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Summary of Work Completed Under the 6.1 Program: Development of Reliable Particle Filters

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SUMMARY OF WORK COMPLETED UNDER THE 6.1 PROGRAM: DEVELOPMENT OF RELIABLE PARTICLE FILTERS

1. INTRODUCTION

The objective of the project was to develop and implement improved methods in non-stochastic particle-filter estimation techniques as well as to study and implement alternative, more traditional, and less computationally complex algorithms for comparison. In addition to publishing the results of the investigations, the programmed algorithms were tested, commented and made available to the larger DoD and intelligence community by incorporation into the Statement C supplement to the Tracker Component Library (TCL) [1]. The full library is posted on the Defense Intelligence Information Enterprise web site at `https://bitbucket.di2e.net/projects/TCL`.

2. THE BENEFITS AND CHALLENGES OF PARTICLE-FLOW FILTERING

Originating in the seminal work of [2], modern particle filters enable computationally feasible Bayesian estimation of very difficult nonlinear dynamic systems. For example, they were the only practicable approach considered when estimating the search region to find the missing Malaysian Airlines Jet MH370 [3]. Particle filtering techniques have shown performance superior to traditional methods in a number of areas including dead reckoning of a car driving in a garage after losing a GPS signal [4], and appear to be a popular approach to acoustic inversion [5]. They have also found use in difficult track-before-detect applications [6] including periscope detection [7].

Traditional stochastic resampling particle filters are subject to a major potential problem: particle collapse [8]. Particle collapse is a catastrophic failure of the filter to properly represent the uncertainty in the model. Particle collapse could call into jeopardy the ability to use a particle filter in many mission critical systems. However, in 2007 [9], a completely different new class of particle filters that is not subject to particle collapse was developed. These are called homotopy particle filters or particle-flow filters. Rather than stochastically resampling particles, the initial versions of the filter moved the particles using deterministic differential equations called "flows," such as the exact flow of [10, 11], the incompressible flow derived in [11], and the geodesic flow of [12]. However, the best performing flow is based on a stochastic rather than a deterministic differential equation. This is the so-called "Gromov" flow [13].

However, the Gromov flow is a stiff stochastic differential equation and it is very difficult to balance finite precision limitations and integration accuracy when choosing a step size. The inability to handle the stiffness of the equations can severely limit the utility of the algorithm. The work of [14] offers an adaptive step-size selection technique as an initial attempt to ameliorate this problem. However, the technique is not scale invariant. Thus, it is really only valid for the very specific problem they have chosen. Changing the units of the quantities in the problem, for example, from kilometers to meters, changes the performance of the problem. Consequently, the two primary publications [15, 16] arising from this 6.1 project provide a

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scale-invariant automatic step-size selection algorithm. Additionally, the literature on particle-flow filtering is very fragmented. Small advances in the field are strewn across a large number of conference papers and a small handful of journal articles. Consequently, [15, 16] provide complete derivations of the particle-flow filtering algorithms from the ground up.

3. PUBLICATIONS AND HOW THEY ADDRESS THE PROBLEM

Publications related to this 6.1 program both advance the state of the art in particle-flow filtering as well as advance other, lower computational complexity algorithms and techniques related to the numeric integration of general stochastic differential equations. These publications are:

1. [15] **Particle-Flow Filters: Biases and Bias Avoidance**
   This paper, winner of the *2019 Jean Pierre Le-Cadre Second Place Best Paper Award* at the 22nd International Conference on Information Fusion, presents a scale-invariant adaptive step-size selection heuristic for the implementation of particle-flow filters. Additionally, biases present when integrating explicit and incompressible particle flows are derived and it is shown that such biases are not present when considering the Gromov flow.

2. [16] **Consideration of Particle-Flow Filter Implementations and Biases**
   This Naval Research laboratory (NRL) memo expands upon the conference paper of [15]. It provides derivations for all aspects of the particle-flow filter that were used in [15], including a derivation of the process noise-covariance matrix of the filter, which has not been previously published. Techniques for integrating the stochastic flow are discussed and it is mentioned that techniques with a higher order than the simple Euler-Maruyama method had poor performance.

3. [17] **Ito-Taylor Expansion Moments for Continuous-Time State Propagation**

   Particle-flow filters can handle continuous-time nonlinear dynamic systems, but so can a number of simpler continuous-discrete filtering algorithms [18], such as the continuous-discrete extended Kalman filter [19, 20] and the cubature Kalman filter [21]. This paper takes the stochastic differential-equation prediction formula described in [22] and provides all of the moments necessary to generalize the cubature Kalman filtering algorithm of [21]. In total, this enables the implementation of nine continuous-discrete cubature Kalman-filter variants, which can function as particle-filter alternatives in some nonlinear dynamic scenarios.

4. [23] **Basic Linear Cartesian Dynamic Models in Local Coordinates**
   In order to test nonlinear estimation algorithms such as particle filters and strong tracking filters, it can be good to have a number of nonlinear dynamic models for practical systems available. This paper derives expressions for the conversions of a linear dynamic model when converted into different measurement coordinate systems. The change of coordinates makes these new systems nonlinear. These systems are good for testing numerical-integration algorithms, because one can always compute the exact solutions for comparison. This is because one knows how to integrate linear systems: the solution is just a change of coordinates of the final linear answer.

   This note derives local-coordinate dynamic models of constant-velocity problems in a variety of 2D polar and r-u coordinate systems and in 3D spherical and r-u-v coordinate systems, thus sparing the reader tedious derivations for simple tracking problems. The conversions for r-u and r-u-v coordinate systems do not appear to have been previously published.

5. [24] **Strong Tracking Filters: Derivation and Improved Heuristic**

   A design choice when choosing a type of model-based estimation algorithm, is how well the algorithm handles mismatches between the model for which it is designed and the real system that it is estimating, which will never be a perfect match. This paper looks at strong tracking filters, which have been widely used by authors in China, but which have been almost completely overlooked by Western researchers. In addition to providing an English-language derivation, the paper offers a new method of adjusting the innovation matrix that allows the filter to be used with measurements of varying dimensionalities over time and analyzes the performance for differing degrees of model mismatch.

6. [25] **An Approximate Bayesian Extended Target-Tracking Algorithm**

   Target-tracking algorithms are often developed under a point-target approximation. However, real targets have a finite width, which can cause a mismatch between the assumed measurement distribution and the true non-Gaussian measurement distribution. This paper looks at the common ellipsoidal point-target approximation used in the Bayesian extended target-tracking algorithm of [26, 27]. A new, simple approximation to the measurement-update step of an extended target-tracking filter as well as a heuristic for track initialization. Like previous work, the filter uses a Gaussian approximation for the center of the target ellipsoid and an inverse-Wishart distribution to represent the uncertainty in the shape of the ellipsoid. A method of initializing the extended target state is provided, since such a routine was hitherto missing from the literature.

4. CODE AVAILABLE TO THE DEFENSE COMMUNITY

As a result of this 6.1 effort, a number of algorithm implementations were made available to the defense community in the full TCL, which is hosted on the Defense Intelligence Information Enterprise's web site. A subset of the most notable function additions shall be subseuqently listed.

Multiple variants of traditional stochastic resampling particle filters as well as particle-flow filters were implemented. The function `demoParticleFilters` demonstrates a sampling-importance-resampling (SIR) particle filter with a particle-flow filter and an extended Kalman filter on a simple track-filtering problem. The individual filters themselves are implemented as separate functions, allowing them to be easily used outside of the example. When considering the particle-flow filter, the functions

- `particleFlowUpdateGauss`
- `particleFlowUpdateGaussStoch`

implement not only the components of the particle-flow filter derived in [15, 16] under this 6.1 effort, but also multiple particle-flow filter components from the literature than can be combined into numerous filter variations based on various options given to the function. For example, the function `particleFlowUpdateGauss` implements three different flows, three different step-size algorithms, and four methods of obtaining a necessary covariance approximation, allowing one to make 36 different algorithm combinations. Similarly, the function `particleFlowUpdateGaussStoch` includes options for six initiation algorithms, three step-size selection algorithms and four approximations of a necessary covariance matrix. This translates to 72 possible algorithm combinations. Many of the other functions added as part of this 6.1 endeavor offer such a variety of choices for algorithm selection.

This 6.1 also added a number of functions related to traditional SIR particle-filter variants. These are

- `logGaussOptImportFun`
- `particleFilterUpdate`
- `resampleParticles`
- `sampleGaussOptImportFun`

which themselves include multiple options.

Particle-flow filters can be simulated forward through continuous-time stochastic dynamic models by applying any routine for simulating continuous-time stochastic differential equations to each of the particles. A number of routines for this. Stochastic Runge-Kutta variants do not require derivatives of the stochastic-process components, whereas Taylor series approximations do. The functions implemented include:

- `implicitWeakRungeKStep`
- `implicitWeakTaylorStep`
- `semiImplicitStrongRungeKStep`
- `semiImplicitStrongTaylorStep`
- `strongRungeKStep`
- `strongRungeKStepJump`
- `strongStochTaylorStep`
- `strongStochTaylorStepJump`
- `weakRungeKStep`
- `weakStochTaylorStep`

Additionally, since one needs to test the above functions against some kind of "truth" to make sure that they are functioning properly, a number of explicit solutions for the Black-Scholes model from finance are implemented as:

- `BlackScholesPred`
- `BlackScholesPredGaussPrior`

- `BlackScholesStep`

As a comparison to the particle-filter routines, moments of the stochastic processes from [17] which can be used in continuous-discrete Gaussian prediction algorithms are implemented in:

- `strongTaylorStepMeanCov`
- `weakTaylorStepMeanCov`

and incorporated into actual state and covariance prediction functions as:

- sqrtStochTaylorCubPredAdd
- stochTaylorCubPred
- stochTaylorCubPredAdd

When considering the use of linear dynamic models in nonlinear coordinate systems for testing a tracking algorithm, the drift functions derived in [23] are implemented as:

- `aCVPolar`
- `aCVRu2D`
- `aCVRuv`
- `aCVSpherical`

For considering tracking with mismatched dynamic models, variants of the prediction and measurement-update step of the strong tracking filter, with multiple algorithmic options as described in [24], are implemented in:

- `cubSTFUpdate`
- `ESTFUpdate`
- `STFOptim`
- `STFUpdate`

The extended target-tracking algorithms of [25] are implemented in:

- `discExtendObjKalPred`
- `extendObjKalUpdate`

- `extendObjOnePointCartInit`
- `region2TargetExtentProb`
- `targetExtentProb2Region`

A number of algorithms from the literature related to tracking with correlated measurements and process noise were implemented, though ultimately not compared with particle-filter performance. These include

- `discKalPredSimulCorr`
- `infoFilterDiscPredSimulCorr`
- `infoFilterPredUpdateDiscCorrMeas`
- `KalmanPredUpdateDiscCorrMeas`
- `KalmanUpdateDiscCorr`

As an alternative to particle filters for various estimation problems, one can attempt to use higher-order Taylor series approximations than the first- or second-order ones that are typically considered. Thus, measurement conversions and necessary derivatives for some standard conversions were implemented in arbitrary-order form as

- `calcCart2DArbPolarDeriv`
- `calcPolarArbDeriv`
- `calcSpherArbDeriv`
- `calcRuvArbDeriv`
- `elArbDerivs`
- `polarAngArbDerivs`
- `rangeArbDeriv`
- `singleDirCosArbDerivs`
- `spherAngArbDerivs`
- `spherAngArbDerivsRot`
- `uvArbDerivs`
- `uvArbDerivsRot`
- `pol2CartTaylorArbOrder`

Additionally, since there is little in the literature on constrained particle filtering, traditional algorithms related to constrained filtering were implemented as part of an investigation. These are:

- adjProcNoise4Constraint
- constrainedStateProj
- KalmanUpdateConst

Finally, there is an overlap between problems relating to surface-wave direction of arrival (DOA)-only localization of emitters and navigation. The uncertainty regions from such estimation problems are very nonlinear and can be good for potential solutions using particle-flow filters. Related navigation routines implemented include:

- `rhumbIntersect`
- `greatCircleTDOALoc`
- `indirectGreatCircleProb`
- `indirectRhumbSpherProblem`
- `directRhumbProblem`
- `directRhumbProbGen`
- `indirectGeodeticProb`
- `geodesicIntersect`
- `nearestGreatCirclePoint`
- `greatCircleIntersect`
- `minTimeIntercept2DCart`
- `minTimeIntercept3DCart`
- `minTimeInterceptEllips`
- `minTimeInterceptSpher`

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