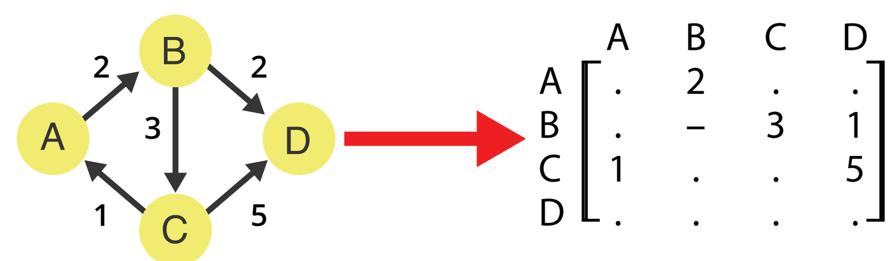
Spiral/AIML: Frontiers of Graph Processing in Linear Algebra

Introduction

Graph algorithms can be expressed as sequences of linear, algebra-like operations through the use of the adjacency matrix. Adjacency matrices are used to represent graphs instead of vertices and edges.



Our work extends the use of linear algebra beyond simple graph traversal.

Writing Graph Algorithms in LA

Many graph primitives can be cast in terms of LA operations.

- Neighbors of a vertex v—vector matrix products.
- Filtering—Hadamard products.
- Semi-rings—to represent Matrix Multiplication like operation (Single-Source Shortest Path uses the min-plus semiring).

This mapping gets rid of the need for "experts" to formulate graph algorithms in LA.

- Formally derived algorithms

Families of Graph Algorithms

Multiple graph algorithms can be enumerated for the same specification.

Our LA approach uses triangle counting:

$\Delta = 1/6\ \Gamma(A\text{^}3)$

This approach allows us to analyze graph algorithms for individual performance characteristics. The algorithm that best-fit the situation is chosen.

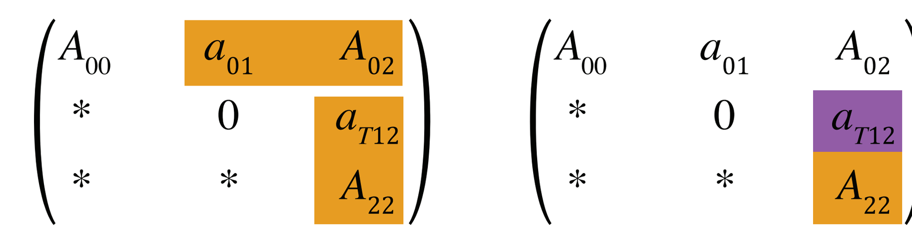
Our **linear algebraic approach** to graph algorithms provides a **flexible framework** that enables high performance code generation for **faster network analysis.**

Writing Graph Algorithms in LA

Operation	**Linear Algebra**
Edge Filtering	$A_b = A > \Delta$ $A_H = A_b \circ A$
Vertex bucketing	$v_{Bi} = (\Delta i \leq tent_{dists} < \Delta(i+1))$ $v'_{Bi} = ((\Delta i \leq new_{dist} < \Delta(i+1)) \circ (new_{dist} < old_{dist}))$
Neighborhood Traversal with Relaxation	$t_{Bi} = (tent_{dists}\ \ v_{Bi})$ $y^T_{out} = t_{Bi}(min.+)\begin{bmatrix} . & 2 & . & . \\ . & . & 3 & 2 \\ 1 & . & . & 5 \\ . & . & . & . \end{bmatrix}$

Busting Myths about Linear Algebraic Approach

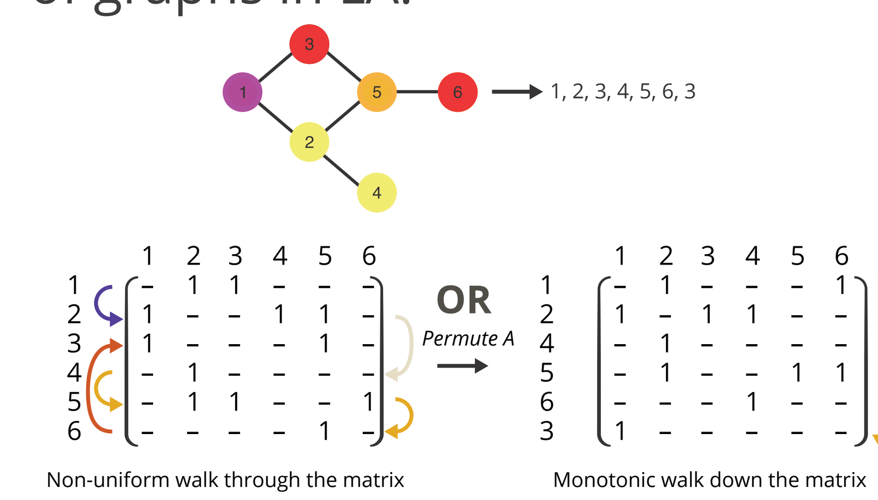
Unifying Edge and Vertex Centric Algorithms



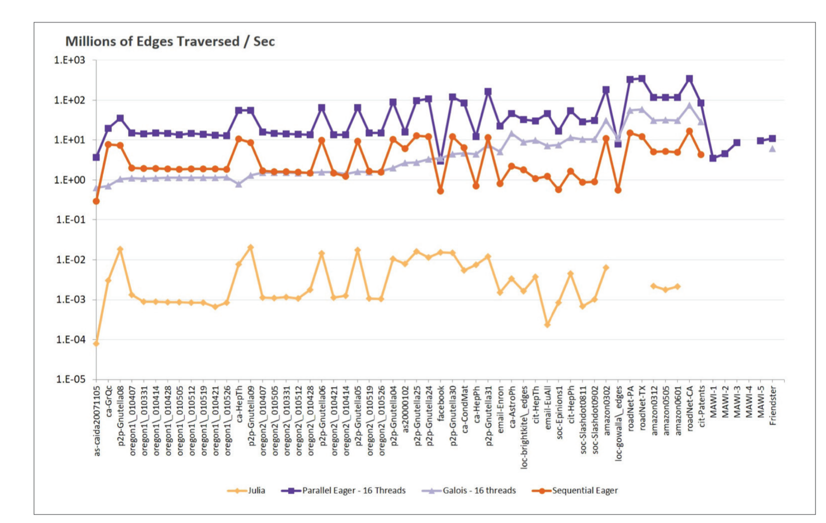
Both classes of algorithms can be expressed using a single linear algebraic framework.

Depth-First Search in Linear Algebra

First look at expressing depth-first traversal of graphs in LA.



Linear Algebraic Approach can Yield Good Performance



2019 Graph Challenge Champion
2018 Graph Challenge Finalist
2017 Graph Challenge Honorable Mention

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