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Force and Moment Accounting Procedure for Vehicles With Air-Breathing Propulsion Systems

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FOREWORD

An engineer in the Aeromechanics and Thermal Analysis Branch at the Naval Air Warfare Center Weapons Division (NAWCWD), China Lake, California, derived a procedure for computing the aerodynamic and propulsive forces and moments acting on vehicles propelled by air-breathing propulsion systems, such as ramjets. This work was performed as part of the fiscal year 2019 Solid Fuel Ramjet Accelerated Flight Demonstration Program. This detailed force and moment accounting procedure is documented herein, making this report a valuable resource that can be utilized on future weapons development projects involving air-breathing propulsion systems.

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1.0 INTRODUCTION

This report presents derivations of the equations describing propulsive forces (uninstalled thrust), aerodynamic forces (axial, normal, and side), and aerodynamic moments (pitching, rolling, and yawing) acting on flight vehicles (e.g., missiles) employing air-breathing propulsion systems (e.g., ramjets). Equations are obtained for computing these forces and moments for each of the three main phases of flight: rocket-boosted flight, air-breathing propulsion flight, and coasting flight. These derivations have been made as general as possible, and should therefore be applicable to most situations or vehicle types.

This report builds upon and supersedes an earlier document (Reference 1) describing the thrust-axial force accounting for simple (no bleed) air-breathing propulsion systems at zero degrees angle of attack. This current document expands upon the earlier work, and now provides an accounting procedure for all propulsive and aerodynamic forces and moments. More complicated propulsion systems involving bleed devices are now accommodated, and the updated accounting procedure is applicable for nonzero values of angle of attack and sideslip angle.

The approach taken in this present work is similar to that followed in the earlier document. Some changes have been made to the nomenclature used, in an attempt to be more consistent. The most significant change to the methodology presented here relates to how the aerodynamic forces and moments caused by pressure are computed. Previously, these were computed based on absolute pressure; in this report, they are computed based on gauge pressure (pressure relative to the freestream pressure). Using gauge pressure allows the aerodynamic forces and moments to be presented more clearly, provides better consistency with the propulsion community, and is more consistent with how wind tunnel test measurements are reported. It is emphasized, however, that the net aerodynamic forces and moments computed using the old and new methodologies remain identical.

For proper force and moment accounting between the propulsion and aerodynamics communities, it is recommended that the procedure outlined in this report be closely followed whenever possible.

1.1 KEY ASSUMPTIONS

The key assumptions underlying the derivations presented in this report are

- The nozzle exit is perpendicular to the axis of the vehicle.
- When the propulsion system is operating, the flow at the nozzle exit plane is uniform, or at least possesses radial symmetry.
- As a consequence, the thrust vector is parallel to the vehicle axis.
- The centroid of the nozzle exit lies on the axis of the vehicle.

- As a consequence, the thrust force does not induce any moments on the vehicle.
- Inlets are operating supercritically, i.e., back pressure does not affect the mass flow rate through the inlet. Spillage at the inlet or bleed slots and its subsequent impact on the vehicle aerodynamics are not affected by changes to back pressure (thrust setting). If the inlet enters subcritical operation, spillage (and thus aerodynamics) will be affected by back pressure (throttle setting).
- The effects of aerodynamic surface (i.e., fin) deflections are not considered separately (fins are just treated as part of the airframe external wetted surfaces).
- Steady-state (or at least quasi-steady) flow exists around and through the vehicle. The equations and procedures presented in this report may not accurately capture highly dynamic, transient events (such as port cover ejection or extremely high rate maneuvers).

1.2 CONTROL VOLUME ANALYSIS

The derivations presented in this report rely extensively on finite control volume analysis (described in most elementary aerodynamics texts, including those by Anderson [References 2 and 3]) of the momentum equation, which is an expression of Newton's second law of motion:

$$\boldsymbol{F} = m\boldsymbol{a} = \frac{d}{dt}(m\mathbf{V}) \tag{1}$$

For steady-state flow, this can be expressed as

$$\text{Force} = \text{Momentum Out} - \text{Momentum In} \tag{2}$$

or mathematically as

$$-\int_{cs} \left(p - p_{ref}\right) \hat{\boldsymbol{n}} dS + \int_{cs} \boldsymbol{\tau} dS = \int_{cs} \rho \boldsymbol{V} \left(\boldsymbol{V} \cdot \hat{\boldsymbol{n}}\right) dS \tag{3}$$

where the individual pressures p and p_{ref} are absolute pressures, and the difference $(p - p_{ref})$ is a gauge pressure. Normal vectors are assumed to point outwards from the control volume. The shear stress vector on a surface can be computed from the flow shear stress tensor according to

$$\boldsymbol{\tau} = -\bar{\tau} \cdot \hat{\boldsymbol{n}} = \begin{bmatrix} \tau_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \tau_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \tau_{zz} \end{bmatrix} \cdot \begin{bmatrix} \hat{n}_x \\ \hat{n}_y \\ \hat{n}_z \end{bmatrix} = \begin{bmatrix} \tau_{xx}\hat{n}_x + \tau_{xy}\hat{n}_y + \tau_{xz}\hat{n}_z \\ \tau_{yx}\hat{n}_x + \tau_{yy}\hat{n}_y + \tau_{yz}\hat{n}_z \\ \tau_{zx}\hat{n}_x + \tau_{zy}\hat{n}_y + \tau_{zz}\hat{n}_z \end{bmatrix} = \begin{bmatrix} \tau_x \\ \tau_y \\ \tau_z \end{bmatrix} \tag{4}$$

When computing moments, the finite control volume analysis is performed considering the conservation of angular momentum (another form of Newton's second law):

$$\boldsymbol{M} = \frac{d}{dt}(m\boldsymbol{r} \times \boldsymbol{V}) \tag{5}$$

For steady-state flow, this physically represents

$$\text{Moment} = \text{Angular Momentum Out} - \text{Angular Momentum In} \tag{6}$$

which is mathematically expressed as

$$-\int_{cs} (p - p_{ref})\,(\boldsymbol{r} \times \hat{\boldsymbol{n}})dS + \int_{cs} (\boldsymbol{r} \times \boldsymbol{\tau})dS = \int_{cs} \rho\,(\boldsymbol{r} \times \boldsymbol{V})(\boldsymbol{V} \cdot \hat{\boldsymbol{n}})dS \tag{7}$$

Note that Equation 7 can be obtained by cross-multiplying the position vector $\boldsymbol{r}$ with the linear momentum equation (Equation 3). Since the cross products appearing in Equation 7 will be used extensively in this derivation, they are expanded here for convenience:

$$\boldsymbol{r} \times \hat{\boldsymbol{n}} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \times \begin{bmatrix} \hat{n}_x \\ \hat{n}_y \\ \hat{n}_z \end{bmatrix} = \begin{bmatrix} y\hat{n}_z - z\hat{n}_y \\ z\hat{n}_x - x\hat{n}_z \\ x\hat{n}_y - y\hat{n}_x \end{bmatrix} \tag{8}$$

$$\boldsymbol{r} \times \boldsymbol{\tau} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \times \begin{bmatrix} \tau_x \\ \tau_y \\ \tau_z \end{bmatrix} = \begin{bmatrix} y\tau_z - z\tau_y \\ z\tau_x - x\tau_z \\ x\tau_y - y\tau_x \end{bmatrix} \tag{9}$$

$$\boldsymbol{r} \times \boldsymbol{V} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \times \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} yw - zv \\ zu - xw \\ xv - yu \end{bmatrix} \tag{10}$$

1.3 COORDINATE SYSTEM

This derivation employs a body-fitted coordinate system, as illustrated in Figure 1. This is the same coordinate system employed by the Missile Datcom program (Reference 4). The x axis is aligned with, and is collinear to, the longitudinal axis of the vehicle. The x axis points in the axial (nose-to-tail) direction. The y axis is perpendicular to the vehicle longitudinal axis and points in the lateral direction. Specifically, the y axis points out the starboard side of the vehicle. The z axis follows the right-hand rule and is perpendicular to the vehicle axis in the vertical direction. The axial placement of the coordinate system origin relative to the vehicle is somewhat flexible, though is usually placed at or near the nose tip.

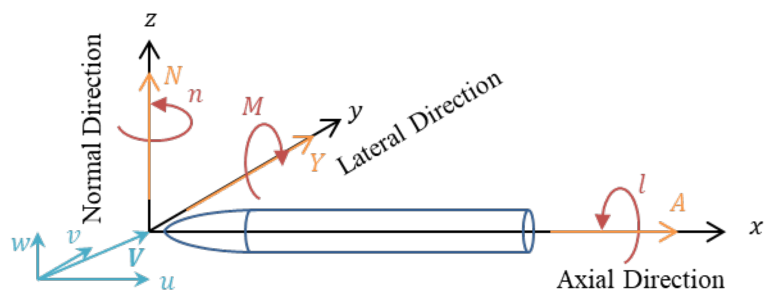

FIGURE 1. Body-Fixed Coordinate System Used in This Work.

The angle of attack is the angle between the incoming flow and the x axis (References 4 and 5):

$$\alpha = \tan^{-1}\frac{w}{u} \tag{11}$$

The sideslip angle is usually defined as (References 4 and 5)

$$\beta = \sin^{-1}\frac{v}{\|\boldsymbol{V}\|} \tag{12}$$

2.0 UNINSTALLED THRUST

Uninstalled thrust is a performance metric commonly used by the propulsion community. It can be easily computed based upon freestream conditions and conditions at the exit plane of the propulsion system. In this section, the expressions for computing uninstalled thrust are derived for the different phases of flight.

2.1 AIR-BREATHING MODE

The control volume useful for deriving uninstalled thrust for an air-breathing propulsion system is the stream tube formed by the flow that passes through the propulsion system combustor (see Figure 2), which specifically excludes any mass flow captured by the inlet, but then lost through the bleed system (if any).

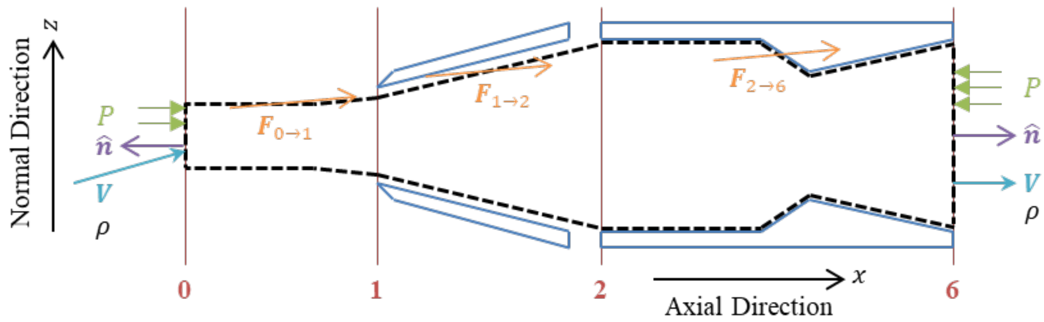

FIGURE 2. Control Volume Used for Computing Uninstalled Thrust.

The upstream boundary (station 0) is taken to be far upstream, where the conditions are at uniform freestream values. The downstream boundary is taken to be the exit plane of the propulsion system (station 6). The downstream control surface (the exit plane of the nozzle) is assumed to be planar and normal to the axial direction. Without loss of generality, the upstream control surface can also be assumed to be planar and perpendicular to the axial direction. Station 1 is the plane marking the inlet/intake to the propulsion system. Station 2 is at the end of the inlet; most importantly, station 2 is downstream of any bleed system such that all air mass flow across station 2 is delivered to the combustor. Note that this control volume lies "inside" of the vehicle geometry and excludes the airflow that gets rejected by the bleed system.

The momentum equation for this control volume can be written as

$$\begin{aligned}\boldsymbol{F}_{0\to1} + \boldsymbol{F}_{1\to2} + \boldsymbol{F}_{2\to6} - \int_0 (p - p_{ref})\,\hat{\boldsymbol{n}}dS - \int_6 (p - p_{ref})\,\hat{\boldsymbol{n}}dS \\ = \int_0 \rho\,\boldsymbol{V}(\boldsymbol{V}\cdot\hat{\boldsymbol{n}})dS + \int_6 \rho\,\boldsymbol{V}(\boldsymbol{V}\cdot\hat{\boldsymbol{n}})dS\end{aligned} \quad (13)$$

Restricting attention to the axial direction, this simplifies to

$$\begin{aligned}A_{0\to1} + A_{1\to2} + A_{2\to6} - \int_0 (p - p_{ref})\,\hat{n}_x dS - \int_6 (p - p_{ref})\,\hat{n}_x dS \\ = \int_0 \rho\,u(\boldsymbol{V}\cdot\hat{\boldsymbol{n}})dS + \int_6 \rho\,u(\boldsymbol{V}\cdot\hat{\boldsymbol{n}})dS\end{aligned} \quad (14)$$

It is then possible to define the uninstalled thrust as

$$T_U \equiv A_{0\to1} + A_{1\to2} + A_{2\to6} \quad (15)$$

such that Equation 14 reduces to

$$T_U = \int_0 (p - p_{ref})\,\hat{n}_x dS + \int_6 (p - p_{ref})\,\hat{n}_x dS + \int_0 \rho\, u(\boldsymbol{V} \cdot \hat{\boldsymbol{n}}) dS + \int_6 \rho\, u(\boldsymbol{V} \cdot \hat{\boldsymbol{n}}) dS \tag{16}$$

Assuming that the flow is uniform at stations 0 and 6, and assuming that the control surfaces are planar and perpendicular to the axial direction (i.e., $\hat{n}_x = -1$ at station 0 and $\hat{n}_x = 1$ at station 6), this equation is simplified to

$$T_U = -(p_0 - p_{ref})S_0 + (p_6 - p_{ref})S_E + \dot{m}_6 u_6 - \dot{m}_0 u_0 \tag{17}$$

where the mass flow rate (out of the control volume) is defined as

$$\dot{m} = \int_{cs} \rho\, (\boldsymbol{V} \cdot \hat{\boldsymbol{n}}) dS \tag{18}$$

Up to this point, the value of the reference pressure p_{ref} has not been specified. In the propulsion community, the reference pressure is generally taken to be the freestream static pressure (i.e., $p_{ref} \equiv p_0$). This causes the first term on the right-hand side of Equation 17 to cancel, leaving

$$T_U = (p_6 - p_0)S_E + \dot{m}_6 u_6 - \dot{m}_0 u_0 \tag{19}$$

It should be noted that $\dot{m}_0$ is the mass flow rate of air delivered to the combustion system, i.e., $\dot{m}_0 = \dot{m}_2 = \dot{m}_{air}$. For vehicles with a bleed system as part of the inlet it should be remembered that $\dot{m}_0 \neq \dot{m}_1$. Finally, the mass flow rate through the nozzle exit (station 6) is not the same as the mass flow rate through stations 0 or 2 ($\dot{m}_0 \neq \dot{m}_6$). This is because of the mass of the fuel that is combusted and added to the flow: $\dot{m}_6 = \dot{m}_{air} + \dot{m}_{fuel}$. With these clarifications in place, Equation 19 can be written as

$$T_U = (p_6 - p_0)S_E + (\dot{m}_{air} + \dot{m}_{fuel})u_6 - \dot{m}_{air} u_0 \tag{20}$$

Equations 19 and 20 are expressions for "uninstalled thrust," the thrust metric that is typically used by the propulsion community. Notwithstanding differences in nomenclature, these expressions are consistent with those for uninstalled thrust as found in standard texts (e.g., Equation 6.8 of Oates [Reference 6] or Equation 2.45 of Anderson [Reference 3]).

As derived here, a positive value of T_U represents a force applied to the flow in the axial direction. Due to Newton's third law of motion, a positive value of T_U represents a force applied to the vehicle opposite to the axial direction (i.e., $T_U > 0$ pushes the vehicle forward).

2.2 ROCKET MODE

Systems employing air-breathing propulsion systems typically begin flight by being boosted by a rocket propulsion system. The expression describing the uninstalled thrust of this mode of flight can be most easily obtained by simplifying Equation 20 with the fact that in rocket boost mode $\dot{m}_{air} = 0$. The resultant expression for uninstalled thrust in rocket mode is

$$T_U = (p_6 - p_0)S_E + \dot{m}_{fuel}u_6 \tag{21}$$

2.3 COAST MODE

When the vehicle is coasting, there is no thrust being provided by the propulsion system:

$$T_U = 0 \tag{22}$$

3.0 AXIAL FORCE

The uninstalled thrust is not the force available to accelerate the vehicle. This uninstalled thrust is partially offset by aerodynamic forces. The resultant "installed thrust" is the force available to accelerate the vehicle in the forward direction and is defined as

$$T_I = T_U - A_{aero} \tag{23}$$

It should be noted that the installed thrust, as computed by this equation, acts in the direction opposite to the positive axial direction. Thus, the net force in the axial direction (the nose-to-tail direction) is

$$A \equiv -T_I = -T_u + A_{aero} \tag{24}$$

The main purpose of this report is to provide a methodology that allows the propulsion and aerodynamic communities to independently compute the force components associated with their respective disciplines such that no force components are double-counted or inadvertently excluded. The propulsive component to the net axial force is the uninstalled thrust as computed by Equation 19 or 20. All other axial forces are considered to be part of the aerodynamic component.

Aerodynamic axial force is related to, and often confused with, drag force. (Drag force is aligned with the velocity vector, and at zero degrees angle of attack and zero degrees sideslip angle axial force and drag force are indeed the same. However, at nonzero angles of attack or sideslip the axial force is not identical to the drag force.) In the literature, the term "drag" will often be used to refer to what is actually an axial force. This should be strongly discouraged and is avoided in this derivation. In this section, expressions and a procedure for computing the aerodynamic component of axial force will be presented. Different procedures are required for the three different phases of flight.

3.1 AIR-BREATHING MODE

The net force (including propulsive and aerodynamic components) on a vehicle can be computed by integrating all the forces on the wetted surfaces (both internal and external) of the vehicle. The control volume, therefore, is as shown in Figure 3, where the control surface can be split into two parts: the external wetted surfaces of the vehicle, and the internal wetted surfaces. The net force can therefore be computed as

$$\boldsymbol{F} = \boldsymbol{F}_A + \boldsymbol{F}_D \tag{25}$$

where $\boldsymbol{F}_A$ is the force on the external wetted surfaces (the "airframe"), and $\boldsymbol{F}_D$ is the force on the internal wetted surfaces (the "duct").

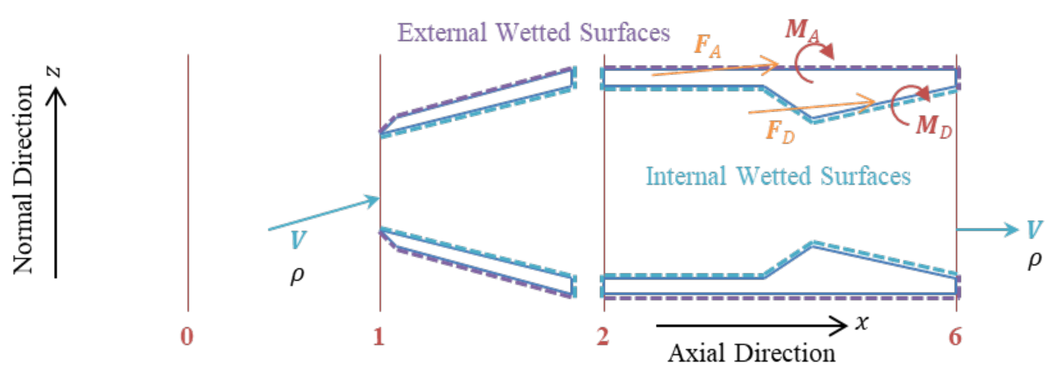

FIGURE 3. Control Volume Enclosing External and Internal Wetted Surfaces of the Vehicle.

The forces on the external wetted surfaces can be directly computed by integrating the pressure and viscous forces on these surfaces:

$$\boldsymbol{F}_A = -\int_A (p - p_{ref})\,\hat{\boldsymbol{n}} dS + \int_A \boldsymbol{\tau}\, dS \tag{26}$$

The control surface corresponding to the external wetted surfaces of the vehicle are illustrated in purple in Figure 4.

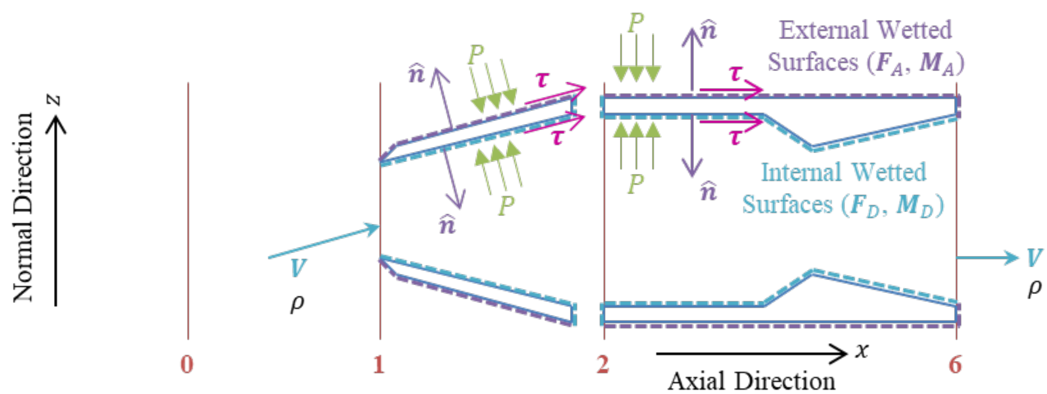

FIGURE 4. Control Volume for Direct Computation of Forces and Moments on Wetted Surfaces of the Vehicle.

Computation of the forces on the internal surfaces is most easily accomplished by considering a control volume enclosing the fluid volume inside the vehicle, as shown in Figure 5.

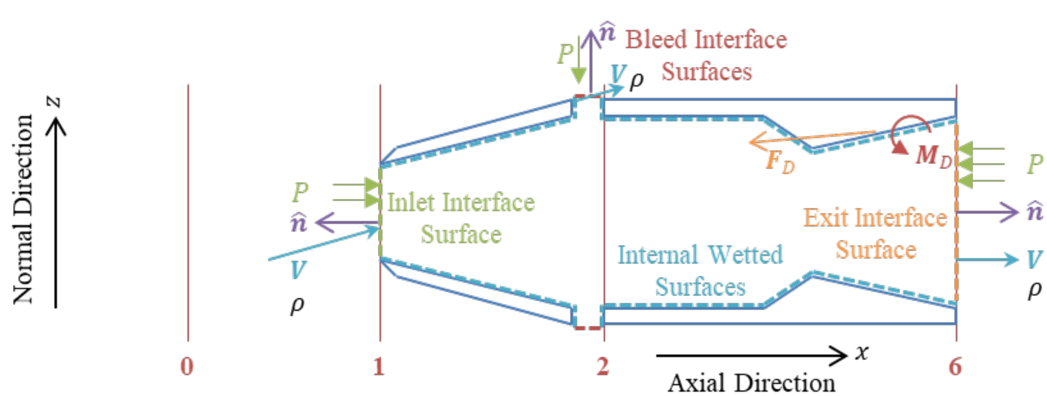

FIGURE 5. Control Volume for Computation of Forces and Moments on Internal Surfaces of the Vehicle.

Noting that the force acting on the fluid by the internal wetted surfaces is equal in magnitude to, but opposite in sign from, the forces $\boldsymbol{F}_D$ on these surfaces, the momentum equation can be written as

$$\begin{aligned} -\boldsymbol{F}_D - \int_1 \left(p - p_{ref}\right) \hat{\boldsymbol{n}} dS - \int_6 \left(p - p_{ref}\right) \hat{\boldsymbol{n}} dS - \int_B \left(p - p_{ref}\right) \hat{\boldsymbol{n}} dS \\ = \int_1 \rho \boldsymbol{V}(\boldsymbol{V} \cdot \hat{\boldsymbol{n}}) dS + \int_6 \rho \boldsymbol{V}(\boldsymbol{V} \cdot \hat{\boldsymbol{n}}) dS + \int_B \rho \boldsymbol{V}(\boldsymbol{V} \cdot \hat{\boldsymbol{n}}) dS \end{aligned} \tag{27}$$

This allows the forces on the internal surfaces to be computed from the flow entering and leaving the vehicle through the inlet, nozzle exit, and bleed:

$$\begin{aligned} \boldsymbol{F}_D = -\int_1 \left(p - p_{ref}\right) \hat{\boldsymbol{n}} dS - \int_6 \left(p - p_{ref}\right) \hat{\boldsymbol{n}} dS - \int_B \left(p - p_{ref}\right) \hat{\boldsymbol{n}} dS \\ - \int_1 \rho \boldsymbol{V}(\boldsymbol{V} \cdot \hat{\boldsymbol{n}}) dS - \int_6 \rho \boldsymbol{V}(\boldsymbol{V} \cdot \hat{\boldsymbol{n}}) dS - \int_B \rho \boldsymbol{V}(\boldsymbol{V} \cdot \hat{\boldsymbol{n}}) dS \end{aligned} \tag{28}$$

Substituting Equations 26 and 28 into Equation 25, the net force on the vehicle can be computed as

$$\begin{aligned} \boldsymbol{F} = -\int_A \left(p - p_{ref}\right) \hat{\boldsymbol{n}} dS + \int_A \boldsymbol{\tau}\, dS - \int_1 \left(p - p_{ref}\right) \hat{\boldsymbol{n}} dS \\ - \int_6 \left(p - p_{ref}\right) \hat{\boldsymbol{n}} dS - \int_B \left(p - p_{ref}\right) \hat{\boldsymbol{n}} dS - \int_1 \rho \boldsymbol{V}(\boldsymbol{V} \cdot \hat{\boldsymbol{n}}) dS \\ - \int_6 \rho \boldsymbol{V}(\boldsymbol{V} \cdot \hat{\boldsymbol{n}}) dS - \int_B \rho \boldsymbol{V}(\boldsymbol{V} \cdot \hat{\boldsymbol{n}}) dS \end{aligned} \tag{29}$$

Restricting consideration to the axial direction results in

$$\begin{aligned} A = -\int_A \left(p - p_{ref}\right) \hat{n}_x dS + \int_A \tau_x\, dS - \int_1 \left(p - p_{ref}\right) \hat{n}_x dS \\ - \int_6 \left(p - p_{ref}\right) \hat{n}_x dS - \int_B \left(p - p_{ref}\right) \hat{n}_x dS \\ - \int_1 \rho u(\boldsymbol{V} \cdot \hat{\boldsymbol{n}}) dS - \int_6 \rho u(\boldsymbol{V} \cdot \hat{\boldsymbol{n}}) dS - \int_B \rho u(\boldsymbol{V} \cdot \hat{\boldsymbol{n}}) dS \end{aligned} \tag{30}$$

Substituting Equations 30 and 16 into Equation 24 and solving for the aerodynamic component of axial force produces

$$\begin{aligned} A_{aero} = A + T_U = &-\int_A \left(p - p_{ref}\right)\hat{n}_x dS + \int_A \tau_x\, dS - \int_1 \left(p - p_{ref}\right)\hat{n}_x dS \\ &-\int_6 \left(p - p_{ref}\right)\hat{n}_x dS - \int_B \left(p - p_{ref}\right)\hat{n}_x dS - \int_1 \rho\, u(\boldsymbol{V}\cdot\hat{\boldsymbol{n}})dS \\ &-\int_6 \rho\, u(\boldsymbol{V}\cdot\hat{\boldsymbol{n}})dS - \int_B \rho\, u(\boldsymbol{V}\cdot\hat{\boldsymbol{n}})dS + \int_0 \left(p - p_{ref}\right)\hat{n}_x dS \\ &+\int_6 \left(p - p_{ref}\right)\hat{n}_x dS + \int_0 \rho\, u(\boldsymbol{V}\cdot\hat{\boldsymbol{n}})dS + \int_6 \rho\, u(\boldsymbol{V}\cdot\hat{\boldsymbol{n}})dS \end{aligned} \tag{31}$$

The terms for station 6 (the nozzle exit plane) cancel, and, with some rearrangement, the aerodynamic component of axial force can be computed as

$$\begin{aligned} A_{aero} = &-\int_A \left(p - p_{ref}\right)\hat{n}_x dS + \int_A \tau_x\, dS + \int_0 \left(p - p_{ref}\right)\hat{n}_x dS \\ &-\int_1 \left(p - p_{ref}\right)\hat{n}_x dS + \int_0 \rho\, u(\boldsymbol{V}\cdot\hat{\boldsymbol{n}})dS - \int_1 \rho\, u(\boldsymbol{V}\cdot\hat{\boldsymbol{n}})dS \\ &-\int_B \left(p - p_{ref}\right)\hat{n}_x dS - \int_B \rho\, u(\boldsymbol{V}\cdot\hat{\boldsymbol{n}})dS \end{aligned} \tag{32}$$

By defining $p_{ref} \equiv p_0$, assuming that the control surface at station 0 (in the freestream) is perpendicular to the axial direction (i.e., $\hat{n}_x = -1$ on station 0), noting that uniform flow exists in the freestream (station 0), and noting that $\dot{m}_0 = \dot{m}_{air}$, Equation 32 reduces to

$$\begin{aligned} A_{aero} = &-\int_A (p - p_0)\,\hat{n}_x dS + \int_A \tau_x\, dS - \int_1 (p - p_0)\,\hat{n}_x dS \\ &-\int_1 \rho\, u(\boldsymbol{V}\cdot\hat{\boldsymbol{n}})dS - \int_B (p - p_0)\,\hat{n}_x dS - \int_B \rho\, u(\boldsymbol{V}\cdot\hat{\boldsymbol{n}})dS \\ &-\dot{m}_{air} u_0 \end{aligned} \tag{33}$$

It is convenient to write this expression using the shorthand notation

$$A_{aero} = A_{A_p} + A_{A_s} + A_{I_p} + A_{I_m} + A_{B_p} + A_{B_m} - A_{ram} \tag{34}$$

which emphasizes the physical interpretation versus the mathematical definition. Each term in Equation 34 is physically interpreted as given in Table 1.

TABLE 1. Definition and Physical Interpretation of Different Components of Aerodynamic Axial Force.

Definition	Physical Interpretation
$A_{A_p} = -\int_A (p - p_0)\,\hat{n}_x dS$	Axial force due to the pressure applied to the external wetted surfaces of the airframe. These are all the wetted surfaces connecting the inlet interface surface and the exit interface surface via a path outside of the propulsion system.
$A_{A_s} = \int_A \tau_x \, dS$	Axial force due to shear stress acting on the external wetted surfaces of the airframe.
$A_{I_p} = -\int_1 (p - p_0)\,\hat{n}_x dS$	Axial force due to the pressure of the upstream flow applied to the inlet interface surface (the imaginary surface forming the interface between the external flow and the internal flow path at the inlet entrance).
$A_{I_m} = -\int_1 \rho u\,(\boldsymbol{V} \cdot \hat{\boldsymbol{n}}) dS$	Axial force associated with the axial component of the momentum of the airflow crossing the inlet interface surface and entering the internal flow path (the propulsion system).
$A_{B_p} = -\int_B (p - p_0)\,\hat{n}_x dS$	Axial force due to the pressure of the flow applied to the bleed interface surface (the imaginary surface forming the interface between the internal flow path and the external flow at the bleed exit).
$A_{B_m} = -\int_B \rho u\,(\boldsymbol{V} \cdot \hat{\boldsymbol{n}}) dS$	Axial force associated with the axial component of the momentum of the airflow crossing the bleed interface surface and entering the external flow.
$A_{ram} = \dot{m}_{air} u_0$	The "ram" axial force, defined as the product of the mass flux entering the propulsion system and the freestream axial velocity. This is an idealization of the momentum entering the propulsion system. It must be subtracted from the aerodynamic axial force, since this quantity is included as part of the uninstalled thrust calculation. (Only required for air-breathing mode.)
$A_{E_p} = -\int_6 (p - p_0)\,\hat{n}_x dS$	Axial force due to the pressure of the flow applied to the nozzle exit interface surface (the imaginary surface forming the interface between the internal flow path and the external flow at the nozzle exit). (Only required for coasting flight mode.)

TABLE 1. (Contd.)

Definition	Physical Interpretation
$A_{E_m} = -\int_6 \rho u\,(\boldsymbol{V} \cdot \hat{\boldsymbol{n}})dS$	Axial force associated with the axial component of the momentum of the airflow crossing the nozzle exit interface surface and entering the external flow. (Only required for coasting flight mode.)
$A_{D_p} = -\int_D (p - p_0)\,\hat{n}_x dS$	Axial force due to the pressure applied to the internal wetted surfaces of the airframe. These are all the wetted surfaces connecting the inlet interface surface and the exit interface surface via a path inside the propulsion system. (Only required for coasting flight mode.)
$A_{D_s} = \int_D \tau_x\, dS$	Axial force due to shear stress acting on the internal wetted surfaces of the airframe. (Only required for coasting flight mode.)

A diagram illustrating the control surfaces used for each component is given in Figure 6 for a normal shock inlet and in Figure 7 for an axisymmetric inlet (which can easily be generalized to any inlet configuration).

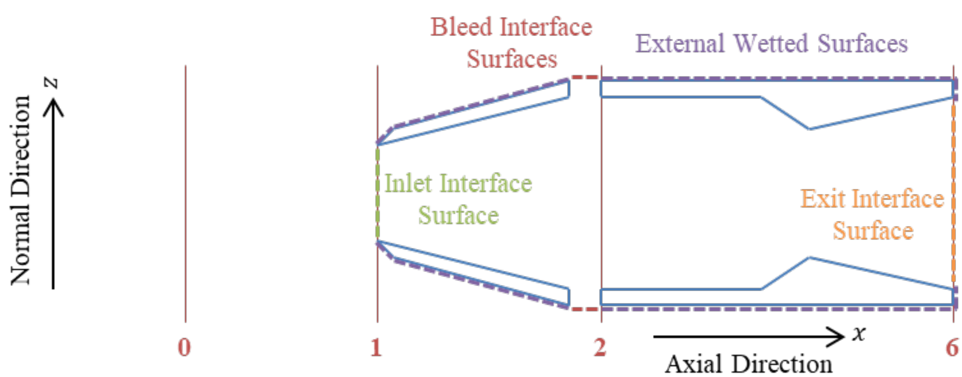

FIGURE 6. Control Surfaces for Vehicle With Normal Shock Inlet.

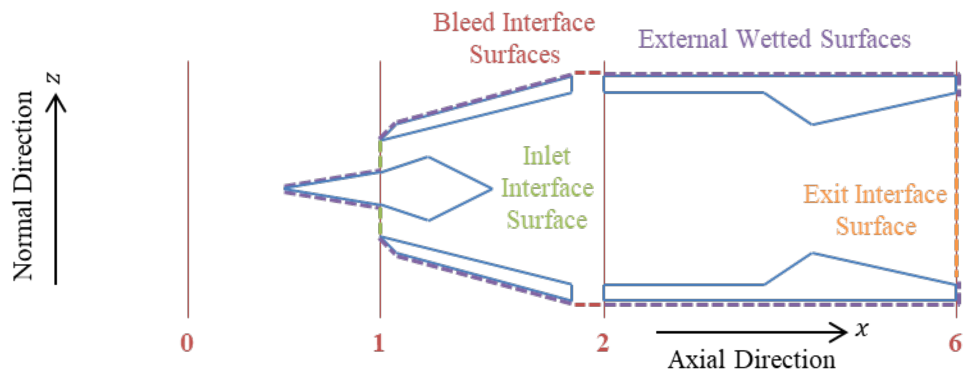

FIGURE 7. Control Surfaces for Vehicle With Axisymmetric Inlet.

3.2 ROCKET MODE

The expression describing the aerodynamic axial force for a vehicle in the rocket-boosted mode of flight can be most easily obtained by simplifying Equation 33 to obtain

$$A_{aero} = -\int_{A} (p - p_0)\,\hat{n}_x dS + \int_{A} \tau_x \, dS \tag{35}$$

$$A_{aero} = A_{A_p} + A_{A_s} \tag{36}$$

This simplification takes advantage of the fact that there is no flow through the inlet to the propulsion system (the inlet flow path is plugged at some point by a port cover). All wetted surfaces of the internal flow path upstream of this blockage then need to be treated as external wetted surfaces of the airframe (see Figure 8).

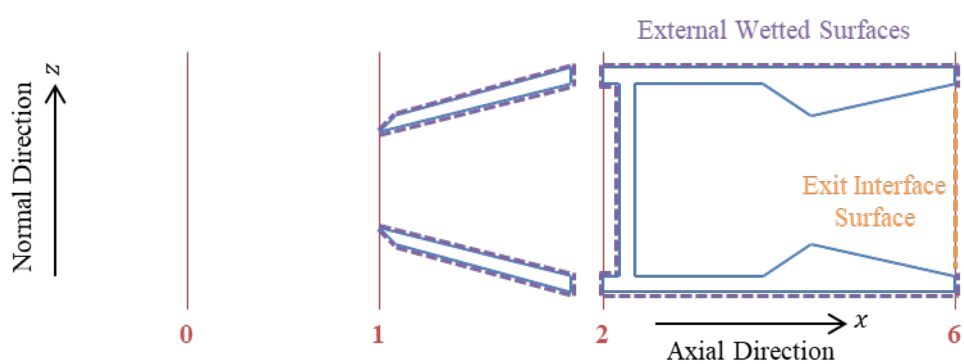

FIGURE 8. Control Surfaces for Vehicle in Rocket-Boosted Flight Mode.

3.3 COAST MODE

In coasting flight, there are no propulsion forces acting on a vehicle; all forces are aerodynamic in nature. However, for air-breathing propulsion systems, there will still be air flowing through the inlet and internal flow path (unless some sort of inlet door device is employed). This internal flow will produce forces on the vehicle, which must be taken into account as part of the vehicle aerodynamics. (These forces due to the internal flow path are also present when the propulsion system is operating but in that case, get absorbed into the calculation of the uninstalled thrust.)

In coast mode, the aerodynamic forces can be split into two main components: the forces on the external wetted surfaces of the airframe and the forces on the internal flow path (see Equation 25). The forces on the external airframe surfaces are computed by directly integrating the pressure and shear (as per Equation 26). However, there are two possible approaches to computing the forces on the internal flow path.

In the first approach, the forces on the internal surfaces are related to the properties of the flow entering and exiting the internal flow path. This is the approach taken to obtain the forces on the vehicle when the propulsion system is operational (see Section 3.1). The net (internal plus external) axial force on the vehicle in coast mode can therefore be computed based on Equation 30). By defining $p_{ref} \equiv p_0$, Equation 30 can be rewritten as

$$\begin{aligned} A_{aero} = A = &-\int_A (p - p_0)\,\hat{n}_x dS + \int_A \tau_x\, dS - \int_1 (p - p_0)\,\hat{n}_x dS \\ &-\int_1 \rho u\,(\boldsymbol{V}\cdot\hat{\boldsymbol{n}})dS - \int_6 (p - p_0)\,\hat{n}_x dS - \int_6 \rho u\,(\boldsymbol{V}\cdot\hat{\boldsymbol{n}})dS \\ &-\int_B (p - p_0)\,\hat{n}_x dS - \int_B \rho u\,(\boldsymbol{V}\cdot\hat{\boldsymbol{n}})dS \end{aligned} \tag{37}$$

It must be noted that when the vehicle is operating in coasting flight mode, it is not possible to assume that the flow at the nozzle exit is uniform. As a result, it is necessary to retain the integrals on the nozzle exit plane. Written in shorthand, Equation 37 becomes

$$A_{aero} = A_{A_p} + A_{A_s} + A_{I_p} + A_{I_m} + A_{E_p} + A_{E_m} + A_{B_p} + A_{B_m} \tag{38}$$

where the different terms are interpreted as given in Table 1.

The alternative approach for handling the forces on the internal flow path is to directly integrate the pressure and shear on these surfaces (see Figure 4):

$$\boldsymbol{F}_D = -\int_D \left(p - p_{ref}\right)\hat{\boldsymbol{n}} dS + \int_D \boldsymbol{\tau}\, dS \tag{39}$$

The net force vector on the vehicle thus becomes

$$\boldsymbol{F} = -\int_{A} \left(p - p_{ref}\right)\hat{\boldsymbol{n}}dS + \int_{A} \boldsymbol{\tau}\, dS - \int_{D} \left(p - p_{ref}\right)\hat{\boldsymbol{n}}dS + \int_{D} \boldsymbol{\tau}\, dS \tag{40}$$

Restricting consideration to the axial direction results in

$$A_{aero} = A = -\int_{A} (p - p_0)\,\hat{n}_x dS + \int_{A} \tau_x\, dS - \int_{D} (p - p_0)\,\hat{n}_x dS + \int_{D} \tau_x\, dS \tag{41}$$

or, written in shorthand,

$$A_{aero} = A_{A_p} + A_{A_s} + A_{D_p} + A_{D_s} \tag{42}$$

The axial force on a vehicle in coasting flight can be computed according to either Equation 37 or 41, providing the same numerical value. It is the recommendation of the author that the axial force be computed both ways and compared, as a way of checking that no errors have been made during the calculations.

4.0 NORMAL FORCE

The forces acting on the vehicle in the normal direction (the vertical direction perpendicular to the vehicle axis) are considered to be exclusively of aerodynamic origin; the propulsion system is considered to make no contribution to the normal force. (This is true so long as the exit plane of the nozzle is perpendicular to the vehicle axis, and if the flow leaving the nozzle is radially uniform. Propulsion systems employing canted nozzles or thrust vector control systems are considered to be outside the scope of this report.) At zero degrees angle of attack and zero degrees sideslip angle, the normal force is equivalent to lift. Different procedures are required to compute normal force for the three different phases of flight.

4.1 AIR-BREATHING MODE

Equation 29 gives the net force vector on a vehicle when the air-breathing propulsion system is activated. Considering only the force component in the normal direction gives

$$\begin{gathered} N_{aero} = N = -\int_A \left(p - p_{ref}\right)\hat{n}_z dS + \int_A \tau_z \, dS \\ -\int_1 \left(p - p_{ref}\right)\hat{n}_z dS - \int_6 \left(p - p_{ref}\right)\hat{n}_z dS - \int_B \left(p - p_{ref}\right)\hat{n}_z dS \\ -\int_1 \rho w \left(\boldsymbol{V}\cdot\hat{\boldsymbol{n}}\right) dS - \int_6 \rho w \left(\boldsymbol{V}\cdot\hat{\boldsymbol{n}}\right) dS - \int_B \rho w \left(\boldsymbol{V}\cdot\hat{\boldsymbol{n}}\right) dS \end{gathered} \tag{43}$$

Since the nozzle exit plane is assumed to be perpendicular to the vehicle axis (thus $\hat{n}_z = 0$ on station 6), and since it is assumed that the flow at the nozzle exit plane is uniform and that the thrust is aligned with the vehicle axis (thus $w = 0$ on station 6), the integrals for station 6 equate to zero. With these observations, and defining the reference pressure to be the freestream static pressure ($p_{ref} \equiv p_0$), the expression for the normal force becomes

$$\begin{gathered} N_{aero} = -\int_A \left(p - p_0\right)\hat{n}_z dS + \int_A \tau_z \, dS \\ -\int_1 \left(p - p_0\right)\hat{n}_z dS - \int_1 \rho w \left(\boldsymbol{V}\cdot\hat{\boldsymbol{n}}\right) dS - \int_B \left(p - p_0\right)\hat{n}_z dS \\ -\int_B \rho w \left(\boldsymbol{V}\cdot\hat{\boldsymbol{n}}\right) dS \end{gathered} \tag{44}$$

This can be written in shorthand notation as

$$N_{aero} = N_{A_p} + N_{A_s} + N_{I_p} + N_{I_m} + N_{B_p} + N_{B_m} \tag{45}$$

Each term in Equation 45 is physically interpreted as given in Table 2.

TABLE 2. Definition and Physical Interpretation of Different Components of Normal Force.

Definition	Physical Interpretation
$N_{A_p} = -\int_A (p - p_0)\,\hat{n}_z dS$	Normal force due to the pressure applied to the external wetted surfaces of the airframe. These are all the wetted surfaces connecting the inlet interface surface and the exit interface surface via a path outside of the propulsion system.
$N_{A_s} = \int_A \tau_z\, dS$	Normal force due to shear stress acting on the external wetted surfaces of the airframe.
$N_{I_p} = -\int_1 (p - p_0)\,\hat{n}_z dS$	Normal force due to the pressure of the upstream flow applied to the inlet interface surface (the imaginary surface forming the interface between the external flow and the internal flow path at the inlet entrance). Note that this term will equate to zero if the inlet interface plane is perpendicular to the vehicle axis ($\hat{n}_z = 0$).
$N_{I_m} = -\int_1 \rho w\,(\boldsymbol{V} \cdot \hat{\boldsymbol{n}}) dS$	Normal force associated with the normal component of the momentum of the airflow crossing the inlet interface surface and entering the internal flow path (the propulsion system).
$N_{B_p} = -\int_B (p - p_0)\,\hat{n}_z dS$	Normal force due to the pressure of the flow applied to the bleed interface surface (the imaginary surface forming the interface between the internal flow path and the external flow at the bleed exit).
$N_{B_m} = -\int_B \rho w\,(\boldsymbol{V} \cdot \hat{\boldsymbol{n}}) dS$	Normal force associated with the normal component of the momentum of the airflow crossing the bleed interface surface and entering the external flow.
$N_{E_m} = -\int_6 \rho w\,(\boldsymbol{V} \cdot \hat{\boldsymbol{n}}) dS$	Normal force associated with the normal component of the momentum of the airflow crossing the nozzle exit interface surface and entering the external flow. (Only required for coasting flight mode.)
$N_{D_p} = -\int_D (p - p_0)\,\hat{n}_z dS$	Normal force due to the pressure applied to the internal wetted surfaces of the airframe. These are all the wetted surfaces connecting the inlet interface surface and the exit interface surface via a path inside the propulsion system. (Only required for coasting flight mode.)
$N_{D_s} = \int_D \tau_z\, dS$	Normal force due to shear stress acting on the internal wetted surfaces of the airframe. (Only required for coasting flight mode.)

4.2 ROCKET MODE

The expression for normal force on a vehicle in the rocket-boosted mode of flight is most easily obtained by simplifying Equation 44 to obtain

$$N_{aero} = -\int_A (p - p_0)\, \hat{n}_z dS + \int_A \tau_z \, dS \tag{46}$$

$$N_{aero} = N_{A_p} + N_{A_s} \tag{47}$$

This simplification has taken advantage of the fact that the inlet flow path is plugged at some point by a port cover, and thus there is no flow through the inlet to the propulsion system. Any wetted surfaces of the internal flow path upstream of this blockage need to be treated as external wetted surfaces of the airframe (see Figure 8).

4.3 COAST MODE

The relevant expression for computing the normal force on a vehicle in coasting flight can be derived following the same procedure as used in Section 3.3 for the coasting flight axial force. For brevity, the details of the derivation will not be repeated, and instead only the final expressions will be given.

If the forces on the internal surfaces are related to the properties of the flow entering and leaving the internal flow path, the normal force during coasting flight can be computed as

$$\begin{gathered} N_{aero} = N = -\int_A (p - p_0)\, \hat{n}_z dS + \int_A \tau_z \, dS \\ -\int_1 (p - p_0)\, \hat{n}_z dS - \int_1 \rho w\, (\boldsymbol{V} \cdot \hat{\boldsymbol{n}}) dS - \int_6 \rho w\, (\boldsymbol{V} \cdot \hat{\boldsymbol{n}}) dS \\ -\int_B (p - p_0)\, \hat{n}_z dS - \int_B \rho w\, (\boldsymbol{V} \cdot \hat{\boldsymbol{n}}) dS \end{gathered} \tag{48}$$

$$N_{aero} = N_{A_p} + N_{A_s} + N_{I_p} + N_{I_m} + N_{E_m} + N_{B_p} + N_{B_m} \tag{49}$$

Here a simplification has been made based on the assumption that the nozzle exit plane is perpendicular to the vehicle axis (i.e., $\hat{n}_z = 0$).

If the pressure and shear on the internal surfaces are directly integrated, the normal force is computed as

$$N_{aero} = N = -\int_A (p - p_0)\,\hat{n}_z dS + \int_A \tau_z\, dS - \int_D (p - p_0)\,\hat{n}_z dS + \int_D \tau_z\, dS \quad (50)$$

$$N_{aero} = N_{A_p} + N_{A_s} + N_{D_p} + N_{D_s} \quad (51)$$

Shorthand terms are interpreted as given in Table 2. It is the recommendation of the author that the normal force be computed according to Equations 48 and 50 and compared, as a way of checking that no errors have been made during the calculations.

5.0 SIDE FORCE

The forces acting on the vehicle in the lateral direction (i.e., forces acting normal to the vehicle axis in the starboard direction) are considered to be exclusively of aerodynamic origin; the propulsion system is considered to make no contribution to the side force. (This is true so long as the exit plane of the nozzle is perpendicular to the vehicle axis, and if the flow leaving the nozzle is radially uniform. Propulsion systems employing canted nozzles or thrust vector control systems are considered to be outside the scope of this report.) Derivation of the side force on the vehicle follows the same procedure as followed for the normal force. As a result, the details of the derivation will be skipped in the interest of brevity, and only the final equations will be presented. Different procedures are required to compute side force for the three different phases of flight.

5.1 AIR-BREATHING MODE

The side force acting on a vehicle with an operating air-breathing propulsion system can be computed as

$$\begin{aligned} Y_{aero} = Y = &-\int_A (p - p_0)\,\hat{n}_y dS + \int_A \tau_y\, dS \\ &-\int_1 (p - p_0)\,\hat{n}_y dS - \int_1 \rho v\,(\boldsymbol{V}\cdot\hat{\boldsymbol{n}}) dS - \int_B (p - p_0)\,\hat{n}_y dS \\ &-\int_B \rho v\,(\boldsymbol{V}\cdot\hat{\boldsymbol{n}}) dS \end{aligned} \quad (52)$$

assuming that the exit plane of the nozzle is perpendicular to the vehicle axis. This can be written in shorthand as

$$Y_{aero} = Y_{A_p} + Y_{A_s} + Y_{I_p} + Y_{I_m} + Y_{B_p} + Y_{B_m} \tag{53}$$

The physical interpretation of each term in Equation 53 is given in Table 3.

TABLE 3. Definition and Physical Interpretation of Different Components of Side Force.

Definition	Physical Interpretation
$Y_{A_p} = -\int_A (p - p_0)\, \hat{n}_y dS$	Side force due to the pressure applied to the external wetted surfaces of the airframe. These are all the wetted surfaces connecting the inlet interface surface and the exit interface surface via a path outside of the propulsion system.
$Y_{A_s} = \int_A \tau_y\, dS$	Side force due to shear stress acting on the external wetted surfaces of the airframe.
$Y_{I_p} = -\int_1 (p - p_0)\, \hat{n}_y dS$	Side force due to the pressure of the upstream flow applied to the inlet interface surface (the imaginary surface forming the interface between the external flow and the internal flow path at the inlet entrance). Note that this term will equate to zero if the inlet interface plane is perpendicular to the vehicle axis ($\hat{n}_y = 0$).
$Y_{I_m} = -\int_1 \rho v\, (\boldsymbol{V} \cdot \hat{\boldsymbol{n}}) dS$	Side force associated with the lateral component of the momentum of the airflow crossing the inlet interface surface and entering the internal flow path (the propulsion system).
$Y_{B_p} = -\int_B (p - p_0)\, \hat{n}_y dS$	Side force due to the pressure of the flow applied to the bleed interface surface (the imaginary surface forming the interface between the internal flow path and the external flow at the bleed exit).
$Y_{B_m} = -\int_B \rho v\, (\boldsymbol{V} \cdot \hat{\boldsymbol{n}}) dS$	Side force associated with the lateral component of the momentum of the airflow crossing the bleed interface surface and entering the external flow.

TABLE 3. (Contd.)

Definition	Physical Interpretation
$Y_{E_m} = -\int_6 \rho v\,(\boldsymbol{V} \cdot \hat{\boldsymbol{n}})dS$	Side force associated with the lateral component of the momentum of the airflow crossing the nozzle exit interface surface and entering the external flow. (Only required for coasting flight mode.)
$Y_{D_p} = -\int_D (p - p_0)\,\hat{n}_y dS$	Side force due to the pressure applied to the internal wetted surfaces of the airframe. These are all the wetted surfaces connecting the inlet interface surface and the exit interface surface via a path inside the propulsion system. (Only required for coasting flight mode.)
$Y_{D_s} = \int_D \tau_y\, dS$	Side force due to shear stress acting on the internal wetted surfaces of the airframe. (Only required for coasting flight mode.)

5.2 ROCKET MODE

By simplifying Equation 52, it is possible to obtain the expression for side force on a vehicle in the rocket-boosted mode of flight:

$$Y_{aero} = -\int_A (p - p_0)\,\hat{n}_y dS + \int_A \tau_y\, dS \tag{54}$$

$$Y_{aero} = Y_{A_p} + Y_{A_s} \tag{55}$$

Any wetted surfaces of the internal flow path upstream of this blockage need to be treated as external wetted surfaces of the airframe (see Figure 8).

5.3 COAST MODE

If the forces on the internal surfaces are related to the properties of the flow entering and leaving the internal flow path, the side force during coasting flight can be computed as

$$
\begin{gathered}
Y_{aero} = Y = -\int_A (p - p_0)\,\hat{n}_y dS + \int_A \tau_y\, dS \\
-\int_1 (p - p_0)\,\hat{n}_y dS - \int_1 \rho v\,(\boldsymbol{V}\cdot\hat{\boldsymbol{n}})dS - \int_6 \rho v\,(\boldsymbol{V}\cdot\hat{\boldsymbol{n}})dS \\
-\int_B (p - p_0)\,\hat{n}_y dS - \int_B \rho v\,(\boldsymbol{V}\cdot\hat{\boldsymbol{n}})dS
\end{gathered}
\tag{56}
$$

$$
Y_{aero} = Y_{A_p} + Y_{A_s} + Y_{I_p} + Y_{I_m} + Y_{E_m} + Y_{B_p} + Y_{B_m} \tag{57}
$$

where a simplification has been made based on the assumption that the nozzle exit plane is perpendicular to the vehicle axis (i.e., $\hat{n}_y = 0$).

If the pressure and shear on the internal surfaces are directly integrated, the side force is computed as

$$
Y_{aero} = Y = -\int_A (p - p_0)\,\hat{n}_y dS + \int_A \tau_y\, dS - \int_D (p - p_0)\,\hat{n}_y dS + \int_D \tau_y\, dS \tag{58}
$$

$$
Y_{aero} = Y_{A_p} + Y_{A_s} + Y_{D_p} + Y_{D_s} \tag{59}
$$

Shorthand terms are interpreted as given in Table 3. It is the recommendation of the author that the side force be computed according to Equations 56 and 58 and compared, for error-checking purposes.

6.0 PITCHING MOMENT

All moments acting on the vehicle are considered to be exclusively of aerodynamic origin; the propulsion system is considered to make no contribution to the moments. (This is true so long as the exit plane of the nozzle is perpendicular to the vehicle axis, the centroid of the nozzle exit plane is aligned with the axis of the vehicle, and the flow leaving the nozzle is radially uniform. Propulsion systems employing canted nozzles or thrust vector control systems are considered to be outside the scope of this document.) The pitching moment acts about the lateral axis of the vehicle to rotate the vehicle in the vertical plane (i.e., move the nose up or down). In the coordinate system used in this derivation, a positive pitching moment rotates the vehicle in the nose-up direction. Different procedures are required to compute the pitching moment for the three different phases of flight.

6.1 AIR-BREATHING MODE

The pitching moment acting on a vehicle can be computed by integrating all the moments on the wetted surfaces (both internal and external) of the vehicle. The control volume therefore is as shown in Figure 3, where the control surface can be split into two parts: the external wetted surfaces of the vehicle, and the internal wetted surfaces. The net moment can therefore be computed as

$$\boldsymbol{M}_{aero} = \boldsymbol{M} = \boldsymbol{M}_A + \boldsymbol{M}_D \tag{60}$$

where $\boldsymbol{M}_A$ is the moment on the external wetted surfaces (the "airframe"), and $\boldsymbol{M}_D$ is the moment on the internal wetted surfaces (the "duct").

The moment on the external wetted surfaces can be directly computed by integrating the moments associated with the pressure and viscous forces on these surfaces:

$$\boldsymbol{M}_A = -\int_A \left(p - p_{ref}\right)(\boldsymbol{r} \times \hat{\boldsymbol{n}})dS + \int_A \boldsymbol{r} \times \boldsymbol{\tau}\, dS \tag{61}$$

The control surface corresponding to the external wetted surfaces of the vehicle are illustrated in Figure 4.

When the propulsion system is operating, it is most convenient to relate the moments on the internal wetted surfaces to the properties of the flow entering and exiting the internal flow path. This can be accomplished by considering a control volume enclosing the fluid volume inside the vehicle, as shown in Figure 5. Noting that the moment acting on the fluid by the internal wetted surfaces is equal in magnitude to, but opposite in sign from, the moments $\boldsymbol{M}_D$ on these surfaces, the angular momentum equation can be written as

$$\begin{aligned} -\boldsymbol{M}_D - \int_1 \left(p - p_{ref}\right)(\boldsymbol{r} \times \hat{\boldsymbol{n}})dS - \int_6 \left(p - p_{ref}\right)(\boldsymbol{r} \times \hat{\boldsymbol{n}})dS \\ - \int_B \left(p - p_{ref}\right)(\boldsymbol{r} \times \hat{\boldsymbol{n}})dS = \int_1 \rho\,(\boldsymbol{r} \times \boldsymbol{V})(\boldsymbol{V} \cdot \hat{\boldsymbol{n}})dS \\ + \int_6 \rho\,(\boldsymbol{r} \times \boldsymbol{V})(\boldsymbol{V} \cdot \hat{\boldsymbol{n}})dS + \int_B \rho\,(\boldsymbol{r} \times \boldsymbol{V})(\boldsymbol{V} \cdot \hat{\boldsymbol{n}})dS \end{aligned} \tag{62}$$

This allows the moments on the internal surfaces to be computed from the flow entering and leaving the vehicle through the inlet, nozzle exit, and bleed:

$$\begin{aligned}\boldsymbol{M}_D = &-\int_1 \left(p - p_{ref}\right)(\boldsymbol{r} \times \widehat{\boldsymbol{n}})dS - \int_6 \left(p - p_{ref}\right)(\boldsymbol{r} \times \widehat{\boldsymbol{n}})dS \\ &- \int_B \left(p - p_{ref}\right)(\boldsymbol{r} \times \widehat{\boldsymbol{n}})dS - \int_1 \rho\,(\boldsymbol{r} \times \boldsymbol{V})(\boldsymbol{V} \cdot \widehat{\boldsymbol{n}})dS \\ &- \int_6 \rho\,(\boldsymbol{r} \times \boldsymbol{V})(\boldsymbol{V} \cdot \widehat{\boldsymbol{n}})dS - \int_B \rho\,(\boldsymbol{r} \times \boldsymbol{V})(\boldsymbol{V} \cdot \widehat{\boldsymbol{n}})dS\end{aligned} \tag{63}$$

By defining $p_{ref} \equiv p_0$, the net moment acting on the vehicle can then be computed as

$$\begin{aligned}\boldsymbol{M}_{aero} = &-\int_A (p - p_0)\,(\boldsymbol{r} \times \widehat{\boldsymbol{n}})dS + \int_A (\boldsymbol{r} \times \boldsymbol{\tau})\,dS \\ &- \int_1 (p - p_0)\,(\boldsymbol{r} \times \widehat{\boldsymbol{n}})dS - \int_6 (p - p_0)\,(\boldsymbol{r} \times \widehat{\boldsymbol{n}})dS - \int_B (p - p_0)\,(\boldsymbol{r} \times \widehat{\boldsymbol{n}})dS \\ &- \int_1 \rho\,(\boldsymbol{r} \times \boldsymbol{V})(\boldsymbol{V} \cdot \widehat{\boldsymbol{n}})dS - \int_6 \rho\,(\boldsymbol{r} \times \boldsymbol{V})(\boldsymbol{V} \cdot \widehat{\boldsymbol{n}})dS - \int_B \rho\,(\boldsymbol{r} \times \boldsymbol{V})(\boldsymbol{V} \cdot \widehat{\boldsymbol{n}})dS\end{aligned} \tag{64}$$

Restricting consideration to the pitching moment (the y component of the moment vector) results in

$$\begin{aligned}M_{aero} = &-\int_A (p - p_0)\,(z\hat{n}_x - x\hat{n}_z)dS + \int_A (z\tau_x - x\tau_z)\,dS \\ &- \int_1 (p - p_0)\,(z\hat{n}_x - x\hat{n}_z)dS - \int_6 (p - p_0)\,(z\hat{n}_x - x\hat{n}_z)dS \\ &- \int_B (p - p_0)\,(z\hat{n}_x - x\hat{n}_z)dS - \int_1 \rho(zu - xw)\,(\boldsymbol{V} \cdot \widehat{\boldsymbol{n}})dS \\ &- \int_6 \rho(zu - zw)\,(\boldsymbol{V} \cdot \widehat{\boldsymbol{n}})dS - \int_B \rho(zu - xw)\,(\boldsymbol{V} \cdot \widehat{\boldsymbol{n}})dS\end{aligned} \tag{65}$$

Under the assumptions made in this derivation for when the propulsion system is active, the integrals associated with the nozzle exit plane (station 6) equate to zero:

$$\begin{aligned}-\int_6 (p - p_0)\,z\hat{n}_x dS + \int_6 (p - p_0)\,x\hat{n}_z dS - \int_6 \rho zu\,(\boldsymbol{V} \cdot \widehat{\boldsymbol{n}})dS \\ + \int_6 \rho xw\,(\boldsymbol{V} \cdot \widehat{\boldsymbol{n}})dS = 0\end{aligned} \tag{66}$$

Specifically, the second term in Equation 66 is equal to zero since the exit plane is assumed perpendicular to the vehicle axis ($\hat{n}_z = 0$). The fourth term is zero because it is additionally assumed that the thrust vector is aligned with the vehicle axis. The first and third terms are zero because it is additionally assumed that the centroid of the nozzle exit plane lies on the axis of the vehicle.

The pitching moment on a vehicle with an operating air-breathing propulsion system can therefore be computed as

$$\begin{aligned} M_{aero} = &-\int_A (p - p_0)\,(z\hat{n}_x - x\hat{n}_z)dS + \int_A (z\tau_x - x\tau_z)\,dS \\ &-\int_1 (p - p_0)\,(z\hat{n}_x - x\hat{n}_z)dS - \int_1 \rho(zu - xw)\,(\boldsymbol{V} \cdot \hat{\boldsymbol{n}})dS \\ &-\int_B (p - p_0)\,(z\hat{n}_x - x\hat{n}_z)dS - \int_B \rho(zu - xw)\,(\boldsymbol{V} \cdot \hat{\boldsymbol{n}})dS \end{aligned} \tag{67}$$

which can be presented in shorthand notation as

$$M_{aero} = M_{A_p} + M_{A_s} + M_{I_p} + M_{I_m} + M_{B_p} + M_{B_m} \tag{68}$$

The physical interpretation of each term in Equation 68 is given in Table 4.

TABLE 4. Definition and Physical Interpretation of Different Components of Pitching Moment.

Definition	Physical Interpretation
$M_{A_p} = -\int_A (p - p_0)\,(z\hat{n}_x - x\hat{n}_z)dS$	Pitching moment due to the pressure applied to the external wetted surfaces of the airframe. These are all the wetted surfaces connecting the inlet interface surface and the exit interface surface via a path outside of the propulsion system.
$M_{A_s} = \int_A (z\tau_x - x\tau_z)\,dS$	Pitching moment due to shear stress acting on the external wetted surfaces of the airframe.
$M_{I_p} = -\int_1 (p - p_0)\,(z\hat{n}_x - x\hat{n}_z)dS$	Pitching moment due to the pressure of the upstream flow applied to the inlet interface surface (the imaginary surface forming the interface between the external flow and the internal flow path at the inlet entrance).

TABLE 4. (Contd.)

Definition	Physical Interpretation
$M_{I_m} = -\int_1 \rho(zu - xw)\,(\boldsymbol{V} \cdot \hat{\boldsymbol{n}})dS$	Pitching moment associated with the axial and normal components of the momentum of the airflow crossing the inlet interface surface and entering the internal flow path (the propulsion system).
$M_{B_p} = -\int_B (p - p_0)\,(z\hat{n}_x - x\hat{n}_z)dS$	Pitching moment due to the pressure of the flow applied to the bleed interface surface (the imaginary surface forming the interface between the internal flow path and the external flow at the bleed exit).
$M_{B_m} = -\int_B \rho(zu - xw)\,(\boldsymbol{V} \cdot \hat{\boldsymbol{n}})dS$	Pitching moment associated with the axial and normal components of the momentum of the airflow crossing the bleed interface surface and entering the external flow.
$M_{E_p} = -\int_6 (p - p_0)\, z\hat{n}_x dS$	Pitching moment due to the pressure of the flow applied to the nozzle exit interface surface (the imaginary surface forming the interface between the internal flow path and the external flow at the nozzle exit). (Only required for coasting flight mode.)
$M_{E_m} = -\int_6 \rho(zu - xw)\,(\boldsymbol{V} \cdot \hat{\boldsymbol{n}})dS$	Pitching moment associated with the axial and normal components of the momentum of the airflow crossing the nozzle exit interface surface and entering the external flow. (Only required for coasting flight mode.)
$M_{D_p} = -\int_D (p - p_0)\,(z\hat{n}_x - x\hat{n}_z)dS$	Pitching moment due to the pressure applied to the internal wetted surfaces of the airframe. These are all the wetted surfaces connecting the inlet interface surface and the exit interface surface via a path inside the propulsion system. (Only required for coasting flight mode.)
$M_{D_s} = \int_D (z\tau_x - x\tau_z)\, dS$	Pitching moment due to shear stress acting on the internal wetted surfaces of the airframe. (Only required for coasting flight mode.)

6.2 ROCKET MODE

The expression for pitching moment during the rocket-boosted portion of flight can be determined by simplifying Equation 67 by removing the terms associated with the airflow through the inlet interface plane and the bleed system:

$$M_{aero} = -\int_A (p - p_0)(z\hat{n}_x - x\hat{n}_z)dS + \int_A (z\tau_x - x\tau_z)\,dS \tag{69}$$

$$M_{aero} = M_{A_p} + M_{A_s} \tag{70}$$

Any wetted surfaces of the internal flow path upstream of the point of blockage need to be treated as external wetted surfaces of the airframe (see Figure 8).

6.3 COAST MODE

In coasting flight, the flow of air through the internal flow path produces moments that need to be captured as part of the aerodynamics of the vehicle. It is possible to compute the moments caused by this internal flow using two different methods (just as was the case with the forces already discussed).

In the first approach, the moments on the internal wetted surfaces are related to the properties of the flow entering and exiting the internal flow path; the relevant expression is Equation 65 derived previously. However, in coasting flight mode, it is not possible to assume that the flow through the nozzle has radial symmetry. Equation 65 can therefore only be slightly simplified based on the assumption that the nozzle exit plane is perpendicular to the vehicle axis ($\hat{n}_z = 0$ on station 6):

$$\begin{aligned} M_{aero} = &-\int_A (p - p_0)(z\hat{n}_x - x\hat{n}_z)dS + \int_A (z\tau_x - x\tau_z)\,dS \\ &-\int_1 (p - p_0)(z\hat{n}_x - x\hat{n}_z)dS - \int_1 \rho(zu - xw)(\boldsymbol{V}\cdot\hat{\boldsymbol{n}})dS \\ &-\int_6 (p - p_0)\,z\hat{n}_x dS - \int_6 \rho(zu - xw)(\boldsymbol{V}\cdot\hat{\boldsymbol{n}})dS \\ &-\int_B (p - p_0)(z\hat{n}_x - x\hat{n}_z)dS - \int_B \rho(zu - xw)(\boldsymbol{V}\cdot\hat{\boldsymbol{n}})dS \end{aligned} \tag{71}$$

$$M_{aero} = M_{A_p} + M_{A_s} + M_{I_p} + M_{I_m} + M_{E_p} + M_{E_m} + M_{B_p} + M_{B_m} \tag{72}$$

The shorthand terms are defined in Table 4.

The alternative approach is to directly integrate the moments caused by pressure and shear acting on the internal flow path surfaces:

$$\boldsymbol{M}_D = -\int_B \left(p - p_{ref}\right)(\boldsymbol{r} \times \widehat{\boldsymbol{n}})dS + \int_D (\boldsymbol{r} \times \boldsymbol{\tau})\, dS \tag{73}$$

The net moment vector on the vehicle therefore becomes

$$\begin{aligned}\boldsymbol{M}_{aero} = -\int_A \left(p - p_{ref}\right)(\boldsymbol{r} \times \widehat{\boldsymbol{n}})dS + \int_A (\boldsymbol{r} \times \boldsymbol{\tau})\, dS \\ - \int_D \left(p - p_{ref}\right)(\boldsymbol{r} \times \widehat{\boldsymbol{n}})dS + \int_D (\boldsymbol{r} \times \boldsymbol{\tau})\, dS\end{aligned} \tag{74}$$

The pitching moment can thus be computed as

$$\begin{aligned}M_{aero} = -\int_A (p - p_0)\,(z\hat{n}_x - x\hat{n}_z)dS + \int_A (z\tau_x - x\tau_z)\, dS \\ - \int_D (p - p_0)\,(z\hat{n}_x - x\hat{n}_z)dS + \int_D (z\tau_x - x\tau_z)\, dS\end{aligned} \tag{75}$$

$$M_{aero} = M_{A_p} + M_{A_s} + M_{D_p} + M_{D_s} \tag{76}$$

Shorthand terms are interpreted as given in Table 4. It is the recommendation of the author that the pitching moment be computed according to Equations 71 and 75 and compared, for error-checking purposes.

7.0 ROLLING MOMENT

The rolling moment acts to rotate the vehicle about its longitudinal axis. In the coordinate system used in this derivation, a positive rolling moment rotates the vehicle such that a fin on the starboard side rises, and a fin on the port side drops. Derivation of the rolling moment follows the same procedure presented in detail for the pitching moment. In the interest of brevity, only the final expressions of interest will be presented for the rolling moment. Different procedures are required to compute the rolling moment for the three different phases of flight.

7.1 AIR-BREATHING MODE

The rolling moment on a vehicle with an operating air-breathing propulsion system can be determined by simplifying the x component of Equation 64:

$$
\begin{aligned}
l_{aero} = &-\int_A (p - p_0)\left(y\hat{n}_z - z\hat{n}_y\right)dS + \int_A \left(y\tau_z - z\tau_y\right)dS \\
&-\int_1 (p - p_0)\left(y\hat{n}_z - z\hat{n}_y\right)dS - \int_1 \rho(yw - zv)\,(\boldsymbol{V}\cdot\hat{\boldsymbol{n}})dS \\
&-\int_B (p - p_0)\left(y\hat{n}_z - z\hat{n}_y\right)dS - \int_B \rho(yw - zv)\,(\boldsymbol{V}\cdot\hat{\boldsymbol{n}})dS
\end{aligned}
\tag{77}
$$

Once again, it has been possible to eliminate the integrals for the nozzle exit plane, as discussed in Section 6.1. In shorthand notation, this can be written as

$$l_{aero} = l_{A_p} + l_{A_s} + l_{I_p} + l_{I_m} + l_{B_p} + l_{B_m} \tag{78}$$

where the various terms are physically interpreted as given in Table 5.

TABLE 5. Definition and Physical Interpretation of Different Components of Rolling Moment.

Definition	Physical Interpretation
$l_{A_p} = -\int_A (p - p_0)\left(y\hat{n}_z - z\hat{n}_y\right)dS$	Rolling moment due to the pressure applied to the external wetted surfaces of the airframe. These are all the wetted surfaces connecting the inlet interface surface and the exit interface surface via a path outside of the propulsion system.
$l_{A_s} = \int_A \left(y\tau_z - z\tau_y\right)dS$	Rolling moment due to shear stress acting on the external wetted surfaces of the airframe.
$l_{I_p} = -\int_1 (p - p_0)\left(y\hat{n}_z - z\hat{n}_y\right)dS$	Rolling moment due to the pressure of the upstream flow applied to the inlet interface surface (the imaginary surface forming the interface between the external flow and the internal flow path at the inlet entrance). Note that this term will equate to zero if the inlet interface plane is perpendicular to the vehicle axis ($\hat{n}_y = \hat{n}_z = 0$).

TABLE 5. (Contd.)

Definition	Physical Interpretation
$l_{I_m} = -\int_1 \rho(yw - zv)\,(\boldsymbol{V} \cdot \hat{\boldsymbol{n}})dS$	Rolling moment associated with the lateral and normal components of the momentum of the airflow crossing the inlet interface surface and entering the internal flow path (the propulsion system).
$l_{B_p} = -\int_B (p - p_0)\left(y\hat{n}_z - z\hat{n}_y\right)dS$	Rolling moment due to the pressure of the flow applied to the bleed interface surface (the imaginary surface forming the interface between the internal flow path and the external flow at the bleed exit).
$l_{B_m} = -\int_B \rho(yw - zv)\,(\boldsymbol{V} \cdot \hat{\boldsymbol{n}})dS$	Rolling moment associated with the lateral and normal components of the momentum of the airflow crossing the bleed interface surface and entering the external flow.
$l_{E_m} = -\int_6 \rho(yw - zv)\,(\boldsymbol{V} \cdot \hat{\boldsymbol{n}})dS$	Rolling moment associated with the lateral and normal components of the momentum of the airflow crossing the nozzle exit interface surface and entering the external flow. (Only required for coasting flight mode.)
$l_{D_p} = -\int_D (p - p_0)\left(y\hat{n}_z - z\hat{n}_y\right)dS$	Rolling moment due to the pressure applied to the internal wetted surfaces of the airframe. These are all the wetted surfaces connecting the inlet interface surface and the exit interface surface via a path inside the propulsion system. (Only required for coasting flight mode.)
$l_{D_s} = \int_D \left(y\tau_z - z\tau_y\right)dS$	Rolling moment due to shear stress acting on the internal wetted surfaces of the airframe. (Only required for coasting flight mode.)

7.2 ROCKET MODE

The expression for the rolling moment acting on a vehicle in rocket-boosted flight can be obtained by simplifying Equation 77:

$$l_{aero} = -\int_A (p - p_0)\left(y\hat{n}_z - z\hat{n}_y\right)dS + \int_A \left(y\tau_z - z\tau_y\right)dS \tag{79}$$

$$l_{aero} = l_{A_p} + l_{A_s} \tag{80}$$

Any wetted surfaces of the internal flow path upstream of the point of blockage need to be treated as external wetted surfaces of the airframe (see Figure 8).

7.3 COAST MODE

If the moments on the internal surfaces are related to the properties of the flow entering or leaving the internal flow path, the rolling moment can be computed as

$$\begin{aligned} l_{aero} = &-\int_A (p-p_0)\left(y\hat{n}_z - z\hat{n}_y\right)dS + \int_A \left(y\tau_z - z\tau_y\right) dS \\ &-\int_1 (p-p_0)\left(y\hat{n}_z - z\hat{n}_y\right)dS - \int_1 \rho(yw - zv)\,(\boldsymbol{V}\cdot\hat{\boldsymbol{n}})dS \\ &-\int_6 \rho(yw - zv)\,(\boldsymbol{V}\cdot\hat{\boldsymbol{n}})dS - \int_B (p-p_0)\left(y\hat{n}_z - z\hat{n}_y\right)dS \\ &-\int_B \rho(yw - zv)\,(\boldsymbol{V}\cdot\hat{\boldsymbol{n}})dS \end{aligned} \tag{81}$$

It has been possible to eliminate the pressure integral for the nozzle exit plane by using the assumption that the nozzle exit is perpendicular to the vehicle axis ($\hat{n}_y = \hat{n}_z = 0$). However, in coasting flight, it is not possible to assume that the flow through the nozzle exit has radial symmetry. Thus, it is necessary to retain the momentum integral for the nozzle exit plane. Equation 81 can be written in shorthand notation as

$$l_{aero} = l_{A_p} + l_{A_s} + l_{I_p} + l_{I_m} + l_{E_m} + l_{B_p} + l_{B_m} \tag{82}$$

If the moments due to pressure and shear on the internal surfaces are directly integrated, the rolling moment can be expressed as

$$\begin{aligned} l_{aero} = &-\int_A (p-p_0)\left(y\hat{n}_z - z\hat{n}_y\right)dS + \int_A \left(y\tau_z - z\tau_y\right) dS \\ &-\int_D (p-p_0)\left(y\hat{n}_z - z\hat{n}_y\right)dS + \int_D \left(y\tau_z - z\tau_y\right) dS \end{aligned} \tag{83}$$

$$l_{aero} = l_{A_p} + l_{A_s} + l_{D_p} + l_{D_s} \tag{84}$$

where the shorthand terms can be interpreted as given in Table 5. It is the recommendation of the author that the rolling moment be computed according to Equations 81 and 83 and compared, to help check for errors.

8.0 YAWING MOMENT

The yawing moment acts about the vertical axis of the vehicle to rotate the vehicle in the horizontal plane (i.e., move the nose left or right). In the coordinate system used in this derivation, a positive yawing moment rotates the vehicle in the nose-left direction. Derivation of the yawing moment follows the same procedure presented in detail for the pitching moment. In the interest of brevity, only the final expressions of interest will be presented for the yawing moment. Different procedures are required to compute the yawing moment for the three different phases of flight.

8.1 AIR-BREATHING MODE

The yawing moment on a vehicle with an operating air-breathing propulsion system can be determined by simplifying the z component of Equation 64:

$$\begin{aligned} n_{aero} = &-\int_A (p - p_0)\left(x\hat{n}_y - y\hat{n}_x\right)dS + \int_A \left(x\tau_y - y\tau_x\right) dS \\ &-\int_1 (p - p_0)\left(x\hat{n}_y - y\hat{n}_x\right)dS - \int_1 \rho(xv - yu)\,(\boldsymbol{V} \cdot \hat{\boldsymbol{n}})dS \\ &-\int_B (p - p_0)\left(x\hat{n}_y - y\hat{n}_x\right)dS - \int_B \rho(xv - yu)\,(\boldsymbol{V} \cdot \hat{\boldsymbol{n}})dS \end{aligned} \tag{85}$$

Once again, it has been possible to eliminate the integrals for the nozzle exit plane, as discussed in Section 6.1. In shorthand notation, this can be written as

$$n_{aero} = n_{A_p} + n_{A_s} + n_{I_p} + n_{I_m} + n_{B_p} + n_{B_m} \tag{86}$$

where the various terms are physically interpreted as given in Table 6.

TABLE 6. Definition and Physical Interpretation of
Different Components of Yawing Moment.

Definition	Physical Interpretation
$n_{A_p} = -\int_A (p - p_0)\left(x\hat{n}_y - y\hat{n}_x\right)dS$	Yawing moment due to the pressure applied to the external wetted surfaces of the airframe. These are all the wetted surfaces connecting the inlet interface surface and the exit interface surface via a path outside of the propulsion system.
$n_{A_s} = \int_A \left(x\tau_y - y\tau_x\right) dS$	Yawing moment due to shear stress acting on the external wetted surfaces of the airframe.
$n_{I_p} = -\int_1 (p - p_0)\left(x\hat{n}_y - y\hat{n}_x\right)dS$	Yawing moment due to the pressure of the upstream flow applied to the inlet interface surface (the imaginary surface forming the interface between the external flow and the internal flow path at the inlet entrance).
$n_{I_m} = -\int_1 \rho(xv - yu)\,(\boldsymbol{V} \cdot \hat{\boldsymbol{n}})dS$	Yawing moment associated with the axial and lateral components of the momentum of the airflow crossing the inlet interface surface and entering the internal flow path (the propulsion system).
$n_{B_p} = -\int_B (p - p_0)\left(x\hat{n}_y - y\hat{n}_x\right)dS$	Yawing moment due to the pressure of the flow applied to the bleed interface surface (the imaginary surface forming the interface between the internal flow path and the external flow at the bleed exit).
$n_{B_m} = -\int_B \rho(xv - yu)\,(\boldsymbol{V} \cdot \hat{\boldsymbol{n}})dS$	Yawing moment associated with the axial and lateral components of the momentum of the airflow crossing the bleed interface surface and entering the external flow.
$n_{E_p} = -\int_6 (p - p_0)\,(-y\hat{n}_x)dS$	Yawing moment due to the pressure of the flow applied to the nozzle exit interface surface (the imaginary surface forming the interface between the internal flow path and the external flow at the nozzle exit). (Only required for coasting flight mode.)

TABLE 6. (Contd.)

Definition	Physical Interpretation
$n_{E_m} = -\int_6 \rho(xv - yu)\,(\boldsymbol{V} \cdot \hat{\boldsymbol{n}})dS$	Yawing moment associated with the axial and lateral components of the momentum of the airflow crossing the nozzle exit interface surface and entering the external flow. (Only required for coasting flight mode.)
$n_{D_p} = -\int_D (p - p_0)\left(x\hat{n}_y - y\hat{n}_x\right)dS$	Yawing moment due to the pressure applied to the internal wetted surfaces of the airframe. These are all the wetted surfaces connecting the inlet interface surface and the exit interface surface via a path inside the propulsion system. (Only required for coasting flight mode.)
$n_{D_s} = \int_D \left(x\tau_y - y\tau_x\right) dS$	Yawing moment due to shear stress acting on the internal wetted surfaces of the airframe. (Only required for coasting flight mode.)

8.2 ROCKET MODE

The expression for the rolling moment acting on a vehicle in rocket-boosted flight can be obtained by simplifying Equation 85:

$$n_{aero} = -\int_A (p - p_0)\left(x\hat{n}_y - y\hat{n}_x\right)dS + \int_A \left(x\tau_y - y\tau_x\right) dS \tag{87}$$

$$n_{aero} = n_{A_p} + n_{A_s} \tag{88}$$

Any wetted surfaces of the internal flow path upstream of the point of blockage need to be treated as external wetted surfaces of the airframe (see Figure 8).

8.3 COAST MODE

If the moments on the internal surfaces are related to the properties of the flow entering or leaving the internal flow path, the yawing moment can be computed as

$$\begin{aligned} n_{aero} = &-\int_A (p-p_0)\left(x\hat{n}_y - y\hat{n}_x\right)dS + \int_A \left(x\tau_y - y\tau_x\right)dS \\ &-\int_1 (p-p_0)\left(x\hat{n}_y - y\hat{n}_x\right)dS - \int_1 \rho(xv-yu)\,(\boldsymbol{V}\cdot\hat{\boldsymbol{n}})dS \\ &-\int_6 (p-p_0)\,(-y\hat{n}_x)dS - \int_6 \rho(xv-yu)\,(\boldsymbol{V}\cdot\hat{\boldsymbol{n}})dS \\ &-\int_B (p-p_0)\left(x\hat{n}_y - y\hat{n}_x\right)dS - \int_B \rho(xv-yu)\,(\boldsymbol{V}\cdot\hat{\boldsymbol{n}})dS \end{aligned} \tag{89}$$

where it has been possible to make a minor simplification based on the assumption that the nozzle exit plane is perpendicular to the vehicle axis ($\hat{n}_y = 0$ on station 6). In shorthand notation this can be written as

$$n_{aero} = n_{A_p} + n_{A_s} + n_{I_p} + n_{I_m} + n_{E_p} + n_{E_m} + n_{B_p} + n_{B_m} \tag{90}$$

where the different terms are interpreted as given in Table 6.

If the moments due to pressure and shear on the internal surfaces are directly integrated, the yawing moment can be expressed as

$$\begin{aligned} n_{aero} = &-\int_A (p-p_0)\left(x\hat{n}_y - y\hat{n}_x\right)dS + \int_A \left(x\tau_y - y\tau_x\right)dS \\ &-\int_D (p-p_0)\left(x\hat{n}_y - y\hat{n}_x\right)dS + \int_D \left(x\tau_y - y\tau_x\right)dS \end{aligned} \tag{91}$$

$$n_{aero} = n_{A_p} + n_{A_s} + n_{D_p} + n_{D_s} \tag{92}$$

where the shorthand terms can be interpreted as given in Table 6. It is the recommendation of the author that the yawing moment be computed according to Equations 89 and 91 and compared, to help check for errors.

9.0 DERIVATIVE QUANTITIES

From the three aerodynamic forces (A, N, Y) and the three aerodynamic moments (M, l, n), it is possible to compute a number of additional useful quantities, as described in this section.

9.1 LIFT AND DRAG

Lift and drag are two alternative forces sometimes used to describe the aerodynamics of a vehicle (in particular aircraft). Drag is aligned with the velocity vector of the airflow and represents the force generally retarding the forward motion of the vehicle. Drag and axial force are similar (and often confused); at zero degrees angle of attack they are identical. Lift acts perpendicular to the direction of the airflow and represents the force generally acting against gravity. Lift and normal force are similar and at zero degrees angle of attack are identical.

For zero sideslip, lift and drag can be computed from axial force and normal force as a function of angle of attack:

$$L = N\cos\alpha - A\sin\alpha \tag{93}$$

$$D = A\cos\alpha + N\sin\alpha \tag{94}$$

The relationship between lift and drag and axial force, normal force, and side force becomes more complicated when sideslip angle is nonzero, and also depends upon the particular convention used. This situation lies outside the scope of the present report.

9.2 REFERENCE POINT AXIAL TRANSLATION

The expressions for moments derived in this report yield the moments relative to the origin of the coordinate system. Often it is desirable to obtain the value of the moment relative to some other axial location (such as the center of gravity). Fortunately, it is fairly straightforward to compute the pitching moment about a "new" reference point based on the pitching moment computed about the "old" reference point, the normal force, and the distance between the new and old reference points:

$$M_{new} = M_{old} + (x_{new} - x_{old})N \tag{95}$$

Similarly, the yawing moment about a new reference point can be computed as

$$n_{new} = n_{old} - (x_{new} - x_{old})Y \tag{96}$$

The rolling moment is unaffected by an axial translation of the reference point.

9.3 CENTER OF PRESSURE

The center of pressure is defined as the reference point location that produces zero pitching moment. Thus, by rearranging Equation 95, the axial position of the center of pressure (relative to the reference point used to the evaluate the aerodynamics) can be computed as

$$x_{cp} = -\frac{M}{N} \tag{97}$$

It should be noted that for symmetrical vehicles both pitching moment and normal force are zero at zero degrees angle of attack. As a result, the computation of center of pressure at exactly zero degrees angle of attack is indeterminate ($\frac{0}{0}$). It is recommended that the center of pressure for a small but finite value of angle of attack be used instead.

9.4 STATIC MARGIN

The static margin is a measure of the static stability of a vehicle. It is defined as the normalized distance between the center of pressure and the center of gravity:

$$SM = \frac{x_{cp} - x_{cg}}{d_{ref}} \tag{98}$$

Normally for missiles, the reference length used for normalization is the nominal fuselage diameter. A positive value for static margin indicates that the center of pressure is aft of the center of gravity; the vehicle is thus statically stable. Conversely, a negative value for static margin corresponds to the center of pressure lying forward of the center of gravity and indicates an unstable configuration.

9.5 AERODYNAMIC COEFFICIENTS

It is common practice to reduce aerodynamic forces and moments to coefficient form. These coefficients can then be used to compute the actual forces and moments on a vehicle over a range of freestream conditions. Force coefficients are computed as

$$C_\xi = \frac{\xi}{q_0 S_{ref}} = \frac{\xi}{\frac{1}{2}\rho_0 V_0^2 S_{ref}} \tag{99}$$

where ξ represents any aerodynamic force of interest. The dynamic pressure is defined as $q \equiv \frac{1}{2}\rho V^2$, and the reference area for missiles is typically taken to be the nominal cross-sectional area of the missile fuselage. Moment coefficients are computed according to

$$C_\xi = \frac{\xi}{q_0 S_{ref} d_{ref}} = \frac{\xi}{\frac{1}{2}\rho_0 V_0^2 S_{ref} d_{ref}} \tag{100}$$

where here ξ represents any aerodynamic moment of interest. For missiles, the characteristic length d_{ref} is commonly taken to be the nominal diameter of the fuselage.

10.0 COMPUTATIONAL PROCEDURE

Table 7 presents a summary of the relevant, final equations describing the computation of uninstalled thrust, the three aerodynamic forces, and the three aerodynamic moments, for the three different modes of flight.

TABLE 7. Summary of Equations for Computing Forces and Moments in Different Phases of Flight.

Quantity	Air-Breathing Mode	Rocket Mode	Coast Mode
Thrust	20	21	22
Axial Force	33	35	37 or 41
Normal Force	44	46	48 or 50
Side Force	52	54	56 or 58
Pitching Moment	67	69	71 or 75
Rolling Moment	77	79	81 or 83
Yawing Moment	85	87	89 or 91

To ensure proper force accounting, the following computational procedure should be adhered to

1. The propulsion team will compute the uninstalled thrust using the pertinent equations.
2. The aerodynamics team will compute the aerodynamic forces and moments according to the pertinent equations summarized in Table 7, making sure that all computations are consistent with the physical interpretations presented in Tables 1 through 6.
3. The propulsion team will compute the installed thrust (the net axial force available to accelerate the vehicle) as per Equation 23. This installed thrust is only used to verify the design of the propulsion system; it is not used for trajectory prediction.

4. The modeling and simulation team will compute the trajectory of the vehicle based upon the uninstalled thrust computed by the propulsion team and the aerodynamic forces and moments computed by the aerodynamics team.

When computing the aerodynamic forces and moments using computational fluid dynamics, the following recommendations and points should be kept in mind:

- The mathematical definitions given in Tables 1 through 6 are based upon an outward-pointing unit normal vector. Care must be taken to ensure that the computed sign of each of the quantities listed in these tables is consistent with the physical interpretation.
- The inlet interface surface should be located downstream of the cowl lip, yet upstream of any bleed devices.
- While not essential, it is recommended that the inlet interface surface be a planar surface normal to the axial direction whenever possible, as this causes certain terms to reduce to zero.
- The exit interface surface is a control surface dividing the propulsion system from the downstream external flow. Wetted surfaces at the base of a vehicle do not form part of the exit surface; these surfaces should usually be included in the external wetted surfaces of the airframe. While the aerodynamics team performs no integrations over this exit plane when the propulsion system is active, it is important for proper force accounting that the area of the exit interface surface as used by the aerodynamics team be the same as that used by the propulsion team.
- It is essential that the external wetted surfaces, the inlet interface surface, the bleed interface surfaces, and the exit interface surface collectively form a manifold (airtight) surface. The external wetted surfaces are those surfaces connecting the inlet surface and the exit surface via a path outside of the propulsion system. All the external wetted surfaces need not be contiguous. For example, the spike of an axisymmetric inlet could be considered an external wetted surface, even though it is not contiguous to the fuselage wetted surfaces (see Figure 7).
- Half-model simulations employing a symmetry plane can often be used for vehicles possessing lateral symmetry (which is normally the case). However, whole-model simulations need to be employed if it is necessary to compute side force, rolling moment, yawing moment or the effects of sideslip angle. When using half-model simulations it is necessary to double the force and moment values extracted from the simulation, in order to obtain values representative of the full vehicle.
- Rotating vehicles (such as gun-launched projectiles) will, in most circumstances, require whole-body simulations.
- Simulations conducted for the sole purpose of obtaining the aerodynamics of the vehicle in air-breathing propulsion flight mode do not require that the full internal flow path be modeled. It is only necessary that the internal flow path be modeled

past any bleed devices (if present) and that a representative flowfield be obtained on the inlet interface surface and any bleed interface surfaces.

- Simulations conducted for the purposes of obtaining the aerodynamics of the vehicle in rocket-boosted flight mode require that the internal flow path be modeled up to the point of blockage. It may be possible to reduce the portion of the internal flow path that is modeled, so long as the flow at the inlet interface surface and any bleed interface surfaces is not significantly impacted. (For example, if an inlet without any bleed devices is blocked with a port cover at the entrance to the combustor, it may be desirable to move the blockage in the simulation to a point closer to the inlet entrance. This would help eliminate unsteady flow or acoustics that would hinder convergence of the solution.) All wetted surfaces of the internal flow path upstream of the blockage need to be treated as external wetted surfaces of the airframe (see Figure 8).
- Simulations conducted for the purposes of obtaining the aerodynamics of the vehicle in coasting flight mode require that the internal flow path be fully modeled.
- Separate simulations are required to compute the aerodynamics of the vehicle in air-breathing propulsion flight mode and rocket-boosted flight mode. However, a single simulation can be used to compute the aerodynamics of the vehicle in air-breathing propulsion flight mode and in coasting flight mode. This requires that the internal flow path be included in the simulation in its entirety (it is not necessary or desirable to model reacting flow in the combustor, however).
- The terms describing the contributions to the forces and moments due to pressure and shear (e.g., A_{I_p}, N_{D_s}, M_{A_p}, etc.) can be computed in the STAR-CCM+ flow solver using the built-in force report and moment report features. Other flow solvers likely have similar features. It is recommended that these built-in features be used to compute these terms (versus performing these integrations manually). However, the terms describing the contributions to the forces and moments due to flow momentum (e.g., A_{I_m}, N_{B_m}, M_{E_m}, etc.) will need to be integrated manually (using the surface integral report feature in STAR-CCM+).
- It is strongly recommended that the forces and moments on the vehicle in coasting flight mode be computed using both approaches presented in this document. All information required for both approaches will be available from a single simulation, and comparing the results from both approaches provides an opportunity for error checking.

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NOMENCLATURE

$\boldsymbol{a}$	Acceleration vector, ft/s^2
A	Axial force, lbf
C	Aerodynamic coefficient
d	Diameter, ft
D	Drag force, lbf
$\boldsymbol{F}$	Force vector, lbf
l	Rolling moment, ft·lbf
L	Lift force, lbf
M	Pitching moment, ft·lbf
m	Mass, slug
$\boldsymbol{M}$	Moment vector, ft·lbf
$\dot{m}$	Mass flow rate, slug/s
N	Normal force, lbf
n	Yawing moment, ft·lbf
$\hat{n}$	Components of the outward-pointing unit normal vector
$\hat{\boldsymbol{n}}$	Outward-pointing unit normal vector
p	Pressure, lbf/ft^2
q	Dynamic pressure, lbf/ft^2
$\boldsymbol{r}$	Position vector, ft
S	Area, ft^2
Y	Side force, lbf
SM	Static margin
t	Time, s
T_I	Installed thrust, lbf
T_U	Uninstalled thrust, lbf
u	Flow velocity in the axial direction, ft/s
$\boldsymbol{V}$	Flow velocity vector, ft/s
v	Flow velocity in the lateral direction, ft/s
w	Flow velocity in the normal direction, ft/s
x	Coordinate in the axial direction, ft
y	Coordinate in the lateral direction, ft
z	Coordinate in the normal direction, ft

Symbols

α	Angle of attack, degrees
β	Sideslip angle, degrees
ξ	Placeholder variable representing any aerodynamic force or moment
ρ	Density, slug/ft^3
τ	Components of the surface shear stress vector, lbf/ft^2
$\bar{\tau}$	Flow shear stress tensor, lbf/ft^2
$\boldsymbol{\tau}$	Surface shear stress vector, lbf/ft^2

Subscripts

0	Freestream
1	Inlet cowl lip
2	End of inlet
6	Nozzle exit
A	Airframe (external wetted surfaces)
$aero$	Aerodynamic
B	Bleed interface surface
cg	Center of gravity
cp	Center of pressure
cs	Control surface
D	Duct (internal wetted surfaces)
E	Nozzle exit interface surface
I	Inlet interface surface
m	Due to momentum
p	Due to pressure
ref	Reference
s	Due to shear
x	Axial direction
y	Lateral direction
z	Normal direction

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