IDA

INSTITUTE FOR DEFENSE ANALYSES

Memetics – Overview and Baseline Models

Arthur Fries, Project Leader Nozer D. Singpurwalla

October 2008 Approved for public release; distribution unlimited. IDA Document D-3599 Log: H 08-001213

This work was conducted under IDA's independent research program. The publication of this IDA document does not indicate endorsement by the Department of Defense, nor should the contents be construed as reflecting the official position of that Agency.

 $^{\odot}$ 2008 Institute for Defense Analyses, 4850 Mark Center Drive, Alexandria, Virginia 22311-1882 \bullet (703) 845-2000.

This material may be reproduced by or for the U.S. Government.

INSTITUTE FOR DEFENSE ANALYSES

IDA Document D-3599

Memetics – Overview and Baseline Models

Arthur Fries, Project Leader Nozer D. Singpurwalla



The Institute for Defense Analyses is a non-profit corporation that administers three federally funded research and development centers to provide objective analyses of

national security issues, particularly those requiring scientific and technical expertise, and conduct related research on other national challenges.

PREFACE

This document was prepared by the Institute for Defense Analyses (IDA) under IDA's independent research program. The objective of this report is twofold: to document key notions and ideas that collectively comprise what is known as *meme theory* or *memetics* – an emerging area of research for potential applicability in the defense, homeland security, and intelligence arenas – and to introduce, as a platform for developing mathematical models in memetics, existing classes of mathematical models for growth, survival, and the dynamics of biological systems.

The IDA Technical Review Committee was chaired by Rear Admiral Richard B. Porterfield, USN (Ret.), and consisted of Dr. Amy A. Alrich of the Intelligence Analyses Division, and Mr. James D. Morgeson and Dr. Caroline F. Ziemke of the Strategy, Forces and Resources Division.

MEMETICS – OVERVIEW AND BASELINE MODELS

EXE	CUT	IVE SUMMARY
	A. B. C.	 Introduction Potential Applicability
I.	ON	N MEMETICS: AN OVERVIEW
	A.	Genes and Genetics
	B.	The Incubation and Spreading of Germs – Epidemiology
	C.	Memes and Memetics
	D.	Meme-Gene Co-Evolution
	E.	The Terrorist Meme and Memetic Engineering
	F.	Summary and Conclusion
II.	G.	References
11.		S GROWTH, SURVIVAL, AND PREDATOR-PREY MODELS
	A. B.	1
	Б. С.	Life Expectancy Curves The Predator-Prey Models of Lotka and Volterra
	C.	1. The Lotka-Volterra Equations
		 Population Equilibrium
	D.	1 1
	Ъ.	1. Preliminaries and the Branching Process Model
		 Probability Generating Functions
		3. Some Examples
	E.	References

List of Figures

II-1.	Predator-Prey Population Dynamics	II-3
II-2.	Schemata of Toxic Meme Generation	II-5

EXECUTIVE SUMMARY

The objective of this report is twofold. The first, addressed in Chapter I of the body of this report and summarized in Section B below, is to document key notions and ideas that collectively comprise what is known as *meme theory* or *memetics* – an emerging area of research for potential applicability in the defense, homeland security, and intelligence arenas. The second, addressed in Chapter II and summarized in Section C below, is to introduce, as a platform for developing mathematical models in memetics, existing classes of mathematical models for growth, survival, and the dynamics of biological systems. Before advancing to these summary sections, Section A presents a background discussion of what we mean by the terminologies *memetics* and *memes*.

A. BACKGROUND

1. Introduction

In this report, we adopt the simple viewpoint that a meme is a self-reproducing and propagating information structure analogous to a gene in biology. Further, we take memetics to be the study of the replication, spread, and change of memes, e.g., how ideas proliferate or how cultures can be characterized in terms of socially transmitted perspectives and traits that evolve other time. There has been considerable debate as to whether memes are truly observable and measurable, and whether memetics constitutes a distinct scientific discipline separate from existing and well-established social science sectors such as cultural anthropology and social psychology.¹ We intentionally eschew these debates and instead concentrate on the advancement of new research frontiers that could serve to promote the utility of socio-cultural modeling. In this regard, the specific focus of this report is on taking steps toward the development of a viable analytical framework for describing and studying memes.²

¹ Auger, R., ed. (2001). *Darwinizing Culture: The Status of Memetics as a Science*. Oxford University Press.

² A broader research agenda is addressed in the companion IDA Document D-3601, *Idea Spread: Toward a Research Program in Socio-Cultural Modeling and Cultural Engineering for Security, Defense, and Intelligence.*

Analytical models of the propagation of memes, both mathematical and statistical, would support applications of memetics in two fundamental ways. The first is to provide systematic and organized vehicles for studying and understanding specific problem settings – via combinations of theoretical abstractions and tractable simulations. The second is to develop dynamic mechanisms for tracking and managing particular implementations of memetic engineering, incorporating qualitative and quantitative insights on the degree of attained "success" (e.g., via direct polling of the populace and/or analysis of indirect performance metrics based on surrogate indicators of public attitudes).

2. Potential Applicability

Well-developed meme theories potentially could be utilized to counter terrorists and insurgents, both before and after they become entrenched in their hostile stances, by influencing their beliefs in quantifiable and measurable ways. Applied as an interventionist strategy, memetic engineering recontextualizes the content of terrorism's message – thereby disengaging the susceptible mind from the terror meme, and thus reducing the potential for acts of terrorism, especially those attributed to mimicry. In order to defeat the terror meme, one essentially needs to reverse-engineer it – first explaining what makes the terror meme attractive, and then identifying the individual components of the terror meme so that they can be manipulated and altered. Specific approaches that could be pursued include:

- Rewriting scripts populated with toxic means and retransmitting them through mass media.
- Editing the terror meme by eliminating its power and thus reducing its attractiveness.
- Altering the cultural and societal architecture that encourages the incubation and transmission of the terror meme.
- Destroying the linkages (or networks) that facilitate transmission of the terror meme.
- Encouraging (though not censoring) the media to portray acts of terror in a different light by not giving the terrorists the dignity of coverage, courage, machismo, and martyrdom.

More broadly, applications of memetics conceptually could be tailored to support other critical defense, security, and intelligence concerns – including peacekeeping missions, psychological and public affairs operations, training, and recruitment. Studies of memetics also could advance scientific frontiers in the areas of brain research, cognition, and socio-cultural modeling.

B. OVERVIEW OF MEMETICS

Memes can be habits, skills, stories, or any other kind of information that is copied from host to host (i.e., person to person). A meme is meant to be understood in rough analogy to a gene. Just as genes contain the instructions for creating the bodies that carry them, memes build the culture that transmits them. Further, just as genes proliferate in the gene pool by leaping from body to body via sperm or eggs, memes propagate in the meme pool by bounding from brain to brain via an imitation process.

Meme theory posits that people are the hosts and transmitters of memes, which may be passed from person to person through text, voice, image, or sound. At times, this transmission can occur rapidly wherein an idea can parasitically infect the mind and alter behavior. On the other hand, some memes can be spread more deliberately, with selection being more closely aligned with Darwin's theory of natural selection so that only the "best" memes survive.

Meme engineering explores how a person may pass the meme on as he or she perceives it with the aim of altering the behavior of others. Some memes are so interconnected that they become a *memplex*, i.e., a set of related memes that are aggregated, copied, and disseminated as a distinct group, such as religious, cultural, or political doctrines.

A memetics perspective on terrorism posits the view that terrorist acts tend to be mimicked behavior caused through the acceptance of information associated with religious fanaticism and corrupted ideologies. The underlying memes that dictate terrorist actions are labeled *terrorist memes*. Within this context, *memetic engineering* is an interventionist strategy for recontextualizing the content of a terrorist's message – thereby disengaging the susceptible mind from the terror meme, and thus reducing the potential for acts of terrorism, especially those attributed to mimicry. In order to defeat the terror meme, one needs to essentially reverse-engineer it – first explaining what makes the terror meme attractive, and then identifying the individual components of the terror meme so that they can be manipulated and altered.

C. BASELINE MODELS FOR STUDYING MEMETICS

One fundamental research goal (well beyond of the scope of this introductory report) is to develop new models for characterizing the growth of memes, the survival

probabilities of memes, and the dynamics between toxic memes and their defeator memes. To this end, we first consider existing sets of models that may relate to memetics, via the parallelism between memetics, genetics, growth processes, and so forth.

Many of the models introduced in Chapter II are familiar to demographers, actuaries, and mathematical biologists, while others are well known to those working in reliability and survival analysis. The classes of models include growth models (exponential, suppressed exponential, and logistic), survival models (Gompertz, Makeham, Weibull, and Pareto), predator-prey models (Lotka-Volterra), and branching process models (Markov Chain). The potential application of branching process methods, suitably modified to account for the transformation of memes throughout their propagation, appears to be a fruitful research path ahead.

I. ON MEMETICS: AN OVERVIEW

The objective of this report is to document, in a conversational and non-technical style, some of the key notions and ideas that collectively comprise what is known as *meme theory* or *memetics*. The literature here is immense (see Finkelstein 2006, 2008a, and 2008b), as the potential applicability of memetics has been discussed in numerous fields – ranging from anthropology to sociology, with recent extensions to the intelligence and counterterrorism domains.

Memetics studies the replication, spread, and change of memes, e.g., how ideas proliferate or how cultures can be characterized in terms of socially transmitted perspectives and traits that evolve over time. Well-developed meme theories potentially could be utilized to counter terrorists and insurgents, both before and after they become entrenched in their hostile stances, by influencing their beliefs in quantifiable and measurable ways. Likewise, applications of memetics conceptually could be tailored to support critical defense, security, and intelligence concerns – including peacekeeping missions, psychological and public affairs operations, training, and recruitment. Studies of memetics also could advance scientific frontiers in the areas of brain research, cognition, and socio-cultural modeling.

While much of the memetics literature is excessively verbose, our review reveals that its essence revolves around a few key ideas. Elucidation of these, postponed until Section D, is intended to create a platform upon which technical formulations and methodological developments can take place – facilitating the conversion of memetics abstractions to tractable analytical tools amenable to practitioners.

Meme theory can be better appreciated if the parallel to gene theory and genetics is understood. Likewise, the similar correspondence with germ theory (i.e., how germs incubate and spread) is illuminating. Accordingly, Sections B and C below present overviews of relevant aspects of genetics and germ theory, along with their connection to Darwinian evolution.

A. GENES AND GENETICS

The biologist Dawkins conceptualized the living world to be parsed into the two primary categories of "replicators" and "vehicles." Humans are on the vehicle side of the ledger, and their genes are the replicators. It is the genes that propagate themselves to the next generation; thus humans can be viewed as survival machines for the genes. Collectively, genes contain instructions for building the bodies that carry them, the vehicles. In other words, genes are the true replicators that act in their own interests – the concept embraced by the title of Richard Dawkins' (1976) famous book *The Selfish Gene*.

The interest of a gene is replication. Those genes that make more copies of themselves, those that reproduce with greater fidelity, and those that have greater longevity will leave more copies for future generations. This is the gene-centered Darwinian view that of all of the evolved species of animals and plants, those forms surviving are the ones best adapted to the environment, i.e., the "survival of the fittest" theory.

Genetics, the study of genes, is the branch of biology that deals with heredity and variation in similar or related animals and plants. Fundamental to genetics are *Mendel's laws*, which prescribe the four principles of hereditary phenomena. These laws specify the means by which traits are passed from parent to offspring, and the causes of similarities and differences between related organisms. There are different kinds of genetics: e.g., clinical genetics, molecular genetics, and population genetics. There are challenging mathematical problems spawned by genetics. These have been formulated and addressed by illustrious individuals such as Malthus, Lotka, Volterra, Fisher, Hardy, Haldane, and Karlin, to name a few. Perspectives on the statistical and probabilistic aspects of genetics can be found in the notable books by Gillespi (2004) and Durret (2008), respectively.

B. THE INCUBATION AND SPREADING OF GERMS – EPIDEMIOLOGY

Here one considers the spreading of a virus (or germ), which can be transported and transmitted under certain conditions that favor or limit its transportation and transmission. These germs can either remain at their source and be sterile, or emerge in the spreading of infection. Thus, for diffusion to take place, we must necessarily have a germ, a carrier, and receptive surroundings. The initial germ cannot travel on its own; it needs a carrier, typically, a human being moving from one place to another, or a mosquito, in the case of the malaria virus. For diffusion to take place, the environment must be receptive; i.e., it must be suitable for the development of the germ, so that it does not remain sterile and cause merely a localized infection. Some parallels between the transmission of genes and the spread of germs are now apparent.

There are different ways to destroy and sterilize a germ, or to inhibit its diffusion. The first is to destroy the germ at its source. This could be difficult, however, because either the germ has not yet been identified, or, if it has been identified, its exact location is unknown. Further, there may be too many germs to destroy completely if the germ has multiplied extensively. The second line of defense consists of suppressing the carrier, or preventing the carrier from penetrating the territory to be preserved, i.e., imposing a quarantine of subjects. The third defensive approach is to take preventive measures via immunization and vaccination, providing immunity to potential hosts.

C. MEMES AND MEMETICS

The material of Sections A and B sets the stage for an appreciation of memes and memetics. This is because the notion of memes is a more recent one. Its development draws upon parallels with genetics and epidemiology, fields that have a mathematically mature science base.

When Dawkins (1976) introduced the idea of a meme, he defined it to be a selfreproducing and propagating unit of cultural information. He was looking for a word to describe a replicator in human culture that mimics the way a gene behaves under evolutionary biology principles. Dawkins shortened the Greek word *mimeme* (which means *something imitated*) to *meme*, because it sounded like a gene and therefore could convey the idea of a unit of cultural transmission. The Oxford English Dictionary (2008) defines a meme as "an element of culture that may be considered to be passed on by nongenetic means, such as imitation." Blackmore (1998) describes the meme in terms of broadly defined instructions for behaviors and communications that can be learned by imitation and that stored in the brain. To Dawkins (1976), memes are informational replicators residing in the brain. Finkelstein (2008b) documents numerous other definitions of a meme, and introduces his own: "information which propagates, persists, and has impact."

Memes can be habits, skills, stories, or any other kind of information that is copied from host to host (i.e., person to person). A meme is meant to be understood in rough analogy to a gene. Just as genes contain the instructions for creating the bodies that carry them, memes build the culture that transmits them. Further, just as genes proliferate in the gene pool by leaping from body to body via sperm or eggs, memes propagate in the meme pool by bounding from brain to brain via a process that we call imitation.

Meme theory posits that people are the hosts and transmitters of memes, which may be passed from person to person through text, voice, image, or sound. At times, this transmission can occur rapidly wherein an idea can parasitically infect the mind and alter behavior. On the other hand, some memes can be spread more deliberately, with selection being more closely aligned with Darwin's theory of natural selection so that only the "best" memes survive.

Dawkins identifies three characteristics of a successful meme replication:

- The more faithful the copy, the more will remain of the initial pattern after several rounds of copying
- The faster the copying, the more the replicator will spread
- The longer any instance of the replicating pattern survives, the more copies can be made of it.

Meme engineering explores how a person may pass the meme on as he or she perceives it with the aim of altering the behavior of others. Some memes are so interconnected that they become a *memplex*, i.e., a set of related memes that are aggregated, copied, and disseminated as a distinct group, such as religious, cultural, or political doctrines.

Memetics also articulates reasons for certain beliefs surviving and spreading:

- Memes survive and spread because they are helpful to the people who store them (e.g., true information about the world).
- Certain memes are numerous because they are good fits to pre-existing genetic dispositions.
- Certain memes spread because they facilitate the replication of genes that make vehicles that are good hosts for these memes (e.g., religious beliefs that urge people to have more children).
- Memes survive and spread because of self-perpetuating properties of the memes themselves.

D. MEME-GENE CO-EVOLUTION

Interactions between genes and memes can occur, for instance, when an individual's propensity to adopt a particular meme depends on his (her) genotype (e.g.,

assertions of gender or racial superiority). Recently, branches of theoretical population genetics have focused on these sorts of phenomena, labeling them *meme-gene co-evolution*.

The works of Cavalli-Sforza and Feldman (1981), Boyd and Richerson (1985), and Feldman and Laland (1996) utilize population genetics models to investigate the evolution and dynamics of cultural traits. They explore phenomena so diverse as linguistics, social values and customs, epidemics, and the diffusion of innovations. The said literature postdates Dawkins (1976), but does not explicitly use the term meme. Instead, it refers to *cultural trait or cultural phenotype*.

E. THE TERRORIST MEME AND MEMETIC ENGINEERING

Currently, an era of growing terrorist threats confronts intelligence and security agencies. A memetics perspective on terrorism posits the view that terrorist acts tend to be mimicked behavior caused through the acceptance of information associated with religious fanaticism and corrupted ideologies. The underlying memes that dictate terrorist actions are labeled *terrorist memes*.

Within this context, *Memetic Engineering* is an interventionist strategy for recontextualizing the content of a terrorist's message – thereby disengaging the susceptible mind from the terror meme, and thus reducing the potential for acts of terrorism, especially those attributed to mimicry. Many factors contribute to the incubation and replication of the terrorist memes. One of these is the advances in information technology and the strong focus of the media in reporting terrorist acts. Another factor for a meme to successfully replicate from mind to mind is *resonance* with the existing mental constructs; i.e., the meme must fit with a human need or desire.

Memetic engineering can be seen as entailing two phases: mapping the threat, and defeating the terror meme. With respect to mapping the threat, three elements have been recognized:

- A person must be *susceptible* to the message of the meme. That is, the person must be open to the intent of the message and the form in which the message is communicated.
- The person must be *immune* to alternative messages that might be inflicted upon the person by different cultures, education, and socialization.
- The meme must *resonate* with some intrinsic emotion or value possessed by the host.

In order to defeat the terror meme, one essentially needs to reverse-engineer it – first explaining what makes the terror meme attractive, and then identifying the individual components of the terror meme so that they can be manipulated and altered. Some of the components of the terror meme include a desire to be part of something larger, venting frustration and anger, finding absolution, fulfilling a fantasy, and fulfilling a need for power.

Defeating the terror meme could entail several approaches, including the following:

- Rewriting scripts populated with toxic memes and retransmitting them through mass media.
- Editing the terror meme by eliminating its power and thus reducing its attractiveness.
- Altering the cultural and societal architecture that encourages the incubation and transmission of the terror meme.
- Destroying the linkages (or networks) that facilitate transmission of the terror meme.
- Encouraging (though not censoring) the media to portray acts of terror in a different light by not giving the terrorists the dignity of coverage, courage, machismo, and martyrdom.

F. SUMMARY AND CONCLUSION

One overall objective of our research is to develop mathematical models of memetics for its use in the intelligence and national security domains. Mathematical models serve several purposes. The first is a systematic and organized vehicle for understanding and unambiguously communicating a problem. The second is to develop mechanisms for managing the problem and putting it to use in practical settings.

The science of memetics is relatively new. As a consequence, a mathematical platform that articulates the essence of the issues underlying memetics remains to be developed. Since memetics has its genesis in genetics, however, the well developed mathematics of genetics could be a starting point for developing the mathematics of memetics. Like genetics, memetics appears to also have parallels with germ theory and the spread of germs in the context of infectious diseases. Here again, we have the mathematics of germ spread (epidemiology), which could be relevant. Then there is the gene-meme co-evolution theory whose mathematics goes under the label of *cultural evolution theory*. The underlying premise here is that there is an interaction between

genes and memes; such interactions produce cultural and sociological traits. In particular, one's disposition to embrace certain memes could be a function of one's genotype. Finally, there is the notion of memetic engineering. This takes a systems engineering-type approach to the memetics of terror, and endeavors to destroy the terror meme by first understanding its constituent elements and then defeating them.

Thus it appears that there is a collection of available mathematical and engineering technologies that can be brought to bear on memetics to begin to transform it from a descriptive field to a more quantitative science. As of this writing, it is not clear to us whether the science of memetics is rich enough to spawn mathematics that is fundamentally new.

G. REFERENCES

- 1. Blackmore, S. (1998). The Meme Machine. Oxford University Press.
- 2. Boyd, R. and Richerson, P. (1985). *Culture and Evolutionary Process*. University of Chicago Press.
- 3. Cavalli-Sforza, L. L. and Feldman, M. W. (1981). *Cultural Transmission and Evolution: A Quantitative Approach*. Princeton University Press.
- 4. Dawkins, R. (1976). The Selfish Gene. Oxford University Press.
- 5. Durret, R. (2004). *Probability Models for DNA Sequence Evolution*. Springer-Verlag.
- 6. Feldman, M. W. and Laland, K. N. (1996). Gene-Culture Co-evolution Theory. *TREE*, 11, pp. 453-457.
- 7. Finkelstein, R. (2006). Military Memetics. Briefing Report.
- 8. Finkelstein, R. (2008). A Memetics Compendium. Briefing Report, DARPA Task Order CA-FIN-3212-024-08.
- Finkelstein, R. (2008). Information Propagation, Impact & Persistence (InfoPip): Defining Memes. Briefing Report, DARPA Task Order CA-FIN-3212-024-08.
- 10. Gillespi, J. H. (2004). *Population Genetics*. The Johns Hopkins University Press.
- 11. Simpson, J. (2008), Ed. Oxford English Dictionary. Oxford University Press.

II. ON GROWTH, SURVIVAL, AND PREDATOR-PREY MODELS

Chapter I provided an introductory overview of memetics and its relationships with cultural evolution theories. It motivates us in this chapter to look at mathematical models for growth, survival, and the dynamics of biological systems as a platform for developing mathematical models in memetics. Of interest here is the growth of memes, the survival probabilities of memes, and the dynamics between toxic memes and their defeator memes.

The models to be described here are familiar to demographers, actuaries, and mathematical biologists; some of these are also recognizable to those working in reliability and survival analysis. These models are intended to bring some synergy to the science of memetics, and it is with this point of view that we give an overview of such models.

A. POPULATION GROWTH MODELS

One of the simplest models for population growth, called the *law of growth*, is prescribed by a differential equation of the form dN(t)/dt = rN(t), where N(*t*)denotes the size of a population at time *t*, and *r* is a constant called the *Malthusian parameter*. Essentially, the model prescribes that the rate of change of a population is proportional to its present size.

The solution of the above differential equation follows by integration. Specifically,

$$\int_{N(0)}^{N(t)} dN(t) / N(t) = \int_{0}^{t} r dt ,$$

where N(0) is the initial size of the population. Thus, $\ln(N(t)/N(0)) = rt$, yielding $N(t) = N(0) \cdot e^{rt}$.

A slightly more complicated version of the exponential growth model attempts to suppress the long run growth. Here, the governing differential equation is of the form

$$dN(t)/dt = N(t) \cdot \left[\frac{(rt-1)}{t} \right],$$

with r > 1. Its solution, also obtained by direct integration, is simply $N(t) = N(0) \cdot e^{rt} / t$, where the *t* in the denominator clearly suppresses the exponential growth of the preceding result.

A third model for population growth, is the *logistic growth model*, popular in biology. Here, the governing differential equation is of the form

$$dN(t)/dt = rN(t)(K - N(t))/K,$$

where *K* is some constant. Its solution is

$$N(t) = \frac{K}{1 + \left(\frac{K}{N(0)} - 1\right) \cdot e^{rt}}$$

B. LIFE EXPECTANCY CURVES

These curves are used in actuarial science to specify mortality laws. They are also used by survival analysts in biomedicine to manage health care, as well as in engineering reliability to facilitate maintenance planning. Their relevance to memetics could be in terms of the amount of time a toxic meme could prevail in a population.

Let S(x) denote the probability that a newborn reaches age *x*; i.e., it survives to *x*. Then, the *Gompertz Law of Survival* (1832) states that

$$S(x) = \exp[-m(C^x - 1)], x \ge 0,$$

for some constants C > 1 and $m \ge 0$. Observe that S(x) = 1 when x = 0, and that $S(x) \downarrow 0$ as $x \uparrow \infty$.

A variation of the above is Makeham's Law of Survival (1860), wherein

$$S(x) = \exp[-Ax - B(C^{x} - 1)], x \ge 0,$$

for some constants C > 1, B > 0, and $A \ge -B$. Here again S(x) = 1 when x = 0, and $S(x) \downarrow 0$ as $x \uparrow \infty$. When B = 0 and A > 0, S(x) yields an exponential survival function commonly used in reliability and life-testing. Some other commonly used survival curves are the Weibull and the Pareto.

C. THE PREDATOR-PREY MODELS OF LOTKA AND VOLTERRA

To describe the dynamics of biological systems, Lotka (1925) and Volterra (1926) proposed a system of non-linear differential equations wherein two species, one a predator and the other a prey, interact within an ecological system. These equations can

be germane to memetics if memes can be categorized as being either toxic or as being antidotes to toxic memes, as is conceptualized in memetic engineering. From the point of view of military intelligence, a toxic meme can be viewed as the prey and its antidote meme considered to be the predator. This makes sense because in a biological context, the prey of a species tends to grow exponentially (in population size) and is supposed to have an unlimited supply of food. The predator of the species is smaller in size and depends on the prey for its supply of food.

1. The Lotka-Volterra Equations

Let x(t) denote the number of the prey of a species at time t, and y(t) the number of its predators at that same point in time t. Then the dynamics of the interaction between the species is described by the following system of differential equations:

$$dx(t)/dt = x(t)[\alpha - \beta y(t)]$$
 and $dy(t)/dt = -y(t)[\gamma - \delta x(t)]$,

where α , β , γ , and δ are parameters characterizing the interaction. The first equation represents the feature that the change in the prey's number is given by its own growth minus the rate at which it is preyed upon. Similarly, the second equation characterizes the change in the predator's population to be its growth minus its natural death, namely $\gamma y(t)$.

The solution of these equations is periodic. It yields a simple harmonic motion with the population of predators following that of prey; see Figure II-1.



Figure II-1. Predator-Prey Population Dynamics

The portrayed dynamics show that predators thrive when prey are plentiful, but then the predators eventually outstrip their food supply and begin to decline. As the predator population shrinks down, the prey population thrives again and the process repeats itself cyclically. There is no prediction whenever x(t) or y(t) is zero.

2. Population Equilibrium

Population equilibrium means the absence of any growth or decay in the population size. This is tantamount to setting dx(t)/dt = 0 = dy(t)/dt, which yields the non-trivial solution $x = \gamma/\delta$ and $y = \alpha/\beta$ for all t > 0. It can be shown that the predator and prey populations cycle and oscillate around the curve

$$y(t)^{\alpha} e^{-\beta y(t)} x(t)^{\gamma} e^{-\delta x(t)} = \text{constant.}$$

The extinction of both species is difficult. It can happen only if the prey are completely eradicated by artificial means; this will cause the predators to die of starvation. If the predators are eradicated, then the prey population grows unchecked (in this model).

D. THE PROPAGATION OF A TOXIC MEME – A BRANCHING PROCESS MODEL

From a historical perspective, the study of branching processes arose from a somewhat sexist context of evaluating the probability that a man's direct male line of descendents eventually dies out. The problem is attributed to Sir Francis Galton and the Rev. H. W. Watson (1874). Its more recent impetus has come from a study of the behavior of "particles," such as bacteria, genes, and neutrons, that generate new particles of the *same* type. Since memes transform themselves as they propagate, studying their behavior via standard branching process theory seems inappropriate. The standard theory needs to be enhanced to account for the generation of the essentials of the technology of the standard theory because it could serve as a foundation for the development of an enhanced theory.

The material that follows is motivated with the above consideration in mind. Section 1 introduces preliminary notation, terminology, and key results from branching processes theory. Section 2 gives an overview of probability generating functions (pgf), a key technical tool for studying standard branching processes.

1. Preliminaries and the Branching Process Model

Consider a toxic meme (e.g., anti-American sentiment), henceforth the *parent meme*, that can spawn ξ_1 offspring memes, all judged to be identical in their toxicity to the parent meme. We assume that ξ_1 is random with $P(\xi_1 = n) = p_n$, n = 0,1,2,...,. The parent meme is said to belong to the 0th generation, and its offspring are said to belong to the 1st generation. Each first generation offspring meme can generate additional offspring, with memes being identical to the parent meme. The additional offspring memes are said to belong to the 2nd generation. This process of memes generating offspring memes continues like the branches of a tree, and thus the label "branching processes." Figure II-2 portrays an illustrative example.



Figure II-2. Schemata of Toxic Meme Generation

In principle, the above process of toxic meme generation can continue indefinitely, so that, after several cycles of meme generation, one could have an essentially infinite population of toxic memes. However, it is possible that not all memes spawn offspring so that the meme tree could be *de facto* pruned. For example, in Figure II-2, the 3rd generation consisting of only two memes does not spawn additional memes; thus the process of toxic meme generation is extinguished.

To model mathematically the above process, let X_j denote the total number of (toxic) memes in the *j*-th generation, j = 0, 1, 2, ..., In our case, $X_0 = 1$, and in Figure II-2 we have $X_1 = 2$, $X_2 = 4$, $X_3 = 2$, and $X_4 = 0$, since the process gets extinguished at the 4th generation. The collection of random variables { X_j , j = 0, 1, 2, ...,} constitutes a stochastic

process, of which the Markov Chain is an important sub-class. Thus, we ask, under what conditions will the process $\{X_j, j = 0, 1, 2, ...,\}$ be a Markov Chain?

It is well known that the sum of independent, identically distributed discrete random variables constitutes a Markov Chain. Thus, if we suppose that meme *i* gives rise to ξ_i memes for the next generation, where ξ_i is a discrete random variable, and if the various ξ_i 's are *independent* and *identically* distributed, then { X_j , j = 0,1,2,...,} will be a Markov Chain. Specifically, ξ_1 is the number of memes spawned by the parent meme, ξ_2 is the number of memes spawned by the first offspring (which necessarily is in the 1st generation of memes), ξ_3 is the number of memes spawned by the second offspring (which may be 1st or 2nd generation of memes), and so on.

The Markov Chain $\{X_j, j = 0, 1, 2, ...,\}$ has the set of non-negative integers as its state space, with 0 as an absorbing state. Its transition function is given by the expression

$$p(x, y) = P(\xi_1 + ... + \xi_x = y),$$

for $x \ge 1$. Here p(x,y) denotes the probability that the chain in state x transitions to state y in one step of the iteration, i.e., a generation with x offspring memes is followed with the next generation having exactly y offspring memes. Interest focuses on ρ , the probability that the parent meme and all its offspring become extinct, i.e., the Markov Chain starting in state 1 gets absorbed in state 0. Once ρ can be determined, the branching process model construction is such that were one to start with x toxic genes, all identical to each other, then the probability that all the x toxic genes and their descendents get extinguished is ρ^x .

We have assumed that the ξ_i 's are independent and identically distributed, and that $P(\xi_i = n) = p(n)$, for n = 0,1,2,..., and all *i*. If p(1) = 1, that is, each meme will generate exactly one offspring meme, then the resulting Markov (Branching) Chain is degenerate, i.e., every state is an absorbing state. Thus, we require that p(1) < 1. When such is the case, 0 is an absorbing state, and every other state is a transient state. A consequence is that the chain is either absorbed at 0 or it approaches $+\infty$, i.e., it explodes. Indeed, it can be shown

$$P_x \left(\lim_{j \to \infty} X_j = +\infty \right) = 1 - \rho^x, \ x = 1, 2, ...$$

where $P_x(A)$ is the probability of some event A of interest when the Markov Chain is in the state x. Thus the probability that a single toxic meme eventually spawns an infinite number of toxic memes is $(1 - \rho)$. Consequently, it is important to know whether $\rho = 1$ or $\rho < 1$. The answer to this question is provided via pgf's.

Specifically, write $\varphi(t)$, the pgf of the random variable ξ_i , as

$$\phi(t) = p(0) + \sum_{y=1}^{\infty} p(y)t^{y}, \ 0 \le t \le 1,$$

and note that $\mu \equiv \sum_{n=0}^{\infty} np(n)$ is the mean of ξ_i . Then, the behavior of ρ is governed by the

roots of the equation $\varphi(\rho) = \rho$.

When $\mu \le 1$, $\varphi(\rho) = \rho$ has no roots in [0,1), and thus $\rho = 1$. This implies that the extinction of a toxic meme is certain if $\mu \le 1$ and p(1) < 1. When $\mu > 1$, $\varphi(\rho) = \rho$ has a unique root ρ_0 in the interval [0,1). Thus to control the spread of toxic memes, we must have $\mu \le 1$, i.e., on average, a toxic mean should spawn at most one offspring toxic meme. Memetic engineering therefore should focus on the control of μ . The case $\mu \le 1$ is known as the *subcritical case*, and the case $\mu > 1$ is referred to as the *supercritical case*.

2. Probability Generating Functions

The pgf of the random variable ξ_i , $\phi(z) = \sum_{n=0}^{\infty} p(n)z^n$, is absolutely convergent for $|z| \leq 1$. Moreover, $\varphi(1) = 1$, $\varphi(0) = p(0)$, $\varphi^1(0) = p(1)$, and, in general, $\varphi^n(0) = n!p(n)$, where φ^n denotes the *n*-th derivative of φ with respect to *z*. Thus, knowing $\varphi(z)$ in some neighborhood of 0, we know p(n) for all *n*.

Also,
$$\phi^1(z) = \sum_{n=0}^{\infty} nz^{n-1} p(n)$$
 implies $\phi^1(1) = \sum_{n=0}^{\infty} np(n) \equiv \mu$, the mean of a ξ_i .
Similarly, we can see that σ^2 , the variance of a ξ_i is $\sigma^2 = \phi^2(1) + \phi^1(1) - (\phi^1(1))^2$.

Accordingly, the pgf encompasses all the key features of ξ_i .

There are some other properties of pgf's that are germane to us in a study of branching processes. For example, suppose that a discrete random variable ξ_1 has pgf $\varphi_1(z)$, and another discrete random variable ξ_2 has pgf $\varphi_2(z)$. Then, if ξ_1 and ξ_2 are independent, the pgf of their sum $\xi_1 + \xi_2$, denoted by say $\varphi_{1,2}(z)$, is $\phi_{1,2}(z) = \phi_1(z) \cdot \phi_2(z)$. An induction argument similarly establishes that the pgf of the sum of *k* independent and identically distributed discrete random variables $\xi_1 + \xi_2 + \dots + \xi_k$ is $(\varphi(z))^k$, where $\varphi(z)$ is the pgf of each ξ_i .

More generally, it can be proved that, if the number of memes in a population has pgf Q(z), and if the number of offspring spawned by each meme has pgf $\varphi(z)$, then the

pgf for the total number of offspring memes is $Q(\varphi(z))$. Thus, if the 0th generation has one toxic meme, then the number of toxic memes spawned by the first toxic meme has pgf $\varphi(z)$, and the total number of toxic memes in the second generation will have pgf $\varphi(\varphi(z))$. Continuing in this fashion, the total number of toxic memes in the *m*-th generation will have pgf

$\phi_m(z) = \phi(\phi(\cdots(\phi(z))\cdots)),$

where the function φ appears exactly m times in the right-hand-side of the equation, $\varphi_m(z) = \varphi(\varphi_{m-1}(z))$, and $\varphi_0(z) = z$.

Let π_m be the probability the offspring of toxic genes spawned by a single 0-th generation toxic gene gets extinguished at or before the *m*-th generation. Then, clearly

$$\pi_0 = 0, \pi_1 = p(0) = \phi(0), \text{ and } \pi_m = \phi_m(0) = \phi(\phi(\cdots(\phi(0))\cdots))$$

Thus $\pi_{m+1} = \varphi(\pi_m)$. However, because $\phi^1(z) = \sum_{n=0}^{\infty} nz^{n-1}p(n) > 0$, it follows that φ is an increasing function of z. Consequently, since $\pi_1 > \pi_0$, it follows by induction that we must have $\pi_{m+1} > \pi_m$. Moreover, π_m is a probability that is bounded above by 1.

Consequently, $\pi_m \to \rho$ for some ρ , and continuity arguments readily establish that $\phi(\rho) = \rho$, a result we used in Section 1.

3. Some Examples

Suppose that each toxic meme spawns offspring memes with their number governed by a Poisson distribution $p(n) = \lambda^n e^{-\lambda} / n!$. Then $\varphi(z) = \exp(\lambda(z - 1))$, and we seek the roots of $z = \exp(\lambda(z-1))$. The mean number of offspring is λ . With $\lambda = 1$, we obtain $\pi_1 = .3679$, $\pi_2 = .6259$, $\pi_{31} = .9411$, $\pi_{127} = .9854$, and $\pi_{\infty} = 1 = \rho$. When $\lambda = 1.01$, the corresponding extinction probabilities are .3642, .6197, .9729, .9729, and .9803.

Lotka fitted the U.S. population via a pgf representation of the form $p(0) = \alpha$ and $p(n) = \beta \gamma^{n-1}$. For this representation, $\phi(z) = \alpha + \beta z (1 - \gamma z)^{-1}$ and $\phi^1(z) = \beta (1 - \gamma z)^{-2}$. Incorporating the earlier general result $\phi(1) = 1$, the equation $\phi(z) = z$ here can be simplified to obtain the relationship $(\gamma z - \alpha)(z - 1) = 0$. Thus, the unique root is the minimum of 1 and α/γ . With Lotka's fitted values $\alpha = .4825$, $\beta = .2126$, and $\gamma = .5893$, the estimated extinction probability is $\alpha/\gamma = .8188$.

E. REFERENCES

1. Galton, F. and H. W. Watson (1874). On the Probability of Extinction of Families. *Journal of the Anthropological Institute*, 3, pp. 308-311.

- 2. Lotka, A.J. (1931). The Extinction of Families. *Journal of the Washington Academy of Sciences*, 21, pp. 377-380 and 453-459.
- 3. Karlin, S. and H.M. Taylor (1975). *A First Course in Stochastic Processes*. Academic Press.
- 4. Gompertz, B. (1825). On the Nature of the Function Expressive of the Law of Human Mortality, and on a New Mode of Determining the Value of Life Contingencies. *Philosophical Transactions of the Royal Society of London*, 115, pp. 513-585.
- 5. Makeham, W.M. (1860). On the Law of Mortality and the Construction of Annuity Tables. *J. Inst. Actuaries and Assur. Mag.* 8, pp. 301-310.
- 6. Malthus, Thomas, A. (1798). An Essay on the Principle of Population. *Proceedings of the Royal Society.*
- 7. Lotka, A.J. (1925). *Elements of Physical Biology*. Williams & Wilkins Co.
- 8. Volterra, V. (1926). Variazioni e fluttuazioni del numero d'individui in specie animali conviventi. *Mem. R. Accad. Naz. dei Lincei*. Ser. VI, 2.

REPORT D		Form Approved								
-	viewing instructions, see	OMB No. 0704-0188								
Public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing this collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing										
this burden to Department of Defense, Washington Headquarters Services, Directorate for Information Operations and Reports (0704-0188), 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202- 4302. Respondents should be aware that notwithstanding any other provision of law, no person shall be subject to any penalty for failing to comply with a collection of information if it does not display a currently										
valid OMB control number. PLEASE DO NOT RETURI 1. REPORT DATE (DD-MM-YYYY)	2. REPORT TYPE	DDRESS.	2	DATES COVERED (From - To)						
31-10-2008	IAD Draft Final			larch 2008 – August 2008						
4. TITLE AND SUBTITLE	In D Druit I mui			. CONTRACT NUMBER						
			D	ASW01-04-C-0003						
Memetics – Overview and Baseline	Models		5k	D. GRANT NUMBER						
			—							
			50	2. PROGRAM ELEMENT NUMBER						
			—							
6. AUTHOR(S)			50	I. PROJECT NUMBER						
Singpurwalla, Nozer D.; Fries, Arthu	r			e. TASK NUMBER						
				C5504						
			51	. WORK UNIT NUMBER						
7. PERFORMING ORGANIZATION NAM				PERFORMING ORGANIZATION REPORT						
				NUMBER						
Institute for Defense Analyses			D	-3599						
4850 Mark Center Drive			Н	08-001213						
Alexandria, Virginia 22311-1882										
9. SPONSORING / MONITORING AGEN		ESS(ES)	10	. SPONSOR/MONITOR'S ACRONYM(S)						
IDA Contracting Officers' Represent	ative (COR)									
Mr. Jay S. Minsky										
4850 Mark Center Drive			11	. SPONSOR/MONITOR'S REPORT						
Suite 9601				NUMBER(S)						
Alexandria, Virginia 22311-1882			—							
12. DISTRIBUTION / AVAILABILITY ST		ing Information and Of	D Studies OU							
Approved for public release; distribu	uon uninnited. Enterpr	ise information and Os	SD Studies, OU	SD(AT&L)/ARA/FFRDC, 03-11-2008.						
13. SUPPLEMENTARY NOTES										
14. ABSTRACT										
This report documents key notions as	nd ideas that collectively	y comprise what is kno	wn as meme the	eory or memetics – an emerging area of						
research for potential applicability in										
mathematical models for growth, sur	vival, and the dynamics	of biological systems	as a platform fo	r developing mathematical models in						
memetics.										
15. SUBJECT TERMS										
Meme, memetics, meme engineering	, memplex, terrorist me	me								
16. SECURITY CLASSIFICATION OF:		17. LIMITATION	18. NUMBER	19a. NAME OF RESPONSIBLE PERSON						
		OF ABSTRACT	OF PAGES	Arthur Fries						
a. REPORT b. ABSTRACT	c. THIS PAGE	Unlimited	28	19b. TELEPHONE NUMBER (include area						
Unclassified Unclassified	Unclassified			code) 703-845-2364						
				Standard Form 298 (Rev. 8-98) Prescribed by ANSI Std. Z39.18						