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Carrier Tracking of Phase Modulated Signals with a Phase-Lock Loop

by Michael L Don

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14. ABSTRACT

Oscillator discrepancies and Doppler shifts can cause a mismatch between transmitter and receiver carrier frequencies in wireless communications. A phase-lock loop is typically employed to recover the carrier and remove any frequency mismatch. This technical note demonstrates the use of a Costas loop for phase recovery of a quadrature phase-shift keying signal, paying special attention to characterization of the frequency offset. Full simulation source code is provided.

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phase-lock loop, Doppler, phase modulation, Costas loop, frequency measurement

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1. Introduction

In wireless communications, radio signals are typically modulated to encode information and up-converted to a desired carrier frequency for transmission. During reception, the signal must be correctly down-converted from the carrier frequency for demodulation. This process is complicated by differences between the transmitter and receiver oscillators, as well as Doppler shifts, which cause a mismatch between the up-conversion and down-conversion frequencies. Typically, a phase-lock loop is employed to accurately recover the carrier frequency and correct this mismatch.[1]

This technical note describes and simulates a simple phase-lock loop for carrier recovery of a quadrature phase-shift keying (QPSK) signal. Unlike many applications where only the resulting corrected signal is of interest, attention is paid to the calculation of the frequency offset for Doppler shift or oscillator mismatch characterization. Simulation code is included in the Appendix.

2. QPSK Signal Model

To demonstrate carrier recovery, a QPSK signal with random data was simulated with a symbol rate of 10 kHz. Figure 1, on the left, shows the typical QPSK constellation diagram of the symbols, each point representing the four possible QPSK phases. These symbols were up-sampled by a factor of 8 using a raised cosine filter, giving the constellation diagram on the right of Fig. 1. Down-conversion carrier mismatch is modeled by shifting the frequency by 1 kHz and channel noise is modeled by a complex Gaussian random variable $CN(0,0.02)$. Figure 2 illustrates the signal in the time domain. The top plot shows the symbols drawn from the set $\{0,1,2,3\}$, the middle plot shows the up-sampled in-phase, quadrature (IQ) data, and the bottom plot shows the shifted signal with noise. Figure 3 shows the up-sampled signal with the shifted signal in the frequency domain. The 1-kHz shift in frequency is clearly visible.

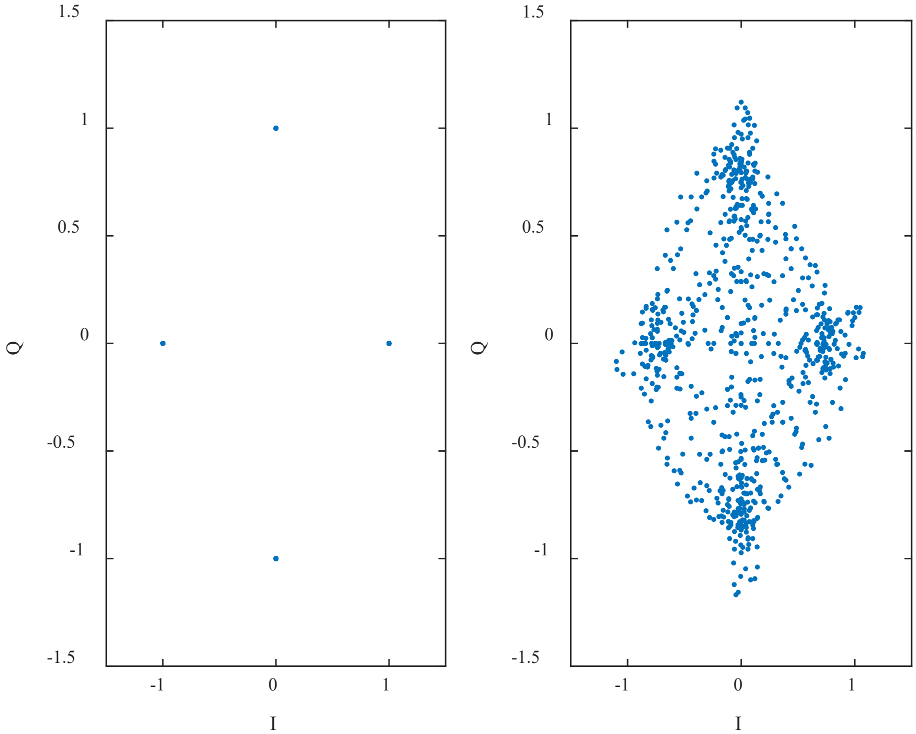

Fig. 1 Constellation diagram of symbols (left) and up-sampled signal (right)

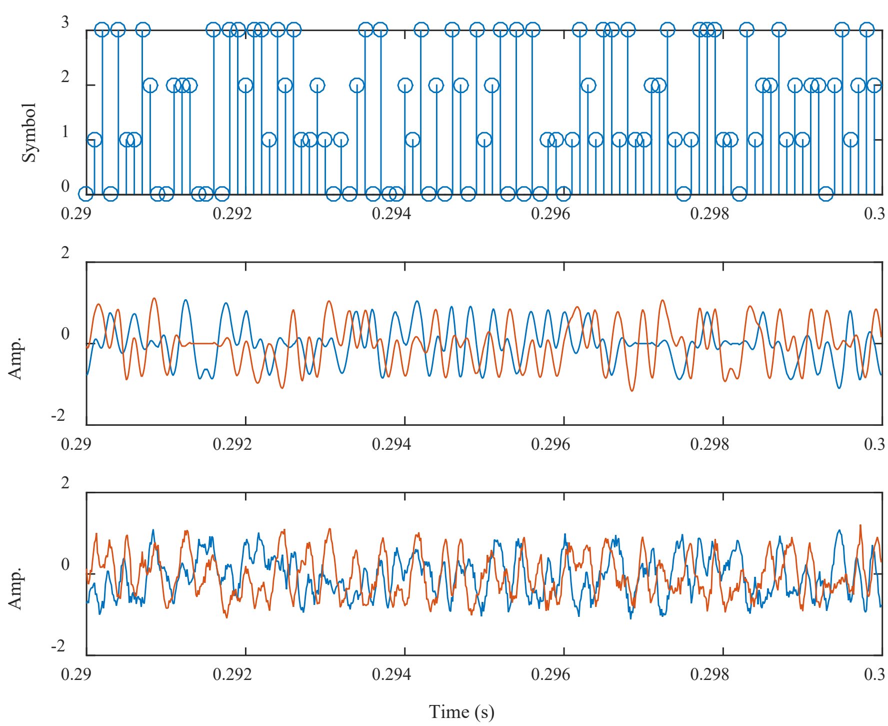

Fig. 2 Symbols (top), IQ data of up-sampled signal (middle), and IQ data of shifted signal with added noise (bottom)

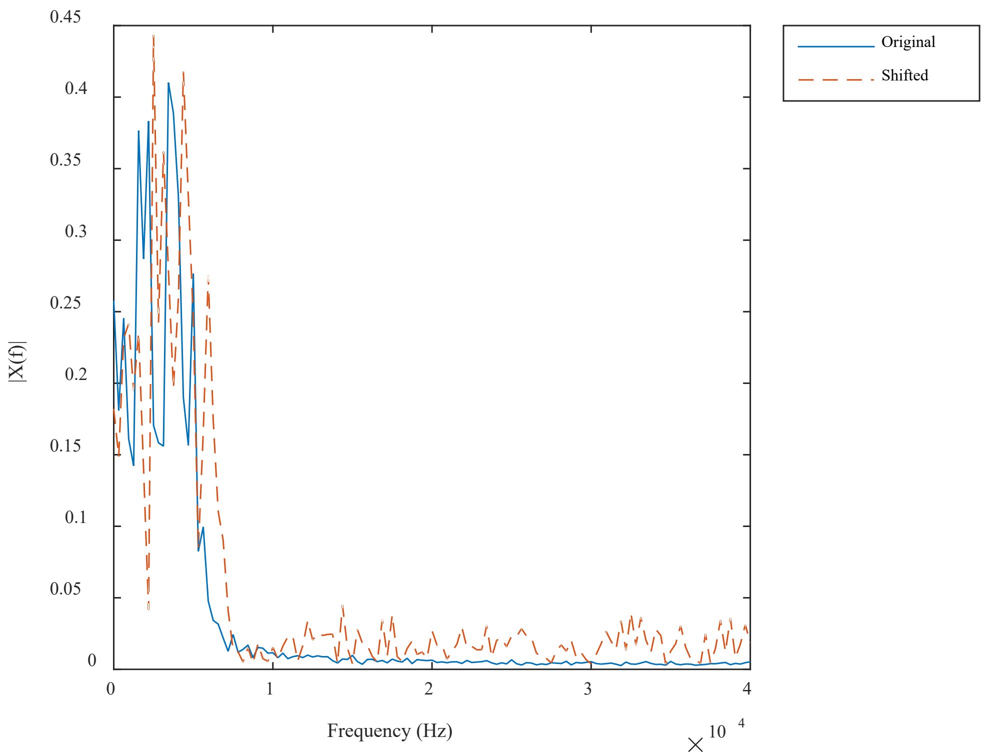

Fig. 3 **Spectrum of original up-sampled signal and the shifted signal**

3. Carrier Recovery

Carrier recovery is implemented here using a Costas loop.[2] For each sample, the recovered signal y_k is calculated by correcting the shifted signal x_k by the current estimated phase $\hat{\phi}_k$

$$y_k = x_k e^{-i\hat{\phi}_k} \,. \tag{1}$$

For the fourth-order signal QPSK, the phase offset estimate is calculated as

$$g_k = \frac{\Im(y_k^4)}{|y_k^4|} \,. \tag{2}$$

The estimated angular frequency update is then

$$\hat{\omega}_{k+1} = \hat{\omega}_k + \beta g_k. \tag{3}$$

The estimated phase update is calculated as

$$\hat{\phi}_{k+1} = \hat{\phi}_k + \alpha g_k + \hat{\omega}_{k+1}. \tag{4}$$

Typically, $\beta = \sqrt{\alpha}$, with $\alpha = 0.015$ for this simulation.

Figure 4 shows the constellation of the shifted signal on the left and the recovered signal on the right. The frequency offset has the effect of rotating the constellation, leaving no visible trace of the original symbols in the plot on the left. Once this offset has been removed through carrier recovery, the symbol grouping can be distinguished in the plot on the right. Figure 5 shows the recovery result in the frequency domain, showing that the spectrum of the recovered signal now matches that of the original signal. The frequency in hertz can be calculated from the angular frequency ω as $\omega F_s/(2\pi)$, where F_s is the sampling frequency. Given the original data rate of 10 kHz and the factor of 8 up-sampling, the sampling frequency is 80 kHz. Using this calculation, the estimated frequency offset history of the Costas loop is plotted in hertz in Fig. 6, showing convergence to the correct value. Figure 7 shows a constellation diagram of the down-sampled signal using a matched filter and Fig. 8 compares the original data to the recovered data. The few outlying points are caused by filtering end effects and can be ignored.

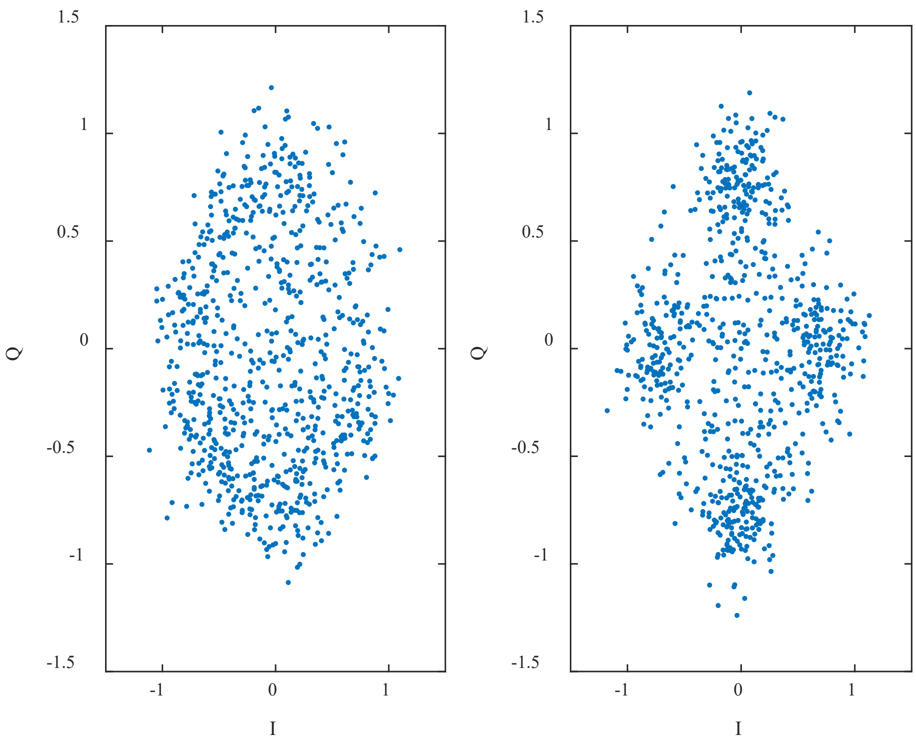

Fig. 4 Constellation diagram of the shifted signal (left) and the recovered signal (right)

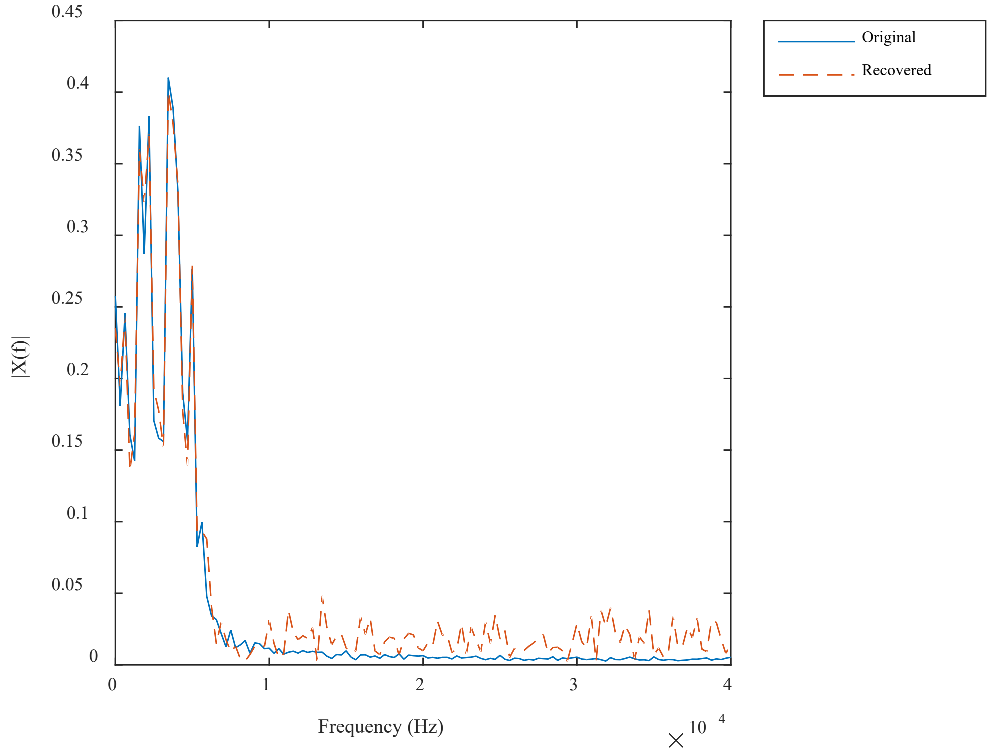

Fig. 5 Spectrum of original up-sampled signal and the recovered signal

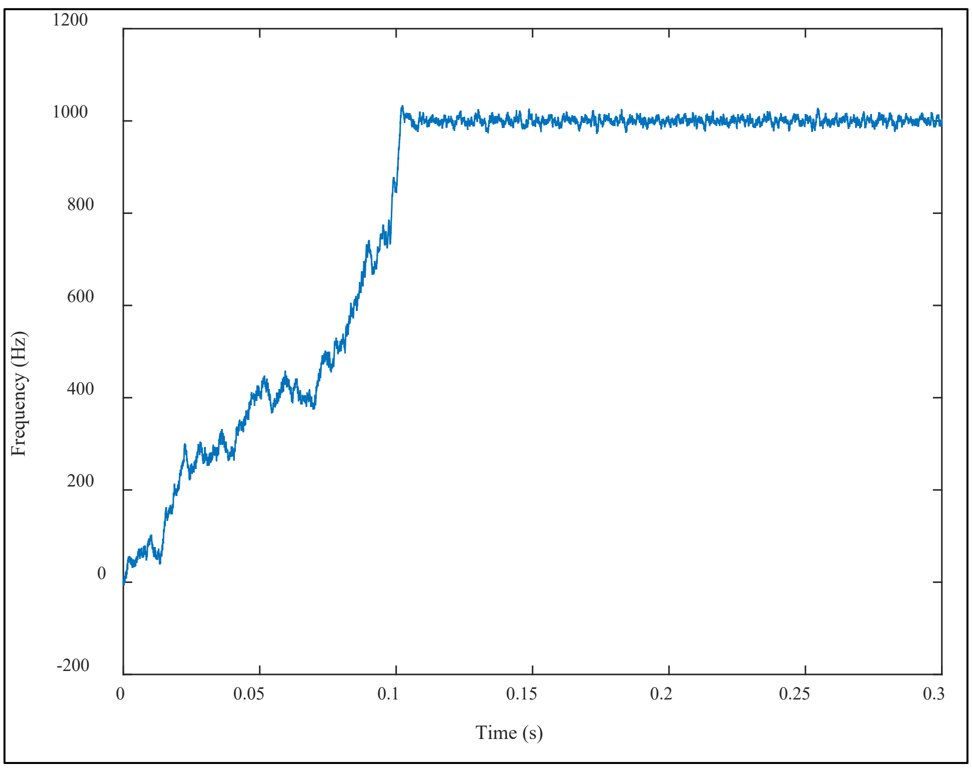

Fig. 6 History of the estimated frequency offset

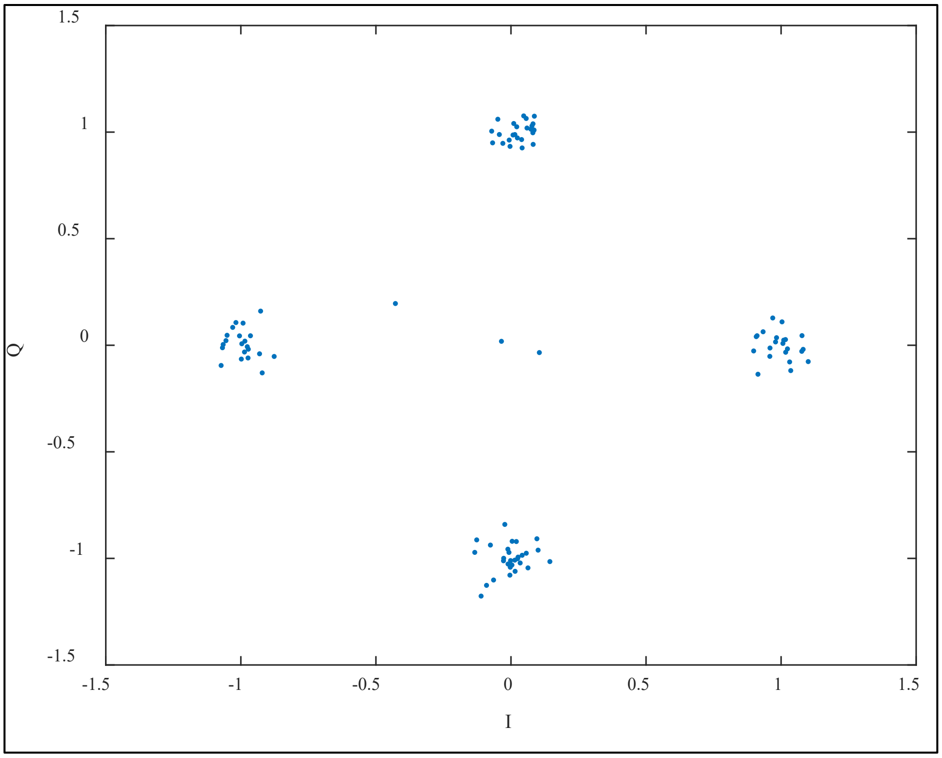

Fig. 7 Recovered constellation diagram

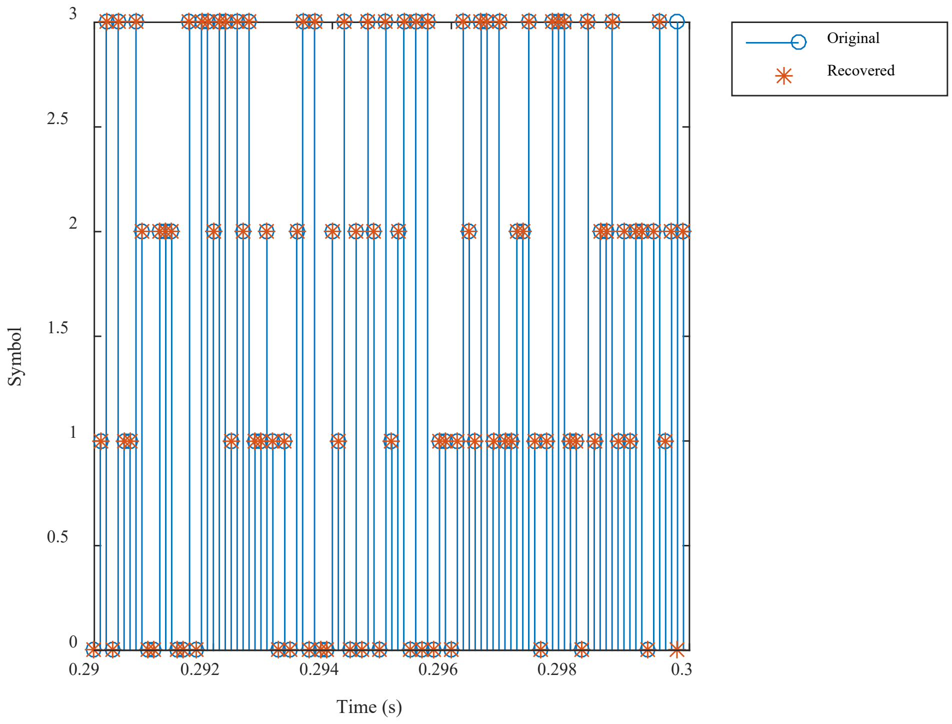

Fig. 8 Original and recovered symbols

Figure 9 shows the Costas loop's convergence to the frequency offset for three values of α. Larger values of α lead to faster convergence as shown in the top plot, but also have greater variance as shown in the bottom plot. The Costas loop can measure and correct time-varying frequency offsets as well as the constant offsets shown in the previous examples. Figure 10 shows the results of tracking a signal with the frequency offset varying between 1 and 2 kHz using $\alpha = 0.02$.

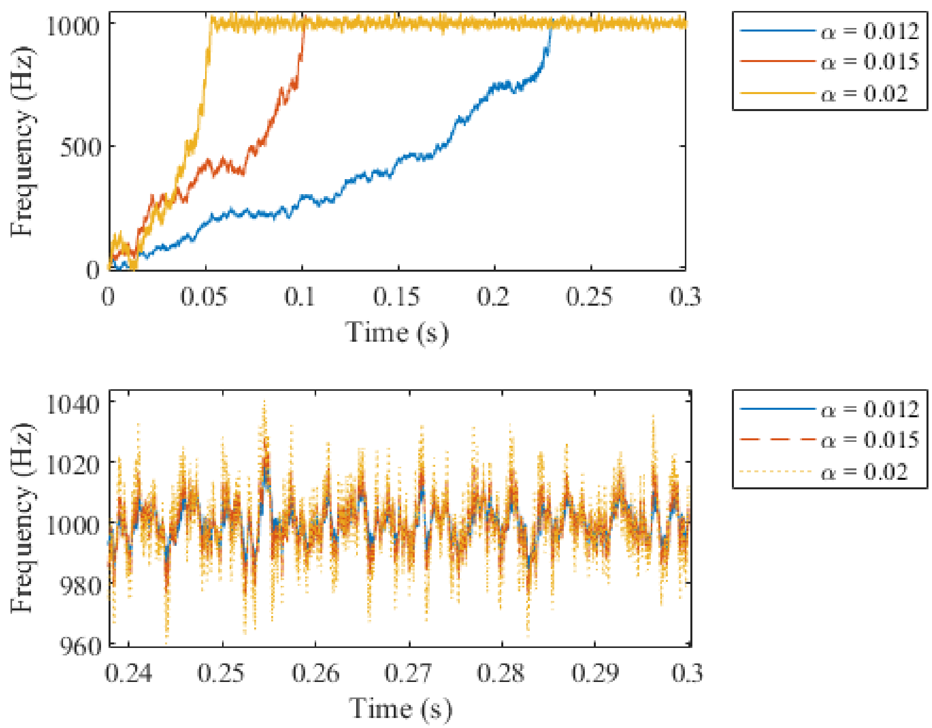

Fig. 9 Comparison of Costas loop convergence for three values of α

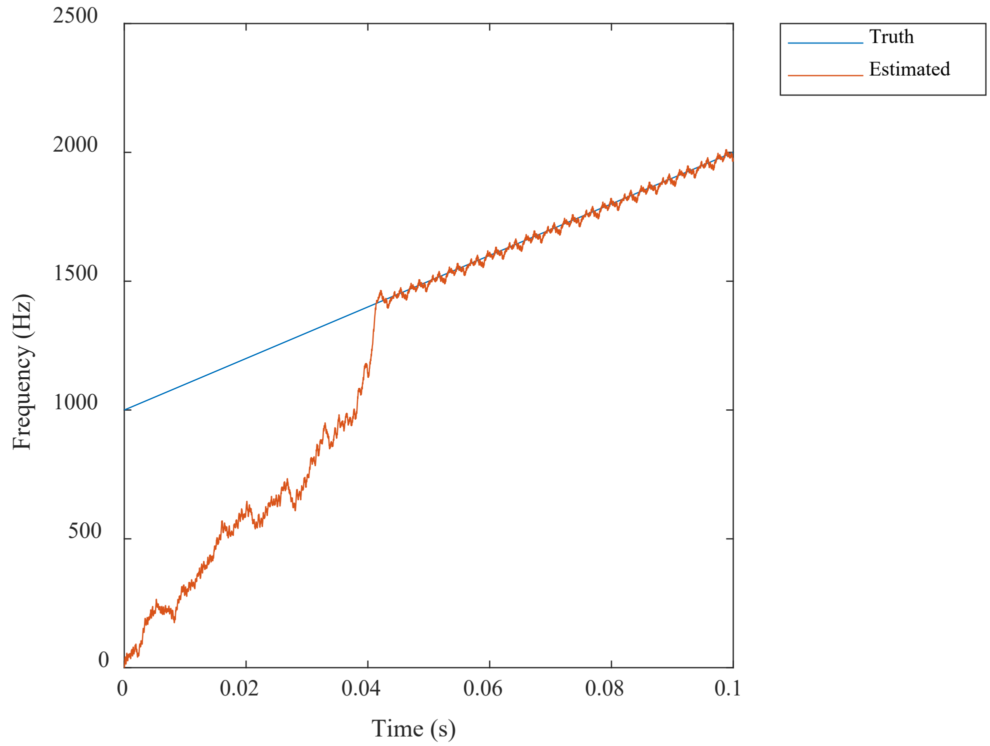

Fig. 10 Example of estimating a time-varying frequency offset

4. Conclusion

Oscillator discrepancies and Doppler shifts can cause a mismatch between the transmitter and receiver carrier frequency in wireless communications. A phase-lock loop is typically employed to recover the carrier and remove any frequency mismatch. This technical note has demonstrated how to use a Costas loop for phase recovery of a QPSK signal, paying special attention to characterization of the frequency offset. Full source code has been provided in the Appendix to reproduce all of the results in this report.

5. References

1. Abramovitch D. Phase-locked loops: a control centric tutorial. In: Proceedings of the 2002 American Control Conference; 2002 May 8. IEEE 2002;1:1–15. IEEE cat. No. CH37301.

2. Costas JP. Synchronous communications. Proc IEEE. 2002 Aug;90(8):1461–1466.

Appendix. Simulation Code

```
%% PLL QPSK
clear
close all;
rng(2) %init random number gen
set(0,'defaultAxesFontName','Times')
set(0,'defaultTextFontName','Times')
set(0,'defaultTextFontSize',12)
set(0,'defaultAxesFontSize',12)

%% Parameters
n=3000; %n symbols
data_rate=10e3;
k=8; %samples/symbol
phi_offset=0; %init phase (rad)
f_offset=1e3; %freq offset (Hz)
w=0.1; %noise
a=0.015; %filter constant
nfft=8; %2^nfft size fft
nplot=100; %number of symbols to plot

fs=data_rate*k; %sampling rate
dt=1/fs; %sample period
ns=k*n; %num samples
ts = 0:1/fs:(ns-1)/fs; %samping time
tsym = 0:1/data_rate:(n-1)/data_rate; %symbol time
omega_offset=f_offset*2*pi/fs; %(rads/sample) offset
samp_plot_index=(ns-nplot*k+1:ns); %index of samples to plot
sym_plot_index=(n-nplot+1:n); %index of symbols to plot

%% Create QPSK signal
s=randi([0 3],1,n); %symbols
x=exp(1i*2*pi*s/4); % QPSK
x_save=x; %save original symbols for later
figure(1) %plot constellation diagram
subplot(1,2,1)
plot(x(sym_plot_index),'.')
xlabel('I')
ylabel('Q')
xlim([-1.5 1.5])
ylim([-1.5 1.5])

%% up-sample with raised cosine filter
x=upfirdn(x,rcosdesign(0.25,6,k),k)*2; %transmit filter
x=x(1:ns);
subplot(1,2,2) %plot constellation diagram
plot(x(samp_plot_index),'.')
xlabel('I')
ylabel('Q')
xlim([-1.5 1.5])
ylim([-1.5 1.5])

figure(2) %plot time domain signals
subplot(3,1,1)
stem(tsym(sym_plot_index),s(sym_plot_index))
ylabel('Symbol')
ylim([0 3])
subplot(3,1,2)
```

```
plot(ts(samp_plot_index),real(x(samp_plot_index)))
hold on
plot(ts(samp_plot_index),imag(x(samp_plot_index)))
ylabel('Amp.')

figure(3) %plot freq. domain
X = fft(x(end-2^nfft+1:end),2^nfft)/2^nfft;
f = fs/2*linspace(0,1,2^nfft/2+1);
plot(f,2*abs(X(1:2^nfft/2+1)))
xlabel('Frequency (Hz)')
ylabel('|X(f)|')

figure(5) %use later to compare recovered results
plot(f,2*abs(X(1:2^nfft/2+1)))
xlabel('Frequency (Hz)')
ylabel('|X(f)|')

%% offset freq. and add noise
x=x.*exp(1i*(phi_offset +
fs*ts*omega_offset))+w*(randn(1,ns)+randn(1,ns)*1i); %freq/phase
offset

figure(2) %plot time domain shifted signal
subplot(3,1,3)
plot(ts(samp_plot_index),real(x(samp_plot_index)))
hold on
plot(ts(samp_plot_index),imag(x(samp_plot_index)))
xlabel('Time (s)')
ylabel('Amp.')

figure(3) %plot freq. domain shifted signal
hold on
X = fft(x(end-2^nfft+1:end),2^nfft)/2^nfft;
f = fs/2*linspace(0,1,2^nfft/2+1);
plot(f,2*abs(X(1:2^nfft/2+1)),'--')
legend('Original','Shifted','Location','NorthEastOutside')

figure(4) %Constellation diagram of shifted signal
subplot(1,2,1)
plot(x(samp_plot_index),'.')
xlim([-1.5 1.5])
ylim([-1.5 1.5])
xlabel('I')
ylabel('Q')

%% PLL
b=a^2; %beta
phi_hat=zeros(1,ns); %pre-allocate estimated phi
phi_error_hat=zeros(1,ns); %pre-allocate estimated phi error
omega_hat=zeros(1,ns); %pre-allocate estimated carrier
y=zeros(1,ns); %pre-allocate output signal
for i=1:ns-1
      y(i)=x(i) * exp(-1i*(phi_hat(i))); %carrier sync
      v=y(i).^4; % 4th-order, detect phase error
      phi_error_hat(i)=imag(v) / abs(v);
      omega_hat(i+1)=omega_hat(i)+b*phi_error_hat(i); %loop
filter
```

```
    phi_hat(i+1)=phi_hat(i)+a*phi_error_hat(i)+omega_hat(i+1);
end

%% Plot Results
figure(4) %constellation diagram
subplot(1,2,2)
plot(y(samp_plot_index),'.')
xlim([-1.5 1.5])
ylim([-1.5 1.5])
xlabel('I')
ylabel('Q')

figure(5) %frequency domain of recovered signal
Y = fft(y(end-2^nfft+1:end),2^nfft)/2^nfft;
hold on
plot(f,2*abs(Y(1:2^nfft/2+1)),'--')
xlabel('Frequency (Hz)')
ylabel('|X(f)|')
legend('Original','Recovered','Location','NorthEastOutside')

figure(6) %pll freq. estimate history
f_hat=omega_hat*fs/(2*pi); %Hz
plot(ts,f_hat)
xlabel('Time (s)')
ylabel('Frequency (Hz)')

%% downsample
y=upfirdn(y,rcosdesign(0.25,6,k),1,k)/2; %receiver filter

figure(7) %downsampled constellation diagram
y=y(7:end);
plot(y(sym_plot_index),'.')
xlabel('I')
ylabel('Q')

figure(8) %data recovery
s2=real(round(4*log(y)./(1i*2*pi)));
s2(s2<0)=4+s2(s2<0); %correct for phase rollover
stem(tsym(sym_plot_index),s(sym_plot_index))
hold on
plot(tsym(sym_plot_index),s2(sym_plot_index),'*')
legend('Original','Recovered','Location','NorthEastOutside')
xlabel('Time (s)')
ylabel('Symbols')
```

List of Symbols, Abbreviations, and Acronyms

ARL	Army Research Laboratory
DEVCOM	US Army Combat Capabilities Development Command
IQ	in-phase, quadrature
QPSK	quadrature phase-shift keying

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