



**AFRL-AFOSR-VA-TR-2021-0097**

---

**Compositional and Hierarchical Design of Network Control Systems**

**Arcak, Murat**  
**REGENTS OF THE UNIVERSITY OF CALIFORNIA**  
**2150 SHATTUCK AVE RM 313**  
**BERKELEY, CA,**  
**US**

---

**08/16/2021**  
**Final Technical Report**

**DISTRIBUTION A: Distribution approved for public release.**

Air Force Research Laboratory  
Air Force Office of Scientific Research  
Arlington, Virginia 22203  
Air Force Materiel Command

**REPORT DOCUMENTATION PAGE**

Form Approved  
OMB No. 0704-0188

The public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing the burden, to Department of Defense, Washington Headquarters Services, Directorate for Information Operations and Reports (0704-0188), 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302. Respondents should be aware that notwithstanding any other provision of law, no person shall be subject to any penalty for failing to comply with a collection of information if it does not display a currently valid OMB control number.  
**PLEASE DO NOT RETURN YOUR FORM TO THE ABOVE ADDRESS.**

<b>1. REPORT DATE (DD-MM-YYYY)</b> 16-08-2021	<b>2. REPORT TYPE</b> Final	<b>3. DATES COVERED (From - To)</b> 01 Jul 2018 - 30 Jun 2021
--	--------------------------------	--

<b>4. TITLE AND SUBTITLE</b> Compositional and Hierarchical Design of Network Control Systems	<b>5a. CONTRACT NUMBER</b>
	<b>5b. GRANT NUMBER</b> FA9550-18-1-0253
	<b>5c. PROGRAM ELEMENT NUMBER</b> 61102F

<b>6. AUTHOR(S)</b> Murat Arcak	<b>5d. PROJECT NUMBER</b>
	<b>5e. TASK NUMBER</b>
	<b>5f. WORK UNIT NUMBER</b>

<b>7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)</b> REGENTS OF THE UNIVERSITY OF CALIFORNIA 2150 SHATTUCK AVE RM 313 BERKELEY, CA US	<b>8. PERFORMING ORGANIZATION REPORT NUMBER</b>
--	---

<b>9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)</b> AF Office of Scientific Research 875 N. Randolph St. Room 3112 Arlington, VA 22203	<b>10. SPONSOR/MONITOR'S ACRONYM(S)</b> AFRL/AFOSR RTA2
	<b>11. SPONSOR/MONITOR'S REPORT NUMBER(S)</b> AFRL-AFOSR-VA-TR-2021-0097

**12. DISTRIBUTION/AVAILABILITY STATEMENT**  
A Distribution Unlimited: PB Public Release

**13. SUPPLEMENTARY NOTES**

**14. ABSTRACT**  
we also introduced a relaxation of this notion that we called "near-equitability," resulting in "practical stability" of the subspace that describes the desired relation between the full and reduced systems. We guaranteed the aforementioned stability and practical stability properties of the aggregate subspace with Lyapunov functions constructed using incremental dissipativity properties of the components.

**15. SUBJECT TERMS**

<b>16. SECURITY CLASSIFICATION OF:</b>			<b>17. LIMITATION OF ABSTRACT</b>	<b>18. NUMBER OF PAGES</b>	<b>19a. NAME OF RESPONSIBLE PERSON</b> FREDERICK LEVE
<b>a. REPORT</b>	<b>b. ABSTRACT</b>	<b>c. THIS PAGE</b>			<b>19b. TELEPHONE NUMBER (Include area code)</b>
U	U	U	UU	6	696-9730

Standard Form 298 (Rev.8/98)  
Prescribed by ANSI Std. Z39.18

## **Project Report for FA9550-18-1-0253**

Compositional and Hierarchical Design of Network Control Systems  
Murat Arcak, Principal Investigator  
Department of Electrical Engineering and Computer Sciences  
University of California, Berkeley

The purpose of this project was to develop a compositional and hierarchical framework to manage the complexity of network control systems. The aim in hierarchical control is to compose a control action for an aggregate model and to refine this action to lower-level controllers. The aim in compositional design is to expose the system as an interconnection of subsystems and to derive guarantees for each subsystem under appropriate assumptions on the others.

### **Aggregation for Hierarchical Control:**

We developed procedures for aggregating the components of a network into a smaller number of groups so that we obtain a reduced model that captures the interaction between these groups. The order reduction allows us to pursue control design methods that satisfy complex requirements, such as those expressed in temporal logic, that do not scale well to large networks. Unlike standard aggregation procedures, which rely on weak coupling assumptions between the groups, we developed a new approach where we represent the interconnection of the components with a graph and leverage the graph-theoretic notion of "equitable partitions" to identify an invariant subspace where the full network model collapses to the reduced model. We then developed a hierarchical controller where the high-level control action regulates the dynamical behavior on this subspace, thus shaping the behavior of each group, and low-level controllers implemented by each component guarantee asymptotic stability of this subspace, thus ensuring conformity with the behavior assigned to each group. Since the equitability condition can be restrictive in practice, we also introduced a relaxation of this notion that we called "near-equitability," resulting in "practical stability" of the subspace that describes the desired relation between the full and reduced systems. We guaranteed the aforementioned stability and practical stability properties of the aggregate subspace with Lyapunov functions constructed using incremental dissipativity properties of the components.

A comprehensive journal paper, [7], summarizing these results was published in *Automatica*. In a follow-up paper presented at the 2019 IEEE Conference on Decision and Control [13], we developed a

numerical procedure to design low-level controllers simultaneously with a Lyapunov function that establishes (practical) stability of the aggregate subspace. In [11], presented at the 2020 American Control Conference we applied this approach to the co-design of motion planning and control. An aggregate model of lower fidelity is used for planning in real time and the high-fidelity model is used to design a tracking controller that bridges the gap between the two models. This result is of great practical interest, because a high-fidelity model is typically too complex for real-time planning.

In [8], published in Nature Communications, we used the network aggregation concepts for model reduction in multicellular biological systems. The analysis of spatio-temporal behaviors in such systems is challenging due to high order and nonlinear differential equations governing them. Our approach greatly simplified this task and enabled design of new synthetic gene networks accomplishing desired pattern formation specifications for gene expression.

### Reachability Analysis:

This project has also contributed significantly to reachability analysis tools, which are of critical importance for the design and verification of control systems when the aim is to reach a target set, to avoid unsafe sets, or to accomplish more complex tasks specified in the form of automata or temporal logic. Addressing the computational bottlenecks that heretofore hindered progress in this area, we have developed a suite of tools to efficiently over-approximate reachable sets with multi-dimensional intervals. These tools, which are summarized in the new book [1], exploit types of monotonicity among other system properties and provide tight over-approximations with great computational efficiency.

Interval over-approximations are of interest because they can be obtained, stored and manipulated with low computational complexity. They are particularly efficient when obtaining discrete abstractions, where the state space is partitioned into intervals which become the discrete states in the abstract model, and the successors of each such state are the intervals that are reachable from the predecessor.

Among the theoretical advances achieved in this project was a new method for interval reachability analysis that makes use of sensitivity matrices, which contain the partial derivatives representing the variations of the system trajectories in response to variations of the

initial states. Using interval arithmetics, we first over-approximate the possible values of the second-order sensitivity at the final time of the reachability problem. Then we exploit these bounds and the evaluation of the first-order sensitivity matrices at a few sampled initial states to obtain an over-approximation of the first-order sensitivity, which is in turn used to over-approximate the reachable set of the initial system. Unlike existing methods relying only on the first-order sensitivity matrix, this new approach provides guaranteed over-approximations of the first-order sensitivity and can also provide such over-approximations with an arbitrary precision by increasing the number of samples. This result was reported in the IFAC World Congress [10] and also subsumed into the book [1].

In another paper, [9], at the same congress we applied the aforementioned interval reachability tools to motion planning and combined the results with the hierarchical control method [11] discussed above to achieve safety in an autonomous docking application.

Another application was to autonomous driving scenarios in [2]. In this application we first leverage our interval reachability analysis tools to obtain a finite-state abstraction of the dynamical model. Next we exploit inherent monotonicity properties of the model to design a safety controller. Monotonicity properties provide major computational simplifications and make it possible to deal with complex scenarios, such as a vehicle following scenario and a left turn scenario.

We have also studied other reachable set representations, such as sublevel sets of polynomial functions [5,6], which require heavier computation but may provide tighter fits to the actual reachable set. A salient feature of these publications is that they study robustness to unmodeled dynamics which, to our knowledge, has not been attempted in reachability analysis before.

### Compositional Design and Verification:

Another thrust in this project was to improve the scalability of control design and verification tools so that they can be applied to higher dimensional systems. In [3] we further developed dissipativity-based compositional analysis tools, introduced earlier by the PI. In particular we made use of a new notion of *delta dissipativity*, originally introduced in a game theoretic context, to derive a compositionality result in general interconnections of dynamical systems. We presented conditions under which a large-scale interconnection of delta

dissipative systems is delta dissipative, and adapted the result to also analyze stability and asymptotic stability of equilibrium points for the interconnection without relying on the knowledge of the location of these points.

An increasingly popular control synthesis approach (already mentioned above in connection with references [2] and [9]) is to first obtain a "finite abstraction" that translates a continuous-state system into a finite state machine that mimicks the original dynamics. However, such abstraction-based procedures suffer from several computational bottlenecks that hinder their applicability to higher order systems. To overcome these bottlenecks we presented a flexible and extensible framework in [14] that decomposes the abstraction-based synthesis algorithm into smaller components and exploits structural properties of the system and the control specification. This framework is grounded in the theory of *relational interfaces* from the Programming Languages literature and provides a principled methodology to seamlessly combine different techniques, such as dynamic precision grids, refining abstractions while synthesizing, or decomposed control predecessors, or to create custom procedures to exploit an application's intrinsic structural properties.

In the other publications, [4] and [12], we started investigating learning-based approaches to achieve further scalability in reachability analysis as well as control synthesis. This line of work will be continued in a subsequent project.

#### Dissemination:

During the project period the PI presented the results at: (1) a semi-plenary at the 2020 IEEE Conference on Decision and Control, (2) the Workshop on Analysis and Control of Large Scale Complex Networks in Grenoble, France, (3) the International Workshop on Societal-Scale Cyber-Physical Transport Systems at the Royal Institute of Technology (KTH) in Stockholm, Sweden, (4) the pre-conference workshop "Computation-aware algorithmic design for cyber-physical systems" at the 2018 IEEE Conference on Decision and Control, and (5) the tutorial session "Monotone systems theory for reachability and safety" at the 2020 IEEE Conference on Decision and Control. He further presented seminars featuring results from the project at: (6) the University of Southern California, (7) University of California at Santa Barbara, (8) University of Washington, and (9) McGill University, Montreal, Canada.

## Publications:

### Book:

[1] Pierre-Jean Meyer, Alex Devonport and Murat Arcak. Interval Reachability Analysis: Bounding Trajectories of Uncertain Systems with Boxes for Control and Verification. Springer, SpringerBriefs in Control, Automation and Robotics, 2021.

### Journal Papers:

[2] Stanley Smith, Adnane Saoud and Murat Arcak. Monotonicity-based symbolic control for safety in driving scenarios. IEEE Control Systems Letters, vol.6, pages 830-835, 2022.

[3] Katherine Schweidel and Murat Arcak. Compositional analysis of interconnected systems using delta dissipativity. IEEE Control Systems Letters, vol.6, pages 662-667, 2022.

[4] He Yin, Peter Seiler, Ming Jin and Murat Arcak. Imitation learning with stability and safety guarantees. IEEE Control Systems Letters, vol.6, pages 409-414, 2022.

[5] He Yin, Peter Seiler and Murat Arcak. Backward reachability using integral quadratic constraints for uncertain nonlinear systems. IEEE Control Systems Letters, vol.5, no.2, pages 707-712, 2021.

[6] He Yin, Andrew Packard, Murat Arcak and Peter Seiler. Reachability analysis using dissipation inequalities for uncertain nonlinear dynamical systems. Systems and Control Letters, vol.142, pages 104736, 2020.

[7] Stanley Smith, Murat Arcak and Majid Zamani. Approximate abstractions of control systems with an application to aggregation. Automatica, vol.119, 2020.

[8] Melinda Perkins, Dirk Benzinger, Murat Arcak and Mustafa Khammash. Cell-in-the-loop pattern formation with optogenetically emulated cell-to-cell signaling. Nature Communications, vol.11, no.1, 2020.

## Conference Papers:

- [9] Pierre-Jean Meyer, He Yin, Astrid H Brodtkorb, Murat Arcak and Asgeir Sorensen. Continuous and discrete abstractions for planning, applied to ship docking. In Proceedings of the 21st IFAC World Congress, July 2020.
- [10] Pierre-Jean Meyer and Murat Arcak. Interval reachability analysis using second-order sensitivity. In Proceedings of the 21st IFAC World Congress, July 2020.
- [11] He Yin, Monimoy Bujarbaruah, Murat Arcak and Andrew Packard. Optimization based planner-tracker design for safety guarantees. In Proceedings of the 2020 American Control Conference, pages 5194-5200, Denver, Colorado, July 2020.
- [12] Alex Devonport and Murat Arcak. Data-driven reachable set computation using adaptive Gaussian process classification and Monte Carlo methods. Proceedings of the 2020 American Control Conference, pages 2629-2634, Denver, Colorado, July 2020.
- [13] Stanley Smith, He Yin and Murat Arcak. Continuous abstraction of nonlinear systems using sum-of-squares programming. In Proceedings of the 58th IEEE Conference on Decision and Control, pages 8093-8098, Nice, France, December 2019.
- [14] Eric Kim, Murat Arcak and Sanjit Seshia. Flexible computational pipelines for robust abstraction-based control synthesis. In CAV 2019: Computer Aided Verification, pages 591-608, New York City, 2019.