and natural frequencies of the supporting structure that may be considered if these mobilities are calculated. There is no limit on the anisotropy which may be considered. It has been assumed here that there is no coupling between the x and y directions, i.e., that a force in the x direction will produce only motion in the x direction. While this seems to be a usually adequate assumption, it would be quite straightforward to extend the analysis by including terms of the form $P_{XY}(\omega_f)F_{Yo}$ in the expression for x_o in Equation (9).

These mobilities will be needed only at discrete values of ω_f, thus a tabulation of numerical values will be perfectly sufficient for the method presented. Of course, the frequencies must be close enough so that no points of instability are missed. The mobilities may be either computed or measured. In either case, the mass of the blades must be lumped at the hub as previously mentioned in the discussion of Equation (1). The dynamics of the blades, in no way, however, enter into the determination of P_X and P_Y. These quantities represent the response of the fuselage or supporting structure as an independent body.

Typical computational procedures would consist of analyzing a rigid fuselage on the landing gear, or a finite element analysis of the fuselage, or a bending analysis of a wing structure supporting a rotor-prop in various positions. Any of these are reasonably standard procedures having no direct connection with the problem at hand except as a source of certain required data.

When the structure in question actually exists, a relatively straightforward test will supply the needed data. The measured response at the hub due to an excitation at the hub as a functin of frequency is required. This test should be performed in each direction and the complex amplitudes (or amplitude and phase) should be recorded.

Now using Equations (8) and (9), x_o and y_o may be expressed as functions of ζ_{Xo}, ζ_{Yo} as follows:

$$\begin{aligned} x_o &= nm_b\omega_f^2 P_X(\omega_f)\zeta_{Xo} \\ y_o &= nm_b\omega_f^2 P_Y(\omega_f)\zeta_{Yo} \end{aligned} \tag{10}$$

This may now be put into Equations (6) and (7) to give

$$\{[\bar{\omega}^2(1 - \Lambda_1) + (1 - \bar{\Lambda}_2 + A_{YR})] + i(\bar{\lambda}_\beta + A_{XI})\}\zeta_{Xo} - \bar{\omega}(\bar{\lambda}_\beta - 2i)\zeta_{Yo} = 0 \quad (11)$$

$$\bar{\omega}(\bar{\lambda}_\beta - 2i)\zeta_{Xo} + \{[\bar{\omega}^2(1 - \Lambda_1) + (1 - \bar{\Lambda}_2 + A_{YR})] + i(\bar{\lambda}_\beta + A_{YI})\}\zeta_{Yo} = 0 \quad (12)$$

where

$$\frac{nm_b}{2(1 + r^2/b^2)}\omega_f^2 P_X = \Lambda_4\omega_f^2 P_X = A_{XR} + iA_{XI} \quad (13)$$

and similarly for P_Y. Note that Λ_4 is related to Coleman's Λ_3 through the effective mass ($\Lambda_3 = \Lambda_4/m_e$). For convenience and to better represent the possible anisotropy, the following parameters are defined.

$$A_R = 1 - \bar{\Lambda}_2 + \frac{1}{2}(A_{XR} + A_{YR})$$

$$A_I = \frac{1}{2}(A_{XI} + A_{YI})$$

$$\Delta_R = \frac{1}{2}(A_{XR} - A_{YR})$$

$$\Delta_I = \frac{1}{2}(A_{XI} - A_{YI})$$

Equations (11) and (12) then become:

$$\{[\bar{\omega}^2(1 - \Lambda_1) + A_R + \Delta_R] + i(\bar{\lambda}_\beta + A_I + \Delta_I)\}\zeta_{Xo} - \bar{\omega}(\lambda_\beta - 2i)\zeta_{Yo} = 0 \quad (14)$$

$$\bar{\omega}(\bar{\lambda}_\beta - 2i)\zeta_{Xo} + \{[\bar{\omega}^2(1 - \Lambda_1) + A_R - \Delta_R] + i(\lambda_\beta + A_I - \Delta_I)\}\zeta_{Yo} = 0 \quad (15)$$

The determinant of the coefficients of Equations (14) and (15) must be zero for a non-trivial solution to exist. This is written

$$\{[\bar{\omega}^2(1 - \Lambda_1) + A_R + i(\bar{\lambda}_\beta + A_I)] + (\Delta_R + i\Delta_I)\}$$

$$\cdot\{[\bar{\omega}^2(1 - \Lambda_1) + A_R + i(\lambda_\beta + A_I)] - (\Delta_R + i\Delta_I)\} + \bar{\omega}^2(\bar{\lambda}_\beta - 2i)^2 = 0$$

or

$$[\bar{\omega}^2(1 - \Lambda_1) + A_R + i(\bar{\lambda}_\beta + A_I)]^2 = (\Delta_R + i\Delta_I)^2 + \bar{\omega}^2(2 + i\bar{\lambda}_\beta)^2 \quad (16)$$

In this equation, all the parameters are real and thus the real and imaginary parts of the equation must separately equal zero. Equation (16) is the general equation describing the steady state, neutraly stable oscillation at a frequency of ω_f. For each value of ω_f, the equation may be solved for λ_β and ω as will be described below.

SPECIAL CASE: ISOTROPIC SUPPORTS

Before proceeding with the solutions of the general equations, it will be instructive to examine the classical but unusual conditions of isotropy. When $P_X = P_Y$, $\Delta_R = \Delta_I = 0$ and Equation (16) can be written

$$[\bar{\omega}^2(1 - \Lambda_1) + A_R + i(\bar{\lambda}_\beta + A_I)]^2 = \bar{\omega}^2(2 + i\bar{\lambda}_\beta)^2$$

Taking the square root of both sides and separating the real and imaginary parts yields

$$\begin{aligned} \bar{\omega}^2(1 - \Lambda_1) - 2\bar{\omega} + A_R &= 0 \\ \bar{\lambda}_\beta(1 - \bar{\omega}) + A_I &= 0 \end{aligned} \quad (17)$$

At each frequency of oscillation, ω_f , A_R and A_I are known and the first equation may be solved for $\bar{\omega}$ and then the second may be solved for $\bar{\lambda}_\beta$. These are the rotor speed and the blade damping required for a neutrally stable oscillation at a frequency ω_f.

In the appendix, it is shown that for single degree-of-freedom isotropy, these equations reduce to forms given by Coleman (Ref. 1) and Warming (Ref. 3).

GENERAL CASE: ANISOTROPIC HUB

For the general case, Equation (16) does not reduce to so simple a form as for the case of isotropy. This equation, when expanded and when the real and imaginary parts are separated, yields two equations as follows:

$$\bar{\omega}^4(1 - \Lambda_1)^2 + \bar{\omega}^2\{\bar{\lambda}_\beta^{\,2} + 2(1 - \Lambda_1)A_R - 4\} + \{(\bar{\lambda}_\beta + A_I)^2 + A_R^{\,2} - (\Delta_R^{\,2} - \Delta_I^{\,2})\} = 0 \qquad (18)$$

$$\bar{\omega}^2\{\bar{\lambda}_\beta(1 + \Lambda_1) + A_I(1 - \Lambda_1)\} + (\bar{\lambda}_\beta + A_I)A_R - \Delta_R\Delta_I = 0 \qquad (19)$$

These equations may be readily solved by the use of the Newton-Raphson method (See Reference (8), for example). It has been found that the solutions for the isotropic case given by Equation (17) are a good starting point for the iteration.

The results can be plotted as in Coleman as ω_f vs ω. A more useful form, however, is a plot of λ_β vs ω, showing the blade damping required for stability at each rotor speed.

ILLUSTRATIVE EXAMPLE

As an illustration of the kinds of results which may be obtained using these procedures, the example shown here is presented. The data is representative of a helicopter on the ground with brakes off. Figure 1 illustrates the amplitudes of P_X and P_Y as a function of forcing frequency for two cases, i.e., with and without damping in the landing gear.

Figure 2 shows ω as a function of ω_f for the two cases. For the condition of no damping, equations (18) and (19) either have no solution when there is an instability or have a solution with $\lambda_\beta = 0$ when the system is stable. This situation is well known[3]. When there is no hub damping, the entire system is either absolutely stable or unstable and blade damping alone cannot change the range of instability.

Figure 3 illustrates the blade damping required as a function of rotor speed. The ranges of instability for any value of λ_β are easily read from the figure. For conditions of lesser gear damping, there will be situations where there will be no solution to the equations. This indicates a condition of absolute instability and only additional damping in the gear can remove the instability.

CONCLUSIONS

A procedure has been presented for applying the theory of mechanical instability to hinged rotors on very general supports. The response of the hub may be computed or measured independently of the rotor dynamics.

The resulting procedure is quite easy to use and may be applied to almost any structure supporting a rotor of three or more hinged blades.

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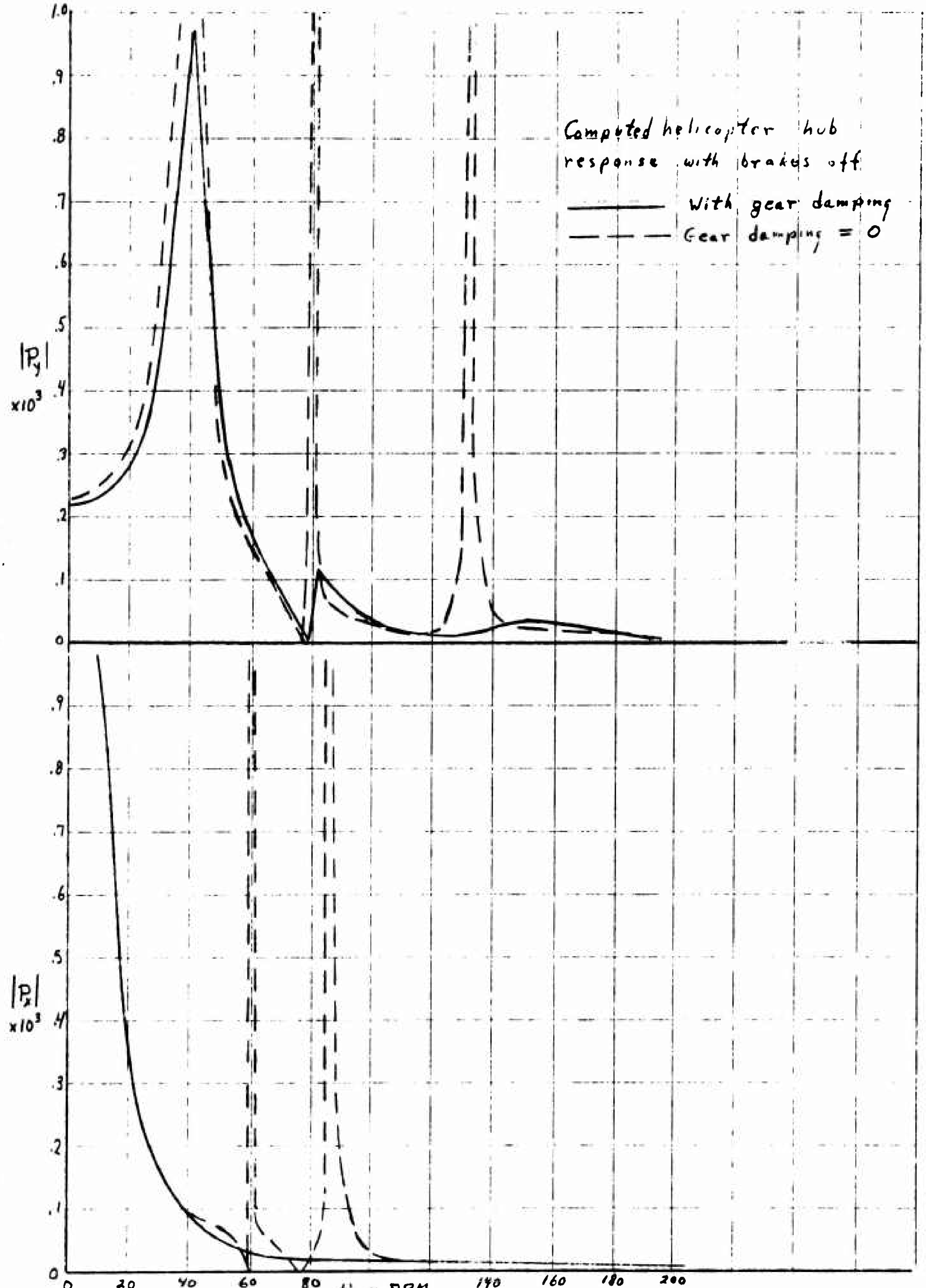

FIG. 1 Amplitudes of Hub Response vs Forcing Frequency

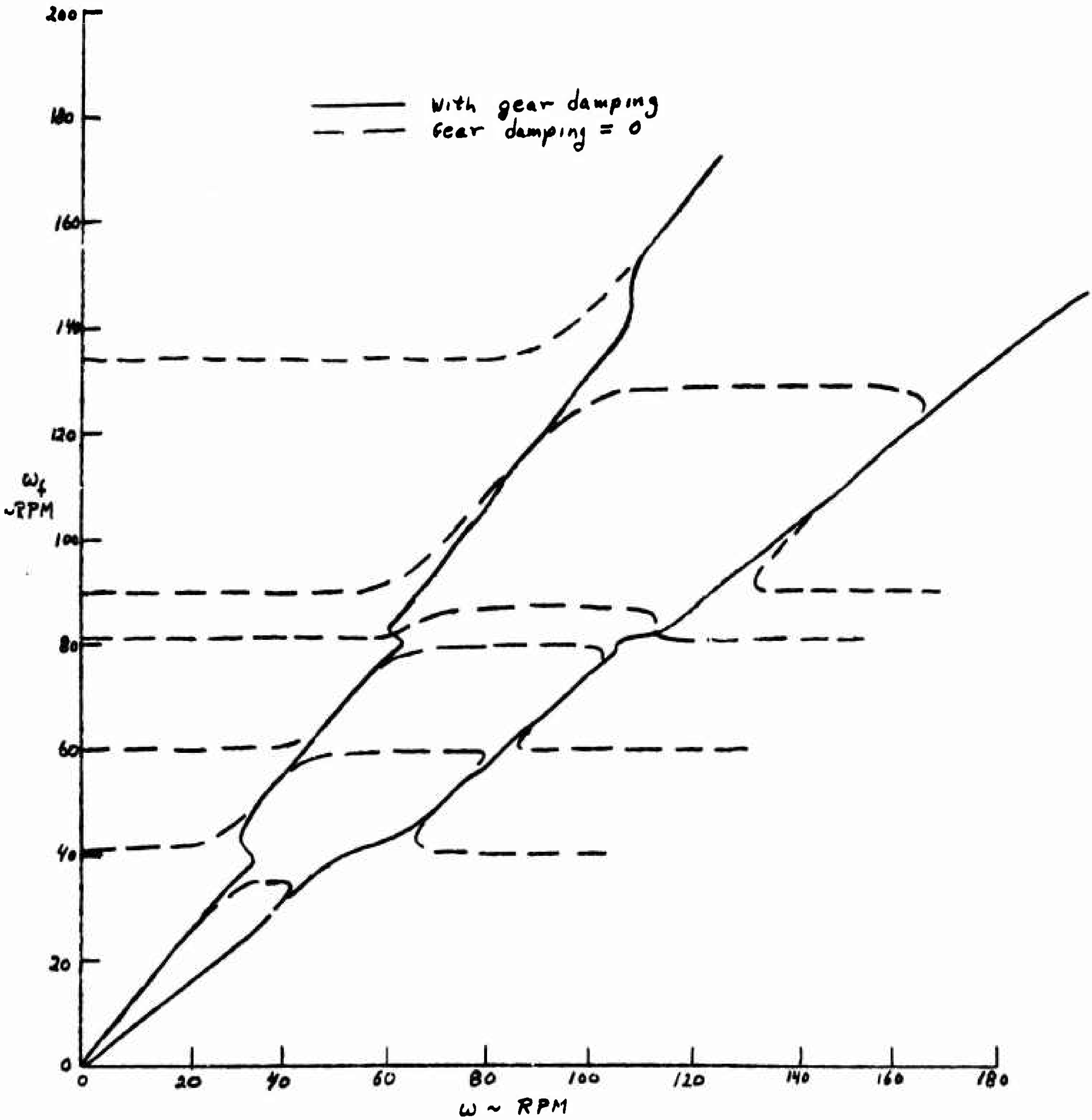

FIG. 2. Whirling Frequency vs Rotor Speed

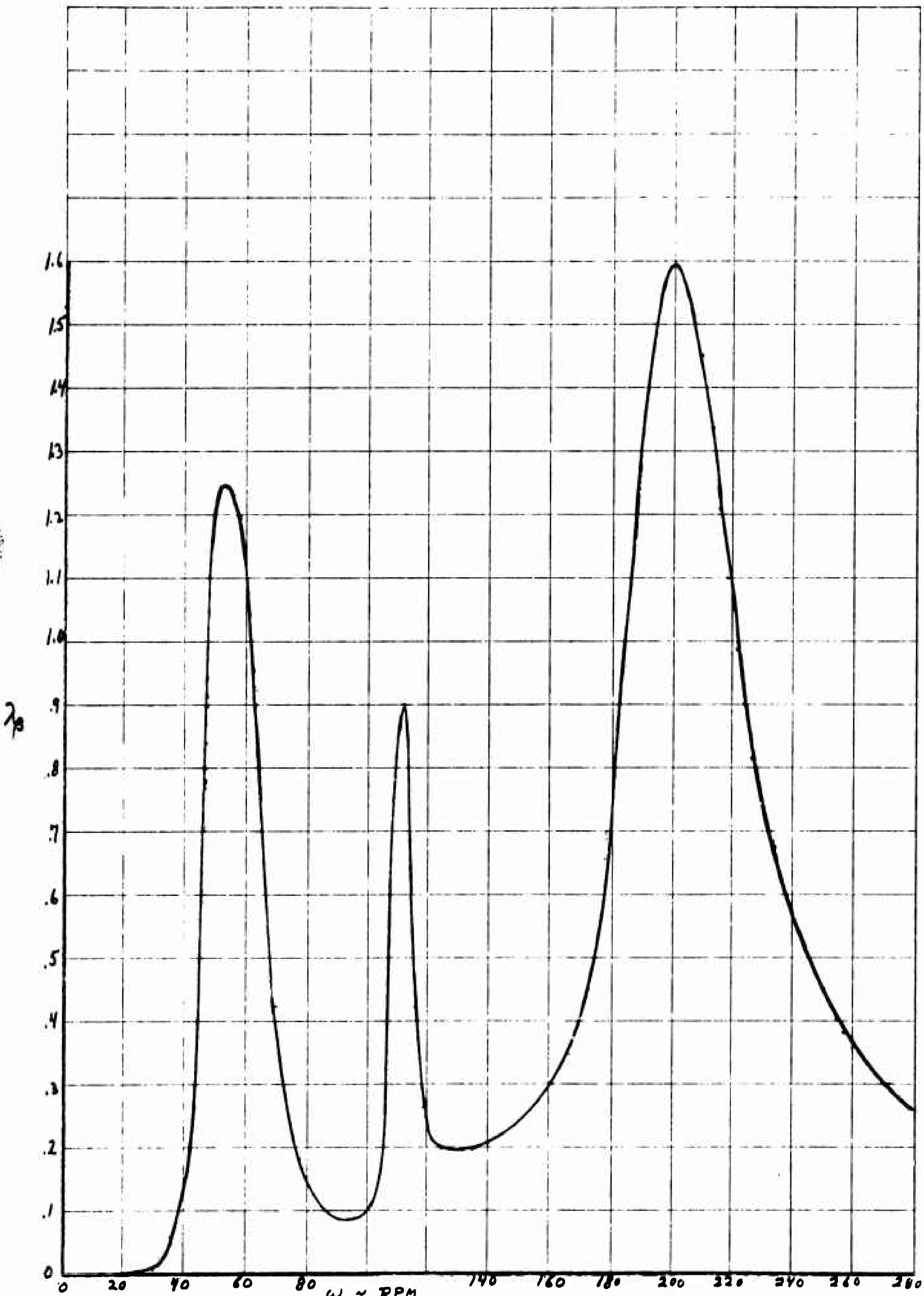

FIG. 3 Blade Damping Required vs. Rotor Speed

APPENDIX

THE ISOTROPIC SINGLE-DEGREE-OF-FREEDOM HUB

The single-degree-of-freedom hub on isotropic supports has been analyzed by Coleman[1] and Warming[3] and many others. For this condition, it may be shown that

$$A_R = 1 + \bar{\Lambda}_2 + \Lambda_3 \frac{\bar{\omega}_r^2 - 1}{[\bar{\omega}_r^2 - 1]^2 + \bar{\lambda}_f^2} \tag{A-1}$$

$$A_I = \Lambda_3 \frac{\lambda_f}{[\bar{\omega}_r^2 - 1]^2 + \bar{\lambda}_f^2} \tag{A-2}$$

where ω_r and λ_f are the natural frequency and damping ratio of the hub and the bars indicate division by ω_f. Substituting Equations (A-1) and (A-2) into Equation (17) results in

$$\bar{\omega}^2(1 - \Lambda_1) - 2\bar{\omega} + \{1 - \bar{\Lambda}_2 + \Lambda_3 \frac{\bar{\omega}_r^2 - 1}{[\bar{\omega}_r^2 - 1]^2 + \lambda_f^2}\} = 0 \tag{A-3}$$

$$\bar{\lambda}_\beta = \frac{\Lambda_3}{\bar{\omega} - 1} \frac{\bar{\lambda}_f}{[\bar{\omega}_r^2 - 1] + \bar{\lambda}_f^2} \tag{A-4}$$

At the "center of instability" where $\bar{\omega}_r = 1$, Equation (A-4) gives

$$\bar{\lambda}_\beta \bar{\lambda}_f = \frac{\Lambda_3}{\bar{\omega} - 1}$$

which is identical to Equation (17) of Reference 3. Also, Equation (A-3) becomes

$$\bar{\omega}^2(1 - \Lambda) - 2\bar{\omega} + (1 - \bar{\Lambda}_2) = 0$$

or

$$\bar{\omega} = \frac{1}{1 - \Lambda_1} \{1 + \sqrt{1 - (1 - \Lambda_1)(1 - \bar{\Lambda}_2)}\}$$

which is identical to Equation (16) of Reference 3.

Equations (34) and (35) of Reference 1 may be written:

$$(1 - \Lambda_1)\bar{\omega}^2 - 2\bar{\omega} + (1 - \bar{\Lambda}_2) - (\bar{\omega} - 1)\frac{\bar{\lambda}_f \bar{\lambda}_\beta}{\bar{\omega}_r^2 - 1} + \frac{\Lambda_3}{\bar{\omega}_r^2 - 1} = 0$$

$$(1 - \Lambda_1)\bar{\omega}^2 - 2\bar{\omega} + (1 - \bar{\Lambda}_2) + (\bar{\omega}^2 - 1)\frac{\bar{\lambda}_\beta(\bar{\omega}_r^2 - 1)}{\lambda_f} = 0$$

These equations can be rearranged to be exactly Equations (A-3) and (A-4) above.

Thus, it is seen that for the special case of the isotropic single-degree-of-freedom hub, the equations reduce to the standard forms obtained by others.