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Equivalence of Single-Frequency Radar Imaging Algorithms

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14. ABSTRACT

We perform mathematical analysis of two single-frequency synthetic aperture radar (SAR) imaging techniques from the open literature. In one approach, developed by Borden and Cheney, the radar signal is first transformed to a two-dimensional Doppler-time distribution via a short-time Fourier-transform, and then matched filtering operations are applied to focus the transformed data. In the second approach, developed by Garry et al, the data are focused by applying a single-frequency matched filter directly to the time-domain data. Our analysis shows that the two techniques are equivalent, under the assumption that the effects of the amplitude of the matched filter are minimal.

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Matt Burfeindt and Margaret Cheney

Background

Doppler synthetic aperture radar (SAR), also known as Doppler-only SAR, is a technique for forming images from single-frequency radar data [1,2]. Range resolution is synthesized by taking advantage of the change in Doppler frequency that occurs as the radar traverses the synthetic aperture. This is distinct from traditional SAR imaging approaches which achieve range resolution by processing over a bandwidth of frequencies. Single-frequency techniques are of interest, as they may allow for SAR imaging using hardware designs of lower complexity.

In the literature, there are several signal processing approaches to forming single-frequency SAR images. In one approach, developed by Borden and Cheney [1], the radar signal is first transformed to a two-dimensional Doppler-time distribution via a short-time Fourier-transform, and then matched filtering operations are applied to focus the transformed data. In the second approach, developed by Garry et al [2], the data are focused by applying a single-frequency matched filter directly to the time-domain data.

In this memo report, we show that these two focusing operations are mathematically equivalent, under the assumption that the amplitude scaling effects of the matched filter in the Borden and Cheney approach are negligible.

Mathematical Development

We define the single-frequency radar signal from a single scatterer as $s(t) = \sigma \exp\left(-\frac{j4\pi}{\lambda}R(t)\right)/R^2(t)$, where t is the slow-time variable, λ is the wavelength of the signal, $R(t)$ is the range between the radar and the scatterer as a function of time, and σ is a scaling factor related to the radar cross-section.

To focus $s(t)$ using the Borden and Cheney method, we first perform the STFT operation given by $d(\tau,\omega) = \int s(t)\Pi(t-\tau)\exp(-j\omega t)\,dt$, where $\Pi(t-\tau)$ is a unit rectangular window centered at τ and ω is the radial frequency. We then perform the matched filtering operation given by

$$I(\mathbf{x}) = \iint d(\tau,\omega) d_0^*(\tau,\omega,\mathbf{x}) d\tau d\omega. \tag{1}$$

In (1), $I(\mathbf{x})$ is the focused image value at the pixel corresponding to image position $\mathbf{x}$ and $d_0^*(\tau,\omega,\mathbf{x})$ is the result of applying the STFT to the matched filter given by $s_0(t,\mathbf{x}) = \exp\left(-\frac{j4\pi}{\lambda}R_0(t,\mathbf{x})\right)$, where $R_0(t,\mathbf{x})$ is the distance between $\mathbf{x}$ and the radar as a function of time. The operation in (1) is somewhat simplified from the description provided in [1] in that it neglects the amplitude scaling of the matched filter, which typically has little effect on SAR resolution.

To focus $s(t)$ using the Garry method, we perform a matched filter operation directly to the time-domain signal. The result is given by

$$I(\mathbf{x}) = \int s(t) s_0^*(t,\mathbf{x}) dt. \tag{2}$$

We show via the following analysis that (1) and (2) are equivalent. We begin by substituting the STFT relationships into (1), which yields after rearranging terms the following:

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$$I(\mathbf{x}) = \iiint \left\{ \int \exp\big(-j\omega(t-t')\big)\, d\omega \right\} s(t) s_0(t') \Pi(t-\tau)\Pi(t'-\tau)\, dt\, dt'\, d\tau. \tag{3}$$

By evaluating the integral in brackets, we can simplify (3) to

$$I(\mathbf{x}) = \iiint \delta\big(t-t'\big) s(t) s_0(t') \Pi(t-\tau)\Pi(t'-\tau)\, dt\, dt'\, d\tau, \tag{4}$$

where $\delta(t)$ is the Dirac delta distribution. Evaluating the integral in t' yields

$$I(\mathbf{x}) = \iint s(t) s_0^*(t) \Pi^2(t-\tau)\, dt\, d\tau. \tag{5}$$

Integrating $\Pi^2(t-\tau)$ over τ yields a constant that does not affect the focusing behavior of the integral, and thus, to a scale factor, (5) reduces to (2).

Conclusion

In the preceding analysis, we showed that the two focusing strategies for Doppler SAR are equivalent. Thus, the simpler implementation of the technique from [2] may be more attractive for some applications.

References

[1] B. Borden and M. Cheney, “Synthetic-aperture imaging from high-Doppler-resolution measurements,” *Inverse Problems*, vol. 21, 2005.

[2] J. Garry, G. Smith, and C. Baker, “Practical implementation of stripmap Doppler imaging,” *IET Radar, Sonar, and Navigation*, vol. 9, no. 8, 2015.