AD/A-000 957

MULTIPLE SATELLITE DATA PROCESSING FOR THE NAVSTAR GLOBAL POSITIONING SYSTEM

Patrick J. Fell

Naval Surface Weapons Center
Dahlgren Laboratory, Virginia

October 1974

NAVAL WEAPONS LABORATORY

Dahlgren, Virginia

22448

C. J. Rorie, Capt., USN
OIC, and Assistant Commander

C. W. Bernard
Deputy Technical Director

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER TR-3210	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER AD/A-000957
4. TITLE (and Subtitle) MULTIPLE SATELLITE DATA PROCESSING FOR THE NAVSTAR GLOBAL POSITIONING SYSTEM		5. TYPE OF REPORT & PERIOD COVERED 6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Patrick J. Fell		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS Naval Surface Weapons Center Dahlgren Laboratory Dahlgren, Va. 22448		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS		12. REPORT DATE October 1974 13. NUMBER OF PAGES 37
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)		15. SECURITY CLASS. (of this report) UNCLASSIFIED 15a. DECLASSIFICATION DOWNGRADING SCHEDULE
16. DISTRIBUTION STATEMENT (of this Report) Approved for Public Release; Distribution unlimited		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		
18. SUPPLEMENTARY NOTES		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number)		

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

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modeling errors in radiation pressure are reduced by the multiple satellite processing scheme. However, adequate time solutions from single satellite processing are achievable if accurate radiation pressure modeling is available.

DL TECHNICAL REPORT TR-3210
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MULTIPLE SATELLITE DATA PROCESSING FOR THE NAVSTAR GLOBAL POSITIONING SYSTEM

Patrick J. Fell
Warfare Analysis Department

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FOREWORD

This report examines a least squares method for the solution of orbital and bias parameters for multiple Global Positioning System satellites. The method includes the solution for timing errors present in the system.

The work described in this report was performed in the Astronautics and Geodesy Division, Warfare Analysis Department. The work was funded by the Naval Research Laboratory under Task #WR-0-0112.

Appreciation is expressed to S. Nichols of the Naval Research Laboratory and to R. W. Hill and S. Meyerhoff of the Astronautics and Geodesy Division.

The report was reviewed by R. J. Anderle, Head, Astronautics and Geodesy Division.

RELEASED BY:

R. A. NIEMANN, Head
Warfare Analysis Department

ABSTRACT

A least squares method is presented for the solution of orbital and bias parameters for multiple Global Positioning System satellites being observed by a common station array. The effects of radiation pressure modelling errors on timing solutions are examined for both multiple and single satellite processing using a time equation consisting of a bias and linear drift term. Results indicate that the errors in orbital and timing solutions caused by modeling errors in radiation pressure are reduced by the multiple satellite processing scheme. However, adequate time solutions from single satellite processing are achievable if accurate radiation pressure modeling is available.

CONTENTS

1. INTRODUCTION

The Navstar Global Positioning System (GPS) is a passive all weather navigation satellite system proposed for the 1980-1990 time frame. The system is currently in the prototype evaluation phase with two experimental models to be launched in the next three years. These prototype models are being designed and built by the Naval Research Laboratory to test further the concept of passive satellite navigation based on highly accurate frequency standards. Timation III-A is the first in this series.

When operational the GPS system will enable users to determine their three dimensional position and time instantaneously. Ranging measurements taken simultaneously from at least four satellites will be reduced to determine these parameters. Anticipated positional accuracies in the horizontal and vertical axes are better than 10 meters 90% of the time.

Obviously there will exist a critical need for precise orbit determination and prediction, and for extremely stable oscillators aboard the satellites. The GPS system as envisioned will employ atomic frequency standards in each satellite to assure high oscillator stability. The Naval Research Laboratory is currently developing the clock technology necessary to meet GPS accuracy goals. Prototype models (Timation series) will test the feasibility of using rubidium, then cesium standards in a space environment.

Current plans call for approximately five operational ground stations to track all system satellites for orbit determination. These tracking stations will also be equipped with high stability oscillators. But even with atomic standards there are offsets and drifts in frequency.

Since frequency (timing) errors at the common tracking stations will be reflected in the computed orbits of all GPS satellites a procedure can be developed which will use data from various satellites to determine system ephemerides and timing errors precisely. Such a procedure is examined in this report. The method models the timing error of each system clock as a bias and linear drift term. The assumption is that this approximation will be accurate for atomic standards over a few day period. This assumption is supported by rubidium frequency data taken at the Naval Research Laboratory (NRL) [Reference 1]. The method may incorporate more sophisticated clock modeling as warranted.

However, precise system time determination depends on the accuracy of the dynamical model used in orbit determination. It has been demonstrated that radiation pressure modeling for GPS altitude satellites is critical [Reference 2]. A radiation pressure bias of one percent may introduce orbit errors of four meters during two day orbit predictions. The effects that radiation errors may have on the timing solutions for GPS satellites are also considered in this report.

II. MULTIPLE SATELLITE PROCESSING

The concept of multiple satellite processing for the GPS system is based on the fact that timing errors at common tracking stations will affect the computed orbits of all satellites. Theoretically a multiple satellite processing scheme should predict the parameters of the time equation better since it utilizes all observations. These timing parameters, which are modeled here as a bias and linear drift, replace range and range drift pass parameters associated with range data (Appendix B). Knowledge of timing errors at one station is required to make timing parameters linearly independent. A better knowledge of system timing errors will allow a better determination of satellite orbital constants.

A mathematical description of least squares processing of multiple satellite data is described in Appendix B. The procedure involves forming the normal equations for each pass of satellite data, formally eliminating the pass dependent parameters, combining normal equations and solving for orbital and timing corrections.

III. SENSITIVITY ANALYSIS

A. Radiation Pressure Modeling

An analysis was conducted to determine what effects errors in radiation pressure modeling would have on timing solutions produced by both individual and multiple satellite processing. The results for all cases assume the following:

(i) System clocks are accurately modelled with a bias and linear drift.

(ii) Tracking station positions are perfectly known.

(iii) Clock errors are perfectly known for one station.

1. An Unmodelled Bias

a. Multiple Satellite Processing

Results have been obtained using synthetic range data from six satellites (Table 1) and five tracking stations (Table 2). The standard error on the range data was 50 centimeters. Again the timing errors at one station were assumed known to a high degree of accuracy in order to make the timing parameters independent. Cases were considered with a radiation pressure bias of one and five percent. No attempt was made to solve for this error.

For each case orbital and timing solutions were made over a two day period using the least squares approach discussed above. The results obtained for the time bias and drift terms are given in Tables 3 and 4 respectively. The average standard error on the timing solutions is also given. Table 5 gives the maximum difference between the true clock time and the time predicted using the multiple satellite solution. This difference is approximately a linear function of radiation modeling error magnitude when the error appears as a bias.

SATELLITE ORBITAL ELEMENTS

Satellite No.	Semi-Major Axis	Eccentricity	Inclination	Mean Anomaly	Argument of Perigee	Right Ascension
1	20270.4 km	0.0°	125°	40°	-26.7°	120°
2	20270.4	0.0°	125°	220°	-26.7°	120°
3	20270.4	0.0°	125°	89°	-13.3°	240°
4	20270.4	0.0°	125°	260°	-13.3°	240°
5	20270.4	0.0°	125°	120°	0.0°	0.0°
6	20270.4	0.0°	125°	300°	0.0°	0.0°

TABLE 1

TRACKING STATION NET

Station	Number	Latitude	Longitude
Panama	1	9.0°	280.0°
Florida	2	25.6°	279.6°
Maryland	3	38.7°	283.5°
Seychelles	4	-4.7°	55.5°
Samoa	5	-14.3°	189.3°

TABLE 2

TIME BIAS SOLUTION
vs.
RADIATION MODEL ERROR

Satellite	True Bias	Radiation Bias 1%	Radiation Bias 5%
1	10.0*	10.0*	10.2*
2	15.0	15.0	14.9
3	-5.0	-4.5	-2.6
4	20.0	20.5	22.5
5	25.0	25.2	26.0
6	-20.0	-19.8	-19.1
Station			
1	5.0	4.9	4.5
2	10.0	9.9	9.3
3	-10.0	-10.2	-11.1
4	15.0	14.6	12.8
5	0.0**	0.0	0.0

Standard Error (1 Sigma) On Time Bias Solution: .07 nanoseconds

* nanoseconds
**station 5 parameters constrained

TABLE 3

TIME DRIFT SOLUTION
vs.
RADIATION MODEL ERROR

Satellite	True Drift	Radiation Bias 1%	Radiation Bias 5%
1	40.0*	40.0*	40.3*
2	25.0	25.2	26.2
3	30.0	29.7	28.4
4	-25.0	-25.2	-26.1
5	30.0	30.0	29.8
6	-20.0	-20.0	-20.4
Station			
1	25.0	25.1	25.6
2	30.0	30.0	29.7
3	35.0	35.0	35.0
4	-20.0	-19.8	-19.0
5	0.0**	0.0	0.0

Standard Error(1 Sigma) On Time Drift Solution: .03 nanosec/day

* nanoseconds/day
**station 5 parameters constrained

TABLE 4

MAXIMUM TIME SOLUTION ERROR
DUE TO RADIATION BIAS

Radiation Error	During Fit Span 1%	5%
Satellite		
1	0.0*	.8*
2	.4	2.3
3	.5	2.4
4	.5	2.5
5	.2	1.0
6	.2	.9
Station		
1	.1	.7
2	.1	1.3
3	.2	1.1
4	.4	2.2
5	0.0	0.0

*nanoseconds

TABLE 5

b. Individual Satellite Processing

Using the synthetic range data generated for the multiple satellite case individual satellite orbit solutions were determined which best fit the data in the least squares sense. Cases were again considered where radiation biases of one and five percent were present. In this processing mode timing errors were solved for as pass bias parameters. The time bias present in the satellite and station clock were combined into a range bias. Time drifts were likewise considered as a range drift. Again unmodelled radiation bias resulted in errors in the range bias solution which were linear with radiation error magnitude. Table 6 gives the average of the pass timing errors due to radiation bias. This value in nanoseconds is equivalent to the error in computed range bias due to radiation error.

c. Comparison of Processing Modes

Since each processing method treats the timing parameters differently a direct comparison of results is difficult. However, a comparison of Table 5 and 6 can reveal information about the size of errors due to radiation bias. For individual satellite processing the average time error for a pass is about 1.2 nanoseconds for 1% radiation bias and 6.6 nanoseconds for 5% bias (Table 6). These error levels are for a pair of clocks. However in multiple processing the maximum error levels for the same radiation biases are smaller. If the worst satellite and station errors in Table 5 are summed the result is smaller than individual processing results.

INDIVIDUAL PROCESSING MODE
AVERAGE TIME ERROR PER PASS

Radiation Bias	1%	5%
Average Pass Time Error (nanoseconds)	1.2	6.6

TABLE 6

Another way to check on the relative accuracy of the two methods is to compare the orbit solutions. Figure 2 graphically illustrates various trajectory differences resulting from a 5% radiation bias. Table 7 lists each case considered.

In cases I and II both radiation bias and timing errors were present in the data. Case III however has no timing errors present. In no case was radiation a parameter of fit.

From Figure 2 it can be seen that with a bias in radiation pressure modeling the multiple satellite processing mode computes an ephemeris which is more consistant with that produced from data with the same radiation error but without timing errors. The coupling between the range bias (timing errors) and the induced error due to radiation bias causes a "poorer" determined orbit with the individual processing mode.

2. <u>An Approximate Radiation Pressure Model</u>

Since radiation pressure modeling is critical for GPS altitude satellites, attempts are being made to model radiation pressure to better than 1%. At the Naval Surface Weapons Center various radiation pressure models are being developed for the Timation III-A satellite. The asymmetry of this satellite prohibits a spherical model with fixed surface to mass ratio to be used in orbit determination. A model incorporating varying exposed surface area and approximating the reflection and absorbtion properties of the satellite is required.

To determine the errors that will be introduced into

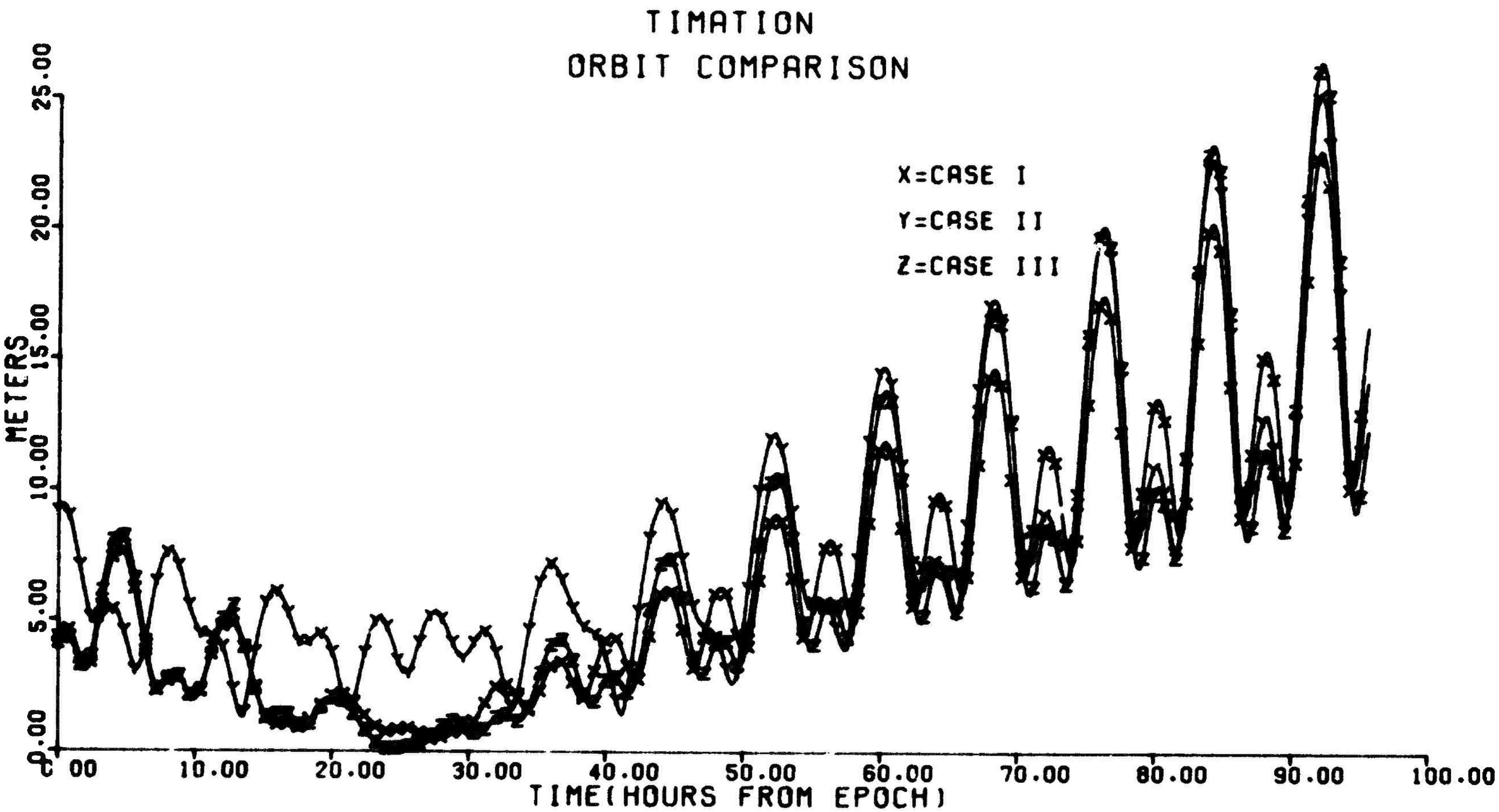
TIMATION
ORBIT COMPARISON
X=CASE I
Y=CASE II
Z=CASE III
METERS
0.00
5.00
10.00
15.00
20.00
25.00
0.00
10.00
20.00
30.00
40.00
50.00
60.00
70.00
80.00
90.00
100.00
TIME(HOURS FROM EPOCH)

Figure 2

ORBIT COMPARISON
(5% Radiation Error)

Case	
I	Combined Solution vs. "True" Trajectory
II	Individual Solution vs. "True" Trajectory
III	Individual Solution (Radiation Bias Only) vs. "True" Trajectory

TABLE 7

timing solutions due to an approximate surface model, an orbit was generated using a rectangular box model with sides of varying surface area, reflectivity and absorbtion. A least squares fit was made using range data with timing errors to determine the satellite's ephemeris and pass timing errors. The radiation model employed in the orbit solution was a sphere. A scaling parameter was determined for this model. The average pass timing error introduced by using the spherical surface model was .6 nanoseconds.

Thus it is likely that a radiation pressure model which approximates the physics of the satellite's orbit in terms of radiation pressure may yield acceptable timing results with individual satellite processing. Incorporating this model into the multiple satellite processing scheme should yield very acceptable results.

B. Station Position Uncertainty

The preceeding results were based on the assumption that station position locations are perfectly known. Increasing the uncertainty in station position directly effects the standard errors on the time bias and drift solutions. Table 8 demonstrates increasing standard errors of the timing solutions as a function of increasing uncertainty in station position for all five stations considered in the multiple satellite case. This correlation between the uncertainty in station position and standard errors on timing solutions necessitates accurate determination of tracking locations for the GPS system.

TIMING SOLUTION UNCERTAINTY
vs.
STATION POSITION UNCERTAINTY

Position Uncertainty (meters)	Bias Uncertainty (nanosec)	Drift Uncertainty (nanosec/day)
0	.07	.03
1	.4	.3
5	1.0	.7

TABLE 8

IV. CONCLUSIONS

This report has examined a least squares method for the solution of orbital and bias parameters for multiple Global Positioning System satellites being observed by a common station net. The method includes the solution for time biasing parameters present in the system. The effects of radiation pressure modeling errors and station position uncertainty on timing solutions has been examined. Results based on the assumption that timing errors at one station are perfectly known indicate the following:

(1) The error in the solution of time bias terms is correlated with radiation pressure modeling errors and appears to be approximately a linear function of its magnitude.

(2) Multiple satellite processing yields a better determination of system timing parameters and satellite orbital parameters than single satellite processing in the presence of unmodelled radiation error. In this case radiation pressure was not a parameter in the solution.

(3) Adequate timing solutions from single satellite processing are achievable if accurate radiation pressure modeling is available.

(4) The uncertainty in station position affects the standard errors on timing solutions. Therefore station position should be known to better than one meter.

REFERENCES

1. Nichols, S. and White, J., "Satellite Applications of a Rubidium Frequency Standard", Presented at the 28th Annual Frequency Control Symposium, May 1974.

2. Swift, E. R., "Analysis of Navigation Error Sources for the Timation DNSS Option", Naval Weapons Laboratory Report TR-3067, November 1973.

APPENDIX A

PASS MATRIX CONCEPT

PASS MATRIX CONCEPT

Data Aggregation

The pass matrix concept as used in orbit determination involves aggregating satellite tracking data on a pass basis. Normal equations are formed from the tracking data of each separate pass over a station. The parameters of fit are the satellite orbital constants and certain bias parameters characteristic to the pass. These bias parameters represent three components of station position, refraction bias, range or frequency bias, and range of frequency drift.

Formation of Pass Normal Equations

After data from a pass has been filtered to eliminate "bad" points, the normal equations are formed:

Let D_{t_i} represent the data taken at time t_i with associated standard error σ_i. The A matrix is then given by

$$A = \begin{bmatrix} \frac{\partial D_{ti}}{\partial P_1} & \cdots\cdots & \frac{\partial D_{ti}}{\partial P_6} & \frac{\partial D_{ti}}{\partial C_D} & \frac{\partial D_{ti}}{\partial BIAS} \\ \vdots & & \vdots & \vdots & \vdots \\ \frac{\partial D_{tn}}{\partial P_1} & \cdots\cdots & \frac{\partial D_{tn}}{\partial P_C} & \frac{\partial D_{tn}}{\partial C_D} & \frac{\partial D_{tn}}{\partial BIAS} \end{bmatrix} \qquad (A-1)$$

where the parameters of fit $P_1, \ldots P_6$, C_D, and bias terms are given in Table A-1. The weight matrix W for the data of a pass is given by

$$W = \begin{bmatrix} \frac{1}{\sigma_1^2} & & 0 \\ & \ddots & \\ 0 & & \frac{1}{\sigma_n^2} \end{bmatrix} \qquad \text{(A-2)}$$

With the A and W matrices defined, the least squares normal equations for a pass are given by:

$$B\overline{\Delta P} = \overline{E} \qquad \text{(A-3)}$$

$$B = A^T W A \qquad \text{(A-4)}$$

$$\overline{E} = A^T W \overline{\delta} \qquad \text{(A-5)}$$

where the n x 1 vector $\overline{\delta}$ contains the observational residuals. The pair $\left[B_j, \overline{E}_j\right]$ are denoted as the pass matrices for the j'th pass.

PARAMETERS OF FIT

NOTATION	PARAMETER
P_1	x or a
P_2	y or e sin ω
P_3	z or e cos ω
P_4	$\dot{x}$ or I
P_5	$\dot{y}$ or L+G
P_6	$\dot{z}$ or Ω
C_D	Drag
Bias Terms	
x,y,z	Station Position
C_R	Refraction Bias
R_B or f_B	Range or Frequency Bias
$\dot{R}_B$ or $\dot{f}_B$	Range or Frequency Drift

TABLE A-1

APPENDIX B

LEAST SQUARES METHOD

LEAST SQUARES METHOD FORMULATION

Range Data

The range data class is given by

$$D(t) = \left| \overline{R}_{sat}(t) - \overline{R}_s(t) \right| + R_B + \dot{R}_B (t - t_o) + \Delta R(1 + C_R) \qquad \text{(B-1)}$$

where the vectors $\overline{R}_{sat}$ and $\overline{R}_s$ are the radius vectors of the satellite and observing station respectively. The terms R_B and $\dot{R}_B$ are respectively the range bias and range drift characteristic to the pass. C_R represents a refraction scaling parameter and ΔR the Hopfield tropospheric refraction correction. The time t_o is the epoch of the fit.

Least Squares Method

Let $[B_i, \overline{E}_i]_{jk}$ denote the pass matrices (Appendix A) formed from the filtered observations from the i'th pass of satellite j over station k. These normal equations

$$B_i(\overline{\Delta p})_{jk} = \overline{E}_i \qquad \text{(B-2)}$$

are formed for the orbital (7) and bias (6) parameters associated with the satellite-station pair jk (Table A-1).

With each timing offset modeled as a bias and linear drift, the range and range drift pass parameters for the i'th pass are written, respectively, as

$$R_B = (t_j + t_k)\, C \qquad \text{(B-3)}$$

and

$$\dot{R}_B = (\dot{t}_j + \dot{t}_k)\, C \qquad \text{(B-4)}$$

where t_j and t_k are the time bias for satellite j and station k and $\dot{t}_j$ and $\dot{t}_k$ are the time drifts. These time bias parameters are not however pass dependent.

With the partial derivatives of the range data D(t) with respect to time bias and time drift:

$$\frac{\partial D(t)}{\partial t_j} = \frac{\partial D(t)}{\partial t_k} = C \qquad \text{(B-5)}$$

$$\frac{\partial D(t)}{\partial \dot{t}_j} = \frac{\partial D(t)}{\partial \dot{t}_k} = C(t-t_o) \qquad \text{(B-6)}$$

the pass matrix pair $\left[B_i, \bar{E}_i\right]_{jk}$ may be expanded with the four time bias parameters replacing the two range bias parameters:

$$\underset{\text{(13 parameters)}}{\left[B_i, \bar{E}_i\right]_{jk}} \longrightarrow \underset{\text{(15 parameters)}}{\left[B_i', \bar{E}_i'\right]_{jk}}. \qquad \text{(B-7)}$$

For each pass i the elements of the pass matrices $\left[B_i', \bar{E}_i'\right]_{jk}$ are re-arranged so that pass dependent parameters (station bias and refraction correction) may be formally eliminated:

$$\left[B_i', \bar{E}_i'\right]_{jk} \longrightarrow \left[\begin{array}{c:cc} B'_{oo} & B'_{ob} & \bar{E}'_o \\ \hdashline B'_{ob} & B'_{bb} & \bar{E}'_b \end{array}\right]_{jk} \qquad \text{(B-8)}$$

where the subscript "b" refers to pass dependent parameters and the subscript "o" refers to all other parameters (orbit, drag and timing).

The equations for the elimination of pass i dependent parameters are

$$B'_{oo} \; (\overline{\Delta P}_o)_{jk} + B'_{ob} \; (\overline{\Delta P}_b)_{jk} = \bar{E}'_o \qquad \text{(B-9a)}$$

$$B'_{bo} \; (\overline{\Delta P}_o)_{jk} + B'_{bb} \; (\overline{\Delta P}_b)_{jk} = \bar{E}'_b. \qquad \text{(B-9b)}$$

Solving equation (B-9b) for $(\Delta P_b)_{jk}$ and substituting equation (B-9a) we obtain

$$(B'_{oo} - B'_{ob} B'^{-1}_{bb} B'_{bo}) \; (\overline{\Delta P}_o)_{jk} = \bar{E}'_o - B'_{ob} B'^{-1}_{bb} \bar{E}'_b \qquad \text{(B-10)}$$

or

$$B''_i \; (\overline{\Delta P}_o)_{jk} = \bar{E}''_i \qquad \text{(B-11)}$$

For each pair jk we now have the eliminated pass matrices $[B''_i, \bar{E}''_i]_{jk}$ for the parameter set $(\overline{\Delta P}_o)_{jk}$. The union of all parameter sets, $(\overline{\Delta P}_o)_{jk}$, consists of 6j orbital parameters, j drags, and 2(j + k) timing parameters. By keeping track of those parameters in each matrix pair $[B''_i, \bar{E}''_i]_{jk}$, the elements from all eliminated pass matrices may now be combined to form the normal equations for all orbital and timing parameters.