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NUCLEAR-POWERED FLIGHT

By

An Informal Committee

of

THE APPLIED PHYSICS LABORATORY

of

THE JOHNS HOPKINS UNIVERSITY

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FIRST PROGRESS REPORT

2015 1

14 January 1947

TO: L. R. Hafstad

FROM: A. E. Ruark

SUBJECT: Transmittal of Progress Report entitled "Nuclear-Powered Flight", by an Informal Committee of the Applied Physics Laboratory of the Johns Hopkins University.

In accordance with your verbal instructions of about 9 June 1946, the Committee has considered the general problem of air vehicles driven by nuclear power. Three copies of the subject report are respectfully submitted herewith. A first draft was submitted October 25, 1946. Since that time many errors have been corrected and much new material has been added. The initial distribution is indicated in the report.

Your comments and those of other interested persons will be appreciated by the Committee. Review by suitable members of APL is hereby requested.

It is believed that any further work on this subject at APL should be carried on by a small staff with fresh instructions, and that the existing large committee should be discharged in the near future.

FOR THE COMMITTEE

Arthur E. Ruark, Chairman;
Technical Supervisor
for Research Laboratory.

AER:rh

Encl. 3 -- Copies 1, 2, and 3 of subject report.

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CHAPTER IV. PRELIMINARY REPORT ON NUCLEAR ENERGY FOR ROCKET PROPULSION

by

F. T. McClure and R. B. Kershner

1. Introduction

In the comparison of fuels for use in rocket propulsion, probably the most significant parameter is the so-called effective gas velocity or, equivalently, the specific impulse. This quantity is defined by the ratio of the thrust to the mass rate of discharge of propulsive gas and is mainly a function of the thermodynamic properties of the gas. A convenient formula for the specific impulse, I, of a gas, is

$$(1) \qquad I \left[\text{lb(force)-sec/lb(mass)} \right] = 9.302 \; I_r \sqrt{nT_c}$$

where n = inverse of the molecular weight of the gas (combustion products), T_c = chamber gas temperature in degrees Kelvin, and I_r, the reduced specific impulse, is a function only of the ratio of specific heats of the gas, the ratio of chamber pressure to atmospheric pressure, and the area expansion ratio of the rocket nozzle. The function I_r is graphed over a range of all three variables in ABL-SR-10 (OSRD No. 5548), "The Reduced Specific Impulse of Ideal Gases", Nancy Marmer and F. T. McClure. Numerically it varies between 1.6 and 2.6 for the usual range of the variables.

Other things being equal a rocket loaded with propellant of high specific impulse has a greater range than a corresponding rocket with a low specific impulse propellant. A major portion of the effort of rocket development work has been aimed at obtaining fuels with a high specific impulse. Fuels in common use now have an impulse of 180 to 250 lb.sec/lb.

Equation (1) shows that a high specific impulse requires high gas temperature and low molecular weight. Since available construction materials seem to place an upper limit on the temperature not much above temperatures obtained with current fuels (3000 to 3500°K, i.e. 5400 - 6300°R) it appears that significant improvements can be attained only by the use of fuels with a lower molecular weight.*

The importance of low molecular weight is well recognized and accordingly considerable attention has been paid to the use of hydrogen as a rocket fuel. The problem is, then, to find a means for heating hydrogen to a temperature in the neighborhood of 3000°K in as economical a manner as possible. The most obvious means for accomplishing this is to burn a portion of the hydrogen, for example with oxygen, to supply the necessary heat. With the use of the hydrogen-oxygen combination an optimum specific impulse may be expected with approximately a 5-to-1 mole ratio. (See "Fuel Systems for Jet Propulsion" by A. W. Lemmon, Jr., Report of the Gilliland Committee, and "Calculated Performance of Hydrogen and Oxygen as Jet Motor Propellant", Aerojet Engineering Corporation, Technical Memorandum No. RTM-23.) With 5-to-1 mole ratio and an operating pressure of 50 atmospheres, an impulse of 395 lb.sec/lb at sea level is predicted. With this ratio of hydrogen to oxygen a temperature of 2760°K is obtained but the mean molecular weight is about 8.4 due to the formation of a considerable amount of water vapor in

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*The upper allowable limit on the gas temperature might be raised still further by improved cooling of the walls by film methods or the like. If, however, the gas is to be heated by heat transfer from a wall (by black radiation or conduction) its temperature cannot be raised above that of that wall, so that cooling does not solve the problem.

the reaction products. With a greater amount of oxygen the temperature is higher but the impulse is lower due to the overbalancing effect of the increase in molecular weight. Conversely, with less oxygen the molecular weight is lower but the decrease in temperature is sufficient to reduce the specific impulse. The combination of hydrogen and oxygen in a mole ration of 5-to-1 gives a higher specific impulse than is predicted for any other fuel so far investigated.

Clearly, a means for heating hydrogen to a high temperature without increasing the molecular weight would give a very significant increase in the specific impulse. In fact, hydrogen alone at a temperature of 2500° Kelvin and a pressure ratio of 50 would give a sea level impulse of about 730 lb.sec/lb. It must be borne in mind, however, that the mechanism for heating the hydrogen constitutes a dead weight in the rocket which somewhat reduces the effectiveness of the gain in specific impulse. In particular, if the weight of the energy source required to produce a certain thrust was greater than the thrust produced, the resulting rocket would not rise in spite of the high specific impulse. The problem is to produce an energy source with very high power per unit weight. Recent developments in nuclear energy reactors suggest consideration of these devices as a promising means for heating hydrogen for rocket propulsion purposes.

In this report a simple quantitative discussion of the advantages of a rocket operated by hydrogen heated by a nuclear energy reactor will be given. For comparison purposes a hydrogen-oxygen rocket will be used as the prototype of "conventional" rockets. While a hydrogen-oxygen propulsion system has not yet been successfully used, it seems amply clear that the problems of development cannot be more difficult than those to be expected in the nuclear energy case. In particular the difficult problem of handling liquid hydrogen is common to both.

One feature of rocket design which reduces slightly the advantage of a low molecular weight fuel is the fact that such a fuel is likely to have low density and thus require a disproportionately large tank and structure weight for its storage in the rocket. Neglect of this point is likely to give a misleading impression of the relative advantages of different fuels. For example, in the case of a bi-fuel rocket with a large discrepancy between the densities of the two fuel components, the optimum ratio of the two fuel components is not the ratio which gives the maximum specific impulse. As mentioned before, the optimum specific impulse with the hydrogen-oxygen rocket is expected with approximately 5-to-1 mole ratio of hydrogen to oxygen. This implies a substantially larger volume of hydrogen than oxygen and, correspondingly, a disproportionately large weight of the hydrogen tanks. While shifting to lower hydrogen-oxygen ratios decreases the specific impulse and thus increases the fuel weight required, it might also overcompensate by decreasing the total fuel volume and hence decreasing the required tank and structure weight. The optimum ratio is that which gives the minimum sum of fuel weight and tank and structure weight.

Determination of this optimum requires a knowledge of the required velocity and an exact relation between tank and structure weight and fuel volume. The last relation is not well established but recent estimates of the Douglas Aircraft Corporation and the Glenn L. Martin Company ("Proposal for structural study of high altitude test vehicle", Glen L. Martin Company, Engineering Report 2373, May, 1946, and "Consideration of a high altitude space vehicle (Hall project)" Report ES-20515, El Segundo Engineering Department, Douglas Aircraft Company, March 28, 1946.) for the design of a satellite rocket have indicated that a tank and structure weight as low as one pound per cubic foot might be obtainable by the use of recent aircraft engineering design. Assuming this value, a rough analysis indicates that the two influences of changing the weight ratio of hydrogen to oxygen almost exactly compensates when fuel ratios are varied from 5-to-1 down to 3-to-1. Fuel ratios in this range lead to almost the same payload-range relation, at least for ranges up to satellite. For an escape rocket which with a single stage hydrogen-oxygen rocket is on the borderline of feasibility, the small effect of varying the hydrogen-oxygen balance may become very significant.

2. Requirements for Long Range Rockets

In this section we compare the design requirements of a hydrogen-oxygen rocket and a hydrogen-nuclear energy rocket to obtain various ranges. The ranges considered are 1000, 5000, and 10,000 miles. This last range is very nearly equivalent to a satellite rocket. In addition, an "escape" rocket is included. In calculating the velocity necessary for attaining these ranges air drag was neglected and effectively instantaneous burning was assumed. As a result the rockets described would not actually attain the ranges given but comparison should still be essentially valid. However, it should be noted that the assumption of sea level impulse throughout burning will make an error which will at least partially compensate those mentioned above.

Table 1 gives, then, the required initial velocities for a drag free shell with the prescribed range. These are obtained from the formula

(2) $$V = 36,670 \sqrt{\frac{\sin \Theta}{1 + \sin \Theta}}$$

where V is the velocity in ft./sec, 36,670 is the escape velocity in ft/sec, and Θ is 1/2 the range in radians.

Table 1

Range (miles)	1000	5000	10,000	escape
Velocity (H/sec)	12,300	22,300	25,600	36,700

The fuel required for a rocket to attain a given velocity is calculated from the well known rocket formula

(3) $$V = 32.16 \ I \ \log_e \left(\frac{\text{weight with fuel}}{\text{weight without fuel}} \right)$$

Values of the specific impulse, I, for a hydrogen-oxygen rocket with a mole ratio of 5-to-1 and for rockets propelled by hydrogen heated (by a nuclear reactor) to 2500°K, 2060°K and 1630°K respectively, are given in Table 2. The operating pressure was taken as 50 atmospheres in all cases.

Table 2

Code Number	A	B	C	D
Fuel	$5H_2$-to-$1O_2$	H_2 + N.E.	H_2 + N.E.	H_2 + N.E.
Gas Temperature (°K)	2760	2500	2060	1630
Specific Impulse (lb.sec/lb)	395	730	665	590

Table 3 gives the percent fuel, the percent tanks and supporting structure, and the remaining percent, β, for rockets of the four types A, B, C, D to attain the velocities given in Table 1. The percent fuel is calculated from (3) and the percent tanks and structure are obtained from the assumption of one pound of tank and structure weight per cubic foot of fuel.

The remainder, β, is the percent of weight available for rocket motor and nozzle, pumps, control, payload and (except in case A) nuclear reactor.

Table 3

Weight distribution of various long-range rockets

Velocity	Code Number	% H_2	% O_2	% Tanks & Structure	β
12300	A	15	47	4	34
	B	41	---	9	50
	C	44	---	10	46
	D	48	---	11	41
22300	A	20	63	5	12
	B	61	---	14	25
	C	65	---	15	20
	D	69	---	16	15
25600	A	21	66	6	7
	B	66	---	15	19
	C	70	---	16	14
	D	74	---	17	9
36700	A	23	71	6	0
	B	79	---	18	3
	C	82	---	19	(impossible)
	D	86	---	19	(impossible)

It will be noticed that the value of β for the nuclear energy rockets is almost always greater than for the "conventional" rocket. The difference between the value of β for cases B, C, D and the value of β for case A represents the weight percentage available for the nuclear reactor, if the nuclear rocket is just to compete with the "conventional" prototype.

3. Energy Considerations

In this section we give a preliminary survey of the energy requirements for a nuclear heated hydrogen rocket. We consider case B in which the hydrogen is heated to 2500°K.

To vaporize one gram of hydrogen at its boiling point and heat the resulting gas to 2500°K, at constant pressure, requires approximately 9400 gm-cals. (see, for example, NDRC Report A-116, "Thermodynamic Properties of Propellant

Gases", J. O. Hirschfelder, F. T. McClure, C. F. Curtiss, D. W. Osborne). Thus the energy required, E, is given by

(4) $E = 3.93 \times 10^{11}$ ergs/gm.

From Table 3 it is seen that the total weight of a rocket using hydrogen-nuclear energy must be at least 1/.16 = 6.25 times the weight of the reactor if the rocket is to out-perform a conventional rocket even at 1000 miles range. Thus, if W is the weight of the nuclear reactor the rocket weight is greater than 6.25W. Allowing an initial thrust of 2 g (over 1 g is required to rise at all) the thrust, F, must exceed 12.5 W. Since thrust equals the product of the specific impulse, I, and the mass rate of discharge, $\dot{m}$, we have

(5) $730 \cdot \dot{m} = 12.5\ W$

where $\dot{m}$ is in grams/sec if W is in grams.

From (4) the mass rate, $\dot{m}$, requires an energy rate of

$$\dot{m}\,E = 3.93\ \dot{m} \times 10^{11} \text{ ergs/sec.}$$

Hence, from (5),

(6) $\frac{\dot{m}\,E}{W} = 6.74 \times 10^{9}$ ergs/sec-gm.

Thus a power output of .674 K.W. per gram of reactor is required. This is equivalent to 305 KW, or 410 horsepower, per pound of reactor.

It is obvious that the power output of nuclear reactions can greatly exceed the above requirement. The problem is to develop a means for transferring the energy produced into the hydrogen gas in the form of heat.

4. The Heat Exchange Problem.

Let the nuclear energy reactor be of the nuclear fission-chain variety (as opposed to radioactive material). Assume it has a uniform cross-section of arbitrary shape and is characterized by the following parameter:

Cross section area of reactor matter: A_r

Length: L

Total effective heat transfer surface: S

Temperature of surface: T

Density: ρ_r.

Then the weight of the reactor is

(7) $W = \rho_r A_r L$

Suppose that the heat transfer mechanism operates through the surface of the reactor and is proportional to S (conduction or radiation). Let j be a suitable average rate of energy transfer per unit surface, so that j S is the rate at which energy is made available to the gas. Then, replacing $\dot{n} E$ in (6) by j S and using (7)

$$j\,S = 6.74 \times 10^9 \rho_r A_r L$$

or

(8) $$j \left(\frac{S/L}{A_r}\right) = 6.74 \times 10^9 \rho_r$$

Now $\left(\frac{S/L}{A_r}\right)$ is a geometrical factor giving the ratio of the perimeter to the area of the reactor cross section. For example, if the reactor consists of a bundle of rods of <u>radius</u> X,

$$\frac{S/L}{A_r} = \frac{2}{X}$$

If the reactor consists of concentric annular cylinders of <u>thickness</u> X then, again,

$$\frac{(S/L)}{A_r} = \frac{2}{X}$$

Thus, for the moment, we rewrite (8) as

(9) $j/X = 3.37 \times 10^9 \rho_r$ ..

It is difficult to decide what density of reactor would be required but we might consider reactors with moderators of BeO or C (graphite), both of which have high melting points. Their densities are about 3 and 2.2 respectively. Actually it seems quite unlikely that BeO would stand up in an atmosphere of hot, high pressure hydrogen. Thermodynamically, graphite also can react with hydrogen but kinetically this heterogeneous reaction may not occur in significant amount during the required operation time. Let us choose, then, the density of graphite for our example (the weight of fissionable and other material is neglected). Then

(10) $j/X = 7.4 \times 10^9$

For large values of X it is easy to show that conduction cannot provide as much heat transfer as is available through radiation if the wall temperature is of the order of 3000°K. Thus we consider first the possibilities of radiative heat transfer. Assuming 100% emissivity of the surface at 3000°K and 100% absorbtion in the gas (neglecting the weight of smoke material necessary to produce high absorbtivity) we are able to obtain a maximum value of X. For, under the above assumptions,

$$j = \sigma(3000)^4$$

where σ is the Stefan-Boltzmann constant ($\sigma = 5.673 \times 10^{-5}$ ergs cm^{-2} $(^\circ K)^{-4}$ sec^{-1}). Then, from (10)

$$X = .62 \text{ cm.}$$

It is seen that radiation will only supply the required energy if X is at most 0.62 cm. The small value of X reflects the requirement of a large ratio of surface to volume of reactor. Considerations of the reactor design imply, however, that the dimensions of the gas spaces within a reactor must be reduced along with the dimensions of the reactor spaces, otherwise the overall density of the reactor would be reduced and the reactor would fall below critical. Thus a high surface to volume for the reactor also implies a high surface to volume for the gas spaces within the reactor. Hence the gas passages are very thin and the absorption of radiation by the gas cannot be expected to approach the 100% assumed above. Allowance for the conceivable absorption attainable, even by the inclusion of smoke in the hydrogen, makes it appear that the possibility of the operation of a nuclear energy rocket depending on radiation for the heat transfer is remote.

On the other hand, with a sufficiently high ratio of surface to volume in the gas passages, heat transfer by conduction exceeds heat transfer by radiation even at 3000°K. In the next sections, therefore, the problem of heat transfer by conduction, in reactors with gas passages with a high surface to volume ratio, is considered in more detail.

5. Heat Exchange by Conduction.

We consider a reactor in the form of a solid cylinder with a number of cylindrical gas passages (pipes) drilled through it lengthwise and arranged in a hexagonal lattice. Then the reactor may be considered as built up from hexagonal cylinders each containing one gas passage (see Fig. 1).

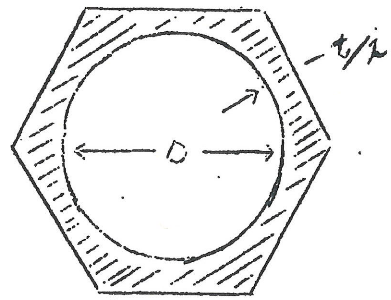

Fig. 1.

Let $A_g = \pi D^2/4$ be the cross-section area of a gas passage and A_r the area of the hexagonal annulus of reactor associated with a single gas passage. Thus A_r is the shaded area in Fig. 1. As usual, let L be the length of the reactor.

We consider the heat transferred by conduction in a single gas passage. We assume a wall temperature of T_w °K and determine the conditions under which the gas will be heated to T°K (from the boiling point) by passage through the pipe. Actually we require that the gas be heated from absolute zero to T°K; the slight additional heating from 0°K to the boiling point largely compensates our neglect of the heat of vaporization of hydrogen.

Under these conditions the equation of energy may be written

$$(11) \qquad \frac{d}{dx}\left[\int_0^T c_p\, dT + \tfrac{1}{2}v^2\right] = \frac{h(T_w - T)}{A_g \rho v}\,\frac{S}{L}$$

where $S = \pi DL$ is the heat transfer surface, h is the heat transfer coefficient, T_w is the wall temperature, and ρ is the gas density and v its velocity so that $\rho A v = \dot{m}$ is constant. For the heat transfer coefficient, h, we use the equation

(12) $$\frac{hD}{k} = 0.023\left(\frac{D\rho v}{\mu}\right)^{.8}\left(\frac{c_p \mu}{k}\right)^{.4}$$

where k is the thermal conductivity of the gas and μ its viscosity so that $\frac{c_p \mu}{k}$ is Prandtl's number (see McAdams, "Heat Transmission", McGraw-Hill (1942), p.168. For hydrogen $c_p \mu / k = .76$ so that (12) becomes

(13) $$h = .027\, c_p\, (\rho v)^{.8}\, \mu^{.2} / D^{.2}$$

Then equation (11) becomes, neglecting the $\frac{1}{2} v^2$ term on the left (which neglect will be justified later),

(14) $$\frac{dT}{dx} = \frac{.027\, (\rho v)^{.8} \mu^{.2}\, S(T_w - T)}{D^{.2}\, A_g \rho v\, L}$$

Note that c_p cancels out in this equation. This fact is important because hydrogen has an exceptionally large specific heat (due to its low molecular weight). As a consequence hydrogen is exceptionally hard to heat by radiation. However, for heating by conduction, the heat transfer coefficient h is also high, in proportion to c_p, so that, for a given mass flow, hydrogen is no harder to heat by conduction than any other gas.

In (14) the viscosity must be considered as function of temperature, T. Over a very wide range of temperature the experimental viscosity data for hydrogen may be represented, with excellent accuracy, by

(15) $$\mu = 1.7 \times 10^{-6}\, T^{.695} \text{ poise.}$$

(See Chapman and Cowling. "The Mathematical Theory of Non-uniform Gases", Cambridge (1939), p.223).

Substituting (15) into (14) and integrating gives

$$Q = \frac{1.9 \times 10^{-3} S}{D^{.2} A_g (\rho v)^{.2}}$$

where Q is defined by

(16) $$Q = \int_o^T \frac{dT}{T^{.139} (T_w - T)} .$$

Hence, using $S/A_g = 4\ L/D$,

(17) $$L = 131\ Q\ D^{1.2} (\rho v)^{.2}$$

This equation gives the length required, for any diameter and rate of flow, in order to heat the gas to T^oK at the exit by heat transfer from a wall at $T_w{}^oK$.

6. The Feasibility of a Nuclear Rocket.

First we consider the requirement that the thrust be sufficient to give a net acceleration of one g in vertical launching (thrust of two g). Let W be the weight of the reactor and ϕW the weight of the entire rocket. Then, since the thrust is $I\ \dot{m}$, we have

(18) $$I\ \dot{m} = 2\phi W = 2\phi A_r\ L \rho_r$$

But $\dot{m} = A_g \rho v$ and $\rho_r = 2.2$ (for graphite) so that (18) may be written

(19) $$\frac{A_g}{A_r} = (4.4/I)\ L\ \phi/(\rho v)$$

The parameter A_g/A_r is of interest in connection with the design of the reactor since it is related to the average "density" of the reactor.

For any choice of T_w, T (and hence I), equations (17) and (19) are the only requirements on the five variables ϕ, L, ρv, A_g/A_r and D in order that a rocket designed as indicated should operate with an initial total acceleration of 2 g (1 g net acceleration in vertical firing). Hence, for any particular choice of T_W and T, a three parameter family of rockets is theoretically possible. However it remains to be determined whether any of these rockets corresponds to realizable values of the parameters. In addition considerations of the nuclear requirements impose some restrictions on the dimensions of the reactor.

To test the feasibility of a nuclear energy rocket we must first choose definite temperatures T and T_W. Choosing $T_W = 3000^oK$ and $T = 2500^oK$ (as in type B) the value of Q is found (by numerical evaluation of the integral in (16)) to be

$$Q = 0.67.$$

Also, for $T = 2500^oK$ we have

$$I = 730$$

from Table 2.

Let us consider the problem of a rocket of type B to carry a payload of 10 tons (payload includes controls, etc.) to a range of 5000 miles. It is seen from Table 3 that for this rocket to be as good as a hydrogen-oxygen rocket we must have $\phi > 1/.13 = 7.7$. Further the payload is 12% of the weight so that the rocket weight is 83.3 tons. For the weight of the reactor, we get W = 10.8 tons. A reasonable assumption on the area ratio might be $A_g/A_r = 1$. Further let us assume that the overall diameter of the reactor is equal to its length. Then, from the weight and density,

$$L = 225 \text{ cm} = 7.4 \text{ ft}.$$

Using these values, we obtain, from (17) and (19)

$$\rho v = 10.5 \text{ gm/cm}^2\text{sec}$$

$$D = 1.49 \text{ cm.}$$

For the thickness, t, between pipes (see Fig. 1) in order to have $A_g/A_r = 1$ we have

$$t = .52 \text{ cm.}$$

Clearly these values are all of reasonable order of magnitude. The roughly 7 ft. diameter of reactor is approximately the body diameter that would be required for an 83 ton rocket in any case. The value $\rho v =$ 10.5 corresponds to a velocity, at the exit of the heating channel of only about 750 ft/sec, of the order expected in ordinary rocket channels. Incidentally the term $\frac{1}{2}v^2$ which was previously neglected in the energy equation is

$$\tfrac{1}{2}v^2 = 2.53 \times 10^8 \ \frac{\text{ergs}}{\text{gm}}$$

to be compared with 3.93×10^{11} (see eqn. 4) for the energy calculated by neglecting this term. Clearly the neglect of $\frac{1}{2}v^2$ was justified.

The question remains as to whether the size of reactor indicated above is reasonable from a nuclear physics standpoint. Referring to Table 2 of Chapter 2, we find that for the optimum shape of a solid reactor the reactor radius, r_c, is 56 cm and the reactor length, h_c, is 103 cm (if $V = 2$). But we have considered a perforated reactor with $A_g/A_r = 1$, corresponding to Γ (see Chapter 2) equal to 0.5. Therefore, the critical diameter of this perforated pile would be

$$\frac{2\,r_c}{1-\Gamma} = 224 \text{ cm.}$$

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and the critical length would be

$$\frac{h_c}{1-\Gamma} = 206 \text{ cm.}$$

Hence, the weight allowed for reactor in the discussion above is apparently ample since the resulting reactor dimensions are somewhat larger than are needed to obtain critical size.

The weight of U_{235} required to operate a reactor of these dimensions can also be obtained from Table 2 of Chapter 2. Assuming $V = 2$ and $T_W = 3000^\circ K$ we get

$$m(1-\Gamma)^2 \approx 9 \text{ Kg}$$

or, for $\Gamma = 0.5$,

$$m \approx 36 \text{ Kg} \approx 80 \text{ lb.}$$

For convenience we summarize the characteristics of the example we have considered,

Weight of fuel (H_2)	50.8 tons
Weight of tanks and structure	11.7 tons
Weight of reactor	10.8 tons
Weight of payload, controls, etc.	10.0 tons
Total weight of rocket	83.3 tons
Range	5000 miles
Weight of U_{235} in reactor	80 lbs.

The above figures represent a rather rough single calculation. However, the results certainly indicate that the possibility of a hydrogen-nuclear energy rocket is sufficiently reasonable to warrant more careful analysis.

7. More Systematic Design.

In the preceding section it was shown that the dimensions of a reactor necessary to operate a rocket are reasonable. In this section the data of Chapter 2 are used, in combination with equations (17) and (19), to design families of rockets showing the effect of varying, ϕ, ρv, and L.

It has been shown, in section 5, that the requirement that hydrogen should be heated from its boiling point to T^oK by passage through a reactor with a wall temperature of $T_w{}^oK$ leads to the following equation:

(17) $$L = 131Q\, D^{1.2}(\rho v)^{.2}$$

It was further shown that the requirement that hydrogen at T^oK should provide thrust sufficient for an initial acceleration of 2 g (in free space) leads to

(19) $$\frac{\Gamma}{1-\Gamma} = \frac{4.4\, L\, \phi}{I(\rho v)}$$

where Γ is the ratio of gas cross-section to total cross-section, so that $\Gamma/(1-\Gamma) = A_g/A_r$, and $\phi =$ rocket weight/reactor weight.

In Chapter 2 it was shown that

(20) $$L = h_c/(1-\Gamma),$$

where h_c is the length of an unperforated reactor of critical dimensions; Values of h_c for U_{235} (graphite moderated) reactors of various shapes and with optimum uranium concentration are given in Chapter 2, Table 2.

Using equation (20), the equations (17) and (19) may be written as follows:

(21) $$D^{1.2} = \frac{h_c}{131Q(1-\Gamma)(\rho v)^{.2}},$$

(22) $\Gamma = (4.4/I) h_c \phi / (\rho v)$

Hence if h_c, ϕ, and ρv are given it is possible to find Γ from equation (22) and then D from equation (21).

In Chapter 2 it was shown that, for a fixed concentration of U_{235}, there is an optimum shape of reactor leading to a minimum mass of U_{235} and hence (since the concentration is fixed) leading to a minimum total reactor mass. This optimum shape occurs for $\mu = .54$, where μ = reactor radius/reactor length. Assuming that the reactor is always designed with this optimum shape it is possible to estimate all the remaining rocket design parameters quite straightforwardly. For example the total reactor weight is given by

(23) $$W = \frac{2.2 \pi h_c^3 \mu^2}{(1 - \Gamma)^2} ,$$

where 2.2 is the density of the reactor material (graphite).

In order to demonstrate the influence of the three parameters h_c, ϕ, and ρv on the final rocket, tables giving the major design values have been prepared corresponding to two values of h_c, two values of ϕ and five values of ρv, for $T_w = 3000^\circ K$ and $T_g = 2500^\circ K$.

The values of h_c chosen were 103 and 88. These were the values of h_c given in Chapter 2, Table 2, for a critical reactor of optimum concentration and optimum shape corresponding to the assumed values 2.0 and 2.4 for V, the average number of neutrons per fission. Thus the effect of variation of h_c displayed by the tables of this chapter may be considered as indicating the advantage of obtaining a large number of neutrons per fission. However it should be emphasized that any method of reducing h_c

would lead to the same results. In particular the use of a concentration of U_{235} above optium would certainly lead to a reduction of h_c (although an increase in the mass of U_{235}).

The values of ϕ chosen were $\phi = 7.7$ and $\phi = 33$. The first of these values, $\phi = 7.7 = 1/.13$, is, as seen from Table 3, the value required for a nuclear energy rocket of type B to compete on exactly even terms with a conventional hydrogen-oxygen rocket at a range of 5000 miles. The other value $\phi = 33 = 1/.03$ is that required for an "escape" rocket with no payload.

The values of ρv chosen were 10, 20, 30, 40, 50. It is very difficult to decide how large a value of ρv may be used. The value $\rho v = 50$ does not lead to an excessive loss of energy but the hydrodynamic pressure drop corresponding to this ρv is approaching 100 lb/in.^{2}. There would be an additional pressure drop due to friction. These effects would interfere with the thrusts in a manner that has not been included in our calculations. Furthermore the pressure drop would impose severe strains on a perforated gfaphite structure at 3000°K. It seems likely that $\rho v = 50$ is as large a value as can be realistic and a careful appraisal of the friction effects may indicate that this value is already unreasonably high.

It should be noticed that the use of equations (20) and (23) is based on the assumption that the reactors are to be just critical in size. It is obvious that two or more such reactors could be operated simultaneously to power a correspondingly larger rocket. In this sense the rockets whose characteristics are given in the following tables are the smallest operable nuclear energy rockets corresponding to the given values of h_c, ϕ, ρv.

Examination of Tables 4 through 7 shows that the rocket characteristics are markedly sensitive to the values of all three parameters. For example, with $\rho v = 10$ a rocket with $\phi = 7.7$ is not excessively heavy but $\phi = 33$ is impossible for any size rocket, at least with $h_c > 88$.

The effect of ρv is, perhaps, more surprising. For small values of ρv an increase in ρv makes possible a great reduction of total rocket weight; at large values of ρv a change in ρv has almost no effect on overall weight. For example, for $\phi = 7.7$ and $h_c = 103$ the rocket weight is reduced from 70 tons to 33 tons by increasing ρv from 10 to 20. Further increase of ρv to as much as 50 only gives a reduction to 23 tons.

The advantage of a low value of h_c is made clear by these tables. For example, if $h_c = 103$ ($V = 2.0$), th minimum rocket weight for $\phi = 7.7$, and $\rho v = 10$ is 70 tons. This weight is reduced to 42 tons if $h_c = 88$($V = 2.4$). It was mentioned above that h_c can be reduced by the use of a concentration of U_{235} above the critical. In view of the great effect of a reduction of h_c it seems likely that U_{235} concentration above the "optimum" would be advantageous for rocket reactors.

Table 4. $\phi = 7.7$, $h_c = 103$, $T_w = 3000^{\circ}K$, $T = 2500^{\circ}K$

ρv	10	20	30	40	50
Γ	.478	.239	.159	.120	.096
D(cm)	1.34	.87	.75	.69	.65
t(cm)	.51	.83	1.04	1.21	1.35
Length of reactor (cm)	197	135	123	117	114
Diameter of reactor (cm)	215	147	133	127	124
Weight of rocket (tons)	70	33	27	24	23
Weight of reactor (tons)	9.0	4.3	3.5	3.2	3.0
Weight of fuel (tons)	42	20	16	15	14
Payload (tons)	8.4	3.9	3.2	2.9	2.8
Burning time (sec)	223	223	223	223	223
Weight of U_{235} (K_g), assuming $V = 2.0$.	33.2	15.6	12.8	11.7	11.1

Table 5. $\phi = 7.7$, $h_c = 88$, $T_w = 3000^{\circ}K$, $T = 2500^{\circ}K$

ρv	10	20	30	40	50
Γ	.409	.204	.136	.102	.082
D(cm)	1.06	.74	.64	.59	.56
t(cm)	.52	.82	1.02	1.18	1.31
Length of reactor (cm)	149	111	102	98	96
Diameter of reactor (cm)	162	121	111	107	105
Weight of rocket (tons)	34	19	16	15	14
Weight of reactor (tons)	4.4	2.4	2.1	1.9	1.8
Weight of fuel (tons)	21	11	10	9	9
Payload (tons)	4.1	2.3	1.9	1.8	1.7
Burning time (sec)	223	223	223	223	223
Weight of U_{235} (K_g), assuming $V = 2.4$	15.2	8.4	7.1	6.6	6.3

Table 6. $\phi = 33$, $h_c = 103$, $T_w = 3000^oK$, $T = 2500^oK$

(Escape rocket with zero payload)

ρv	10	20	30	40	50
Γ	over 1	over 1	.683	.512	.410
D(cm)	--	--	1.69	1.13	.93
t(cm)	--	--	.26	.37	.45
Length of reactor (cm)	--	--	325	211	175
Diameter of reactor (cm)	--	--	354	230	190
Weight of rocket (tons)	--	--	809	342	233
Weight of reactor (tons)	--	--	24.5	10.4	7.1
Weight of fuel (tons)	--	--	639	270	184
Burning time (sec)	--	--	288	288	288
Weight of U_{235} (Kg), assuming V = 2.0	--	--	90	38	26

Table 7. $\phi = 33$, $h_c = 88$, $T_w = 3000^oK$, $T = 2500^oK$

(Escape rocket with zero payload)

ρv	10	20	30	40	50
Γ	over 1	.876	.584	.438	.350
D(cm)	--	4.91	1.18	.88	.71
t(cm)	--	.09	.29	.39	.43
Length of reactor (cm)	--	707	211	157	126
Diameter of reactor (cm)	--	772	231	171	137
Weight of rocket (tons)	--	3300	294	161	104
Weight of reactor (tons)	--	100	8.9	4.9	3.2
Weight of fuel (tons)	--	2600	232	127	82
Burning time (sec)	--	288	288	288	288
Weight of U_{235} (Kg), assuming V = 2.4.	--	344	31	17	11

The wall temperature of $3000^{\circ}K$, used in the preceding examples, is admittedly a stringent requirement. For this reason it is of interest to examine the effect of operating with lower wall temperatures. Accordingly, calculations have been performed giving the rocket dimensions for 5000 mile rockets with wall temperatures of $2500^{\circ}K$ and $2000^{\circ}K$. In order to make the comparison under conditions where the heat transfer problem is unchanged the gas temperatures are chosen as $2060^{\circ}K$ and $1630^{\circ}K$, respectively, since these values result in the same value of Q ($Q = .67$) as is given by $T = 2500^{\circ}K$, $T_w = 3000^{\circ}K$.

The gas temperatures of $2060^{\circ}K$ and $1630^{\circ}K$ were those considered in rockets C and D in Tables 2 and 3. From Table 3 it is found that, for a 5000 mile rocket competing with a conventional (Type A) rocket, the proper values of ϕ are 12.5, and 33 for $T = 2060^{\circ}K$ and $T = 1630^{\circ}K$, respectively. These changed values of ϕ reflect the reduction in specific impulse.

The results of the calculations on rockets of type C and D are summarized in Tables 8 and 9. Comparison of Tables 4, 8, 9 indicates how very important a high operating temperature is. It is emphasized that the changed values of ϕ correspond to a <u>fixed</u> range requirement (5000 miles). Note that, for example, with $\rho v = 30$ the weight of the rocket is 27 tons, 60 tons, and 3400 tons with gas temperatures of $2500^{\circ}K$, $2060^{\circ}K$ and $1630^{\circ}K$ respectively. For $\rho v = 10$ or $\rho v = 20$ the 5000 mile rocket is impossible with the lowest of these gas temperatures, and even at $\rho v = 50$ (which may be impractically high) the rocket weight becomes 340 tons at this gas temperature.

Reference to Table 3 shows that an escape rocket cannot be made with a gas temperature as low as $2000^{\circ}K$, for any value of ρv.

Table 8. $\phi = 12.5$, $h_c = 103$, $T_W = 2500^{\circ}K$, $T = 2060^{\circ}K$

ρ^v	10	20	30	40	50
Γ	.851	.426	.284	.213	.170
D(cm)	3.81	1.10	.86	.76	.70
t(cm)	.12	.51	.68	.80	.91
Length of reactor (cm)	691	179	144	131	124
Diameter of reactor (cm)	752	195	156	142	135
Weight of rocket (tons)	1380	93	60	50	45
Weight of reactor (tons)	111	7.5	4.8	4.0	3.6
Weight of fuel (tons)	900	61	39	32	29
Payload (tons)	166	11.2	7.2	6.0	5.4
Burning time (sec)	216	216	216	216	216
Weight of U_{235} (K_g), assuming $\bar{V} = 2.0$.	365	25	16	13	12

Table 9. $\phi = 33$, $h_c = 103$, $T_W = 2000^{\circ}K$, $T = 1630^{\circ}K$.

ρ^v	10	20	30	40	50
Γ	over 1	over 1	.845	.634	.507
D(cm)	--	--	3.08	1.43	1.08
t(cm)	--	--	.11	.28	.36
Length of reactor (cm)	--	--	665	281	209
Diameter of reactor (cm)	--	--	724	306	227
Weight of rocket (tons)	--	--	3400	606	334
Weight of reactor (tons)	--	--	103	18.4	10.1
Weight of fuel (tons)	--	--	2300	418	231
Payload (tons)	--	--	407	73	40
Burning time (sec)	--	--	204	204	204
Weight of U_{235} (K_g), assuming $\bar{V} = 2.4$	--	--	295	53	29

Tables 8 and 9 showed the effect of changing both the gas and wall temperatures in such a way as to maintain the heat transfer integral, Q, at a fixed value. It is of interest to examine the effect of changing the value of Q while maintaining a fixed specific impulse. This may be done by varying the wall temperature, T_w, while leaving the gas temperature, T, unchanged.

Since the gas temperature and hence, I, is unchanged the value of ϕ appropriate for a given range is also unchanged. Further, it is seen from Chapter 2, Table 2, that the critical dimension h_c, and shape parameter μ are essentially independent of the reactor temperature. Thus, from equation (22), Γ is unchanged and so the major weights and dimensions are independent of the wall temperature for a fixed gas temperature. The only entries in previous tables which are changed by a change in T_w alone, are the pipe diameter, D, wall thickness, t, and weight of U_{235}.

Thus, if the gas temperature is chosen the wall temperature must be determined so that the resulting values of D and t are feasible structurally. Table 10 shows the values of D, t, and U_{235} weight for rockets with a gas temperature of 2060°K, $\phi = 12.5$ (5000 mile range), and $h_c = 103$, and with wall temperatures of 2300°K, 2500°K and 2700°K. The other weights and dimensions would be essentially identical with those in Table 8. It is seen that as the wall temperature approaches the gas temperature the size of pipes must be decreased and the number correspondingly increased to preserve the same total free area. This increase in the number of pipes causes a decrease in the wall thickness between pipes. Thus wall temperature too near the gas temperature leads to a reactor shape which is completely impractical structurally.

Higher wall temperature leads to geometrically stronger reactor designs, however this improvement must be balanced against the adverse effect of the increased temperature on the material strength.

Table 10. Gas Temperature 2060°K . $\phi = 12.5$, $h_c = 103$

T_w	ρ_v	D(cm)	t(cm)	wt. U_{235} (K_g)
2300	10	3.07	.10	348
	20	.89	.41	23
	30	.69	.54	115
	40	.61	.65	12
	50	.56	.73	11
2500	10	3.81	.12	365
	20	1.10	.51	25
	30	.86	.68	16
	40	.76	.80	13
	50	.70	.91	12
2700	10	4.43	.14	382
	20	1.28	.59	26
	30	1.00	.78	17
	40	.88	.93	14
	50	.81	1.06	12

It should be remarked that, in Tables 8, 9, and 10, the weight of U_{235} was based on a linear interpolation between the values of $m(1 - \Gamma)^2$ given, in Table 2 of Chapter 2, for reactor temperatures of 300°K and 3000°K. For this reason these values may not be properly consistent with the values for reactors at 3000°K.

8. Designs to Carry Fixed Payload

The tables of the preceding section give an indication of the relative importance of the various free parameters on the dimensions of the minimum size nuclear energy rockets comparable with a hydrogen-oxygen conventional rocket. However, direct comparison of the various cases is obscured by the fact that the payload in these tables was not constant. It is of interest, then, to examine the problem of designing a nuclear rocket to carry a given payload for a given range.

To approach this problem, let β, as in Table 3, be the percent of rocket weight available for payload and reactor. Thus, if the payload is denoted by P,

(24) $$100(W + P) = \beta \phi W.$$

Now eliminating Γ between equations (22) and (23) gives

(25) $$W = \frac{2.2 \pi h_c^3 \mu^2}{\left[1 - (4.4/I) h_c \phi/(\rho v)\right]^2}$$

Substituting this expression for W into (24) gives an equation for $\beta(\rho v)$ in terms of $\phi/(\rho v)$, with μ, h_c, P, I as parameters. Thus for fixed μ, h_c, P, I a curve representing $\beta(\rho v)$ as a function of $\phi/(\rho v)$ can be drawn. Then given β and ρv, ϕ is immediately determined. Then Γ is given by equation (22) and the remaining parameters are determined as heretofore.

To give a definite illustration of this method, let us consider a rocket with a reactor of "optimum" concentration and optimum shape, and assume $V = 2.0$. Then, from Chapter 2, Table 3, $\mu = .542$, $h_c = 103$. (Recall that this is an unfavorable case for a nuclear energy rocket; a smaller value of h_c is certainly attainable). Further let $T = 2500°K$ so that $I = 730$. Finally, let

P = payload in tons.

Then elimination of W between (24) and (25) leads to

(26) $$\beta(\rho v) = \left[100 + 41.2\, P\left(1 - .621 \frac{\phi}{\rho v}\right)^2\right] / \left(\frac{\phi}{\rho v}\right)$$

Values of $\beta(\rho v)$ less than 62.1 are not admissible. This leads to the interesting fact that in order to make an escape rocket ($\beta = 3$) it is necessary to use a value of ρv greater than $62.1/3 = 20.7$. This is in agreement with Table 6 where it was shown that $\rho v = 20$ did not lead to an escape rocket even with zero payload. Fig. 2 gives the graph of relation (26) for various assumed payloads.

The method of using Fig. 2 for designing specific nuclear energy rockets will now be described. Let us first consider a rocket with a 10 ton payload, a 5000 mile range, and $\rho v = 10$. From Table 3 we have $\beta = 25$. Thus

$$\beta(\rho v) = 25 \times 10 = 250$$

From Fig. 2 we read, corresponding to $P = 10$, $\beta(\rho v) = 250$

$$\phi / \rho v = 0.81$$

Then from equation (22)

$$\Gamma = (4.4/730) \times 103 \times 0.81 = 0.503$$

Then, from equation (25), the reactor weight is

$$W = 9.8 \text{ tons}$$

Finally the weight of the entire rocket is given by

$$W = 8.1 \times 9.8 = 79. \text{ tons.}$$

The rocket dimensions obtained in this way agree fairly closely with those given in Table 4 under $\rho v = 10$ since the payload for that rocket was fortuitously fairly near 10 tons.

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The following tables were prepared by the methods described in the preceding paragraph. Since the payload is the same for all the cases presented the effect of the parameters ρv and β (range) are displayed a little more clearly than in Tables 4 through 7.

Table 11. Effect of ρv on rocket design.

(10 ton payload, 5000 mile rocket, $\beta = 25$)

ρv (gm/cm^2-sec)	β	$\beta\rho v$	$\phi/\rho v$	ϕ	Γ	Reactor Wt. W(tons)	Rocket Wt. ϕW(tons)
10	25	250	.81	8.1	.503	9.8	79
40	25	1000	.35	14.0	.217	4.0	56

Table 12. Effect of range on rocket design.

(10 ton payload, $\rho v = 40$)

Range (miles)	β	$\beta\rho v$	$\phi/\rho v$	ϕ	Γ	Reactor Wt. W(tons)	Rocket Wt. ϕW(tons)
5000	25	1000	.35	14.0	.217	4.0	56
10000	19	760	.42	16.8	.261	4.4	75
Escape	3	120	1.12	44.8	.696	26.2	1170

Table 13. Effect of payload on rocket design.

($\rho v = 40$, 5000 mile rocket, $\beta = 25$)

P	$\beta\rho v$	$\phi/\rho v$	ϕ	Γ	Reactor Wt. W(tons)	Rocket Wt. ϕW(tons)
0	1000	.10	4.0	.062	2.8	11
5	1000	.25	10.0	.155	3.4	34
10	1000	.35	14.0	.217	4.0	56
20	1000	.49	19.6	.304	5.0	99

From Table 11 it is seen that increasing ρv from 10 to 40 makes it possible to reduce the reactor weight nearly 60% and still carry a 10 ton payload 5000 miles. A striking fact observable in Table 12 is that 10000 miles is very little more difficult, from the nuclear standpoint, than 5000 miles. In particular the reactor need be only 10% heavier. This is particularly interesting in view of the fact that a 10000 mile rocket is very nearly a satellite. Of course, it must be recalled that these conclusions are based on the correspondence between β and range given by Table 3 and hence depend on the assumption that the tank and structure weight can be held to one pound per cubic foot of fuel. Also air drag was neglected and the trajectories were calculated on the assumption that the theoretical velocity is instantaneously acquired. Table 13 shows that an increase in payload causes an almost proportional increase in total rocket weight but a very small increase in reactor weight.

9. Conclusions

The calculations made in this chapter have indicated that a rocket propelled by hydrogen heated in pipes pierced in a U_{235} graphite reactor is feasible, in principle. Enough calculations have been given to indicate some of the important parameters and their rough effect on final dimensions. In general it is indicated that such a rocket must be operated at very high reactor temperatures (in the neighborhood of $3000^{\circ}K$) in order to have any appreciable advantage over foreseeable conventional fuels.

While the feasibility in principle has been indicated, many detailed engineering considerations have been neglected. These considerations certainly will alter the overall dimensions considerably, and modify the effect of various parameters. For example, it has been indicated that the total dimensions of rocket to perform a given duty decrease with an increase in the mass rate of gas flow (ρv), at first markedly and then slowly. However, the strain on the reactor structure will increase steadily with increased ρv and probably will necessitate additional supporting structure which may remove the apparent advantage of operating at high ρv.

In general the whole question of supporting a graphite reactor mechanically under the high temperature and high flow rate conditions envisaged in this discussion have been given little consideration and no weight allowance has been made for any supporting structure (except as part of the payload). A careful engineering study of this problem, including the effects of pressure drop, skin friction, thermal stresses, etc, (which would require more basic data than seems now available) might result in a major revision of rocket dimensions.

It should be emphasized again that the weight allowances for "payload" were assumed to include all necessary pumps, control equipment and shielding. This must be remembered in interpreting the tables for "constant payload"; for example, a higher value of ρv may require an increased pump weight. In general it was assumed that shielding weight would be low in the unmanned missile considered. However, some protection probably will be required for control circuits and to prevent excessive boiling off of fuel by the absorption of radiation energy.

It has been assumed that both physical and chemical erosion of the reactor structure by hot hydrogen will be unimportant in the short operation time required. This problem requires considerable attention before it is safe to draw final conclusions on the feasibility of the proposed type of reactor structure.

Another design assumption which requires experimental verification is the extrapolation of the engineering heat transfer relation (Eq. 12).

Throughout, the specific impulse has been taken as the theoretical value corresponding to the temperature at the end of the reactor and no allowance was made for heat loss or departure from ideal flow in the nozzle. These effects may lower the specific impulse by 5% to 10%.

In addition to the engineering factors, mentioned above, which were neglected in our analysis, there are further sources of error entering into the nuclear reactor calculations of Chapter 2. In particular we mention again the use of 2.2 as the density of graphite. Commercial graphites generally have much lower densities. We repeat the fact that the relevant atomic data (cross-sections, etc.) were obtained from unclassified sources and are not as complete as could be desired.

With all these sources of error in mind we feel that the weights and dimensions given in this discussion are better considered as fair orders of magnitude rather than as accurate design figures.

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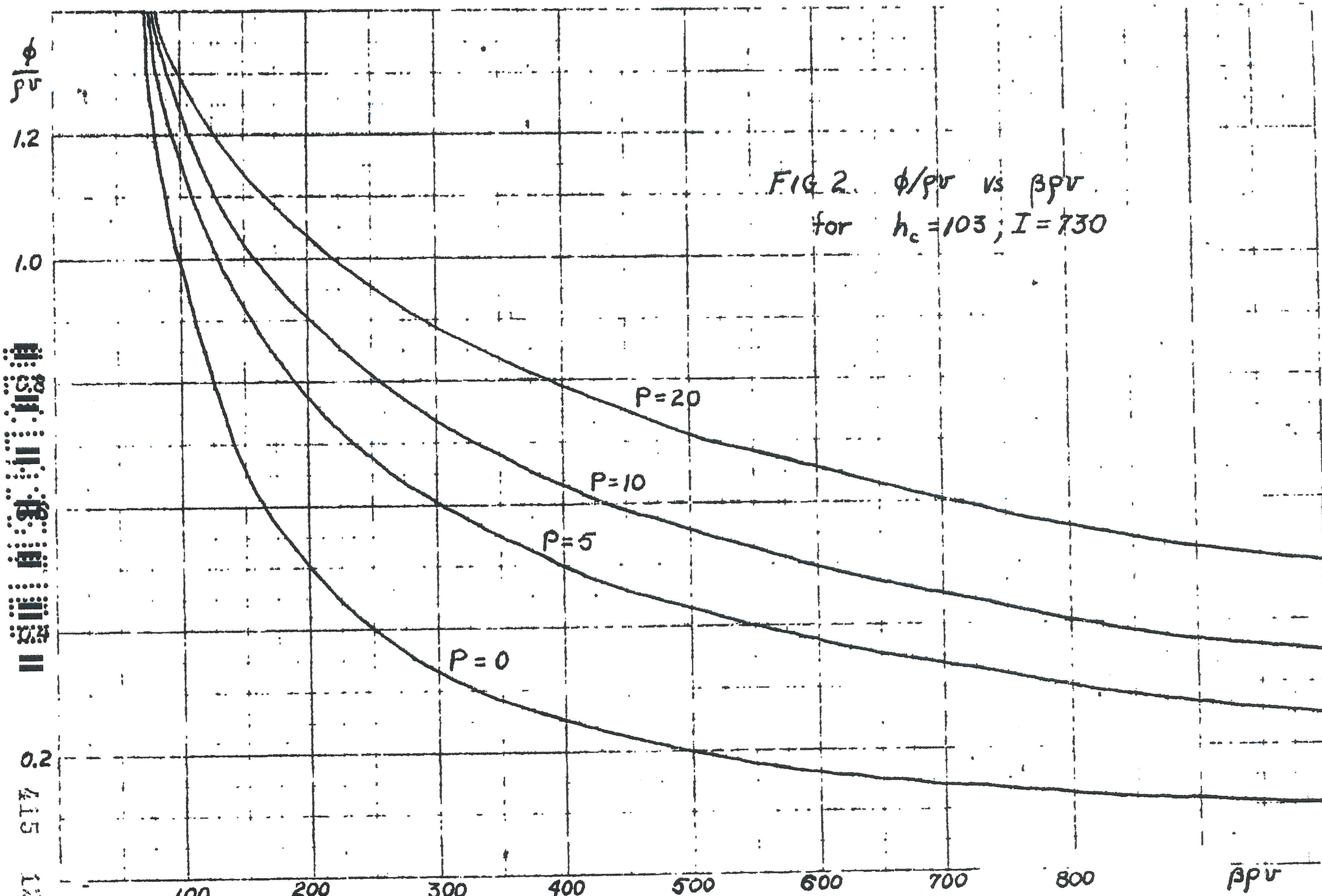

FIG 2. $\phi/\rho v$ vs $\beta\rho v$ for $h_c = 103$; $I = 730$

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