The Use of Game Theory at the Operational Level

A Monograph

by

MAJ Nathan A. Lunde
US Army

School of Advanced Military Studies
US Army Command and General Staff College
Fort Leavenworth, KS

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Political scientists and national level policymakers have used game theory at the strategic level for decades, but there exist minimal reviews of its use at the operational level. Three main challenges have traditionally prevented planners and analysts from using game theory at the operational level, the complex operational environment, the dynamic interactions of the actors and, the use of complex mathematical skills that most Army staff officers do not possess. This monograph demonstrates that these challenges can be overcome, and game theory can provide novel insight during the planning process. Army staff planners can fruitfully use basic game theory with simple math at the operational level for understanding the operational environment, understanding actors and their motivations, and comparing courses of action during the military decision-making process. This monograph shows how to avoid the cumbersome mathematical procedures advanced game theory uses to solve theoretical problems and instead focuses on using basic game theory to provide value during the operations process. It demonstrates the utility of game theory by reviewing the uses of game theory at the strategic level, teaching basic game theory, and covering a few basic games. Then it examines a historic campaign to show how the use of game theory would have arrived at an alternate recommended course of action and outcome, perhaps altering the course of history. Finally, it provides a guide to use game theory by applying it to an exercise at two steps of the military decision-making process, mission analysis, and course of action development.

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Approved by:

_____, Monograph Director
Philip Hultquist, PhD

_____, Seminar Leader
Leroy Bryant Butler, LtCol

_____, Director, School of Advanced Military Studies
Brian A. Payne, COL

Accepted this 21st day of May 2020 by:

_____, Acting Director, Office of Degree Programs
Prisco R. Hernandez, PhD

Abstract

The Use of Game Theory at the Operational Level, by MAJ Nathan A. Lunde, 47 pages.

Political scientists and national level policymakers have used game theory at the strategic level for decades, but there exist minimal reviews of its use at the operational level. Three main challenges have traditionally prevented planners and analysts from using game theory at the operational level, the complex operational environment, the dynamic interactions of the actors and, the use of complex mathematical skills that most Army staff officers do not possess. This monograph demonstrates that these challenges can be overcome, and game theory can provide novel insight during the planning process. Army staff planners can fruitfully use basic game theory with simple math at the operational level for understanding the operational environment, understanding actors and their motivations, and comparing courses of action during the military decision-making process. This monograph shows how to avoid the cumbersome mathematical procedures advanced game theory uses to solve theoretical problems and instead focuses on using basic game theory to provide value during the operations process. It demonstrates the utility of game theory by reviewing the uses of game theory at the strategic level, teaching basic game theory, and covering a few basic games. Then it examines a historic campaign to show how the use of game theory would have arrived at an alternate recommended course of action and outcome, perhaps altering the course of history. Finally, it provides a guide to use game theory by applying it to an exercise at two steps of the military decision-making process, mission analysis, and course of action development.

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Abbreviations

NATO	North Atlantic Treaty Organization
US	United States
USSR	Union of Soviet Socialist Republics

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Introduction

Strategic planners and policy makers usefully applied game theory at the strategic level for decades, but military practitioners tend to not use it at the operational level. When John von Neumann and Oskar Morgenstern developed game theory in the early 1940s while working for RAND, they pursued a mathematical approach to provide solutions to areas of conflict, especially economic conflict. They published their groundbreaking work, *The Theory of Games and Economic Behavior,* in 1944.[1]

Game theory allows for the analysis of decision making by modeling scenarios into simplified games. Game theory attempts to define the players, strategies—or choices available to them—and the expected payoffs from the games' outcomes. It attempts to clarify the uncertainty due to the players' choices. Its primary utility is that it recognizes that outcomes are mutually determined through the interaction of multiple actors, not simply the result of one's own decisions and it allows for an analysis of what opponents are likely to do.[2] For these reasons, policy makers and strategists use game theory to understand strategic problems, such as nuclear adversaries, trade practices, civil war settlements and disarmament, and the lack of international cooperation, resulting in the development of recommendations for policy to help solve these problems.[3]

Whether or not operational level planners can usefully apply game theory is still an open question. Critics of using game theory at the operational level highlight the complexity of dynamic interaction. They note the large time requirement it takes to train officers in the basic concepts of game theory and distill the complexity of operational level problems into a basic game.

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[1] Lawrence Freedman, *Strategy* (New York: Oxford University Press, 2013), 151.

[2] Scott Wolford, *The Politics of the First World War: A Course in Game Theory and International Security* (Cambridge, UK: Cambridge University Press, 2019), 25-26.

[3] Freedman, *Strategy*, 151-177.

This monograph argues that game theory provides a valuable framework, best applied to understanding the players in the environment during the mission analysis and course of action development steps of the Military Decision-Making Process. Game theory seeks to provide understanding about a situation. This requires understanding the actors and their incentives for their potential plans or strategies. Game theory delivers a rational approach to how actors develop their strategies and the basis for their motivations. From this, the commander and staff can gain understanding and then overlay additional factors which include courses of action and potential outcomes. It provides a rational and straightforward approach to simplify complex issues. Thus, game theory gives operational planners another tool to use to gain an understanding of the operational environment.

This monograph focuses on the historical use and application of game theory at the strategic level, current planning process doctrine, and relevant frameworks to answer whether operational planners can use game theory fruitfully at the operational level. This monograph analyzes game theory's use at the strategic level primarily through the prisoner's dilemma as applied to the cold war, international trade, and price cuts during a price war. The New Jersey Campaign in 1777 provides a historical example to apply game theory and understand the competitive environment between Generals George Washington and Charles Cornwallis. Finally, it demonstrates how and where to implement game theoretic tools into current planning processes used by the United States Army. The game theory used is a basic applied approach and not advanced academic game theory which becomes overly complex and unhelpful. Simple games can bring clarity to complex operational situations. The research reviews Army planning doctrine to concentrate on understanding the operational environment and the problem. Mission analysis seeks to understand the actors in the environment and the source of conflict between them.[4] This

[4] US Department of the Army, Field Manual 6-0, *Commander and Staff Organization and Operations* (Washington, DC: Government Printing Office, 2014), 9-6.

research focuses on gaining insights into both adversarial and neutral participants, incentives, potential courses of action, and payoffs. The monograph traces the differences between the strategic and operational application of game theory to understand what elements are consistent while illustrating the differences. Last, it will discuss how to overcome potential challenges to implementation.

What is Game Theory?

Game theory uses rigorous mathematical approaches to analyze conflict and cooperation. It uses an abstract game to analyze a competitive situation with at least two actors. The analysis identifies how actors would rationally approach a situation, assuming simply that they want to earn a higher payoff. Straffin explains four characteristics of games.[5] First, two or more players interact with each other. Second, each player possesses at least two courses of action, strategies from which to choose. Third, the combined strategies of each player lead to the results of the games. Finally, after the game, each player receives a payoff determined by the combined strategies. Dixit, Skeath and Reiley define a strategy as a complete plan of action for an actor given another actor's complete set of actions.[6] Beyond these four characteristics, some game theorists add three other basic assumptions. One is the idea that the actors are rational. The second is each player prefers a higher payoff than lower in the absolute, not relative terms. The third is each player decides with full information about the game.[7] Additionally, games either move sequentially or simultaneously, which significantly determines a player's strategy and payoff.[8]

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[5] Philip D. Straffin, *Game Theory and Strategy* (Washington, DC: The Mathematical Association of America, 1993), 3.

[6] Avinash Dixit, Susan Skeath, and David Reiley, *Games of Strategy* (New York, NY: W. W. Norton and Company, 2015), 710.

[7] Van Pham, "Game Theory: Nash Equilibria," (lecture for ECON 5318 Game Theory, Baylor University Master of Business Administration, Waco, TX, January 12, 2017).

[8] Matthew O. Jackson, "Papers," SSRN, December 5, 2011, accessed July 13, 2019, https://ssrn.com/abstract=1968579.

Simultaneous Games

Wolford provides an excellent introductory game to analyze the above characteristics using two countries on the brink of war in a simultaneous game. There are two players, France led by Jomini, and Prussia, led by Clausewitz. Imagine France demands a portion of Prussia, Alsace and Lorraine. Also, Prussia now faces the choice of ceding land or not. Jomini can ask for all the property, Alsace and Lorraine, or half of it, just Alsace. Further, Prussia can cede half the land, Alsace, or all of it, both Alsace and Lorraine. Thus, there are now two strategies for each player, all, or half. Because each player possesses two strategies, there exist four outcomes. They are, as expressed by Jomini first then Clausewitz; half/half, half/all, all/half, and all/all.

Next, a qualitative analysis allows for understanding how each player assesses each outcome. Clausewitz does not want to cede any more territory than he must. Likewise, Jomini prefers to gain as much land as he can. If Jomini demands only half of the land and Clausewitz agrees to give half, then they reach an agreement. The top left quadrant shows this outcome. If Jomini asks for half and Clausewitz is willing to give all, then Jomini gets half, but could have received more. The top right quadrant depicts this outcome. But, if Jomini asks for all and Clausewitz is only willing to give half, then the two go to war which the bottom left quadrant highlights. Finally, if Jomini asks for all and Clausewitz willingly cedes all then again, Jomini receives all and receives his highest payoff. Table 1, below, shows the qualitative analysis.

Table 1. Qualitative Explanation of Outcomes

Qualitative Explanation of Outcomes		**Clausewitz**	
		Half	All
Jomini	Half	Agree to terms Jomini receives half	Agree to terms, Jomini receives half but could have received more
	All	War, Jomini asks more than Clausewitz is willing to give	Jomini gets all he asked for, Clausewitz willfully cedes all land

Source: Created by the Author.

This analysis allows for the assigning of payoffs to each of the actors' mutually arrived outcomes. The first number in each quadrant shows Jomini's payoff and the second number shows Clausewitz's payoffs. Jomini prefers to receive all that he asks, indicated by his highest payoff of four. But he does not want war with Clausewitz, which is very costly, the lowest payoff of one displays this. And if he asks for half, regardless of what Clausewitz is willing to give, he will receive half and a payoff of two for each of those two outcomes. Likewise, Clausewitz also has preferences. He prefers to only give up half of the disputed territory. Therefore, he receives his highest payoff of four in each of the top quadrants. If Jomini only asks for half, Clausewitz only gives up half. But he does not want to give up all his land, as indicated by the payoff of one. Finally, Clausewitz is willing to fight for half of the land if Jomini should ask for all resulting in a payoff of three. The lower left quadrant indicates the lowest cumulative payoff because it factors in the cost of each country going to war. Jomini wins the war and takes the territory, but the war's cost outweighs the gain of territory which is why he has the lowest payoff in that quadrant. And, Clausewitz only gives up half his territory but had to fight to not lose all of it resulting in a lower score than the top two quadrants. But since he prevented losing all the territory, he still gets a higher payoff than if he had given all the territory. Table 2, below, shows a simultaneously played game in the strategic form.[9]

Table 2. Quantitative Explanation of Outcomes

Game Theory Introduction		**Clausewitz**	
		Half (Alsace)	All (Alsace and Lorraine)
Jomini	Half (Alsace)	2, 4	2, 4
	All (Alsace and Lorraine)	1, 3	4, 1

Source: Scott Wolford, *The Politics of the First World War: A Course in Game Theory and International Security* (Cambridge, UK: Cambridge University Press, 2019), 28-31.

[9] Wolford, *The Politics of the First World War,* 28-31.

This matrix in strategic form, now allows for the analysis of the game and how each player should rationally approach the situation. Players now look for dominant strategies and strategies to eliminate. A dominant strategy exists where a player knows that he/she will always play a strategy, regardless of what the other player chooses. Conversely, a dominated strategy exists when a player knows that he/she will never play a strategy regardless of what the other player chooses. Using the above example, Jomini does not have any dominant strategies. Going further, if Clausewitz chooses half then Jomini receives either a two, if he chooses half, or one if he chooses all. Therefore, when Clausewitz chooses half, Jomini chooses half. But, if Clausewitz chooses all, then Jomini gets a four if he chooses all and a two if he chooses half. In this case he would choose all, not half as he did if Clausewitz chooses half. This exhibits that Jomini's strategy is dependent on what he thinks Clausewitz chooses. However, Clausewitz does possess a weakly dominant strategy. Clausewitz will always play half regardless of Jomini's choice. That is because Clausewitz gets either a four or a three with half and a four and a one with all. Also, if Clausewitz does decide to play all because it still results in a payoff of four for him, it would never occur. That is because Jomini would never play half to Clausewitz's all. Which is why both playing half is a stable equilibrium outcome.[10]

Looking for equilibrium outcomes is the next step after analyzing players, strategies, and results. An equilibrium outcome means that the two players reach an outcome owing to strategies that neither player is willing to deviate from, or has an incentive to change.[11] The equilibrium outcome for the above game is that each player chooses half resulting in Jomini receiving two points and Clausewitz receiving four points.

This equilibrium outcome is a Nash equilibrium. Dixit and Nalebuff define a Nash equilibrium as "an outcome where each player in the game chooses the strategy that best serves

[10] Drew Fudenberg and Jean Tirole, *Game Theory* (Cambridge, MA: The MIT Press, 1995), 6-9.

[11] Straffin, *Game Theory and Strategy*, 8.

his or her interest, in response to the other's strategy."[12] The Nash equilibrium has three characteristics. First, it is stable, which leads into the second, it constitutes mutually best responses. The first two characteristics reinforce each other. Third, ex-ante beliefs prove ex-post correct. The third characteristic means that prior assumptions, or beliefs, about the situation prove themselves correct after the players interact.[13] Neither player can do any better than choosing half/half in this situation. If a player decided to deviate, then the other player will change their strategy as well. For instance, if Jomini agreed to play all hoping to receive four and leaving Clausewitz with one, then Clausewitz will play half because he can win three, leaving Jomini with one. However, then Jomini knowing Clausewitz chooses half will also choose half because he can go from one to two points, leaving them back at the equilibrium choice of half/half. Further, Jomini knows that Clausewitz possesses a dominant strategy of choosing half and therefore bases his strategy selection on that fact.

Sequential Move Games

The above game showed Jomini and Clausewitz acting simultaneously. A simultaneous game provides the context of the information each player possesses and how they work, knowing what the other player knows and knowing how the other player works based on what they know. But not all games or situations occur where each player moves at the same time. A player acts, and the other players must respond with the best choice given the first player's move. The extensive (game tree) form illustrates the strategies, players, and payoffs of a game played sequentially.[14]

Additionally, the extensive form shows six pieces of information about the game. The six pieces of information are: the actors involved, the sequence of the game, the payoffs to the

[12] Dixit and Nalebuff, *The Art of Strategy*, 104.

[13] Pham, "Game Theory: Nash Equilibria."

[14] Wolford, *The Politics of the First World War*, 297.

players based on the sequence, the choices a player faces based on their order in the series, the information available to the players, and the probability each player chooses a strategy based on outside inputs, or exogenous effects.[15] The last characteristic is beyond the scope of this monograph.

The above scenario between Jomini and Clausewitz still provides a useful example for sequential games. The players, strategies, and payoffs all remain the same. In this case, Jomini moves first and either demands both Alsace and Lorraine or just Alsace. Now, Clausewitz, faced with these two strategies, must decide which best leads to his highest payoff. Clausewitz now must choose to cede both or only, Alsace. To determine the best course of action, the players use backward induction to solve the game or determine the subgame perfect equilibria in the game. From this, they derive an equilibrium strategy leading to a mutually beneficial choice. Backward induction allows players to analyze a game based on the final player's set of options. They determine which action the last mover would choose for each subgame and then move back a step to determine what the previous player would optimally choose knowing what the final actor played.[16] Wolford defines a subgame perfect equilibrium as the strategy each actor can do the best given what has already occurred in the game.[17] A subgame contains the choices available to a player at a single node in the game.[18] Table 3, below, shows a game in its extensive, sequential, form. The first number represents Jomini's payoff and the second number represents Clausewitz's payoff.

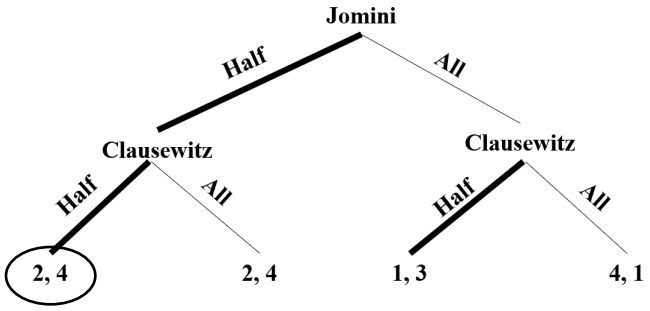
[15] Fudenberg and Tirole, *Game Theory*, 77.

[16] Ibid., 68-69.

[17] Wolford, *The Politics of the First World War*, 297.

[18] Ibid., 297.

Table 3. Extensive Form



Source: Created by Author.

The Jomini-Clausewitz game possesses three subgames. First, Clausewitz can play half or all based on Jomini playing all. Second, Clausewitz plays half or all based on Jomini playing half. The final subgame is of Jomini choosing between half or all. With this information, Clausewitz always plays half in each of his two subgames, as denoted by the bold lines in table 3. The payoffs for his choices indicate this. Additionally, if he chooses all to Jomini's half, that would lead to a self-harming payoff. He would be giving up both territories when Jomini requested only one and is not a rational choice. The last stated strategy further supports the point made above about him playing half as a dominant strategy.

Further, since Jomini expects that Clausewitz always plays half, Jomini then always chooses to play half. This result leads to the Nash Equilibrium determined above in the simultaneous game. Further this gives the subgame perfect equilibrium of Jomini still playing half and Clausewitz again playing half. Each playing half is because if Jomini played all knowing Clausewitz plays half he receives a payoff of one when he could have won two if he had played half.

The relevance of showing games in sequential moves is that it shows games in a more realistic manner, and it demonstrates why some strategies are not credible. Given that Clausewitz possesses a dominant strategy of playing half, it is not credible that he would ever play all. Further, because of this knowledge using backward induction, Jomini plays half, and it is not credible that he would choose all. If the two negotiate this process, it is not credible that

Clausewitz offers both territories, the all strategy, at any time.[19] In a real-world application, it allows for the analysis of people's choices and whether they would execute a strategy just because they have it.

Game theory literature supports using the theory for analyzing competitors and the competitive environment while addressing the limitations of the theory. There are strengths and weaknesses to using game theory. The following sections examine them by looking at arguments by game theory's proponents and detractors.

Literature Review

Many professionals use game theory for competitive situations and scenarios involving conflict. It provides understanding of incentives, payoffs, and strategies of the different players or opponents. International relations strategists use it for national-level policy, and business professionals use it for business strategy. The literature supporting the use of game theory for decision making and understanding the environment in which an organization operates, mainly comes from business and economic strategies. However, some research shows game theory's lack of utility due to cumbersome math and its overreliance on the principle of the perfectly rational human, homo economicus.[20] Lastly, and importantly, military planners researched the use of game theory at the operational level during the 1950s, with little research specifically at that level of operations until recently. The literature about game theory and its application divides it into four areas: pro-game theory at the strategic level, pro-game theory at the operational level, anti-game theory at the strategic level and anti-game theory at the operational level, see table 4. The first set of research to review supports game theory as a tool for understanding the environment and making decisions.

[19] Dixit and Nalebuff, *The Art of Strategy*, 201-208.

[20] John T. Hanley Jr. "Planning for the Kamikazes: Toward a Theory and Practice of Repeated Operational Games," *Naval War College Review* 70, no. 2 (2017): 32.

Table 4. Literature Review

Literature Review	Pro-Game Theory	Anti-Game Theory
Strategic Level	von Neumann (1944) Freedman (2013) Schelling (1960 & 1966) Wolford (2019) Dixit and Nalebuff (2008) Straffin (1993) Jackson (2011)	Freedman (2013) Brodie (1974) Coyne and Horn (2009)
Operational Level	Haywood (1950) Beebe (1957) Rubinstein (1989) Martin (1978) Wolford (2019) Hanley (2017)	Martin (1978)

Source: Created by the Author.

Pro Game Theory

The first set of literature examines how planners can apply game theory for military operational planning. During the 1950s, shortly after the initial development of game theory, US Army Colonel Oliver Haywood and US Navy Captain Robert Beebe conducted research showing the usefulness of game theory in operational level decisions. Haywood showed how game theory fits into the then current decision-making process called, "Standard Armed Forces Form for the Estimate of the Situation." He expressed how the theory provides an analytical mathematical tool to assist commanders to decide between multiple courses of action.[21] Beebe applied game theory to naval operations. He also merged the use of the theory with the estimate of the situation as did Haywood. Further, Beebe recognized that military commanders make decisions in "conflict situation" with each side in direct opposition to each other.[22] Finally in 2017, John Hanley researched the use of game theory in the Naval War College's Halsey war games. He found that it highlights structural details in the operational games which can aid in the formulation of strategy.

[21] Oliver G Haywood, Jr., "Military Doctrine of Decision and the Von Neumann Theory of Games," (student thesis, The Air War College, Maxwell Air Force Base, Alabama, March 20, 1950), 1.

[22] Robert P. Beebe, "Military Decision from the Viewpoint of Game Theory," *Naval War College Review* 10, no. 2 (1957): 28, 46.

Also, it provides a language and a methodology to analyze elements of the game such as choices and outcomes.[23]

Brian Martin describes game theory for war, military operations, and policy. He states that game theory's ability to abstract itself from the social construct of the players and their beliefs becomes a benefit of its usefulness.[24] Further, Martin describes three other positives of game theory, new theory with little deviations, developed by a single person making it coherent, and focused on conflict.[25] Martin goes on to state game theory's usefulness as a tool to analyze decisions.[26]

Ariel Rubinstein describes a coordination game and the implications of standard information in his article, "*Almost Common Knowledge*." He uses the game of two division commanders coordinating an attack along exterior lines against an enemy in the defense with interior lines. He describes the coordination required and the latency of information between the two commanders. This article describes the importance of understanding the enemy's incentives and objectives as well as the motivations and goals of friendly units. Further, it explores the notion of what information everyone possesses and what information gaps exist. Finally, it looks at what the implications of the information gaps mean for the different players.[27]

Scott Wolford, a professor of Political Science at the University of Texas, uses game theory to explain the strategic and political interactions of World War One. He claims game theory allows for the study of history and strategic interactions by being analytical. It avoids polemic answers to why previous leaders chose their actions. He states it is an "exercise in the

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[23] Hanley Jr., "Planning for the Kamikazes," 33, 35.

[24] Brian Martin, "The Selective Usefulness of Game Theory," *Social Studies of Science* 8, no. 1 (1978): 87.

[25] Ibid., 87.

[26] Ibid., 95.

[27] Ariel Rubinstein, "The Electronic Mail Game: Strategic Behavior Under 'Almost Common Knowledge,'" *The American Economic Review* 79, no. 3 (1989): 385-391.

analytical equivalent of empathy."[28] It morally detaches the practitioner from decision making and avoids ideological or superstitious tendencies.[29] Further, because of the analytical nature of game theory, it abstracts from the emotional and messy situations in which most practitioners exist.[30] Wolford serves as a bridge between the operational to the strategic use of game theory.

The next set of literature examines the strategic use of game theory. Lawrence Freedman offers many examples of how strategists used game theory in the past in both international relations and business. Freedman discusses the beginnings of game theory from John von Neumann and its evolution through nuclear strategy, its misuse, and overreliance on complicated math and calculus. He states that von Neumann thought of game theory to develop intelligent strategies, understanding that chance and uncertainty exist in competitive interactions.[31] Freedman discusses how the economist and nuclear strategist, Thomas Schelling, used analogies and then expressed the analogy through game theory.[32] Schelling further asserted that the heart of game theory included conflict, cooperation, and coercion. He stated that the ability to do harm with military force and communicate that ability to the opponent gave the user impressive military power.[33]

Avinash Dixit and Barry Nalebuff, both professors of economics, wrote "The Art of Strategy: A Game Theorist's Guide to Success in Business and Life," about game theory and its usefulness in making decisions for everyday life and business. They discuss that people form judgments in the face of conflict, and while facing opponents. They make decisions based on the information they possess and try to anticipate what the opposition intends to do.[34] They discuss

[28] Wolford, *The Politics of the First World War*, 7.

[29] Ibid., 7.

[30] Ibid., 8.

[31] Freedman, *Strategy*, 151.

[32] Freedman, *Strategy*, 160.

[33] Ibid., 163.

[34] Dixit and Nalebuff, *The Art of Strategy*, xv.

signaling, screening, information, and information asymmetries. Signaling and information and asymmetries become relevant during information operations and in gaining an understanding of the environment.[35]

Philip Straffin lays out a good definition and use of the game called the prisoner's dilemma in "Game Theory and Strategy." He states this game provides usefulness because much of human behavior and interaction ends up in a prisoner's dilemma scenario, and it allows for the study of outcomes and understanding of the actor's motives.[36] Further, regarding information and signaling, he states prisoner's dilemma games require a commitment device, or as he calls it, "a promise."[37] The idea of the promise builds off the concept of information sharing, and asymmetries addressed in Rubinstein's article. Straffin also discusses the prisoner's dilemma in a repeated play game, which reflects reality better than single-game analysis.[38]

Anti-Game Theory

Lawrence Freedman also supplies some critiques of game theory and how practitioners misused or misunderstood it. Freedman explains game theory's use during the cold war as a tool to understand the bipolar nuclear world of the North Atlantic Treaty Organization competing with the Soviet Union. He states the limited value game theory possesses at developing strategies beyond a bipolar world or when analyzing fighting wars limited to only conventional means and not escalating beyond to nuclear weapons.[39] Game theory does not account for ideology and takes a dispassionate view of the situation. This dispassion becomes a further critique of game theory during the Cold War, a mostly ideological fight between the liberal and democratic west

[35] Dixit and Nalebuff, *The Art of Strategy*, 235-245.

[36] Straffin, *Game Theory and Strategy*, 73-74.

[37] Ibid.

[38] Ibid.

[39] Freedman, *Strategy*, 155.

and the communist Soviet Union.[40] Finally, Thomas Schelling states game theory exists on a spectrum of conflict. The two extremes of the spectrum are no hope for cooperation and the other being conflict does not exist. The theory fails if the scenario moves to either of the extremes.[41]

Much like Freedman, Martin also provides criticisms of game theory. He addresses the most significant criticism with the title of his article, "The Selective Usefulness of Game Theory."[42] For Martin, game theory oversimplifies situations. It simplifies to adapt the case to a mathematical model or game. This oversimplification can leave out pertinent details, and it can introduce bias on the part of the individual who creates the game.[43] Further, game theory's use as an ex post facto analysis tool lends itself to justification. Practitioners can change values in the game to get the results they desire and show how a policy or course of action delivered the desired results.[44]

Despite these valid critiques and shortfalls, game theory still provides value to the operational planner if used properly. First, planners must understand the limitations mentioned above to mitigate any shortfalls. Also, planners must acknowledge the challenges and not overly rely on game theory to act as a panacea and always provide the solution. Game theory is one tool among many in the planner's kitbag. Second, the game theory described below and the approach to using it as a military planning tool avoids the overly complicated math required in academic game theory.

Strategic Level Application and Utility with the Prisoner's Dilemma

The prisoner's dilemma provides one of the first games to analyze a strategic level application of game theory. Merrill Flood and Melvin Dresher developed the prisoner's dilemma

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[40] Freedman, *Strategy*, 162.

[41] Thomas C. Schelling, *The Strategy of Conflict* (Cambridge, MA: Harvard University Press, 1960), 15.

[42] Martin, "The Selective Usefulness of Game Theory," Title.

[43] Ibid., 95.

[44] Ibid., 95, 103.

while working at RAND in 1950.[45] Generally, the prisoner's dilemma, a non-zero-sum game, exhibits a situation where the players mutually arrive at a suboptimal solution even though each possess better options. The scenario starts with the police interrogating two suspects of a crime individually. Each suspect can choose one of two possibilities: cooperate with the other criminal and not snitch or defect and snitch to the police officers. The suspects face prison sentences depending on the combination of their two choices, and each has an internal motivation to minimize their sentence. Both suspects realize if they do not cooperate and the other also does not cooperate, they will receive the lightest sentence, they get a more severe punishment if they both defect, and if the partner defects and the other does not, then the one who defects receives the longest sentence. Table 5, depicts the game in matrix form:

Table 5. Prisoner's Dilemma Matrix

Prisoner's Dilemma Matrix		Jomini	
		Cooperate	Defect
Clausewitz	Cooperate	0, 0	-2, 1
	Defect	1, -2	-1, -1

Source: Created by Author.

As the table indicates, both players do the best if both cooperate. However, both have an incentive to defect if they think their counterpart will defect. Therefore, they both decide to defect, and the stable equilibrium outcome is both defecting and serving a moderate sentence as indicated by the payoff negative one and negative one.[46] The prisoner's dilemma allows for the analysis of situations where two actors can work together, cooperate, or work against each other, defect.[47]

[45] Straffin, *Game Theory and Strategy*, 73.

[46] Straffin, *Game Theory and Strategy*, 73.

[47] Freedman, *Strategy*, 155.

Nuclear Situations

Thomas Schelling used game theory to analyze the Cold War nuclear situation between the United States and the Union of Soviet Socialist Republics. He viewed the conflict between the United States and the USSR as a nonzero-sum game. He thought this because the nature of the conflict between the two nuclear powers was "mixed with mutual dependence."[48] Schelling goes on to explain that mutual dependence exists, despite the focus of the conflict, because of the mutual desire to avoid disaster or complete destruction. He grouped the nuclear conflict with other games of mutual dependence such as strikes and negotiations, class and race war, and working in a bureaucracy. He found that in his research traditional game theory did not adequately explain the nature and outcomes of such interactions.[49] He focused his research and analysis on threats, promises, enforcing threats and communication to help explain the above nonzero-sum games.[50]

The prisoner's dilemma provides a tool to look at the national strategic policy where two countries have the option to cooperate or not. It becomes a tool to analyze the nonproliferation of nuclear weapons. Specifically, it allows for analysis in the nuclear strategies in the middle of the twentieth century between the United States of America and the Union of Soviet Socialist Republics. To use the game to study nuclear policy, one must replace cooperate/do not cooperate with using/not using nuclear weapons. Using the logic of the game each country does better not to use nuclear weapons. However, knowing the reluctance to use nuclear weapons, the other country decides to use them. Continuing, then the country first reluctant to use the weapons chooses to use them as well, resulting in both countries using nuclear weapons and arriving at a less optimal payoff and outcome to the situation. The United States and the Union of Soviet Socialist

[48] Schelling, *The Strategy of Conflict*, 83.

[49] Ibid., 83.

[50] Schelling, *The Strategy of Conflict*, 121.

Republics did not exist in a single turn world. They also faced a threat they may not survive a nuclear war. They competed in a repeated play environment, which led to them adopting the mutually beneficial strategy of mutual deterrence and ensuring peace between the two countries.

International Trade and Price Cuts

International trade provides another lens to analyze the use of the prisoner's dilemma regarding national strategy because the elements of national power include economics. This scenario considers two countries that export the same good. Each country faces two options, hold prices steady (cooperate), or cut prices (defect). Each realizes that if they cut the cost of their product, they can gain more customers and therefore earn more revenue. The prospect of earning more revenue means a mutual incentive exists to lower rates. Each possessing the temptation to lower prices leads to a price war between the two countries, meaning they drop rates to the point where they end up losing revenue. This scenario again, as with the two nuclear-armed countries leads to a suboptimal payoff because if they had both agreed not to cut prices, they could still earn more than they are currently.[51]

The prisoner's dilemma gave the strategists a formalized approach to study the vexing and overwhelming thought of nuclear war. It did not provide specific strategies on how to deal with a nuclear-armed enemy. However, it merely provided a framework to abstract the scenario and made it workable in a rigid and systematic method.[52]

Negatives of Using Game Theory at Strategic Level

Game theory possesses some faults when applying to strategic level interactions. First, if a practitioner assumes all players act according to the rational outcomes of the game, they discount the players' other motivations or other nonrational factors. Further, game theory assumes that all players only use the methods of game theory. This assumption creates a situation where

[51] Straffin, *Game Theory and Strategy*, 74.

[52] Freedman, *Strategy*, 155.

the analyst falls victim to the fallacy of the hasty generalization that all players use the rules of game theory. Finally, Coyne and Horn identify four reasons why game theory often becomes overly cumbersome. They are the number of choices, ascribing reason to the options, understanding how opponents value the outcome and, the mathematics involved in determining the strategies all lead to the cumbersome nature of game theory.[53]

Freedman offers a critique of the uses of game theory during the Cold War between the United States and the Soviet Union. He mentions that game theory provided an understanding of outcomes. However, it reached the edge of its usefulness when it came to developing policy and strategy which the governments and planners could use. It also did not provide any clarity for ambiguous options such as a limited approach to the Cold War. It only analyzed outcomes in a binary sense between not using nuclear weapons or using nuclear weapons.[54] A mixed strategy option could explain the results. However, the mixed strategy still only contains using the two options, using nuclear weapons or not. But once the countries decide to use nuclear weapons then not using nuclear weapons is off the table.

Bernard Brodie casts a broader net with his critique of the use of economic models and systems analysis. Economists often group game theory into this type of study at the national strategic level. He states that these tools provided solutions and methods for weapons procurement but lacked strategic value. Systems analysis and similar types of study do not account, and in some cases is intolerant of, the intricacies of politics and ideology.[55]

Historical Example: 1777 New Jersey Campaign

Historical examples allow students and practitioners to examine events and run through iterations of planning to gain insight into decisions and outcomes. If students can access enough

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[53] Kevin P. Coyne and John Horn, "Predicting Your Competitor's Reaction," *Harvard Business Review* 87, no. 4 (April 2009): 92.

[54] Freedman, *Strategy*, 155.

[55] Bernard Brodie, *War and Politics* (London: Cassell, 1974), 474-475.

information and context, then they can replicate events to practice using modern planning processes. Carl von Clausewitz states the benefits and warnings of using historical examples to help clarify theory.[56] He mentions that historical examples help explain an idea and how to apply the concept, but it does not prove the theory. He goes on to warn that too often historical examples do not provide enough detail, or the reader does not have enough context.[57] Thomas Schelling echoes Clausewitz's warning about not providing enough context or detail. He states, "we change the character of the game when we drastically alter the amount of contextual detail that it contains or when we eliminate such complicating factors as the players' uncertainties about each other's value systems."[58]

The New Jersey Campaign of 1777 during the American Revolutionary War provides an excellent historical example of using game theory to analyze the elements of a campaign. Specifically, the one phase of the campaign examined using game theory is after George Washington decides to defend at Trenton after his successful raid against the Hessians on Christmas of 1776. General Washington decided to defend Trenton because he anticipated a counter-attack by the British General Charles Cornwallis. The battle ended with each army arrayed along the Assunpink Creek east of Trenton. The British army defended on the northwest bank, and the Continental Army defended on the southeast bank, see figure 1.[59] The two armies already fought a battle here the previous week after Washington crossed the Delaware River and raided the Hessian mercenaries on Christmas Eve in 1776. The British reinforced into wooded terrain to the north and east of Trenton with the intent of outflanking Washington. The

[56] Carl von Clausewitz, "Historical Examples," in *On War*, eds. and trans. Michael Howard and Peter Paret (Princeton, NJ: Princeton University Press, 1989), 170-173.

[57] Ibid.

[58] Schelling, *The Strategy of Conflict*, 162.

[59] David Hackett Fischer, *Washington's Crossing* (New York, NY: Oxford University Press, 2004), 308-310.

Continentals prepared for this by strengthening their right flank and building fortifications.[60] Washington's forces began to realize the options formulating due to their situation and the geography which they controlled.[61]

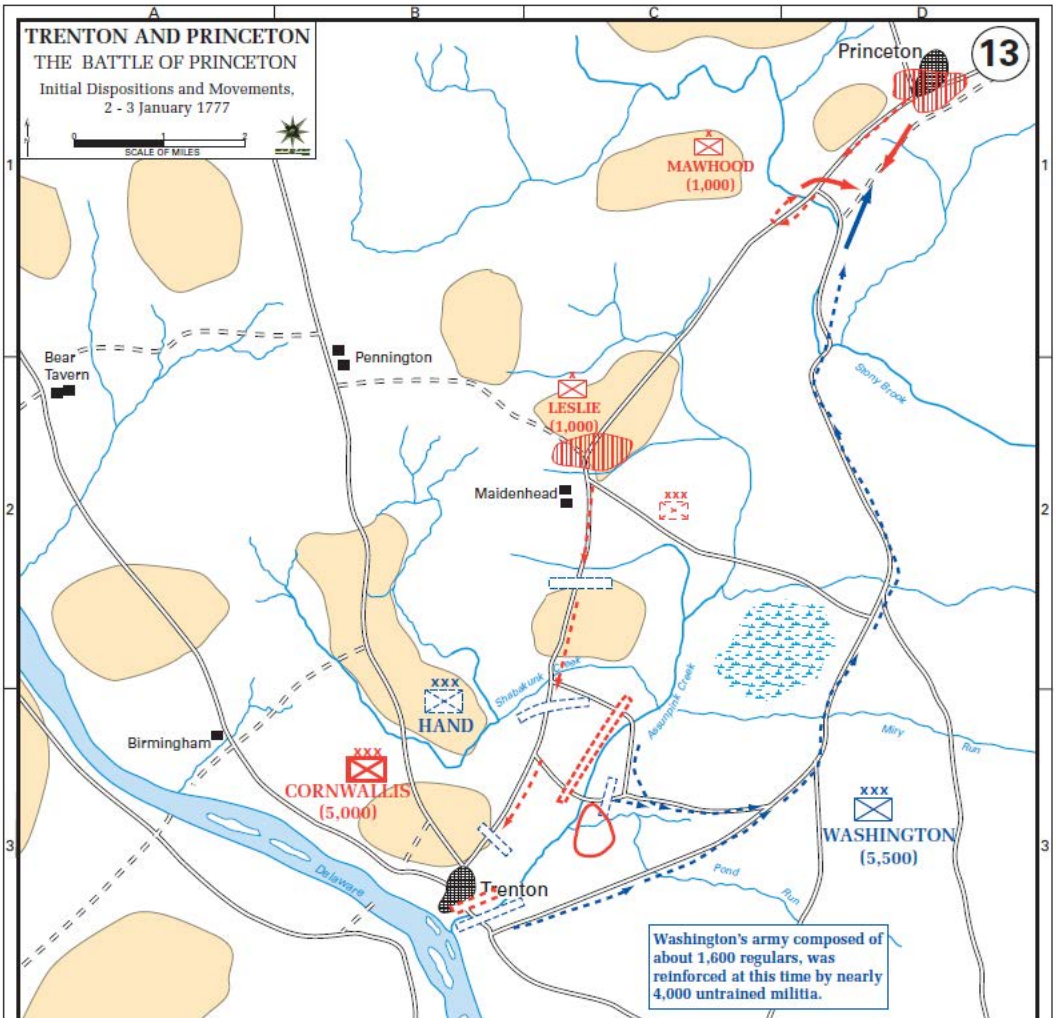

Figure 1. The Battle of Princeton.
Frank Martini "Trenton and Princeton: The Battle of Princeton," United States Military Academy History Atlases, October 30, 2019, accessed October 30, 2019, https://westpoint.edu/sites/default/files/inline-images/academics/academic_departments/history/Am%20Rev/13PrincetonBattle.pdf.

[60] Fischer, *Washington's Crossing*, 310.

[61] Richard M. Ketchum, *Winter Soldiers* (Garden City, NY: Doubleday and Company, Inc., 1973), 337.

Each commander called a council of war with their subordinate commanders and generals.[62] At their respective councils, they had to assess the environment, the disposition of their troops, and potential courses of action as well as the placement of the enemy and his possible courses of action. General Cornwallis knew that he outnumbered the Continental Army with his approximately 8,000 regular soldiers against Washington's 1,400 regular troops and 3,400 militia.[63] Further, because of the professional nature of his army, he expected a victory over Washington the next day.[64] Cornwallis also understood the risk-reward balance at play in the war against the American rebels. He and his generals knew they could win little glory and fame against the Americans, but they could lose much honor if they did not win the war. This fact played into Cornwallis' decision making as he weighed the advice given by his subordinates.[65]

The British quartermaster general, Sir William Erskine, and Major General James Grant were the chief advisors to General Cornwallis. General Erskine advocated for an aggressive night attack into the Continental defenses. He believed Washington would retreat with his army during the night and feared that the British "will see nothing of them in the morning."[66] General Grant provided the counterargument and advocated against a night attack and instead pushed for an attack in the morning. He felt the Continental Army did not possess a line of retreat and would defend the position through the night. Cornwallis agreed with Grant and did not want to risk a night attack in unfamiliar terrain against a flank he had not wholly located.[67]

Washington also called in his subordinate commanders to include Nathanael Greene, James Sullivan, Henry Knox, Arthur St. Clair, and his adjutant Joseph Reed.[68] The rebels feared a

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[62] Fischer, *Washington's Crossing*, 310.

[63] Ketchum, *Winter Soldiers*, 336-337.

[64] Fischer, *Washington's Crossing*, 310.

[65] Ibid., 312.

[66] Ibid., 312.

[67] Ketchum, *Winter Soldiers*, 344.

[68] Fischer, *Washington's Crossing*, 313.

defeat if they remained in their positions, and if they retreated, it would also prove disastrous.[69] The perceived inevitability of a negative outcome comes into play for Washington's decision making. Beyond these two dismal options, St. Clair recommended a third course of action. He recommended withdrawal from Trenton during the night and then advance on the rear and support elements of Cornwallis' army at Princeton.[70] Ketchum argues that beyond St. Clair's recommendation, Washington intended on striking Cornwallis' rear elements keeping in line with his guerrilla style of warfare. This attack on Princeton prevented Washington from having to face a defeat at Trenton or a loss of support and faith from the colonials or the Continental Congress.[71]

From these two councils of war, distinct characteristics and elements of design and game theory, which a commander must weigh, become known. Both commanders must assess the environment and their enemy. They must understand potential strategies and incentives for why the adversary may choose a plan. Finally, they must realize potential payoffs.

Cornwallis' understands his physical environment. He thinks Washington cannot retreat and will "bag him in the morning."[72] He knows the force ratios and quality of each army. He also understands his incentives for defeating the Continental Army. Still, he must do so without losing too much honor or glory and that the risk versus reward for a potential win does not equal the possible loss. Washington knows the physical environment he defends, along a creek with his left flank, and avenue of retreat, blocked by the Delaware River. He knows that Cornwallis possesses a better trained and more massive army. Further, he can surmise that Cornwallis will attack, but it is a matter of when. Finally, Washington's incentives include maintaining his army to continue the fight against the British, keep the support of the colonials and the Continental Congress. He understands the potential outcome of a continued defense is the defeat of his army. Loss of

[69] Fischer, *Washington's Crossing*, 313-314.

[70] Fischer, *Washington's Crossing*, 314.

[71] Ketchum, *Winter Soldiers*, 344.

[72] Ibid., 344.

popular support results from a retreat. But he can potentially garner more support, inflict damage on the enemy, and maintain his command if he can mount an attack on the enemy's rear and support areas. The conclusion of the historical context leads to the next section, which discusses how to apply basic game theory to this historical example.

Applying Game Theory to The Battle of Trenton/Princeton

Game Theory allows for an understanding of a rational outcome but is not deterministic of what should or did happen. As Freedman stated, game theory does not say what someone should do but describes the logical result.[73] When used as a tool to analyze a historical campaign, equilibria found by game theory may be different from the outcome the actors chose. And then, analysts can dig deeper to gain a better understanding of why the difference occurred and look further into motivations and underlying factors.

First, to review the essential elements of the game. Generals Washington and Cornwallis are the actors in the game. Washington possesses three courses of action or strategies. He can retreat into Pennsylvania, defend Trenton, or steal away into the night and attack Princeton. Next, Cornwallis must consider his two strategies, a night attack or wait until morning for a daytime attack. The next step is to qualitatively infer what the outcome is for the interaction of each of the commander's actions.

Table 6 shows the result of each potential interaction. It is not a predictor or deterministic of what will happen, only the analyst's estimation of what is likely to happen. The first step is to analyze all outcomes given a Cornwallis night attack. If Washington decides to retreat, the probable outcome is that Washington loses the confidence of the rebellious colonials, and the Continental Congress and Cornwallis occupy Trenton. If Washington defends Trenton against a night attack, the likely outcome becomes a costly win for Cornwallis. If Washington attacks

[73] Freedman, *Strategy*, 514.

Princeton, Cornwallis conducts his attack, pursues, and divides Washington's forces, leading to a probable defeat for Washington.

The next step is to analyze Washington's choices against a Cornwallis morning attack. Again, if Washington retreats, he probably loses the confidence of the rebellious colonials, and the Continental Congress and Cornwallis occupy Trenton. If he defends against a morning attack, he probably faces defeat, and Cornwallis holds Trenton, but it is a less costly operation for Cornwallis. Lastly, if Washington withdraws in the night with a covering force at Trenton, he probably successfully attacks Princeton and maintains popular support from Congress and the colonials.

Table 6. Qualitative Explanation of Outcomes.

Qualitative Explanation of Outcomes		Washington		
		Retreat	Defend	Attack Princeton
Cornwallis	Night Attack	Washington loses support, Cornwallis gains Trenton	Probable costly win for Cornwallis	Probable Cornwallis pursuit and defeat of Washington
	Morning Attack	Washington loses support, Cornwallis gains Trenton	Probable Win for Cornwallis	Probable successful Washington attack on Princeton

Source: Created by Author.

The next step applies a numerical value to each outcome for each actor, followed by an analysis of the game. A simple rank ordering of each outcome provides a simple way of adding a value for a payoff to each actor. The table below uses a reverse rank order where the higher number is the preferable outcome. Washington prefers to attack Princeton and hopes that Cornwallis waits until morning for his attack, therefore he receives a value of six in that square. Conversely, Cornwallis prefers this outcome the least because Washington's force would escape him and seize Princeton, he receives a one. Washington's second-best option is to attack Princeton against a Cornwallis night attack hoping that some of his forces would at least out pace Cornwallis' attack reach Princeton or another safe location giving him a payoff of five. This is

Cornwallis' third least preferred option giving him a payoff of 3. This works its way through until all squares receive a payoff based on the mutual outcomes shown in table 7.

Table 7. NJ Campaign with reversed rank ordered payoffs.

NJ Campaign 1777		Washington		
		Retreat	Defend	Attack Princeton
Cornwallis	Night Attack	5, 1	2, 4	3, 5
	Morning Attack	6, 2	4, 3	1, 6

Source: Created by Author.

One method to find the equilibrium outcome is to find dominant strategies. First, Washington never chooses to retreat because this is a strictly dominated strategy. The other two strategies give higher payoffs than this strategy, regardless of what Cornwallis wants. Washington can gain nothing by retreating other than the preservation of his army. Washington must preserve his army, but he must also take action to maintain popular support for the revolution. But he loses ground, and he loses support from the people and the congress. Table 8 depicts eliminating retreat from Washington's possible strategies. Second, Washington would never choose to defend either. After deciding never to retreat, the choice to defend is the second strictly dominated strategy. Again, this is because regardless of what Cornwallis chooses, attacking Princeton always yields a higher payoff than defending Trenton. Therefore, while Washington does have three choices, two of them are not credible, and the only rational choice for him is to attack Princeton, shown in table 9.

Table 8. Retreat dominated and eliminated.

Retreat Dominated		Washington		
		Retreat	Defend	Attack Princeton
Cornwallis	Night Attack		2, 4	3, 5
	Morning Attack		4, 3	1, 6

Source: Created by Author.

Table 9. Retreat and Defend dominated and eliminated.

Retreat and Defend Dominated		Washington		
		Retreat	Defend	Attack Princeton
Cornwallis	Night Attack			3, 5
	Morning Attack			1, 6

Source: Created by Author.

First, Cornwallis also looks for any dominant strategies. He does not possess any dominant or dominated strategies. Next, Cornwallis assesses the situation and realizes that Washington's only credible plan is attacking Princeton. Given this information, his rational choice becomes a night attack with a pursuit of Washington's forces. Therefore, the Nash Equilibrium outcome is a Cornwallis night attack and a Washington attack on Princeton yielding a payoff of three for Cornwallis and five for Washington. This outcome fits all three characteristics of a Nash Equilibrium, as previously explained. First, neither Washington nor Cornwallis want to deviate from this outcome; if they do so, it creates an unstable result because they both have the incentive to keep deviating. Second, this outcome provides a mutually best-reply result, which reinforces the first characteristic. Third, the finding confirms the assumptions of each commander going into the interaction. Cornwallis assesses the environment and Washington's options, and Washington likewise evaluates the environment and Cornwallis' options. From this, they can determine the rational outcome of this situation.[74]

The benefit of applying a game-theoretic lens lays not in the actual payoffs or scores, but the analysis of the situation. A staff still benefits from analyzing the operational environment during The Battle of Trenton even if the analyst changes the values of the payoffs. The value comes from understanding the motivations and incentives of the different actors in the game. By changing the values of Washington's and Cornwallis' payoffs, the game still highlights how their motivations determine their decisions. A potential criticism of game theory is the arbitrariness of

[74] Pham, "Game Theory: Nash Equilibria."

the payoffs. The next game uses randomly selected payoffs according to the initial assessment of each actor's rank ordering of the potential courses of action. The use of randomly chosen payoffs highlights the importance of the analysis of each actor's wishes and what they want to accomplish, not the significance of the payoffs. Refer to table 10 to see adjusted payoffs. Again, Washington never chooses retreat because it is still a dominated strategy. With only two remaining options, Washington still chooses to attack Princeton, because again it dominates defending regardless of what Cornwallis chooses. Therefore, Cornwallis makes his decisions based on the fact Washington attacks Princeton. Finally, as above, Cornwallis should choose to attack Washington at night. The result remains the same regardless of the payoffs because each commanders' incentives stay the same in both games. For Washington, retreating is never an option, which then leads to his next decision of never choosing to defend. The payoffs allow for an easy quantifiable feature to analyze. But the value is in the qualitative understanding of the environment.[75]

Table 10. Altered Payoffs Through Eliminating Dominated Strategies.

NJ Campaign 1777 Altered Payoffs		Washington		
		Retreat	Defend	Attack Princeton
Cornwallis	Night Attack	8, -10	-5, -5	0, 5
	Morning Attack	10, -10	5, -8	-10, 10

Retreat Dominated		Washington		
		Retreat	Defend	Attack Princeton
Cornwallis	Night Attack		-5, -5	0, 5
	Morning Attack		5, -8	-10, 10

Defend Dominated		Washington		
		Retreat	Defend	Attack Princeton
Cornwallis	Night Attack			0, 5
	Morning Attack			-10, 10

Source: Created by Author.

[75] The payoffs chosen matter more if the game results in a mixed-strategy equilibrium, because the payoffs are used to calculate the probability of choosing each strategy. Since this monograph emphasizes keeping game theory simple for use in planning, it has avoided games resulting in mixed strategy equilibria.

Applying game theory to The Battles of Trenton and Princeton demonstrates that game theory would have been a useful tool during operational planning. It allows us to analyze the situation and see what the theory describes as a rational decision as compared to what occurred. Cornwallis waited until morning to attack and realized that Washington only left a small rear-guard force, and Washington moved from Trenton in the middle of the night to mount a successful attack on Princeton. The actual outcome differs from what the game shows as a rational outcome. The game describes a situation where Washington still attacks Princeton in the middle of the night. Still, Cornwallis also mounts a night attack, which may result in Cornwallis pursuing Washington's forces and defeating him in detail. Cornwallis attacking at night could have led to a drastically different outcome in the war since the Continental Army could cease to exist, and much of the popular support for the revolution would dissipate as a result.

Implementation of Game Theory into the Military Decision-Making Process

Planners can use game theory tools during the Military Decision-Making Process, specifically during mission analysis for a different perspective on understanding the operational environment and course of action development as a check against undiscovered assumptions. The game theory tools do not replace the existing steps and tools in the military decision-making process but supplement them. Field Manual 6-0 explains that commanders and staffs use mission analysis to understand better the operational environment and the problem the unit faces.[76] Next, planners use mission analysis to develop assumptions to fill knowledge gaps. Finally, given the nature of game theory to understand competition, mission analysis also helps to understand how friendly and enemy forces may interact.[77] The Course of Action Development process provides an objective way to look at multiple potential plans. In the historical example above, Generals Washington and Cornwallis need to understand their potential actions and what they think the

[76] US Department of the Army, Army Field Manual (FM) 6-0, C2, *Commander and Staff Organization and Operations* (Washington, DC: Government Printing Office, 2016), 9-6.

[77] Ibid., 9-8 to 9-9.

enemy commander may do. In a way, the generals in the historical example could have used game theory during their course of action development to check their assumptions. The development starts with a narrative or qualitative assessment and then moves to a quantifiable assessment with weighted scores for each plan.[78] Game Theory allows for an alternative perspective to assess the potential plans. The following thought experiment provides an example of how a staff could use some game theory tools during mission planning.

The exercise is as follows: The United States debates increasing military presence in a friendly country located close to a near-peer adversary, which is meant to act as a deterrent to the adversary from invading a friendly country. The corps staff understands the national policymakers' debate about increasing military presence in a region. Further, they know if the national leadership pursues escalation, the corps is an element of that escalation. The staff works to understand the operational environment and understand national-level priorities and incentives, so they can make recommendations higher for options and prepare for expected courses of action. Second, they grapple with understanding the enemy's motivations and plans of action. The enemy also faces the prospect of increasing its military presence in the area or maintain the status quo. Both powers possess nuclear weapons and do not want to engage in all-out war. Lastly, the population in the area in which both powers could move does not want occupation by a foreign power. The strategic decisions the national policymakers face have operational level implications.

As stated above, mission analysis provides an understanding of the situation and problem. During mission analysis, the staff starts to develop a sense of the actors' motivation and incentives. The intelligence preparation of the battlefield provides a key step during mission analysis. The staff makes assumptions on how the friendly force and enemy forces interact in the environment. From this, the staff develops potential options that each actor could use during the

[78] US Army, FM 6-0, C2, 9-40.

upcoming operation.[79] Also, the intelligence preparation step identifies gaps in the commander's and the staff officer's knowledge. These gaps leads to the development of intelligence requirements to gain information.[80] As stated during the literature review, people make decisions based on the information they possess and anticipate the actions of their competitors. These steps do not replace or negate any of the steps of the Military Decision-Making Process, they are merely a recommendation on how and where to implement game theory tools in the process.

Given this scenario, the staff starts to develop enemy courses of action. These courses of action become the enemy's strategies when applied to a game matrix. The enemy can either militarize the area with one of their divisions or corps or choose not to militarize. The choice of militarizing or not creates two discrete strategies for the enemy. The second step looks at outcomes from each of the strategies. If both militarize, then they face war. If neither militarizes, then they maintain the status quo. If one country militarizes and the other does not, then the country militarizing does so in an uncontested environment. Table 11 displays the outcomes of this scenario.

Table 11. Qualitative Outcomes

Explanation of Outcomes		**USA**	
		Militarize	Do Not Militarize
Enemy	Militarize	War	Enemy uncontested militarization
	Do Not Militarize	USA's uncontested militarization	Status Quo

Source: Created by Author.

The third step requires the staff to look at the enemy's incentives to create a qualitative analysis of their choices then. The enemy wants to militarize the region without the United States also deciding to militarize the region. This creates an uncontested environment for them. Next, they value neither themselves nor the United States militarizing the area, which is the status quo.

[79] US Army, FM 6-0, C2, 9-8.

[80] Ibid., 9-8.

The third desirable outcome is the United States militarizes, but the enemy does not, meaning the United States has an uncontested militarization. Lastly, the enemy does not want to escalate to war and does not want to militarize the region if the United States also militarizes. The staff can now rank order the enemy's courses of action by preference. The operations and intelligence staff can leverage collection assets and develop a collection plan to identify any indicators about the enemy's plan, such as massing forces in the region. The information collection plan helps answer information requirements and assists with effective planning.[81]

The staff now moves into the course of action development. The generating options step gives broad outlines of the choices available to the commander and staff. The staff develops options that can feasibly defeat the enemy's courses of action and then prioritizes them.[82] The staff also generates two broad options. They can militarize or not militarize. The staff can now rank order their courses of action because of the outcomes of each actors' strategies. The commander and staff prefer to maintain the status quo. If the United States moves to militarize the region, it could potentially upset the local, national governments and population. Therefore, a United States militarization of the region and an enemy not militarizing is the second preference. This option means the United States has an uncontested militarization, but as stated, the local government is upset. Third, in the ranking is the United States not militarizing, but the enemy does militarize, giving them an uncontested advantage. Lastly, the United States does not want war, which occurs if both the United States and the enemy both militarize.

Next, the staff develops the game into the matrix or strategic form. First, they conduct the qualitative analysis stating the likely outcome of each engagement, see table 12. Then the staff rank orders the outcomes from the perspective of each commander to generate the quantitative analysis and payoffs, reflected in table 13. This table shows the payoffs with the enemy's firsts

[81] US Army, FM 6-0, C2, 9-10.

[82] Ibid., 9-19.

and the United States second. Using reverse rank order, the lowest number payoff indicates the least preferred option, and the higher the number, the preferred option. Each combatant is a near-peer, and therefore the staff assumes that engagement will favor the side with the initiative.

Table 12. Qualitative Analysis

Qualitative Explanation of Outcomes		**USA**	
		Militarize	Do Not Militarize
Enemy	Militarize	**Friendly**: War **Enemy**: War Least preferred outcome.	**Friendly**: At a disadvantage **Enemy**: Takes initiative and uncontested militarization
	Do Not Militarize	**Friendly**: Secures the area with uncontested militarization, but loses popular support. **Enemy**: At a disadvantage	**Friendly**: Status Quo, prefers this option over all others **Enemy**: Status Quo

Source: Created by Author.

Table 13. Quantitative Outcomes

Quantitative Explanation of Reverse Ordered Outcomes		**USA**	
		Militarize	Do Not Militarize
Enemy	Militarize	1, 1	4, 2
	Do Not Militarize	2, 3	3, 4

Source: Created by Author.

The value of these two products lies in the analysis the staff conducts to grasp an understanding of potential future outcomes. It provides a concise deliverable product that a staff planner can hand to the commander or the chief of staff on one sheet of paper for future reference or reflection as the commander and staff start to weigh options during future steps of the military decision-making process. This analysis provides a moment for the staff to think about what they are doing and what outcomes may result from their planning. This is an example of what Schön calls reflection in practice. As he says, it allows for people to think about what they are doing and then shape what they do, as they are executing the task.[83]

[83] Donald A. Schoen, *Educating the Reflective Practitioner: Toward a New Design for Teaching and Learning in the Professions* (San Francisco, CA: Jossey-Bass, 1987), 26.

The next step requires the staff to narrow down the available options to only the credible options available to the commander. The staff looks for any dominated strategies which a commander would never use. The enemy commander does not possess any dominated strategies and has both strategies available to him. But the United States would never choose to militarize in the game because not militarizing dominates it regardless of what the enemy chooses. Table 14 highlights in bold which option dominates for the United States. For instance, if the enemy decides to militarize, the United States receives a payoff of one if it decides to militarize and a payoff of two if it does not. Therefore, in this case, the United States would choose not to militarize. Likewise, if the enemy does not militarize, then the United States receives a payoff of three if it militarizes and a payoff of four if it does not militarize, and again the United States would choose not to militarize. Therefore, the staff eliminates that as an option.

Table 14. Dominating Payoffs for the USA highlighted in Bold

Quantitative Explanation of Reverse Ordered Outcomes		**USA**	
		Militarize	Do Not Militarize
Enemy	Militarize	1, 1	4, **2**
	Do Not Militarize	2, 3	3, **4**

Source: Created by Author.

Now that the staff understands the United States does not have an incentive to militarize, it can then look at what the enemy may do as a response. The enemy knows the United States does not want to militarize and seeks to maximize its outcome. Therefore, the enemy chooses to militarize since that gives a better payoff than not militarizing. This arrives at the Nash Equilibrium of the enemy militarizing and receiving their payoff of four and the United States not militarizing and receiving their third-best payoff of two. Table 15 shows the resulting Nash Equilibrium circled.

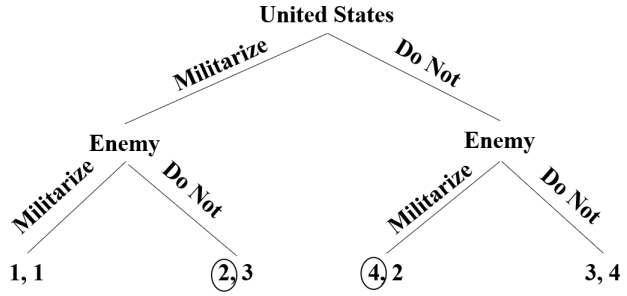
Table 15. Militarize Dominated

Militarization Dominated by Do Not Militarize		USA	
		Militarize	Do Not Militarize
Enemy	Militarize		4, **2**
	Do Not Militarize		3, **4**

Source: Created by Author.

But real-life situations do not always coincide; a party usually acts first compelling the other party to make a decision. In the above scenario, the United States grapples with the decision to militarize the region. Their decision then forces the enemy to make a decision. The next step looks at how the situation unfolds in a sequential move game and if the Nash Equilibrium changes in an analysis of the decision. Refer to table 16 for the sequential game. The table shows the enemy's payoffs first and the United States' payoffs second.

Table 16. Sequential Move Game

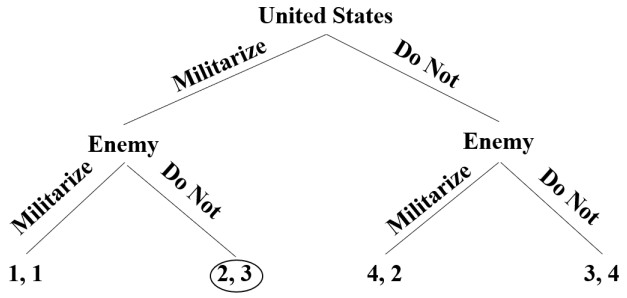


Source: Created by Author.

The actor's choices and payoffs for each outcome remain the same. The only difference is the United States moves first, and the enemy must react. The staff must use subgame analysis to analyze this game and its outcome. The enemy has the second move, and therefore analysis starts with their prospective steps. The two actors know that the enemy will choose not to militarize if the United States elects to militarize because a payoff of two is better than one. And the enemy would want to militarize if the United States chooses not to militarize because four is better than three. Table 16 above indicates this behavior by circling each of the enemy's preferred choices, given what the United States chooses. Now that the United States knows which choices

the enemy would make based on the United States choices, they choose between the two. The United States elects to militarize, knowing the enemy would not militarize, resulting in a payoff of three for the United States. The United States militarizing is better than not militarizing and receiving a payoff of two, knowing the enemy would choose to militarize. Therefore, the Nash Equilibrium becomes the United States militarizing and the enemy not militarizing with a payoff of two to the enemy and three to the United States, see table 17.

Table 17. Nash Equilibrium



Source: Created by Author.

The consecutive game resulted in a different Nash Equilibrium than the simultaneous match, why? Each game results in one party militarizing and the other party not militarizing. In the simultaneous game, the enemy received the most beneficial payoff by militarizing, and the United States knew that and therefore chose not to militarize. However, in the sequential game, the United States decides first. They receive their highest payoff if they do not militarize, and the enemy also chooses not to militarize. Both nations not militarizing does not happen because the enemy has an incentive to militarize if the United States does not. The United States realizes this and therefore sees their next best option is to militarize knowing the enemy will not militarize because it would force a war between the two actors. This game provides an example of a first-mover advantage. If the enemy were to choose first, they too would have an incentive to militarize.

A sequential move game reflects a more realistic situation. But running both types of games provides analytical value to the staff to understand motivations and potential actions. The

staff can see how sequencing actions may change the outcome. As stated above, the value of using this approach lies in the analysis. The staff can create a brief narrative of each outcome as laid out in the matrix form. They can then see that one of their options is not a viable choice. They then looked at a quantitative assessment and determined that an equilibrium outcome is available. The qualitative analysis conducted reiterates Thomas Schelling's point that the math of game theory does not always solve the conflict, and one should not overly rely on math. It is the thinking through the problem which adds value.

Conclusion

Game theory provides an analytical tool to look at competitive situations. It allows analysts to understand potential plans of action, incentives, and payoffs or outcomes. Further, it can highlight information gaps and areas which need further understanding. During the middle of the twentieth century, strategic level planners used it to understand better the competition between the United States and the United Soviet Socialist Republic regarding the use of nuclear weapons and atomic war. Analysts outside the Department of Defense used it to understand trade disputes and price-cutting between competing firms.

At the operational level, game theory allows for the same type of analysis and understanding of potential plans, incentives, and outcomes. This monograph examined the history of game theory and explored basic game theory, establishing the usefulness of game theory at analyzing situations of conflict. The literature review revealed the strengths and weaknesses of game theory, which informed how to use it best to maximize its potential. Examining strategic level decisions such as nuclear situations and international trade provided the context of how previous efforts fruitfully applied game theory. The application of game theory to the American Revolutionary Battles' of Trenton and Princeton arrived at an alternate course of action than what the commanders pursued, demonstrating how using game theory can provide unique insights that were not obvious to a seasoned general like Cornwallis. Finally, the monograph showed how a corps-level staff could use game theory to understand how strategic-level decisions impact

operational level actions, demonstrating the utility of comparing simultaneous and sequential games. The last section provides a basic framework, which a staff could use to approach an operational problem by applying game theory to mission analysis and course of action development.

The use of game theory is not limited to the military decision-making process. Game theory fits well with existing planning processes that the Department of Defense and the United States Army currently use. Planners can use game-theoretic tools during the Joint Operational Design process and the Army Design Methodology. Specifically, during joint design, game theory tools best fit with understanding strategic guidance and understanding the operational environment. During army design, it fits best with framing the operational environment and understanding the problem. Game theory is another useful tool in the staff officer's or planning team's kitbag. When applied through the military decision-making process or the design process, game-theoretical analysis pairs well with other tools to provide a greater understanding of the operational environment.

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