ARL-TR-9271 • AUG 2021

A Crater Expansion Model for High-Velocity Penetration

by Steven B Segletes

NOTICES

Disclaimers

ARL-TR-9271 • AUG 2021

A Crater Expansion Model for High-Velocity Penetration

by Steven B Segletes
Weapons and Materials Research Directorate, DEVCOM Army Research Laboratory

REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1. REPORT DATE *(DD-MM-YYYY)*
August 2021

2. REPORT TYPE
Technical Report

3. DATES COVERED (From - To)
August 2021

4. TITLE AND SUBTITLE
A Crater Expansion Model for High-Velocity Penetration

5a. CONTRACT NUMBER

5b. GRANT NUMBER

5c. PROGRAM ELEMENT NUMBER

6. AUTHOR(S)
Steven B Segletes

5d. PROJECT NUMBER

5e. TASK NUMBER

5f. WORK UNIT NUMBER

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
DEVCOM Army Research Laboratory
ATTN: FCDD-RLW-TC
Aberdeen Proving Ground, MD 21005-5066

8. PERFORMING ORGANIZATION REPORT NUMBER
ARL-TR-9271

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)

10. SPONSOR/MONITOR'S ACRONYM(S)

11. SPONSOR/MONITOR'S REPORT NUMBER(S)

12. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for public release; distribution is unlimited.

13. SUPPLEMENTARY NOTES

14. ABSTRACT
This report explores the possibility of applying the recently developed model for line explosion to the problem of crater formation and growth during the course of eroding penetration. A one-to-one correspondence is shown between each of the five input parameters of the line-explosion model to inputs known during an eroding-penetration event. With such a correspondence, a model to predict the time-dependent crater growth during a penetration event amounts to a direct application of the line-explosion model, using a suitably transformed set of input parameters, derived from the local conditions of the penetration event.

15. SUBJECT TERMS
line explosion, crater expansion, axisymmetry, energy conservation

16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
					Steven B Segletes
a. REPORT	b. ABSTRACT	c. THIS PAGE			19b. TELEPHONE NUMBER (Include area code)
Unclassified	Unclassified	Unclassified	UU	25	410-278-6010

Standard Form 298 (Rev. 8/98)
Prescribed by ANSI Std. Z39.18

Contents

List of Figures

List of Tables

Acknowledgments

The author is extremely grateful to his colleague and friend Dr Michael Grinfeld for providing a prompt technical review to this manuscript, given the time constraint under which the author finds himself. Ms Carol Johnson, editor of this manuscript, is to be commended for her forbearance in the face of this, the 5th manuscript she has reviewed by this author over the course of 6 short weeks. Her unflappable demeanor and consistent work quality earns her the undying respect and admiration of the author.

1. Introduction

In a recent report, ARL-MR-1037,[1] the author studied the problem of an axisymmetric instantaneous line explosion in incompressible fluid media.* In such a model, a line of distributed explosive is situated along the axis of a radially finite, axisymmetric fluid shell. Because the explosive is taken to be concentrated along a 1-D line, the explosion of the line has the characteristic of converting 100% of its chemical energy into kinetic energy at time $t = 0$. Apart from the explosive energy released (per unit axial length), the physical properties of the explosive material are immaterial to the analysis. The explosive serves purely as a source of energy deposition into the fluid cylindrical shell.

In the line-explosion model, the outer boundary of the cylindrical fluid shell is subjected to a constant external pressure provided by an idealized reservoir. This reservoir pressure performs continual work on the expanding fluid shell, until such time that the kinetic energy in the expanding shell is dissipated and the expanding motion of the shell is fully arrested.

In this report, the axisymmetric line-explosion solution[1] becomes the vehicle for formulating a crater expansion model for high-velocity penetration. While the line-explosion problem entails the full axis of explosive detonating instantaneously at time $t = 0$, the axisymmetric penetration problem is one in which a penetrator, through the process of erosion (and in the more general case, deceleration), provides a source of energy deposition into the target that will vary over time in terms of both magnitude as well as the axial location of the deposition. And yet, if the time-rate of energy deposition into the target is constant or only slowly varying in time, it may be reasonable to ignore axial gradients in the crater-expansion process and directly apply the axisymmetric crater expansion model of the line explosion to each successive cross-sectional slice of the target, as it interacts with the eroding penetrator. In this fashion, the line-explosion solution may facilitate a model associated with the problem of time-dependent penetration and crater expansion.

*The axisymmetric line-explosion analysis of ARL-MR-1037[1] was, quite literally, an axisymmetric reinterpretation of the spherical point-explosion solution developed in ARL-TR-9247.[2,3]

2. Penetrator Energy Deposition per Unit Target Depth

For a line explosion, the parameter that wholly governs the energy initially transferred into the fluid is dE_c/dz, the explosive energy deposited per unit length of fluid cylinder. If we desire to apply that model to the problem of eroding penetration, a means to quantify the amount of energy dE that gets deposited into a given cross-sectional slice of target (of thickness dz) is required.

For simplicity, we consider in this report the case where the impact velocity is large enough relative to the target's material strength that the axial penetration occurs in what can be characterized as a *hydrodynamic* manner, such as that of a shaped-charge jet. Note that nothing theoretically precludes a more sophisticated treatment for lower-velocity impacts, involving the strengths of the jet and the target. In a cylindrical penetrator such as a jet, the linear energy density (kinetic energy per unit length L) of the jet is simply

$$\frac{dE}{dL} = \frac{1}{2}\rho_j V_j^2 \pi r_j^2 \quad , \tag{1}$$

where dE/dL is the jet's linear energy density, ρ_j is the jet density, r_j is the jet radius, and V_j is the jet velocity for the impact of interest.

In the hydrodynamic limit, the interface pressure p_0 is characterized by way of a Bernoulli balance as

$$p_0 = \frac{1}{2}\rho_j (V_j - U)^2 = \frac{1}{2}\rho_t U^2 \quad , \tag{2}$$

where U is the rate of penetration into the target and ρ_t is the target density. From Eq. 2, we derive the kinematic relationship between V_j and U that

$$\frac{V_j}{U} = (1 + \gamma) \quad , \tag{3}$$

where $\gamma = \sqrt{\rho_t/\rho_j}$. Intrinsic to Eq. 2 is the notion that the velocity of the jet relative to the jet–target interface is $V_j - U$ and the velocity of the target relative to the interface is U. Thus, the ratio of jet-consumption rate* to target-penetration rate is

$$\frac{dL}{dz} = \frac{V_j - U}{U} = \gamma \quad . \tag{4}$$

*Here, L is taken as jet length consumed rather than the more standard approach of jet length remaining. In the alternate convention, the signs on Eqs. 1 and 4 would be made negative.

We may combine Eqs. 1 and 4 to obtain the rate at which jet energy is consumed (deposited into the target) per unit penetration depth:

$$\frac{dE}{dz} = \frac{dE}{dL}\frac{dL}{dz} = \gamma\frac{dE}{dL} = \frac{\gamma}{2}\rho_j V_j^2\,\pi r_j^2 \quad . \tag{5}$$

The model we adopt is that a fraction of the jet's energy is required to axially penetrate into the target, *creating an initial crater indentation the same diameter as the jet*, and the remaining jet energy is available for the work of radial crater expansion. In mathematical terms,

$$\frac{dE}{dz} = \frac{dE_a}{dz} + \frac{dE_c}{dz} \quad , \tag{6}$$

where dE_a/dz is the linear energy density required for axial penetration and, as in the line-explosion model, dE_c/dz is the linear energy density available for crater expansion. The differential work of axial penetration dE_a is defined here as the interface force (pressure × area) moving through a distance of penetration, dz. Thus,

$$\frac{dE_a}{dz} = p_0\,\pi r_j^2 = \frac{1}{2}\rho_j(V_j - U)^2\pi r_j^2 \quad . \tag{7}$$

We can determine from Eqs. 5 and 7 that

$$\frac{dE_a}{dz} = \frac{\gamma}{(1+\gamma)^2}\cdot\frac{dE}{dz} \quad , \tag{8}$$

as derived from the following chain that also makes use of Eqs. 3 and 4:

$$\begin{aligned}
\frac{dE_a/dz}{dE/dz} &= \frac{\frac{1}{2}\rho_j(V_j-U)^2\pi r_j^2}{\frac{1}{2}\rho_j V_j^2\,\pi r_j^2\gamma} \\
&= \frac{(V_j-U)^2}{V_j^2\,\gamma} \\
&= \left(\frac{U}{V_j}\right)^2\left(\frac{V_j-U}{U}\right)^2\frac{1}{\gamma} \\
&= \frac{1}{(1+\gamma)^2}\,\gamma^2\frac{1}{\gamma} \\
&= \frac{\gamma}{(1+\gamma)^2} \quad .
\end{aligned}$$

We may now use Eqs. 8 and 6 to solve for the linear energy density available for radial crater expansion,

$$\frac{dE_c}{dz} = \frac{1+\gamma+\gamma^2}{(1+\gamma)^2} \cdot \frac{dE}{dz} \quad , \tag{9}$$

as shown in the following chain:

$$\begin{aligned}\frac{dE_c/dz}{dE/dz} &= 1 - \frac{dE_a/dz}{dE/dz} \\ &= 1 - \frac{\gamma}{(1+\gamma)^2} \\ &= \frac{1+\gamma+\gamma^2}{(1+\gamma)^2} \quad .\end{aligned}$$

Examining the fractional term on the right-hand side of Eq. 9, we observe that the target's linear energy density available for crater expansion is always greater than or equal to 75% of the target's total linear energy density. Its value approaches 100% for very small as well as very large values of γ and drops to 75% when $\gamma = 1$.

Relating these results back to the axisymmetric line-explosion model[1] of ARL-MR-1037, for the target cross section under consideration, we start with a crater of initial radius $R_*^\circ = r_j$, the jet radius. The linear energy density of the "explosion" to create radial motion in the target, which in the case of penetration is supplied by the eroding jet, is obtained by substituting Eq. 5 into Eq. 9:

$$\frac{dE_c}{dz} = \frac{1+\gamma+\gamma^2}{(1+\gamma)^2} \cdot \frac{\gamma}{2} \rho_j V_j^2 \, \pi r_j^2 \quad . \tag{10}$$

3. Plastic Work Performed by the Target

In the axisymmetric line-explosion model,[1] the target material expands radially subject to an external pressure p_∞ applied to the outer boundary of the target cylindrical shell. There is no pressure applied to the inner crater wall for $t > 0$, during the period of crater expansion. The work done by this external pressure, given as a function of the target's time-dependent external radius R_{out}, is

$$\frac{dW_{\text{ext}}(R_{\text{out}})}{dz} = \pi (R_{\text{out}}^2 - R_{\text{out}}^{\circ 2}) p_\infty \quad . \tag{11}$$

In contrast, for the crater-expansion problem in penetration mechanics, there is no external pressure p_∞ applied. Rather, we understand that plastic work accomplished in the solid target dissipates the kinetic energy of the expansion. The process is often visualized in terms of a target resistance, H, having units of strength (*i.e.*, force per unit area). This resistance H is not merely, for example, the uniaxial strength of the target material. Rather, it is the integrated effect of the stress field throughout the target's plastic zone, surrounding the expanding crater, brought about by the strain being imposed during the expansion. For eroding penetration, the value of H typically lies in the range of 3–5 times the ultimate strength of the target material.

This resistance, cumulatively acting through the crater's plastic zone, is taken to oppose the crater expansion with a magnitude H, such that energy dissipated through plastic work, W_p, is oft characterized as

$$\frac{dW_p}{d\mathcal{V}} = H \quad , \tag{12}$$

where $\mathcal{V}$ is the crater volume created as a result of target expansion. In the case of an axisymmetric crater expansion from initial ($t = 0$) radius $r = R_*^\circ$ to an expansion $r = R_*$, we would have

$$\frac{d\mathcal{V}(R_*)}{dz} = \pi(R_*^2 - R_*^{\circ 2}) \quad , \tag{13}$$

so that, combining Eqs. 12 and 13, we obtain

$$\frac{dW_p(R_*)}{dz} = \frac{dW_p}{d\mathcal{V}}\frac{d\mathcal{V}}{dz} = \pi(R_*^2 - R_*^{\circ 2})H \quad . \tag{14}$$

While one could (and many have) merely equated the linear energy density deposited in the target by the jet (for example, Eq. 10) with the linear plastic work density accomplished in the target (Eq. 14), and taking the initial crater radius R_*° as 0, such an approach will provide an estimate of the maximum crater radius $R_*^{\max}$ only, but no information about expansion velocities, pressure field, event duration, or time history in any form. In contrast, if we can adapt the penetrating crater expansion problem to the framework of the line explosion, all of these missing pieces will become available. So, let us proceed to do that.

Comparing the form of Eq. 11 with that of Eq. 14, we note an overpowering similarity. Further, for a line-explosion fluid that is assumed incompressible, axisymmetric expansion of the target shell is governed by the continuity relation

$$R_{\text{out}}^2 - R_{\text{out}}^{\circ 2} = R_*^2 - R_*^{\circ 2} \quad . \tag{15}$$

One may, thus, substitute Eq. 15 into the line explosion's linear work density, Eq. 11, to obtain

$$\frac{dW_{\text{ext}}(R_*)}{dz} = \pi(R_*^2 - R_*^{\circ 2})p_\infty \quad . \tag{16}$$

Finally, we see a direct correspondence between the work performed by an external reservoir at a pressure of p_∞ upon a liquid target shell (Eq. 16) and the plastic work accomplished by a solid target possessing a resistance of H (Eq. 14). In particular, by replacing the exterior boundary pressure applied to the fluid shell (p_∞) with the target resistance of the solid target (H), the full apparatus of line-explosion solution may be brought to bear on the problem!

4. Outer Radius of Radial Stress Application

The final detail needing attention is the fact that, in the line-explosion model, the pressure at $r = R_{\text{out}}$ is p_∞. In the solid target block, the pressure at the outer boundary of the target is exactly zero. Therefore, we must reinterpret the meaning of R_{out} for the penetration problem. Since the "outer" component of pressure must, in the current interpretation, equal H, in accordance with Eq. 14, the meaning we attach to R_{out} is not the outer radius of the target block, but (with some hand waving) the outer radius of the plastic zone in the target. At this radius, the target strain is presumed such that the deformation has placed the material in an incipient plastic state with a maximum radial stress equal to H (for a solid, the stress making up the radially directed stress H comprises both a pressure and a deviatoric component; however, to use the line-explosion model, we must abide by the "fluid" formulation allowing only a pressure).

Let us see if we can provide some analytical guidance on the size of the plastic zone. In a plane-strain situation such as this, the *deviatoric* portion* of the isotropic constitutive relation, in the absence of shear strain, is given as

$$\begin{bmatrix} s_r \\ s_\theta \end{bmatrix} = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-\nu & \nu \\ \nu & 1-\nu \end{bmatrix} \begin{bmatrix} e_r \\ e_\theta \end{bmatrix} \quad ,$$

where r and θ denote, respectively, the radial and circumferential directions of the cylindrical coordinate system; s and e are the deviatoric stress and strain, respectively; E the elastic modulus; and ν the Poisson ratio of the material. For our

*For our particular case of an incompressible material, the *hydrostatic* portion of the constitutive relation, $\bar{\sigma} = \bar{\sigma}(\bar{\epsilon})$, is indeterminate, resulting in a $\bar{\epsilon}/(1-2\nu) = 0/0$ situation.

special case of incompressible target material, we know further that $\nu = 0.5$ and that $e_r = -e_\theta$. These substitutions lead to

$$s_r = -s_\theta \tag{17}$$

$$s_\theta = 2E/3 \cdot e_\theta \tag{18}$$

$$(\text{thus, } s_z = 0) \quad .$$

If we adopt the Tresca yield condition, yield occurs when s_θ reaches a magnitude, such that

$$Y = s_\theta - s_r \quad \rightsquigarrow \quad Y = 2s_\theta \quad , \tag{19}$$

Y being the yield strength of the target. The corresponding hoop strain ϵ_θ, evaluated at the outer edge of the target's plastic zone is, for our axisymmetric situation,

$$\epsilon_\theta^{\text{yield}} = \int_{R_{\text{out}}^\circ}^{R_{\text{out}}^{\max}} dr/r = \ln(R_{\text{out}}^{\max}/R_{\text{out}}^\circ) \quad . \tag{20}$$

Substituting Eqs. 20 and 18 into Eq. 19 leads to the yield condition in terms of the strain at the edge of the plastic zone,

$$Y = \frac{2E}{3} \ln\left(\frac{R_{\text{out}}^{\max 2}}{R_{\text{out}}^{\circ 2}}\right) \tag{21}$$

by way of the following chain:

$$\begin{aligned} Y &= 2s_\theta \\ &= 2(2E/3)\epsilon_\theta^{\text{yield}} \\ &= (2E/3)2\ln(R_{\text{out}}^{\max}/R_{\text{out}}^\circ) \\ &= (2E/3)\ln(R_{\text{out}}^{\max 2}/R_{\text{out}}^{\circ 2}) \quad . \end{aligned}$$

Incompressible continuity connects the expansion of the plastic zone radius to the expansion of the crater itself:

$$R_{\text{out}}^{\max 2} - R_{\text{out}}^{\circ 2} = R_*^{\max 2} - R_*^{\circ 2} \quad . \tag{22}$$

Equation 22 can be re-expressed as

$$\left(\frac{R_{\text{out}}^{\max 2}}{R_{\text{out}}^{\circ 2}}\right) = 1 + \frac{R_{*}^{\max 2} - R_{*}^{\circ 2}}{R_{\text{out}}^{\circ 2}} \quad , \tag{23}$$

which can then be substituted into Eq. 21 to obtain

$$\frac{3Y}{2E} = \ln\left(\frac{R_{*}^{\max 2} - R_{*}^{\circ 2}}{R_{\text{out}}^{\circ 2}}\right) \quad . \tag{24}$$

At this point, one may solve for the value of R_{out}° as

$$R_{\text{out}}^{\circ} = \sqrt{\frac{R_{*}^{\max 2} - R_{*}^{\circ 2}}{e^{(3Y/2E)} - 1}} \tag{25}$$

Quantities like Y and E are known material properties of the target. The quantity R_{*}° is the initial crater radius, which we take as r_j, the known radius of the impacting jet. Lastly, the final crater radius $R_{*}^{\max}$ is known from the line-explosion solution, given (using the target resistance H in lieu of the boundary pressure p_∞) as

$$R_{*}^{\max} = \sqrt{R_{*}^{\circ 2} + \frac{dE_c/dz}{\pi H}} \quad . \tag{26}$$

Equation 26 is also expressed in terms of known quantities so that, substituting Eq. 26 into Eq. 25, we obtain, finally,

$$R_{\text{out}}^{\circ} = \sqrt{\frac{dE_c/dz}{\pi H(e^{(3Y/2E)} - 1)}} \quad . \tag{27}$$

Naturally, when employed in the context of crater expansion during a penetration event, the value of dE_c/dz will be evaluated with Eq. 10. Interestingly, the plastic zone size is not a function of the jet diameter directly, but only indirectly through the rate at which the jet deposits energy into the target (dE_c/dz). For the situation when $Y \ll E$, as is often the case, Eq. 27 may be approximated as

$$R_{\text{out}}^{\circ} \approx \sqrt{\frac{2E}{3Y} \cdot \frac{dE_c/dz}{\pi H}} \quad ,$$

through the use of a first-order Maclaurin-series expansion.

5. A Model for Crater Expansion

The purpose of this report is to adapt the axisymmetric, instantaneous line-explosion model[1] to the problem of crater expansion under high-speed penetration. In the latter case, the energy deposition into the target is not *instantaneous*, but *progressive* in time and space. Nonetheless, because the progression is rapid, we hope that each successive cross section in the axisymmetric target may be subject to a deformation that, in most regards, retains the plane strain character of the line-explosion model.

The report's derivations, to this point, have focused on establishing a direct one-to-one correspondence between input parameters to the line-explosion model and inputs to the ballistic-penetration problem. The parameter inputs to the line-explosion model include ρ, R°_*, $R^\circ_{\rm out}$, dE_c/dz, and p_∞. In each case, we are able to find a direct one-to-one correspondence to the ballistic problem—between the following:

1. the density (ρ) of the fluid subjected to the line explosion and the density of the solid ballistic target (ρ_t),
2. the initial inner radius of the line-explosion fluid shell (R°_*) and the radius of the jet penetrator (r_j),
3. the initial outer radius of the line-explosion fluid shell ($R^\circ_{\rm out}$) and the radius of the solid target's plastic zone (Eq. 27),
4. the magnitude of the linear energy density of a line explosion (dE_c/dz) and the linear energy density deposited by an eroding penetrator (Eq. 10), and
5. the work performed by an external reservoir operating at an elevated pressure (p_∞) on a fluid shell and the internal plastic work accomplished by the target resistance (H) in the plastic zone of a deforming solid target.

Thus, we are able, through direct substitution, to recast the problem of line explosion to address the problem of penetrating crater expansion (Table 1).

6. Example Problem

Let us set about demonstrating this adaptation of the line-explosion solution to the problem of jet penetration. Consider the problem of a copper jet segment impacting a monolithic rolled homogeneous armor target (Brinell hardness 321). For the purposes of demonstration, we consider the jet segment to be traveling at a constant velocity of $V_j = 5$ km/s. We take the remaining jet parameters as $\rho_j = 8900$ kg/m^3 and $r_j = 2$ mm (4-mm diameter).

Table 1 One-to-one correspondence of the line explosion and ballistic penetration parameters

Line-explosion parameter	Corresponding ballistic-penetration parameter
ρ	ρ_t
R_*°	r_j
R_{out}°	$\sqrt{\dfrac{dE_c/dz}{\pi H(e^{(3Y/2E)}-1)}}$
dE_c/dz	$\dfrac{1+\gamma+\gamma^2}{(1+\gamma)^2}\cdot\dfrac{\gamma}{2}\rho_j V_j^2\,\pi r_j^2$
p_∞	H

For the target, we take the parameters as $\rho_t = 7850\,\text{kg/m}^3$, $Y = 1.07\,\text{GPa}$, $H = 5.4\,\text{GPa}$, and $E = 205\,\text{GPa}$. The density ratio implies that $\gamma = 0.9392$. Following the tenets of Table 1 (using Eqs. 10 and 27), the equivalent line-explosion parameters may be calculated (in the mks unit system) as $\rho = 7850\,\text{kg/m}^3$, $R_*^\circ = 0.002\,\text{m}$, $R_{\text{out}}^\circ = 0.0859\,\text{m}$, $p_\infty = 5.4\times10^9\,\text{Pa}$, and $dE_c/dz = 985{,}040\,\text{J/m}$.

From these parameters, we may use Eq. 26 to calculate the final crater size as $R_*^{\text{max}} = 0.0079\,\text{m}$ (15.8-mm crater diameter). For further comparison, the normalized plastic-zone extent, $(R_{\text{out}}^\circ - R_*^{\text{max}})/(2R_*^{\text{max}})$, had been previously estimated[4] for steel targets as 3.5, using a purely empirical approach for lower impact velocities. For our current problem, we obtain a value of $(0.0859-0.0079)/(2\cdot0.0079) = 4.9$.

Let us proceed to solve the problem. Recall the line-explosion solution[1]:

$$\frac{dR_*(t)}{dt} = G(R_*) \quad , \tag{28}$$

where

$$G(R_*) = \sqrt{\frac{dE_c/dz - p_\infty(R_*^2 - R_*^{\circ 2})\pi}{\pi\rho}\cdot\frac{R_*^{-2}}{\ln\left(\sqrt{R_*^2 + R_{\text{out}}^{\circ 2} - R_*^{\circ 2}}/R_*\right)}} \quad . \tag{29}$$

A 1-D discretized integration (*i.e.*, summation) has been devised to solve Eq. 28 for time t as a function of current crater radius R_* and the function $G(R_*)$, as given in Eq. 29:

$$t(R_*) = \int_{R_*^\circ}^{R_*}\frac{dr}{G(r)} = \sum_{i=1}^{n}\frac{\Delta r}{G(\bar{r}_i)} \quad ,$$

where $\Delta r = (R_* - R_*^\circ)/n$ and $\bar{r}_i = R_*^\circ + \left(i - \frac{1}{2}\right)\Delta r$. In the present use case, n is arbitrarily chosen as 1000, with $R_* = R_*^{\max}$. Because our hypothetical jet is traveling at constant velocity, the penetration rate is constant and the resulting time t can be directly converted into an axial distance from the jet–target interface, through $\Delta z = U \Delta t$. This allows for immediate reconstruction of the crater profile, as depicted in Fig. 1(a).

In the figure, showing a side view of the axisymmetric impact event, the red-outlined cylindrical jet travels downward, creating an expanding crater as it axially erodes the target. The energy for this process is donated by the eroding penetrator. The target crater grows radially, as a result of the energy deposited at the jet–target interface. The vertical distance from the interface is a measure of not only the axial position z in the target block, but also the time that has elapsed since that particular cross section suffered impact and began its radial growth. Maximum crater expansion, for any given cross section, occurs at 14.75 µs after the expansion commences, corresponding to an axial distance of 3.8 cm from the jet–target interface, as shown in Fig. 1(a). The radial position of the crater wall indicates the crater growth that has transpired for the particular duration associated with each successive cross section.

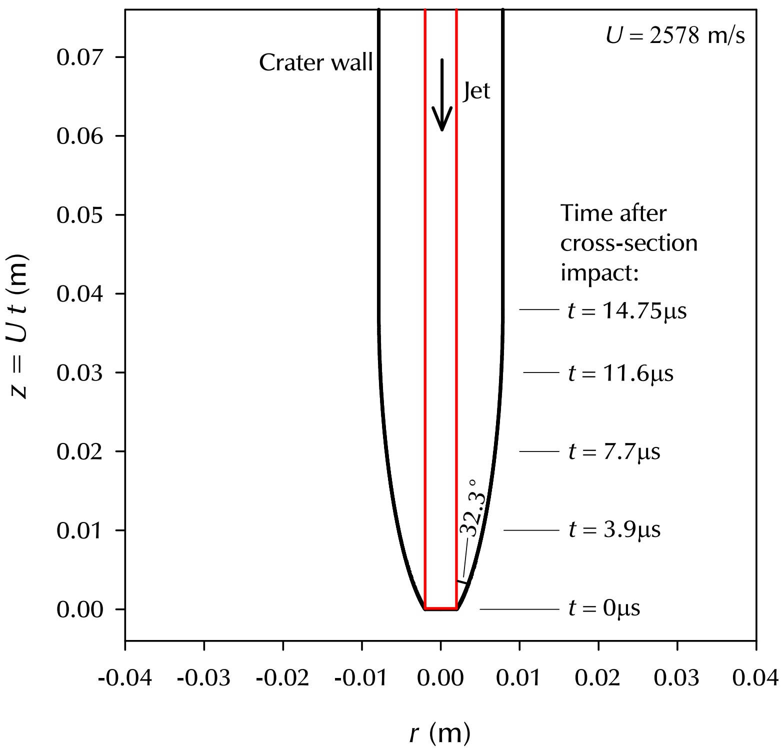
Note, however, that such a solution does not automatically accommodate (*i.e.*, make room for) the eroded penetrator material, which could otherwise force an adjustment to the crater profile. Generally, however, such an adjustment would be limited to a very small region in the vicinity of the interface, shown for the current situation in Fig. 2, where the jet deposition radius on the crater wall, R_j, may deduced from incompressible continuity as $R_j = \sqrt{R_*^2 - r_j^2}$.

As presented in ARL-MR-1037,[1] the spatial distribution of pressure and radial velocity is analytically available as a function of the current crater radius. To reiterate from that source,

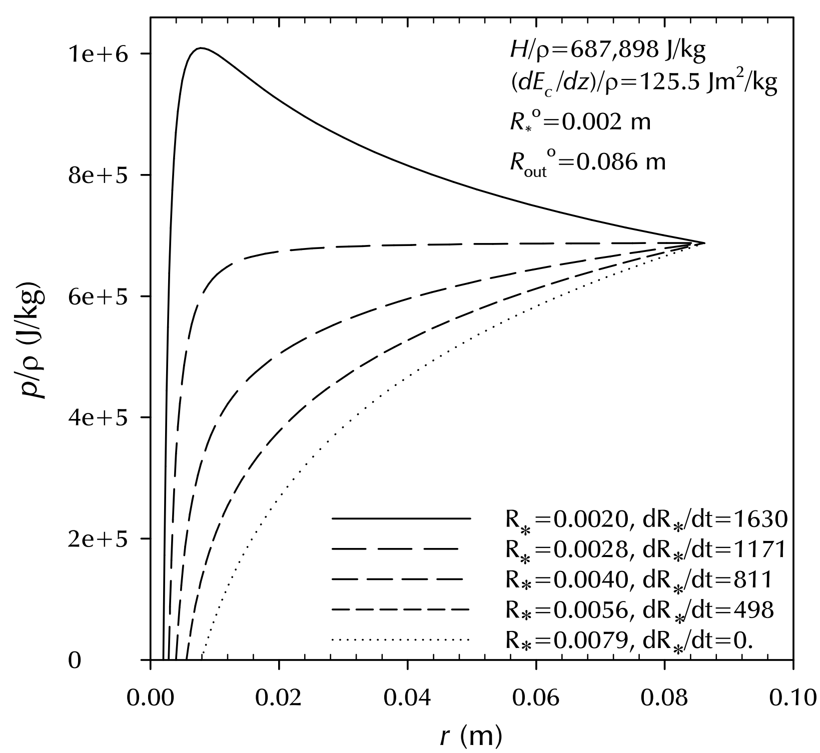
$$\frac{p\big(r, R_*(t)\big)}{\rho} = -\ln\!\left(\frac{r}{R_*}\right)\frac{d\big(R_*\,G(R_*)\big)}{dR_*}G(R_*) + \frac{1}{2}\frac{r^2 - R_*^2}{r^2}G^2(R_*) \qquad (30)$$

$$v\big(r, R_*(t)\big) = G(R_*)\,R_*/r \quad .$$

Equation 30 has been evaluated for five values over the range of crater expansion and the resulting pressure distributions are presented in Fig. 1(b). In this presentation, the radius r is presented on a linear scale. The location on the right where all the



(*a*)



(*b*)

Fig. 1 Application of the line-explosion model to the problem of a copper jet (ρ_j = 8900 kg/m^3, r_j = 2 mm, V_j = 5000 m/s) penetrating rolled homogeneous armor (ρ_t = 7850 kg/m^3, Y = 1.07 GPa, H = 5.4 GPa, E = 205 GPa): (a) the crater profile, (b) the pressure distribution

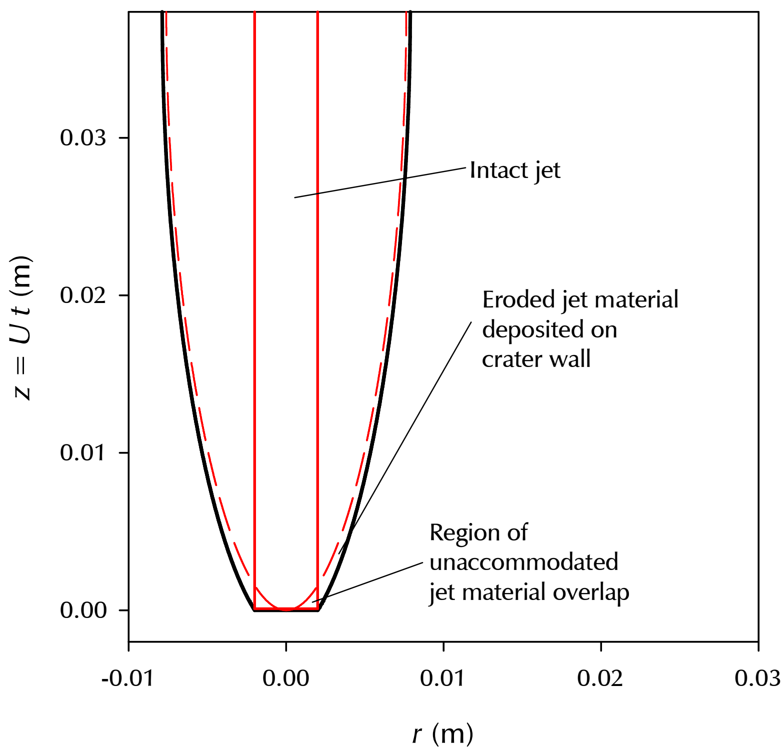


Fig. 2 An illustration of the unaccommodated overlap of intact and eroded jet near the jet–target interface

curves converge occurs at the outer extent of the plastic zone, $r = R^{\circ}_{\text{out}}$. According to the tenets of the model, up until the time when the crater expansion ceases (14.75 μs after each successive cross section impact), the pressure at this plastic-zone periphery corresponds to the target resistance H (normalized by ρ_t in the graph).

The initial growth rate of the crater may be analytically obtained by evaluating Eq. 29 at the initial radius R°_{*}:

$$\left.\frac{dR_*}{dt}\right|_{R_*=R^{\circ}_*} = G(R^{\circ}_*) = \sqrt{\frac{dE_c/dz}{\pi\rho} \cdot \frac{R_*^{\circ-2}}{\ln\left(R^{\circ}_{\text{out}}/R^{\circ}_*\right)}} \quad . \tag{31}$$

Employing Eq. 31 with our equivalent line-explosion parameters, we compute the initial rate of crater expansion when $R_* = 0.002$ as $dR_*/dt = 1630$ m/s (*cf.* the jet velocity of 5000 m/s). Given that the steady-state penetration velocity of the jet–target interface, from Eq. 3, is $U = 2578$ m/s, simple trigonometry shows that the crater's angle of departure from the rod is $\arctan(1630/2578) = 32.3°$ where the crater wall meets the jet–target interface, as seen in Fig. 1(a).

7. Conclusion

In this report, the recently developed line-explosion model presented in ARL-MR-1037[1] was adapted to describe the time-dependent crater growth associated with high-velocity impact and eroding penetration. The line-explosion model was, itself, an axisymmetric interpretation of the more commonly understood point-explosion model.[2,3]

Whereas an idealized line explosion occurs instantly along an infinite length, this report explores the notion of treating the line explosion as an axial progression, rather than an instantaneous event. In so doing, a direct correspondence can be drawn between the *progressive* line explosion and the crater expansion associated with a traditional eroding-penetration event. This report lays out the full one-to-one correspondence of these two seemingly disparate situations, providing the variables and/or the equations for how to convert the conditions of an eroding-penetration event into the equivalent parameters for a line explosion.

An example is solved for a copper jet striking an armor target at 5-km/s velocity. The resulting crater and its time-dependent growth are part of the solution offered, as are the time and spatially dependent fields of pressure and radial velocity inside the target's plastic zone. The example credibly demonstrates the usefulness of this approach.

While, for simplicity, this report addresses a case where the rate of penetration is considered hydrodynamic and the conditions at the penetrator–target interface remained invariant in time, there is no theoretical hindrance to extending the use of this approach to handle lower-velocity impacts, where rod and target strengths play a role. In addition, treating either the deceleration of a finite rod or a velocity gradient in a jet penetrator may also be addressed, with the addition of more complexity to the model.

8. References

1. Segletes SB. Instantaneous axisymmetric line explosion in incompressible fluid-like media. DEVCOM Army Research Laboratory (US); 2021 Aug. Report No.: ARL-MR-1037.

2. Grinfeld MA, Segletes SB. Instantaneous point explosion in incompressible fluid-like media. DEVCOM Army Research Laboratory (US); 2021 Aug. Report No.: ARL-TR-9247.

3. Segletes SB. Verifying the outer boundary condition of the point explosion. DEVCOM Army Research Laboratory (US); 2021 Aug. Report No.: ARL-TN-1074.

4. Segletes SB, Grote R, Polesne J. Improving the rod-penetration algorithm for tomorrow's armors. In: Proceedings of the 22nd Army Science Conference; 2000 Dec 11–13; Baltimore, MD.

1 (PDF)	DEFENSE TECHNICAL INFORMATION CTR DTIC OCA	
1 (PDF)	DEVCOM ARL FCDD RLD DCI TECH LIB	
1 (PDF)	LOS ALAMOS NATIONAL LAB L G HILL	
3 (PDF)	DE TECHNOLOGIES R CICCARELLI W FLIS W CLARK	
4 (PDF)	SANDIA NATL LAB J NIEDERHAUS A ROBINSON C SIEFERT A RODRIGUEZ	
2 (PDF)	DREXEL UNIVERSITY B FAROUK P GRINFELD	
72 (PDF)	DEVCOM ARL FCDD RLD P BAKER FCDD RLD D T ROSENBERGER FCDD RLD FC T SANO FCDD RLD FR M TSCHOPP FCDD RLW S SCHOENFELD FCDD RLW B R BECKER C HOPPEL A TONGE FCDD RLW MB B LOVE D HOPKINS G GAZONAS B POWERS FCDD RLW MG J ANDZELM FCDD RLW T R FRANCART J HOGGE R YEAGER	FCDD RLW TA SR BILYK C ADAMS P BERNING M COPPINGER M J GRAHAM M GREENFIELD W C UHLIG FCDD RLW TB S SATAPATHY T WEERASOORIYA FCDD RLW TC J CAZAMIAS R BECKER D CASEM J CLAYTON M FERMEN-COKER B LEAVY J LLOYD S SEGLETES L SHANNAHAN A TONGE C WILLIAMS FCDD RLW TD A BARD R DONEY M KEELE D KLEPONIS F MURPHY D PETTY C RANDOW S SCHRAML K STOFFEL G VUNNI V WAGONER M ZELLNER FCDD RLW TE P SWOBODA P BARTKOWSKI D GALLARDY D HORNBAKER E HORWATH J HOUSKAMP E KLIER C KRAUTHAUSER M LOVE FCDD RLW TF T EHLERS E KENNEDY L MAGNESS C MEYER B SORENSEN R SUMMERS

FCDD RLW TG
N GNIAZDOWSKI
C CUMMINS
E FIORAVANTE
D FOX
R GUPTA
S HUG
S KUKUCK
C PECORA
J STEWART

2 (PDF) DEVCOM DAC
FCDD DAA A
D HOWLE
FCDD DAW S
J AUTEN