# THREE-DIMENSIONAL MODELING OF NUCLEAR EXPLOSIONS USING A MICRO-MECHANICAL DAMAGE MODEL

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## 1. SUMMARY

We have implemented the latest version of the micro-mechanical damage model developed by Dr. Charles Sammis and associates in a Fortran 90 module that can be incorporated into a 1D, 2D or 3D code. We have added the module to our 1D spherically symmetric nonlinear finite difference code SKIPPER and our 3D finite element code CRAM3D. During the first year of this project we performed an extensive set of calculations using the 1D code, and compared the results with near-field data from US and former Soviet Union underground nuclear explosions. The new model gives a better data fit to the Degelen data than the quasistatic damage model used in Stevens et al (2003), and also gives a fairly good data fit to the Piledriver data without changing any parameters. The results of the calculations will define the physical models and parameters that will be used in year 2 in the 3D code.

## 2. INTRODUCTION

For many years, we have been modeling seismic waves from underground nuclear explosions using numerical calculations in one, two and three dimensions. Under AFRL support, we have developed CRAM3D, a finite element code specifically designed for calculating underground nuclear explosions. Currently CRAM3D uses a continuum model for material strength and damage. For the past two decades, Dr. Charles Sammis and associates at USC have been developing a micromechanical damage model, also developed under AFRL support, which explicitly takes into account damage due to microcracks and failure due to crack coalescence (Ashby and Sammis (1990), Deshpande and Evans (2008), Bhat, Rosakis, and Sammis (2012), Thomas et al, (2017)). We previously implemented an early version of this model in our one-dimensional spherically symmetric code SKIPPER (Rimer et al, 1998,1999; Stevens et al, 2001, 2002, 2003). Here we implement the latest version of the micromechanical damage model first in SKIPPER and then in CRAM3D, and to use it to model the damage caused by the explosion and the effect of this damage on seismic waves.

In the following sections of the report, we:

- 1. Describe the current version of the micromechanical damage model and the equations that are incorporated into the numerical codes
- 2. Describe the other nonlinear mechanisms that are included in our codes and how they interact with the micromechanical damage model
- 3. Give the results of an extensive set of spherically symmetric calculations; compare the results of the calculations to near-field data from the former Nevada Test Site, Fallon, Nevada and Degelen Mountain in the former Soviet Union.
- 4. Describe how the micromechanical damage model is incorporated into the SKIPPER and CRAM3D codes.

## **3. TECHNICAL APPROACH**

## 3.1 The micromechanical damage model

We use the micromechanical damage mechanics originally formulated by Ashby and Sammis (1990) for compressive loading and generalized by Deshpande and Evans (2008) to also allow tensile stress states. As illustrated in Figure 1, frictional sliding on optimally oriented initial flaws nucleates tensile wing-cracks, which, at high stress, coalesce to pulverize the rock. If the initial fracture distribution is isotropic then there will always be optimally oriented starter flaws at any azimuth and the resultant damage pattern will have spherical symmetry.



**Figure 1. Geometry in the micromechanical damage mechanics model.** Sliding on an array of penny-shaped cracks having volume density of  $N_V$  and radius a produces a wedging force  $F_w$  that drives tensile wing cracks to open in the direction of the smallest principal stress  $\sigma_3$  and propagate parallel to the largest principal stress  $\sigma_1$ . Growth of wing cracks is enhanced by  $\sigma_1$ , retarded by  $\sigma_3$ , and enhanced by a global interaction that produces a medin tensile stress  $\sigma\sigma$ . The positive feedback provided by this tensile interaction stress leads to a run-away growth of the wing cracks and ultimate macroscopic failure.

The micromechanical damage mechanics formulated by Ashby and Sammis (1990) models the nucleation, growth, and interaction of a mono-sized distribution of cracks having a fixed orientation. Bhat et al. (2012) used this model as a template to develop their dynamic damage model, which we use in this study. The main features/assumptions of the Ashby/Sammis formulation were:

(1) The starter cracks all have the same radius a and are all oriented at the same angle  $\Psi$  relative to the axis of principal compression  $\sigma_1$ . If the orientation of  $\sigma_1$  changes with time so does the populations of activated cracks. There are  $N_V$  such optimally oriented starter flaws per unit volume for any orientation of  $\sigma_1$ . Sliding on the starter cracks is inhibited by Coulomb friction characterized by coefficient f.

(2) All additional crack damage is in the form of tensile "wing cracks" that nucleate at the tips of the starter flaws and grow parallel to the  $\sigma_1$  axis. They open in the direction of the least compressive principal stress  $\sigma_3$ .

(3) The stress at which the wing cracks nucleate is taken from results in the literature and may be expressed as

$$\sigma_{1c} = \left(\frac{\sqrt{1+f^2}+f}{\sqrt{1+f^2}-f}\right)\sigma_3 - \left(\frac{\sqrt{3}}{\sqrt{1+f^2}-f}\right)\frac{K_{IC}}{\sqrt{\pi a}}$$

where  $K_{IC}$  is the critical stress intensity factor (fracture toughness), which is a material property.

(1)

(4) The stress intensity factor  $K_l$  at the tips of the growing wing cracks (of length *l*) is approximated as that at the tip of a tensile crack of radius  $l + \alpha a$  that is loaded by a point force at its center. The point force is taken as the  $\sigma_3$  component of force generated by frictional sliding on the starter flaw. The geometrical factor  $\alpha$  is the projection of the starter flaw on the  $\sigma_1$  axis. This approximation was tested against the numerical solution and found to be poor for small *l* but asymptotically better as *l* increases. An adjustable parameter  $\beta$  was introduced to improve the fit of the approximate  $K_I$ to the numerical calculations of  $K_I$  at small values of *l*.

(5) The failure stress  $\sigma_1^*(\sigma_3)$  is defined as the maximum value of the  $\sigma_1$  versus *l* curve at constant  $\sigma_3$ .

There are thus four crack related parameters in the model:

(1)  $\alpha = cos\Psi$  where  $\Psi$  is the angle between an optimally oriented starter crack and the  $\sigma_1$  axis.

(2)  $\beta$  adjusted to make the approximate expression for  $K_I$  agree with the full numerical simulation when the wing cracks are short.

(3) a = radius of the starter flaws. It is found by fitting the nucleation equation above to the observed onset of nonlinearity in the stress strain curve or the onset of acoustic emissions.

(4)  $N_V$  = number of favorably oriented (active) starter flaws per unit volume. It is found by fitting the uniaxial strength.

Despite these apparently crude approximations, Ashby and Sammis (1990) found that this approximate model gave a very good fit to the failure surfaces,  $\sigma_1^*(\sigma_3)$ , of a wide range of rocks for reasonable values of *a* (approximately the grain size) and  $N_V$ . It was later shown by Bhat et al. (2012) that the inclusion of a size distribution of cracks and allowance for multiple crack orientations did not significantly effect the failure strength predicted by the model.

By approximating the interaction between growing wing cracks Ashby and Sammis (1990) found a positive feedback that led to mechanical instability and failure. They demonstrated that their model gave an adequate description of the failure envelope ( $\sigma_1$  versus  $\sigma_3$  at failure) for a wide range of rocks loaded in triaxial compression ( $\sigma_1 < \sigma_2 = \sigma_3$ , where compression is taken as negative). However since quasi-static crack growth was assumed (the stress intensity factor was always at its critical value) their formulation does not include effects of loading rate. Bhat et al. (2012) extended the damage mechanics formulation in by incorporating theoretical and experimental dynamic crack growth laws that have been shown to be valid over a wide range of loading rates. They compared their model with uniaxial experiments in marble and were able to predict failure stress over a wide range of loading rates ( $\dot{\varepsilon} \sim 10^{-6}$  to  $10^3 \text{ sec}^{-1}$ ).

#### **3.1.1 Development of the Constitutive Model**

In most brittle materials, micro-crack nucleation, growth and coalescence is driven by local sliding at micro-cracks or grain boundaries as shown schematically in Fig.1. This micro-crack physics produces inelastic dilatancy, modulus reduction, and strain-rate sensitivity of failure strength. Thus any realistic constitutive model of brittle materials should take into account the micromechanics of fracture.

#### Continuum Constitutive Model From Micro-Scale Deformations

Bhat et al. [2012] used an energy-based approach to determine the constitutive relationship of the damaged solid. If S denotes the current damaged state of the material in the sense that variations in stress at constant S induce a purely elastic response, then the stress-strain relationship and the compliance tensor can be written in terms of a Gibbs free energy function, W, under isothermal conditions as:

$$\varepsilon_{ij} = \frac{\partial W(\boldsymbol{\sigma}, S)}{\partial \sigma_{ij}}$$

$$M_{ijkl} = \frac{\partial^2 [W(\boldsymbol{\sigma}, S)]}{\partial \sigma_{ij} \partial \sigma_{kl}}$$
(2)

where W is symmetrized in the components of  $\sigma$  (notations in bold represent tensor quantities).

Let dW denote the change in the free energy function when the solid undergoes deformation that takes it from the state S to S + dS at constant  $\sigma_{ij}$ . Therefore, the inelastic strain associated with dW is given by

$$d\varepsilon_{ij} = \frac{\partial(dW)}{\partial\sigma_{ij}} \tag{3}$$

The Gibbs free energy is written as the sum of the elastic contribution and an inelastic one due to the presence of micro-cracks, which is written in terms of the stress intensity factors  $K_I, K_{II}, K_{III}$ , where subscripts I, II, and III denote the three modes of loading at the crack tip.

$$W(\sigma, S) = W^{e}(\sigma) + \frac{1-\nu^{2}}{E} \int_{\Gamma} \left[ K_{I}^{2}(\sigma, S) + K_{II}^{2}(\sigma, S) + \frac{K_{III}^{2}(\sigma, S)}{(1-\nu)} \right] ds \, d\Gamma$$
(4)

In this expression  $W^e$  is the elastic strain energy,  $\Gamma$  is the locus of all crack fronts in the damaged solid, and *ds* is a function of position along  $\Gamma$  describing the amount of local advance of the micro-cracks. The stress-strain relation and the compliance tensor are then given by Eq. 2.

#### **3.1.2 Evaluation of the Stress Intensity Factors**

The framework describe above was used to evaluate the Gibbs free energy, Eq. 4, and hence the stress-strain relationship and the compliance tensor, for the micro-crack model formulated by Ashby and Sammis [1990]. As discussed above, this model considers an isotropic elastic solid that contains an array of penny shaped cracks all of radius *a* (micro-cracks or grain boundaries) all aligned at an angle  $\Psi$  to the largest (most negative) remote compressive stress  $\sigma_l$  (Fig. 1b). Only those cracks that are optimally oriented for sliding are considered. The population of optimal cracks that exist prior to loading has the volume density Nv, which remains fixed during loading - no new sliding cracks appear during loading. The size and density of these initial flaws are characterized by an initial damage defined through the scalar variable

$$D_0 = \frac{4}{3}\pi N_V (\alpha a)^3 \tag{5}$$

where  $\alpha a$  is the projection of the crack radius in a vertical plane parallel to the direction of  $\sigma_l$ ,  $\alpha = cos\Psi$ . As the wing cracks extend a distance *l* in the  $x_1$  direction (see Fig. 1) the damage increases as

$$D = \frac{4}{3}\pi N_V (l + \alpha a)^3 \tag{6}$$

The stress intensity factors are found by calculating the shear and normal stresses on each sliding crack from the remote compressive stress field. Three deformation regimes for the microcracked solid were identified based on the remote loading state. In *Regime I*, the remote loading is compressive and is not large enough to overcome the frictional resistance on the sliding cracks. The solid thus behaves like an isotropic linear elastic solid. In *Regime II* the frictional resistance on the sliding cracks is overcome by the remote compressive load leading to the nucleation and growth of wing-cracks. In *Regime III* the remote loading stress in the  $x_3$  direction turns tensile leading to the opening of both the angled sliding cracks and their wing-crack extensions.

Once the wing cracks nucleate, the Mode-I stress intensity factor,  $K_I$ , at the tip of the wing-cracks (of length *l*) is evaluated. It has three contributions: (1) Sliding on the angle cracks leads to a wedging force,  $F_w$ , on the wing-cracks. This wedging force is simply the component of the sliding force resolved in the direction of the minimum principal stress  $\sigma_3$ . (2) The remote confining stress, characterized by  $\sigma_3$  tends to close the wing-cracks and, (3) opening of the wing cracks creates tension  $\sigma^{(i)}$  on the unbroken ligaments between neighboring wing-cracks (see Fig. 1), which is represented as an average global interaction between the wing cracks that enhances their growth.

#### 3.1.3 Constitutive Relationship and Damage Evolution

The Gibbs free energy function in Regime's I, II, and III was evaluated using the stress intensity factors calculated as indicated in the previous section and Eq. 4. The Gibbs free energy function was then differenced as shown in Eq. 2 to obtain the stress-strain relationship and the compliance and modulus tensors. The equations derived using this technique are given in Appendix A.

To complete the constitutive model described above, we need an evolution law for the scalar damage parameter, D. Differentiating D in Eq. 6 with respect to time gives

$$\frac{dD}{dt} = \left(\frac{3D^{2/3}D_0^{1/3}}{\alpha a}\right)\frac{dl}{dt} \tag{7}$$

where dl/dt = v is the instantaneous wing-crack tip speed. This is a geometric relation connecting the wing crack tip speed with the evolution of the damage parameter D. Completing this process requires additional physics relating dl/dt to local stress conditions in the vicinity of the microcracks. This is done by making the stress intensity factor and its critical value (toughness) loadingrate dependent for both the nucleation and growth of the wing cracks.

To solve this problem, we need the state of stress around a crack-tip (both stationary and propagating) under various loading conditions. These values then need to be compared with experimentally determined fracture toughness of the material, under similar conditions, to develop crack initiation and growth criteria. The most common form for such criteria is the requirement that the crack must grow in such a way that some parameter (e.g. the dynamic stress intensity factor,  $K_I^d$ ) defined as part of the crack-tip field maintains a value that is specific to the material. This value, representing the resistance of the material to the advance of the crack, is called the dynamic fracture toughness ( $K_{IC}^d$ ) of the material, and it can be determined through experimental measurements only. It can be represented with the following functional form

$$K_{IC}^{d} = K_{IC}^{SS} \left\{ \frac{1 + (\nu/\nu_m)^5}{\sqrt{1 - \nu/c_p}} \right\}$$
(8)

where  $v_m$  is the experimentally determined branching speed.

The dynamic stress intensity factor for the growing crack is given by

$$K_{I}^{d}(v) = \frac{K_{I}(1 - v/c_{R})}{\sqrt{1 - v/c_{p}}}$$
(9)

where  $K_I$  is the quasi-static stress intensity factor of an equivalent crack of the same length but growing at zero speed. The limiting speed of a mode I crack is thus the Rayleigh speed at which point  $K_I^d(c_R) = 0$ .

Setting the dynamic stress intensity factor (Eq. 8) equal to the dynamic fracture toughness (Eq. 9) gives the following non-linear equation for the crack speed:

$$\frac{K_I(1-\nu/c_R)}{\sqrt{1-\nu/c_p}} = K_{IC}^{SS} \left\{ \frac{1+(\nu/\nu_m)^5}{\sqrt{1-\nu/c_p}} \right\}$$
(10)

This expression is solved to obtain the crack speed, which is then used in Eq. 7 to complete the damage evolution equation.

## **3.2 Implementation in SKIPPER and CRAM3D**

Rimer et al (1998) implemented an early version of the micromechanical damage model in SKIPPER, the onedimensional spherically symmetric implementation code. Although in CRAM3D is somewhat more complicated, the procedure is essentially the same. Figure 2 shows a description of a single computational cycle. At the start of each time step, we calculate the accelerations from nodal forces, and update velocities, displacement, strains and strain rates. From this, we calculate new "trial" stresses and stress rates. "Trial" means that the stresses may be adjusted according to nonlinear constitutive models. If the strength of the material is exceeded, then the stresses are reduced according to a plastic flow rule. If a tensile crack opens, then the tensile stress is reduced to zero.



Figure 2. CRAM3D and SKIPPER Computational Cycle

The micromechanical damage model is another type of constitutive model, and it is updated at each time step after the deformation from the previous cycle is calculated. At each time step, and in each element, the code checks to see if the stress state allows cracks to extend, as discussed in the previous section. If it does, then we calculate the speed of crack extension and increase the crack length by the corresponding amount over a time step. If damage increases beyond a critical level failure occurs and we enter a post-failure constitutive model. In the quasistatic model used in the 1998-2003 studies, the code checked to see if an increase in shear stress caused a decrease in crack length, in which case unstable failure occurred. With the new dynamic model, the material does not necessarily fail immediately at the quasistatic failure point because the speed of crack growth is limited by an empirical maximum speed. We use D=1 as the failure condition. The main difference between the 1D and 3D implementations is that in 3D the principal stress directions change and need to be recalculated at each time step, while in 1D the principal stress directions always correspond to the radial and axial directions.

The following sections describe the other components of the numerical calculation.

#### **3.2.1 Equation of State**

The equation of state is specified by empirical constants for pressure vs. volume, plus an optional crush curve for porous media. The equation of state is defined by the equation  $P = K\mu + B\mu^2$  where *P* is pressure, *K* and *B* define the bulk modulus and  $\mu$  is the volumetric change defined by  $\mu = \rho / \rho_0 - 1$  where  $\rho$  and  $\rho_0$  are the current and initial densities, respectively. Granite has a very small porosity and so does not require a crush curve.

#### 3.2.2 Shear Strength

Shear strength is defined by a failure (plastic yielding) condition expressed as a function of deviatoric stress invariants. In its simplest form, shear strength can be stated as a function of shear stress vs. pressure, with the strength increasing with increasing confining pressure. However, the material surrounding the explosion goes through some complicated strain states, initially compressed radially and stretched axially, so a more general relation is needed. We use the formulation developed by Peyton (1983). The failure surface is defined by (Figure 3):

$$\frac{J_2}{\tau^2} - \beta \frac{J_3}{\tau^3} > 1$$
 (11)





where  $J_2$  is the second deviatoric stress invariant stresses. and  $J_3$  is the third deviatoric stress invariant.  $\tau$  is defined by an input curve as a function of pressure. B is a shape factor equal to 0.875, and we also require  $J_2 \leq 3\tau^2$ ,  $\tau$  is specified as a quadratic

 $\beta$  is a shape factor equal to 0.875, and we also require  $J_2 < 3\tau^2$ .  $\tau$  is specified as a quadratic function of the form,

$$\tau = \tau_0 + \tau_m \frac{P}{P_m} \left( 2 - \frac{P}{P_m} \right), \tag{12}$$

where  $\tau = \tau_0 + \tau_m$  for  $P > P_m$ .

#### 3.2.3 Tensile cracking

Tensile cracking is based on the formulation of Maenchen and Sack (1964), and can be a very important effect. When the normal stress in any direction becomes tensile, the material strength in that direction is set to zero. Tensile cracks can heal after compression is reapplied. Tensile cracking is turned on in the granite model. Tensile cracking typically occurs soon after failure from the micromechanical damage model.

#### 3.2.4 Shear strength reduction

Several input parameters are required to define the strength reduction model. The first, *cramepw*, is the amount of plastic work accumulated before strength weakening begins. For the Shoal granite



Figure 4. Granite strength model. "Initial" line is the initial granite strength, "Failed" is the strength immediately after failure, "Weakened" is the final shear strength.

model it is 500 Joules/m<sup>3</sup> (5000 erg/cm<sup>3</sup>). The amount of strength reduction is determined by the shear strain, with limits *deinit*=0.001 and *defull*=0.06. The strength is weakened linearly between these values from the "Failed" strength to the "Weakened" strength. The "Failed" and "Weakened" lines can be thought of as corresponding to static and dynamic friction, which in this case have coefficients of friction of 0.6 and 0.02 (equivalent  $\tau = 0.528$  and 0.017), respectively. The minimum failed strength is defined by a cohesive strength *taucoh*, which was set equal to the zero pressure strength of the rock  $\tau_0$ . The strength states for this model are shown in Figure 4.

#### 3.2.5 Shear modulus reduction

The shear modulus decreases as a function of shear strain, from *epinit* to *epfull*, from its initial value to its reduced value *Gdam*. For the Shoal granite model *epinit*= $4x10^{-4}$ , *epfull*= $1.0*10^{-3}$ , and *Gdam*= $1.0x10^{10}$  Pa ( $10x10^{10}$  dyne/cm<sup>2</sup>).

#### 3.2.6 Options and Order of Application

The nonlinear codes SKIPPER and CRAM3D allow for any or all of these constitutive models to be used, but there is a prescribed order to how they are used, as follows:

- 1. The elastic calculation is done first, generating the trial stresses.
- 2. The micromechanical damage model is done next, taking the material from its elastic state through unstable crack growth.
- 3. Shear failure is calculated next, causing at least a small amount of plastic flow to occur before other nonlinear effects are allowed. The amount of shear failure is measured by the plastic work accumulated. There is an option to allow shear failure before failure from the micromechanical damage model. This may be appropriate near the cavity where stresses are very high and brittle failure is unlikely.
- 4. Tensile cracking can occur once plastic work is nonzero.
- 5. Shear modulus reduction can occur once plastic work is nonzero and the maximum shear strain in an element has exceeded epinit.
- 6. Strength reduction can occur once plastic work exceeds cramepw and the maximum shear strain exceeds deinit.

If the micromechanical damage model is not used, then the requirement that shear failure has occurred before shear modulus reduction can occur is removed. Otherwise, the same order applies.

## 4. RESULTS AND DISCUSSION

# 4.1 Comparison of Spherically Symmetric Granite Runs with and without the Sammis Micromechanical Damage Model

We have implemented the Sammis Micromechanical Damage Model in our 1D spherically symmetric nonlinear finite difference code SKIPPER. We compare the results with the strain damage model that has been used for calculations of Shoal, North Korea and other explosions (Stevens and Thompson, 2015). These calculations were all done with the parameters used for Shoal, a 12.5 kt explosion in granite at a depth of 367.4 m. In the following, we start with properties common to all calculations (Table 1), then discuss the nonlinear effects modeled in the original granite Shoal calculation, and then including the Sammis model with a range of variants on modeling parameters.

Table 1. Properties of granite used in SKIPPER calculations. Numbers below are in SI an	d
CGS units. CGS units are used in SKIPPER, SI in CRAM3D	

Property	Value (SI)	Value (CGS)	Variable Name
Bulk modulus	37.9x10 <sup>9</sup> Pa	$379 \times 10^9$ dyne/cm <sup>2</sup>	K/aks/capa
Shear modulus	23.8x10 <sup>9</sup> Pa	$238 \times 10^9$ dyne/cm <sup>2</sup>	G/amu
Density	$2600 \text{ kg/m}^3$	$2.6 \text{ g/cm}^3$	rho
Melt energy	$5.2 \times 10^9 \text{ J/m}^3$	$5.2 \times 10^{10} \text{ erg/cm}^3$	em/emlt
$\tau_0$ - Strength at P=0	8.96x10 <sup>6</sup> Pa	$8.96 \times 10^7 \text{ dyne/cm}^2$	tauz
$\tau_m$ - Strength at Pmax	3.313x10 <sup>8</sup> Pa	$3.313 \times 10^9 \text{ dyne/cm}^2$	taum
Pmax	5.0x10 <sup>8</sup> Pa	$5.0 \mathrm{x} 10^9 \mathrm{dyne/cm}^2$	pyld

#### 4.1.1 Micromechanical Test Cases

The Sammis micromechanical damage model has several parameters, some of which can be determined experimentally. P and S velocities are derived from the properties in Table 1.

Property	Value (SI)	Value (CGS)	Definition
vp	5175 m/s	$5.175 \times 10^5 \text{ cm/s}$	P velocity
VS	3026 m/s	$3.026 \times 10^5 \text{ cm/s}$	S velocity
vr	2723 m/s	$2.723 \times 10^5 \text{ cm/s}$	Rayleigh velocity
vm	1000 m/s	$1.000 \times 10^5 \text{ cm/s}$	Max crack speed
K1C	$1.0 \mathrm{x} 10^{6} \mathrm{Pa} \mathrm{-m}^{1/2}$	$1.0 \times 10^8 \text{ dyne/cm}^2 \text{-cm}^{1/2}$	Stress intensity factor
a	0.0001 m	0.01 cm	Crack half-width
f	0.6	0.6	Coefficient of friction
$D_0$	0.1	0.1	Initial damage

 Table 2. Sammis model properties for granite

Other properties needed for calculations can be derived from these:

$$\Psi = 0.5 * \tan^{-1}(1/f)$$
  
$$\alpha = \cos(\Psi)$$

(13)

$$N_{\nu} = \frac{3}{4\pi} \frac{D_0}{\left(a\alpha\right)^3} \tag{14}$$

 $N_{\nu}$  is the density of initial cracks that are optimally oriented for sliding – cracks per unit volume. For the parameters above,  $N_{\nu}$  is 36 cracks/cm<sup>3</sup> (36x10<sup>6</sup>/m<sup>3</sup>).  $\Psi$  is the angle of the optimally oriented crack measured from the direction of the maximum principal stress.

The properties used here are those used in Ashby and Sammis for granite, except that  $D_0$  is increased to 0.1 from 0.01 and a is 1 mm instead of 0.5 mm and the coefficient of friction is 0.60 instead of 0.64.

Of these properties, the two most likely to vary are the crack size a and the initial damage  $D_0$ . For the following parameter study we left  $D_0$  fixed. Since the crack size is very likely to scale with problem size, we performed tests with a values of one millimeter, one centimeter, 10 centimeters and one meter. Note that any change in *a* also changes the crack density according to the equation above. Changing a from 0.1 mm to 1 m decreases Nv by  $10^{12}$  which is one crack per 30,000 m3 (about a 30 meter/side block).

For a spherically symmetric problem, we can take the principal stress  $\sigma_1$  to be in the radial direction.  $\sigma_2$  and  $\sigma_3$  are equal. In an explosion, there is an initial pulse of high pressure, which causes a radial compressive strain and compressive stress. In the axial direction, there is compressive stress, but tensile strain as the material is forced to expand axially.

In addition to varying the initial crack size, we also vary the other nonlinear effects that can occur during the explosion. We ran 9 variations (Table 3); each of these were run four times for the different crack sizes. All cases allowed tensile cracking after, but not before, plastic yielding had occurred. Three of them used the reduced strength model described above. Three of the cases used the elastic moduli throughout and six used the damage dependent moduli calculated using the Sammis model.

Case #	Shear modulus	Plastic yielding	Sammis moduli	Weakened
	reduction	allowed before	instead of elastic	shear strength
		crack failure	moduli	
1	Off	Off	Off	Off
2	On	Off	Off	Off
3	On	On	Off	Off
4	Off	Off	On	Off
5	On	Off	On	Off
6	On	On	On	Off
7	Off	Off	On	On
8	On	Off	On	On
9	On	On	On	On

 Table 3. Nonlinear options used in test runs

Calculated cavity radii are listed in Table 4. Table 5 shows calculated reduced displacement potential (RDP). Table 6 shows the maximum extent of plastic yielding. Table 7 shows the maximum extent of full damage and Table 8 the maximum extent of partial damage. For the new calculations "partial damage" means that the cracks in the cell have extended according to the micromechanical damage model. "full damage" means that cracks have reached unstable failure. For the Shoal model, "damage" means that enough plastic work has accumulated to reduce the

strength in the cell, "full damage" means that the strength is reduced continuously from the origin, "partial damage" is discontinuous beyond the full damage extent. The measured cavity radius for Shoal was 25.6 meters (Beers, 1964).

Case #	N/A	a=1mm	a=1cm	a=10cm	a=1m
Shoal	20.58				
1		16.28	16.42	16.67	16.40
2		16.62	16.57	16.81	16.80
3		15.38	16.82	17.24	17.26
4		15.96	15.61	14.65	13.24
5		16.22	15.70	15.17	13.99
6		13.91	14.74	15.02	14.42
7		25.42	23.00	21.35	20.36
8		25.77	23.17	22.23	21.85
9		21.71	21.79	21.87	21.42

Table 4. Calculated final cavity radii (meters)

Case #	N/A	a=1mm	a=1cm	a=10cm	a=1m
Shoal	1760				
1		772	789	824	789
2		821	810	846	849
3		649	850	916	920
4		818	778	780	725
5		862	790	802	790
6		781	1494	1215	914
7		3160	2625	2754	2985
8		3254	2617	2999	3041
9		2992	2810	2625	2393

#### Table 6. Radius of plastic yielding (m)

Case #	N/A	a=1mm	a=1cm	a=10cm	a=1m
Shoal	266.6				
1		140.5	162.5	186.4	171.3
2		145.8	162.5	189.5	177.2
3		156.8	162.5	192.7	192.7
4		143.1	162.5	180.2	168.3
5		151.3	162.5	180.2	168.3
6		180.2	208.9	208.9	199.1
7		278.5	266.7	303.4	286.7
8		278.5	270.6	307.7	290.8
9		262.8	215.7	212.3	199.1

Case #	N/A	a=1mm	a=1cm	a=10cm	a=1m
Shoal	137.9				
1		140.5	162.5	186.4	171.3
2		145.8	162.5	189.5	177.2
3		156.8	162.5	192.7	192.7
4		143.1	162.5	180.2	168.3
5		151.3	162.5	180.2	168.3
6		180.2	208.9	208.9	199.1
7		278.5	266.7	303.4	286.7
8		278.5	270.6	307.7	290.8
9		262.8	215.7	212.3	199.1

## Table 7. Radius of full damage (m)

## Table 8. Radius of partial damage (m)

Case #	N/A	a=1mm	a=1cm	a=10cm	a=1m
Shoal	148.6				
1		140.5	180.2	219.0	233.0
2		145.8	183.3	219.0	243.9
3		156.8	183.3	226.0	243.9
4		143.1	183.3	240.2	357.8
5		151.3	186.4	240.2	367.5
6		180.2	236.6	290.8	521.2
7		282.6	307.7	403.1	626.9
8		282.6	307.7	408.4	589.9
9		262.8	286.7	329.8	527.8

## Table 9. Minimum P velocity

Case #	N/A	a=1mm	a=1cm	a=10cm	a=1m
Shoal	4438				
1		5170	5170	5170	5171
2		4434	4435	4435	4435
3		4435	4435	4435	4435
4		5172	5171	5173	5180
5		4436	4452	4437	4443
6		4638	4466	4453	4485
7		5175	5173	5173	5182
8		4439	4437	4438	4444
9		4457	4468	4453	4450

#### 4.1.2 Comparison with Shoal

Shoal had three shot level stations each located 120 degrees apart (Figure 5) at a distance approximately to 590 meters from the shot. Stevens and Thompson (2015) modeled this explosion using the 3D finite element code CRAM3D, and including tectonic stresses. Stevens and Thompson were able to match the general character of the waveforms including variability from tectonic release, however they were not able to match the waveforms exactly. In particular, although the calculations clearly showed an increase in amplitude in the direction of PM-3, they could not match the very large observed amplitude at that station. The data at these three stations and the results of that calculation are shown in Figure 6.



Figure 5. Location of the Shoal explosion and the three nearfield shot level recording stations and the direction of the local stress state



Figure 6. Left: radial velocity and displacement recorded at the three stations. Right: calculation at zero and 90 degrees. Zero degrees is the direction of PM-1. Note the similarity between the PM-1 displacement and the zero degree calculated waveform, and PM-3 and the 90 degree calculated waveform.

In 1D we cannot include tectonic stresses or other 3D effects, but it is still useful to compare with the data. The Shoal calculation in the tables above used identical material properties to the CRAM3D calculation, with a constant pressure corresponding to the overburden pressure at the shot depth. A comparison of the near-field shot-depth waveforms is shown in Figure 7. Excluding the anomalous station 3, which clearly shows the effect of tectonic stresses, the main difference between the calculated and observed data is that the calculated velocity has a sharper initial peak,

and both the observed velocity and displacement are spread out over a longer period of time. This most likely means that the Shoal granite is weaker than the granite model.



Figure 7. Comparison of SKIPPER calculations of Shoal at 590 meters (red) with recorded data (blue). Observed displacements are all integrated velocity.

The Shoal granite model was derived by combining two granite models, one for Piledriver and one for Degelen Mountain that was developed in an earlier study (Stevens et al, 2003). The Degelen Mountain granite was stronger than the Piledriver granite (NTS Climax Stock Granite). As discussed above, the shear failure model has three parts: an initial strength model corresponding to the laboratory strength of fractured granite, a post-failure model of weakened, crushed granite, and a model based on shear strain that defines how the strength reduction occurs (Figure 4).

In the same report, we studied several other material models. This model was referred to as the Yield/Failure Surface/Shock Damage (YF-SS) model. In the same study, we implemented an early version of the Sammis damage mechanics model. The damage mechanics model was used until unstable crack growth occurred, after which it was replaced by the shock damage model (SA-SS). We also studied acoustic fluidization as a post-failure model. We compared the results with data from Piledriver and Degelen Mountain and found that we achieved the best results with the SA-SS model. Figure 8 shows the comparison with near-field Piledriver waveforms. Although the calculated waveforms are low compared to the data at the closest two stations, the waveforms are quite good at the next two stations. The calculations also have the long negative velocities observed in the data. As shown in Stevens et al (1986), Rimer et al (1987) and earlier reports, an effective stress model was developed that matched the first two stations quite well, but did not do as well farther out. Figure 9 shows calculated and observed waveforms from one Degelen Mountain explosion with a very good data fit at four locations. The main difference between the Degelen and Piledriver models is a weakened coefficient of friction of 0.2 vs. 0.02, respectively.



Figure 8. Calculated (red) and observed (blue) waveforms from the 62 kt explosion Piledriver using the SA-SS model



Figure 8 (continued). Calculated (red) and observed (blue) waveforms from the 62 kt explosion Piledriver using the SA-SS model



Figure 9. Calculated (red) and observed (blue) waveforms from the 78 kt 1987/07/17 Degelen Mountain explosion using the SA-SS model

Plots were made for all of the cases above, and we show a subset here. Figure 10 shows all waveforms for case #8 with a=1 cm. The waveforms are very similar to the Shoal calculation. Figure 11 shows a comparison of the Shoal waveforms with the other crack sizes, also for case #8. The 1 cm case has slightly larger amplitudes than the others, with the smallest amplitudes at 1 mm and 1 m.



Figure 11. Calculated velocity waveforms (red) compared with Shoal data (blue) for case 7. Top row a=1mm, middle row a=10cm, bottom row a=1m

One reason calculation #8 is so similar to the Shoal calculations is that case #8 includes most of the same nonlinear effects, particularly strain weakening and shear modulus reduction. The main difference are that case #8 uses the Sammis model and damage-dependent moduli pre-failure. Velocity waveforms for all cases are shown together in Figure 12.



Figure 11 (continued). Calculated velocity waveforms (red) compared with Shoal data (blue) for case 7. Top row a=1mm, middle row a=10cm, bottom row a=1m



Figure 12. From top to bottom cases 1,2,3,4,5,6,7,8,9, velocity waveforms. All for 1 cm crack size

A few conclusions can be drawn from these. First only cases 7 and 8 have the broadening of the waveform and long negative pulse that is observed in the data. Case 9 has a broader waveform, but a much sharper first arrival. The difference between cases 8 and 9 is that Case 9 allows shear failure before damage due to cracks is complete. The motivation for including this is that near the cavity where stresses are very high and shear failure is physically reasonable, but it results in a high amplitude initial arrival not seen in the data. After the initial peak, however, it has the long duration negative pulse seen in the data. The difference between cases 7 and 8 is shear modulus reduction after failure. One of the motivations for this is that compressional velocities were observed to decrease near the cavity in the Degelen explosions, but it makes surprisingly little difference in the waveforms, at least at this distance range.

#### 4.1.4 Calculated Damage

The calculations led to a variety of damage distributions. For the cases where plastic yielding was allowed before crack failure (3, 6, 9), yielding dominated close to the source and full damage did not occur in that region, but occurred farther out. There are also significant differences in the range of the damage depending on initial crack size (Figure 13, Figure 14).



Figure 13. Final damage for case #8. Top left: a=1mm, top right a=1cm, bottom left a=10cm, bottom right a=1m



Figure 14. Final damage for all cases, a=1 cm. Top: case 1 (left), case 2 (right); row 2: case 3 (left), case 4 (right), row 3: case 5 (left), case 6 (right), bottom: case 7 (left), case 9 (right) Approved for public release; distribution is unlimited.



Figure 14 (continued). Final damage for all cases, a=1 cm. Top: case 1 (left), case 2 (right); row 2: case 3 (left), case 4 (right), row 3: case 5 (left), case 6 (right), bottom: case 7 (left), case 9 (right)

More results from these calculations are shown in Appendix B.

#### 4.2 Comparison with Degelen Mountain Near-Field Data

In Stevens et al (2003), we compared calculations from an early version of the micromechanical damage model with shot level data from underground nuclear explosions at the former Soviet test site at Degelen Mountain. One example was shown earlier in Figure 9. Here we compare the same data set with the results of Shoal calculation #8 with a=1cm. To make this comparison we scale all of the data, both calculated and observed to a common yield of 12.5 kilotons. The waveforms shown are the closest distance used in the calculation to the scaled distance in the observation. The legend in each figure shows the actual distance (e.g. x90), the distance scaled to 12.5 kt, following that number, and the calculated distance shown as "ACE station# calculated distance". This figure can be compared with figure 29 from Stevens et al (2003), which shows the same data together with calculations using the older quasistatic model. Here we have also integrated the velocity data to get displacement data. The data fit is at least as good as the 2003 report, which is quite remarkable since we were not explicitly trying to fit it. Furthermore it is a better data fit than we got to the Shoal data, which we were trying to model.



Figure 15. Shot level data from Degelen Mountain explosions compared to Shoal calculation
#8 The original data is the velocity data shown in blue in the upper half of each figure. The displacement data was obtained by integrating the velocity.



Figure 15 (continued). Shot level data from Degelen Mountain explosions compared to Shoal calculation #8 The original data is the velocity data shown in blue in the upper half of each figure. The displacement data was obtained by integrating the velocity.



Figure 15 (continued). Shot level data from Degelen Mountain explosions compared to Shoal calculation #8 The original data is the velocity data shown in blue in the upper half of each figure. The displacement data was obtained by integrating the velocity.



Figure 15 (continued). Shot level data from Degelen Mountain explosions compared to Shoal calculation #8 The original data is the velocity data shown in blue in the upper half of each figure. The displacement data was obtained by integrating the velocity.

### 4.3 Comparison with Piledriver Data

In the 2003 study, we also compared our calculations with Piledriver, using a slightly different model. See Figure 24 of the 2003 report and Figure 8 above. As in the earlier study, the amplitudes at the first two stations are low, but the calculation has the same long negative velocity pulse as the data, and the data fit is fairly good at the two more distant stations.



Figure 16. Comparison Piledriver near-field data with Calculation #8

## 5. CONCLUSIONS

We have implemented the latest version of the Sammis micromechanical damage model in our spherically symmetric nonlinear finite difference code SKIPPER, using a Fortran 90 module that can be added unchanged to our CRAM3D nonlinear finite element code. We have run an extensive set of calculations using the spherically symmetric code and compared the results with data from nuclear explosions Shoal and Piledriver as well as a series of explosions at the Degelen Mountain test site. We have used this to determine a baseline case, described as Case #8 above, which is most consistent with all of the data. Remarkably, the new model gives a better data fit to the Degelen data than the 2003 model, and also gives a reasonably good data fit to the Piledriver data without changing any parameters. We will use this model as our starting model in the CRAM3D calculations in year two of this project.

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#### **APPENDIX A. MICROMECHANICAL CONSTITUTIVE EQUATIONS**

Bhat et al. (2012) extended the Deshpande and Evans (2008) formulation to incorporate an experimentally motivated new crack growth (damage evolution) law that is valid over a wide range of loading rates. This law is sensitive to both the crack tip stress field and its time derivative. Incorporating this feature produces strain-rate sensitivity in the constitutive response. Bhat et al. (2012) expressed the failure condition in terms of stress invariants, however because stress invariants are isotropic, we have reformulated the equations to allow the inherent anisotropy of the cracks to remain.

We follow Deshpande and Evans (2008) but use the Ashby and Sammis (1999) expression for  $K_{I}$ . When the cracks are shut and have no influence on the elastic response of the solid, the strain energy density W is that for an uncracked solid:

$$W = W_0 = \frac{1}{2E} (\sigma_{11}^2 + \sigma_{22}^2 + \sigma_{33}^2) - \frac{\nu}{E} (\sigma_{11}\sigma_{22} + \sigma_{22}\sigma_{33} + \sigma_{33}\sigma_{11}) + \frac{1}{2G} (\sigma_{11}^2 + \sigma_{23}^2 + \sigma_{31}^2)$$
(1)

Where the elastic moduli are those of the uncracked solid.

When the wing cracks are activated the energy density can be found by a two-step process. First, seal the cracks ( $N_V$  per unit volume) and apply the external loads. The strain energy density is given by  $W_0$  as above. Then, without changing the loads, release the cracks. This results in a change  $\Delta W$  in the strain energy per crack, whereupon the strain energy becomes

$$W = W_0 + N_V \Delta W \tag{2}$$

The energy for a fixed value of  $\ell/a$  is

$$\Delta W = \frac{1}{E} \int_0^a K_I^2 \, 2\pi a da \tag{3}$$

We wish to put the Ashby and Sammis expression for the stress intensity factor in the form

$$K_I = \sqrt{\pi a} (A\sigma_1 + B\sigma_3) \tag{4}$$

so that we can use it in the energy equation and stress-strain relations we have already found following the methodology in Bhat et al. and Thomas et al. Ashby and Sammis wrote the stress intensity factor at the tips of the wing cracks as

$$K_{I} = \frac{F_{3}}{\left(\pi(l+\beta a)\right)^{3/2}} + \frac{2}{\sqrt{\pi}} \left(\sigma_{3} + \sigma_{3}^{i}\right) \sqrt{l}$$
(5)

where the wedging force is

$$F_3 = -(A_1\sigma_1 - A_3\sigma_3)a^2$$
 (6)

The internal stress in (5) is

$$\sigma_3^i = \frac{F_3}{A_{crack} - \pi (l + \alpha a)^2} \tag{7}$$

and the average area per crack in (7) is

$$A_{crack} = \pi^{1/3} \left(\frac{3}{4N_V}\right)^{2/3} = \frac{\pi(\alpha a)^2}{D_0^{2/3}}$$
(8)

The constants are

$$A_1 = \frac{\pi\sqrt{\beta}}{\sqrt{3}} \left( (1+f^2)^{1/2} - f \right)$$
(9)

$$A_3 = A_1 \left\{ \frac{(1+f^2)^{1/2} + f}{(1+f^2)^{1/2} - f} \right\}$$
(10)

In the 1D model  $\beta$  was set to  $\beta = 0.1$  to fit the numerical simulations. In the 3D model we treat  $\beta$  as an adjustable parameter. Based on the fits to a wide variety of rock data in Ashby and Sammis (1999) we take  $\beta = 0.45$ .

Ashby and Sammis [eqn. (26)] give the following equation for the stress intensity factor

$$K_{I} = \frac{-A_{1}\sigma_{1}\sqrt{\pi a}}{\pi^{2}\alpha^{3/2}((D/D_{0})^{1/3} - 1 + \beta/\alpha)^{3/2}} \left\{ \left(1 - \frac{A_{3}}{A_{1}}\lambda\right) \left[1 + 2\left((D/D_{0})^{1/3} - 1\right)^{2} \left(\frac{D_{0}^{2/3}}{1 - D^{2/3}}\right)\right] - \frac{2\lambda}{A_{1}}\alpha^{2}\pi^{2}\left((D/D_{0})^{1/3} - 1\right)^{2} \right\}$$
(11)

where

$$\lambda = \frac{\sigma_3}{\sigma_1} \tag{12}$$

Writing  $K_{\rm I}$  explicitly in terms of  $\sigma_1$  and  $\sigma_3$ 

$$K_{I} = \frac{-A_{1}\sigma_{1}\sqrt{\pi a}}{\pi^{2}\alpha^{3/2}((D/D_{0})^{1/3} - 1 + \beta/\alpha)^{3/2}} \left\{ \left(1 - \frac{A_{3}}{A_{1}}\frac{\sigma_{3}}{\sigma_{1}}\right) \left[1 + 2\left((D/D_{0})^{1/3} - 1\right)^{2} \left(\frac{D_{0}^{2/3}}{1 - D^{2/3}}\right)\right] - \frac{2}{A_{1}}\frac{\sigma_{3}}{\sigma_{1}}\alpha^{2}\pi^{2}\left((D/D_{0})^{1/3} - 1\right)^{2} \right\}$$

$$(13)$$

Define

$$c_1 = \frac{1}{\pi^2 \alpha^{3/2} \left( (D/D_0)^{1/3} - 1 + \beta/\alpha \right)^{3/2}}$$
(14)

$$c_{2} = 1 + 2\left((D/D_{0})^{1/3} - 1\right)^{2} \left(\frac{D_{0}^{2/3}}{1 - D^{2/3}}\right)$$
(15)

$$c_3 = 2\alpha^2 \pi^2 \left( (D/D_0)^{1/3} - 1 \right)^2 \tag{16}$$

as in Deshpande and Evans (eqn. 9). Then

$$K_{I} = -A_{1}c_{1}\sigma_{1}\sqrt{\pi a} \left\{ c_{2} \left( 1 - \frac{A_{3}}{A_{1}}\frac{\sigma_{3}}{\sigma_{1}} \right) - c_{3} \frac{1}{A_{1}}\frac{\sigma_{3}}{\sigma_{1}} \right\}$$
(17)

which can be written

$$K_{I} = -A_{1}c_{1}\sigma_{1}\sqrt{\pi a} \left\{ c_{2} - c_{2}\frac{A_{3}}{A_{1}}\frac{\sigma_{3}}{\sigma_{1}} - c_{3}\frac{1}{A_{1}}\frac{\sigma_{3}}{\sigma_{1}} \right\}$$
(18)

or

$$K_{I} = -c_{1}c_{2}A_{1}\sigma_{1}\sqrt{\pi a} + c_{1}c_{2}A_{3}\sigma_{3}\sqrt{\pi a} + c_{1}c_{3}\sigma_{3}\sqrt{\pi a}$$
(19)

This is in the required form

$$K_I = \sqrt{\pi a} (A\sigma_1 + B\sigma_3) \tag{20}$$

$$A = -c_1 c_2 A_1 \tag{21}$$

$$B = c_1 c_2 A_3 + c_1 c_3 \tag{22}$$

Now set  $K_I = K_{Ic}$  to get an equation for  $\sigma_1$  as a function of  $\sigma_3$  and D. This is the equation we need to solve at each time step for crack growth. We can continue to calculate the effective moduli for the cracked solid.

$$\Delta W = \frac{1}{E} \int_0^a K_I^2 2\pi a da = \frac{1}{E} \int_0^a \pi a (A\sigma_1 + B\sigma_3)^2 2\pi a da = \frac{2\pi^2 a^3}{3E} (A\sigma_1 + B\sigma_3)^2$$
(23)

Using  $E = 2(1 + \nu)G$  and  $D_0 = \frac{4}{3}\pi N_V(\alpha a)^3$ 

$$N_{V}\Delta W = \frac{\pi D_{0}}{4\alpha^{3}G(1+\nu)} (A\sigma_{1} + B\sigma_{3})^{2}$$
(24)

The stress-strain expression is found by differentiation W

$$\varepsilon_{ij} = \frac{\partial W}{\partial \sigma_{ij}} \tag{26}$$

In the principal coordinate system

$$\varepsilon_1 = \frac{\partial W}{\partial \sigma_1} = \frac{\partial W_0}{\partial \sigma_1} + \frac{\pi D_0}{4\alpha^3 G(1+\nu)} 2A(A\sigma_1 + B\sigma_3)$$
(27)

$$\varepsilon_2 = \frac{\partial W}{\partial \sigma_2} = \frac{\partial W_0}{\partial \sigma_2} \tag{28}$$

$$\varepsilon_3 = \frac{\partial W}{\partial \sigma_3} = \frac{\partial W_0}{\partial \sigma_3} + \frac{\pi D_0}{4\alpha^3 G(1+\nu)} 2B(A\sigma_1 + B\sigma_3)$$
(29)

For the elastic components we use the isotropic compliance matrix

$$\varepsilon_{11} = \frac{1}{2(1+\nu)G} [\sigma_{11} - \nu(\sigma_{22} + \sigma_{33})]$$
(30)

$$\varepsilon_{22} = \frac{1}{2(1+\nu)G} \left[ \sigma_{22} - \nu (\sigma_{11} + \sigma_{33}) \right]$$
(31)

$$\varepsilon_{33} = \frac{1}{2(1+\nu)G} \left[ \sigma_{33} - \nu (\sigma_{11} + \sigma_{22}) \right]$$
(32)

$$\varepsilon_{23} = \frac{\sigma_{23}}{2G} \qquad \varepsilon_{13} = \frac{\sigma_{13}}{2G} \qquad \varepsilon_{12} = \frac{\sigma_{12}}{2G}$$
(33)

Combining the elastic and damage contributions

$$\varepsilon_{1} = \frac{1}{2G(1+\nu)} \left\{ \left[ \sigma_{1} - \nu(\sigma_{2} + \sigma_{3}) \right] + \frac{\pi D_{0}}{\alpha^{3}} A(A\sigma_{1} + B\sigma_{3}) \right\}$$
(34)

$$\varepsilon_2 = \frac{1}{2G(1+\nu)} [\sigma_2 - \nu(\sigma_1 + \sigma_3)]$$
(35)

$$\varepsilon_{3} = \frac{1}{2G(1+\nu)} \left\{ \left[ \sigma_{3} - \nu(\sigma_{1} + \sigma_{2}) \right] + \frac{\pi D_{0}}{\alpha^{3}} B(A\sigma_{1} + B\sigma_{3}) \right\}$$
(36)

In matrix form

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \end{bmatrix} = \frac{1}{2G(1+\nu)} \begin{bmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \end{bmatrix}$$
(37)

where

$$M_{11} = \left(1 + \frac{\pi D_0}{\alpha^3} A^2\right)$$
(38)

$$M_{12} = -v \tag{38}$$

$$M_{13} = \left(-\nu + \frac{\pi D_0}{\alpha^3} AB\right) \tag{39}$$

$$M_{21} = -v$$
  $M_{22} = 1$   $M_{23} = -v$  (40)

$$M_{31} = \left(-\nu + \frac{\pi D_0}{\alpha^3} AB\right) \qquad M_{32} = -\nu \qquad M_{33} = \left(1 + \frac{\pi D_0}{\alpha^3} B^2\right)$$
(41)

In matrix form

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \end{bmatrix} = \left(\frac{1}{2G(1+\nu)}\right) \begin{bmatrix} \left(1 + \frac{\pi D_0}{\alpha^3} A^2\right) & -\nu & \left(-\nu + \frac{\pi D_0}{\alpha^3} AB\right) \\ -\nu & 1 & -\nu \\ \left(-\nu + \frac{\pi D_0}{\alpha^3} AB\right) & -\nu & \left(1 + \frac{\pi D_0}{\alpha^3} B^2\right) \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \end{bmatrix}$$
(42)

It remains to invert *M* to find expressions for  $\sigma_1$ ,  $\sigma_2$ , and  $\sigma_3$  that can be used to update the stress state.

Let

$$\bar{A}_{1} = \left(1 + \frac{\pi D_{0}}{\alpha^{3}}A^{2}\right) \quad \bar{A}_{2} = \left(-\nu + \frac{\pi D_{0}}{\alpha^{3}}AB\right) \quad \bar{A}_{3} = \left(1 + \frac{\pi D_{0}}{\alpha^{3}}B^{2}\right) \tag{43}$$

$$\bar{G}_1 = \left(\frac{1}{2G(1+v)}\right) \tag{44}$$

Then

$$M = \bar{G}_1 \begin{bmatrix} \bar{A}_1 & -\nu & \bar{A}_2 \\ -\nu & 1 & -\nu \\ \bar{A}_2 & -\nu & \bar{A}_3 \end{bmatrix}$$
(46)

and

$$\det(M) = \bar{G}_1^3 [\bar{A}_1 \bar{A}_3 - \bar{A}_2^2 + v^2 (-\bar{A}_1 + 2\bar{A}_2 - \bar{A}_3)]$$
(47)

$$M^{-1} = \frac{1}{\bar{G}_1} \frac{\begin{bmatrix} (\bar{A}_3 - v^2) & -(-v\bar{A}_3 + v\bar{A}_2) & (v^2 - \bar{A}_2) \\ -(-v\bar{A}_3 + v\bar{A}_2) & (\bar{A}_1\bar{A}_3 - \bar{A}_2^2) & -(-v\bar{A}_1 + v\bar{A}_2) \\ (v^2 - \bar{A}_2) & -(-v\bar{A}_1 + v\bar{A}_2) & (\bar{A}_1 - v^2) \end{bmatrix}}{[\bar{A}_1\bar{A}_3 - \bar{A}_2^2 + v^2(-\bar{A}_1 + 2\bar{A}_2 - \bar{A}_3)]}$$
(48)

Note that these are all written in terms of the principal stresses and strains, so we need to rotate to the directions of the principal stresses in order to use the equations. For spherically symmetric problems, we always have  $\sigma_1$  in the radial direction and  $\sigma_2=\sigma_3$ . We assume that cracks are randomly oriented around the radial direction. We can then use an axisymmetric form for the equations above:

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \end{bmatrix} = \left(\frac{1}{2G(1+\upsilon)}\right) \begin{bmatrix} \left(1+2\frac{\pi D_0}{\alpha^3}A^2\right) & \left(-\nu+\frac{\pi D_0}{\alpha^3}AB\right) & \left(-\nu+\frac{\pi D_0}{\alpha^3}AB\right) \\ \left(-\nu+\frac{\pi D_0}{\alpha^3}AB\right) & \left(1+\frac{\pi D_0}{\alpha^3}B^2\right) & -\upsilon \\ \left(-\nu+\frac{\pi D_0}{\alpha^3}AB\right) & -\upsilon & \left(1+\frac{\pi D_0}{\alpha^3}B^2\right) \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \end{bmatrix}$$

And its inverse, defining  $A_{1s} = 1 + 2 \frac{\pi D_0}{\alpha^3} A^2$ 

$$M^{-1} = \frac{1}{\bar{G}_1} \frac{\begin{bmatrix} (\bar{A}_3^2 - v^2) & -(\bar{A}_2\bar{A}_3 + v\bar{A}_2) & -(\bar{A}_2\bar{A}_3 + v\bar{A}_2) \\ -(\bar{A}_2\bar{A}_3 + v\bar{A}_2) & (A_{1s}\bar{A}_3 - \bar{A}_2^2) & (vA_{1s} + \bar{A}_2^2) \\ -(\bar{A}_2\bar{A}_3 + v\bar{A}_2) & (vA_{1s} + \bar{A}_2^2) & (A_{1s}\bar{A}_3 - \bar{A}_2^2) \end{bmatrix}}{\begin{bmatrix} A_{1s}\bar{A}_3^2 - 2v\bar{A}_2^2 - 2\bar{A}_3\bar{A}_2^2 - v^2A_{1s} \end{bmatrix}}$$
(49)

# **APPENDIX B – DIAGNOSTIC DATA FROM THE SHOAL CALCULAITONS**

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# APPENDIX B - DIAGNOSTIC DATA FROM THE SHOAL CALCULATIONS

To aid in understanding the calculations and the differences in results due to different parameters, we collected an extensive amount of diagnostic data and generated many plots. A sampling of these are shown here.





Figure B-1. K1R. Units are MPa-sqrt(m). K1R must exceed 1.0 MPa-m<sup>1/2</sup> for crack growth to occur.



Figure B-2.  $d(K1R)/dt^*2.e-5/K1C$ . This is the factor that controls crack growth speed. Negative spikes happen when a tensile crack opens.



Figure B-3. Dependence of crack velocity on dK1R.



Figure B-4. Crack growth speed.



Figure B-5. Damage at 0.02 seconds. Top: 1mm (1), 1cm (r). Bottom: 10cm (1). 1m (r).





Figure B-7. Strain in the first 0.1 s.



Figure B-8. Stress in the first 0.1 s.



ShoalSammis8/1cm Station 3, 99.03 m



Figure B-9. Radial displacement and velocity.





Figure B-10. RVP and RDP

#### **B.2** Damage vs. Time



Figures B-11 to B-14 show damage vs. time for Case#8 with a increasing from 1 mm to 1 m.

Figure B-11. Damage vs. Time for Case 8, a=1mm



Figure B-12. Case 8, a=1cm

Approved for public release; distribution is unlimited.



Approved for public release; distribution is unlimited.



100

300

Figure B-14. Case 8, a=1m

Approved for public release; distribution is unlimited.



Figure B-15. Stress vs. time at a distance of 99m in the first 1.0 s. Top row: a=1mm, a=1cm; bottom row a=10cm, a=1m. 0-1s.

Matprops:	Matprops:
nsammis matprop= 1	nsammis matprop= 1
K= 3.790E+11	K= 3.790E+11
G= 2.380E+11	G= 2.380E+11
rho= 2.600E+00	rho= 2.600E+00
E= 5.904E+11	E= 5.904E+11
pois = 2.404E - 01	pois = 2.404E - 01
lambda = 2.203E+11	lambda = 2.203E+11
$x_{rp} = 5.175E \pm 05$	$x_{D} = 5.175E + 05$
$v_{\rm S} = 3.026E \pm 0.5$	$v_{P} = 3.026E \pm 0.5$
$VS = 3.020 \pm 103$	$v_{3} = 2.020 \pm 0.03$
VI- 2.725E+05	VI- 2.725E+05
VIII- 1.000E+03	VIII- 1.000E+03
KIC= 1.000E+08	KIC= 1.000E+08
acrack= 1.000E-01	acrack= 1.000E+00
iric= 6.000E-01	fric= 6.000E-01
D0 = 1.000E - 01	D0 = 1.000E - 01
phi= 5.152E-01	phi= 5.152E-01
alpha= 8.702E-01	alpha= 8.702E-01
Nv= 3.623E+01	Nv= 3.623E-02
beta= 4.500E-01	beta= 4.500E-01
shapefac= 2.000E+00 (Not used)	shapefac= 2.000E+00 (Not used)
End Matprops	End Matprops
Damage Dependent Quantities	Damage Dependent Quantities
D= 1.000E-01	D= 1.000E-01
c1 = 3.356E - 01	c1 = 3.356E - 01
$c^2 = 1.000E \pm 0.0$	$c^2 = 1.000E \pm 0.0$
$c^{3} = 0.000E \pm 00$	$c^{3} = 0.000E + 00$
$\Delta 1 = 6.889 \text{E} - 01$	$\Delta 1 = 6.889 \text{F} - 01$
$\lambda_{3-2} = 1.49F \pm 0.000$	$\lambda_{3} = 2.149E \pm 0.000$
AS = 2.149E100 A Sammia = 2.312E - 01	$AJ = 2 \cdot I + JE + 00$
$A_{\text{Sammin}} = 7.212E_{\text{O}}$	$A_{3}$
$B_{anulis} = 1.052 \pm 00$	$B_{\text{Sallulis}} = 1.052 \pm 00$
E_Sammis= 1.053E+00	E_Sammis= 1.053E+00
AIS= 1.051E+00	AIS= 1.051E+00
AISsym= 0.000E+00	AISsym= 0.000E+00
A2S=-3.199E-01	A2S=-3.199E-01
A3S= 1.248E+00	A3S= 1.248E+00
A4S= 2.088E-01	A4S= 2.088E+00
ASmat(3,3) = 6.963E+11 2.205E+11	ASmat(3,3) = 6.963E+11 2.205E+11
2.209E+11 2.205E+11 6.889E+11 1.892E+11	2.209E+11 2.205E+11 6.889E+11 1.892E+11
2.209E+11 1.892E+11 5.661E+11	2.209E+11 1.892E+11 5.661E+11
MSmat(3,3)= 1.737E-12 -4.071E-13 -5.418E-	MSmat(3,3) = 1.737E-12 -4.071E-13 -5.418E-
13 -4.071E-13 1.694E-12 -4.071E-13 -5.418E-13	13 -4.071E-13 1.694E-12 -4.071E-13 -5.418E-13
-4.071E-13 2.114E-12	-4.071E-13 2.114E-12
ASmatT(3,3) = 6.826E+11 2.066E+11	ASmatT(3,3) = 6.468E+11 1.708E+11
1.768E+11 2.066E+11 6.826E+11 1.768E+11	6.363E+10 1.708E+11 6.468E+11 6.363E+10
1.768E+11 1.768E+11 5.587E+11	6.363E+10 6.363E+10 2.011E+11
MSmatr(3 3) = 1 694E - 12 - 4 071E - 13 - 4 071E - 13	MSma+T(3,3) = 1,694E-12,-4,071E-13,-4,071E-
13 - 4 071E - 13 1 694E - 12 - 4 071E - 13 - 4 071E - 13	13 - 4 071E - 13 1 694E - 12 - 4 071E - 13 - 4 071E - 13
-4 0.71 $E = 13 2 0.47$ $E = 12 - 3.071$ $E = 13 - 3.071$ $E = 13$	-4 071E-13 5 230E-12
$3.07 \pm 0.12$ $2.037 \pm 12$ $3.0m \pm 0.037 \pm 0.037 \pm 11$ $2.0107 \pm 11$	$1.0710 \pm 0.2000 \pm 2$ $\lambda G_{ma} \pm G_{Tm} (3, 3) = 6.063 \pm 11.0.010 \pm 11.0.010 \pm 10.000 \pm 10.0000 \pm 10.0000000000$
ASIMacoym(S,S) = 0.305ETTT 2.210ETTT	AOI(aCOY)(3,3) = 0.303ETTT 2.210ETTT 2.210ET
2.2IUETII 2.2IUETII 3.0ISETII 1.048ETII 2.2IUETII 1.048ETII	2.2IUETII 2.2IUETII 3.0ISETII 1.048ETII 2.2IOELII 1.048ETII 5.0ISELII
2.21UE+11 1.648E+11 5.615E+11	2.2IUE+II 1.648E+II 5.615E+II
MSmatSym(3,3) = 1.780E-12 -5.418E-13 -5.418E-	MSmatSym(3,3) = 1.780E-12 -5.418E-13 -5.418E-
13 -5.418E-13 2.114E-12 -4.071E-13 -5.418E-13	13 -5.418E-13 2.114E-12 -4.071E-13 -5.418E-13
-4.071E-13 2.114E-12	-4.071E-13 2.114E-12
End Damage Dependent Quantities	End Damage Dependent Quantities
1 mm (cgs units)	1 cm (cgs units)

Matprops:	Matprops:
nsammis matprop= 1	nsammis matprop= 1
K= 3.790E+11	K= 3.790E+11
G= 2.380E+11	G= 2.380E+11
rho= 2.600E+00	rho= 2.600E+00
E= 5.904E+11	E= 5.904E+11
pois= 2.404E-01	pois= 2.404E-01
lambda= 2.203E+11	lambda= 2.203E+11
vp = 5.175E + 05	vp = 5.175E + 05
vs = 3.026E + 05	vs = 3.026E + 05
vr = 2.723E + 0.5	$vr = 2.723E \pm 0.5$
vm = 1.000E + 0.5	vm = 1.000E + 0.5
$K1C = 1.000E \pm 0.8$	$K1C = 1.000E \pm 0.8$
$acrack = 1.000E \pm 01$	$acrack = 1.000E \pm 02$
fric= 6.000E-01	fric= 6.000E-01
D0 = 1 000E - 01	$D0 = 1 \ 0.00E - 01$
$phi = 5 \ 152E - 01$	$phi = 5 \ 152E - 01$
alpha = 8,702E-01	$p_{\rm MI} = 8.702E - 01$
$N_{V} = 3.623E - 05$	$N_{V} = 3.623E - 0.8$
hv = 3.025H = 03	hv = 3.025H = 00
peta = 1.500E 01 shapeface 2 000F+00 (Not used)	peta = 4.500E 01 shapeface 2 000E+00 (Not used)
End Mathrong	End Matprops
Damage Dependent Quantities	Damage Dependent Quantities
Damage Dependent Quantities $D = 1.000 E = 01$	Damage Dependent Quantities $D = 1.000 E = 01$
D = 1.000E = 01	D = 1.000E = 01
$c_1 = 3.330E = 01$	$c_{1-}$ 3.330E-01
C2 = 1.000E+00	C2 = 1.000E+00
$C_{3} = 0.000E_{100}$	$C_{3} = 0.000E_{100}$
AI- 0.009E-01	AI- 0.009E-01
AJ = 2.149ETUU	AJ = 2.149ETUU
$A_{\text{Sammins}} = -2.312E - 01$	A Sammis=-2.312E-01
$B_{\text{Sammin}} = 1.052 \pm 00$	$B_{\text{Sammin}} = 1.052 \pm 00$
E_Sammis= 1.053E+00	E_Sammis= 1.053E+00
AIS= 1.051E+00	AIS= 1.051E+00
AISSYM= 0.000E+00	AISSYM= 0.000E+00
A25=-3.199E-01	A25=-5.199E-01
ASS= 1.248E+00	A35= 1.248E+00
$A4S = 2.088E \pm 01$	$A4S = 2.088E \pm 02$
ASmat(3,3) = 6.963E+11 2.205E+11	ASmat(3,3) = 6.963E+11 2.205E+11
2.209E+11 2.205E+11 6.889E+11 1.892E+11	2.209E+11 2.205E+11 6.889E+11 1.892E+11
2.209E+11 1.892E+11 5.661E+11	2.209E+11 1.892E+11 5.661E+11
MSmat(3,3) = 1./3/E - 12 - 4.0/1E - 13 - 5.418E -	MSmat(3,3) = 1./3/E - 12 - 4.0/1E - 13 - 5.418E -
13 -4.0/1E-13 1.694E-12 -4.0/1E-13 -5.418E-13	13 -4.071E-13 1.694E-12 -4.071E-13 -5.418E-13
-4.071E-13 2.114E-12	-4.071E-13 2.114E-12
ASmatT(3,3) = 6.293E+11 1.533E+11	ASmatT(3,3) = 6.269E+11 1.509E+11
8.598E+09 1.533E+11 6.293E+11 8.598E+09	8.911E+08 1.509E+11 6.269E+11 8.911E+08
8.598E+09 8.598E+09 2.717E+10	8.911E+08 8.911E+08 2.816E+09
MSmatT(3,3) = 1.694E - 12 - 4.071E - 13 - 4.071E - 13	MSmatT(3,3) = 1.694E - 12 - 4.071E - 13 - 4.071E - 13
13 -4.071E-13 1.694E-12 -4.071E-13 -4.071E-13	13 -4.071E-13 1.694E-12 -4.071E-13 -4.071E-13
-4.071E-13 3.706E-11	-4.071E-13 3.553E-10
ASmatSym(3,3) = 6.963E+11 2.210E+11	ASmatSym(3,3) = 6.963E+11 2.210E+11
2.210E+11 2.210E+11 5.615E+11 1.648E+11	2.210E+11 2.210E+11 5.615E+11 1.648E+11
2.210E+11 1.648E+11 5.615E+11	2.210E+11 1.648E+11 5.615E+11
MSmatSym(3,3) = 1.780E-12 -5.418E-13 -5.418E-	MSmatSym(3,3) = 1.780E-12 -5.418E-13 -5.418E-
13 -5.418E-13 2.114E-12 -4.071E-13 -5.418E-13	13 -5.418E-13 2.114E-12 -4.071E-13 -5.418E-13
-4.071E-13 2.114E-12	-4.071E-13 2.114E-12 End Damage Dependent
End Damage Dependent Quantities	Quantities
10 cm (cgs units)	1 m (cgs units)

Figure B-16. Calculated quantities for Sammis micromechanical damage model for four values of the initial crack size acrack.

# List of Symbols, Abbreviations, and Acronyms

AFRL	Air Force Research Laboratory
NTS	Nevada Test Site
USC	University of Southern California

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