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PARTICIPANTS:

Participant Type: Faculty
Participant: Filippo Radicchi
Person Months Worked: | **Funding Support:**
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Participant Type: Postdoctoral (scholar, fellow or other postdoctoral position)
Participant: Ali Faqeeh
Person Months Worked: | **Funding Support:**
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Participant: Dario Mazzilli
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Date Submitted: 8/22/16 12:00AM Date Published: 6/1/16 8:00AM
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Article Title: Leveraging percolation theory to single out influential spreaders in networks
Authors: Filippo Radicchi, Claudio Castellano
Keywords: complex networks, influential spreaders
Abstract: Among the consequences of the disordered interaction topology underlying many social, technological, and biological systems, a particularly important one is that some nodes, just because of their position in the network, may have a disproportionate effect on dynamical processes mediated by the complex interaction pattern. For example, the early adoption of a commercial product by an opinion leader in a social network may change its fate or just a few superspreaders may determine the virality of a meme in social media. Despite many recent efforts, the formulation of an accurate method to optimally identify influential nodes in complex network topologies remains an unsolved challenge. Here, we present the exact solution of the problem for the specific, but highly relevant, case of the susceptible-infected-removed (SIR) model for epidemic spreading at criticality. By exploiting the mapping between bond percolation and the static properties of the SIR model, we prove that the recently intr
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Publication Location:
Article Title: Observability transition in real networks
Authors: Yang Yang, Filippo Radicchi
Keywords: complex networks, observability transition
Abstract: We consider the observability model in networks with arbitrary topologies. We introduce a system of coupled nonlinear equations, valid under the locally tree-like ansatz, to describe the size of the largest observable cluster as a function of the fraction of directly observable nodes present in the network. We perform a systematic analysis on 95 real-world graphs and compare our theoretical predictions with numerical simulations of the observability model. Our method provides almost perfect predictions in the majority of the cases, even for networks with very large values of the clustering coefficient. Potential applications of our theory include the development of efficient and scalable algorithms for real-time surveillance of social networks, and monitoring of technological networks.
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Date Submitted: 6/25/17 12:00AM Date Published: 1/1/17 5:00AM
Publication Location:
Article Title: Redundant Interdependencies Boost the Robustness of Multiplex Networks
Authors: Filippo Radicchi, Ginestra Bianconi
Keywords: multilayer networks, percolation
Abstract: In the analysis of the robustness of multiplex networks, it is commonly assumed that a node is functioning only if its interdependent nodes are simultaneously functioning. According to this model, a multiplex network becomes more and more fragile as the number of layers increases. In this respect, the addition of a new layer of interdependent nodes to a preexisting multiplex network will never improve its robustness. Whereas such a model seems appropriate to understand the effect of interdependencies in the simplest scenario of a network composed of only two layers, it may seem unsuitable to characterize the robustness of real systems formed by multiple network layers. In fact, it seems unrealistic that a real system evolved, through the development of multiple layers of interactions, towards a fragile structure. In this paper, we introduce a model of percolation where the condition that makes a node functional is that the node is functioning in at least two of the layers of the networ
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Date Submitted: 6/25/17 12:00AM Date Published: 1/1/17 3:00PM
Publication Location:
Article Title: Fundamental difference between superblockers and superspreaders in networks
Authors: Filippo Radicchi, Claudio Castellano
Keywords: Influence maximization, complex networks
Abstract: Two important problems regarding spreading phenomena in complex topologies are the optimal selection of node sets either to minimize or maximize the extent of outbreaks. Both problems are nontrivial when a small fraction of the nodes in the network can be used to achieve the desired goal. The minimization problem is equivalent to a structural optimization. The “superblockers,” i.e., the nodes that should be removed from the network to minimize the size of outbreaks, are those nodes that make connected components as small as possible. “Superspreaders” are instead the nodes such that, if chosen as initiators, they maximize the average size of outbreaks. The identity of superspreaders is expected to depend not just on the topology, but also on the specific dynamics considered. Recently, it has been conjectured that the two optimization problems might be equivalent, in the sense that superblockers act also as superspreaders. In spite of its potential groundbreaking importance, no empirical
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Publication Location:
Article Title: Percolation in real multiplex networks
Authors: Ginestra Bianconi, Filippo Radicchi
Keywords: complex networks, percolation
Abstract: We present an exact mathematical framework able to describe site-percolation transitions in real multiplex networks. Specifically, we consider the average percolation diagram valid over an infinite number of random configurations where nodes are present in the system with given probability. The approach relies on the locally treelike ansatz, so that it is expected to accurately reproduce the true percolation diagram of sparse multiplex networks with negligible number of short loops. The performance of our theory is tested in social, biological, and transportation multiplex graphs. When compared against previously introduced methods, we observe improvements in the prediction of the percolation diagrams in all networks analyzed. Results from our method confirm previous claims about the robustness of real multiplex networks, in the sense that the average connectedness of the system does not exhibit any significant abrupt change as its individual components are randomly destroyed.
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Journal: Nature Communications
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Volume: 8 Issue: First Page #: 1540
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Article Title: Optimal percolation on multiplex networks
Authors: Saeed Osat, Ali Faqeeh, Filippo Radicchi
Keywords: Optimal percolation, multilayer networks
Abstract: Optimal percolation is the problem of finding the minimal set of nodes such that if the members of this set are removed from a network, the network is fragmented into non-extensive disconnected clusters. The solution of the optimal percolation problem has direct applicability in strategies of immunization in disease spreading processes, and influence maximization for certain classes of opinion dynamical models. In this paper, we consider the problem of optimal percolation on multiplex networks. The multiplex scenario serves to realistically model various technological, biological, and social networks. We find that the multilayer nature of these systems, and more precisely multiplex characteristics such as edge overlap and interlayer degree-degree correlation, profoundly changes the properties of the set of nodes identified as the solution of the optimal percolation problem.
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Date Submitted: 10/4/18 12:00AM Date Published: 2/1/18 3:00PM
Publication Location:
Article Title: Decoding communities in networks
Authors: Filippo Radicchi
Keywords: community structure, networks
Abstract: According to a recent information-theoretical proposal, the problem of defining and identifying communities in networks can be interpreted as a classical communication task over a noisy channel: memberships of nodes are information bits erased by the channel, edges and nonedges in the network are parity bits introduced by the encoder but degraded through the channel, and a community identification algorithm is a decoder. The interpretation is perfectly equivalent to the one at the basis of well-known statistical inference algorithms for community detection. The only difference in the interpretation is that a noisy channel replaces a stochastic network model. However, the different perspective gives the opportunity to take advantage of the rich set of tools of coding theory to generate novel insights on the problem of community detection. In this paper, we illustrate two main applications of standard coding-theoretical methods to community detection. First, we leverage a state-of-the-ar
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Publication Location:
Article Title: Observability transition in multiplex networks
Authors: Saeed Osat, Filippo Radicchi
Keywords: observability, multiplex networks
Abstract: We extend the observability model to multiplex networks composed of two network layers. We present mathematical frameworks, valid under the treelike ansatz, able to describe the emergence of the macroscopic cluster of mutually observable nodes in both synthetic and real-world multiplex networks. We show that the observability transition in synthetic multiplex networks is discontinuous. In real-world multiplex networks instead, edge overlap among layers is responsible for the disappearance of any sign of abruptness in the emergence of the macroscopic cluster of mutually observable nodes.
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Article Title: Uncertainty Reduction for Stochastic Processes on Complex Networks
Authors: Filippo Radicchi, Claudio Castellano
Keywords: stochastic processes, complex networks
Abstract: Many real-world systems are characterized by stochastic dynamical rules where a complex network of interactions among individual elements probabilistically determines their state. Even with full knowledge of the network structure and of the stochastic rules, the ability to predict system configurations is generally characterized by a large uncertainty. Selecting a fraction of the nodes and observing their state may help to reduce the uncertainty about the unobserved nodes. However, choosing these points of observation in an optimal way is a highly nontrivial task, depending on the nature of the stochastic process and on the structure of the underlying interaction pattern. In this paper, we introduce a computationally efficient algorithm to determine quasioptimal solutions to the problem. The method leverages network sparsity to reduce computational complexity from exponential to almost quadratic, thus allowing the straightforward application of the method to mid-to-large-size systems.
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Article Title: Characterizing the Analogy Between Hyperbolic Embedding and Community Structure of Complex Networks
Authors: Ali Faqeeh, Saeed Osat, Filippo Radicchi
Keywords: low-dimensional embedding, multiplex networks
Abstract: We show that the community structure of a network can be used as a coarse version of its embedding in a hidden space with hyperbolic geometry. The finding emerges from a systematic analysis of several real-world and synthetic networks. We take advantage of the analogy for reinterpreting results originally obtained through network hyperbolic embedding in terms of community structure only. First, we show that the robustness of a multiplex network can be controlled by tuning the correlation between the community structures across different layers. Second, we deploy an efficient greedy protocol for network navigability that makes use of routing tables based on community structure.
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Volume: Issue: First Page #:
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Article Title: Identifying influential spreaders in noisy networks
Authors: ?irag Erkol, Ali Faqeeh, Filippo Radicchi
Keywords: Influence maximization, spreading, networks
Abstract: We consider the problem of identifying the most influential nodes for a spreading process on a network when prior knowledge about structure and dynamics of the system is incomplete or erroneous. Specifically, we perform a numerical analysis where the set of top spreaders is determined on the basis of prior information that is artificially altered by a certain level of noise. We then measure the optimality of the chosen set by measuring its spreading impact in the true system. Whereas we find that the identification of top spreaders is optimal when prior knowledge is complete and free of mistakes, we also find that the quality of the top spreaders identified using noisy information does not necessarily decrease as the noise level increases. For instance, we show that it is generally possible to compensate for erroneous information about dynamical parameters by adding synthetic errors in the structure of the network. Further, we show that, in some dynamical regimes, even completely losin
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Article Title: Weight Thresholding on Complex Networks
Authors: Xiaoran Yan, Lucas Jeub, Alessandro Flammini, Filippo Radicchi, Santo Fortunato
Keywords: thresholding, networks
Abstract: Weight thresholding is a simple technique that aims at reducing the number of edges in weighted networks that are otherwise too dense for the application of standard graph-theoretical methods. We show that the community structure of real weighted networks is very robust under weight thresholding, as it is maintained even when most of the edges are removed. This is due to the correlation between topology and weight that characterizes real networks. On the other hand, the behaviour of other properties is generally system dependent.
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Article Title: Controlling the uncertain response of real multiplex networks to random damage
Authors: Francesco Coghi, Filippo Radicchi, Ginestra Bianconi
Keywords: robustness, multiplex networks
Abstract: We reveal large fluctuations in the response of real multiplex networks to random damage of nodes. These results indicate that the average response to random damage, traditionally considered in mean-field approaches to percolation, is a poor metric of system robustness. We show instead that a large deviation approach to percolation provides a more accurate characterization of system robustness. We identify an effective percolation threshold at which we observe a clear abrupt transition separating two distinct regimes in which the most likely response to damage is either a functional or a dismantled multiplex network. We leverage our findings to propose a new metric, named safeguard centrality, able to single out the nodes that control the response of the entire multiplex network to random damage. We show that safeguarding the function of top-scoring nodes is sufficient to prevent system collapse.
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Article Title: Discordant attributes of structural and functional connectivity in a two-layer multiplex network
Authors: Sol Lim, Filippo Radicchi, Martijn P van den Heuvel, Olaf Sporns
Keywords: Brain networks, Robustness, Multiplex networks
Abstract: Several studies have suggested that functional connectivity (FC) is constrained by the underlying structural connectivity (SC) and mutually correlated. However, not many studies have focused on differences in the network organization of SC and FC, and on how these differences may inform us about their mutual interaction. To explore this issue, we adopt a multi-layer framework, with SC and FC, constructed using Magnetic Resonance Imaging (MRI) data from the Human Connectome Project, forming a two-layer multiplex network. In particular, we examine whether node strength assortativity within and between the SC and FC layer may confer increased robustness against structural failure. We find that, in general, SC is organized assortatively, indicating brain regions are on average connected to other brain regions with similar node strengths. On the other hand, FC shows disassortative mixing. This discrepancy is apparent also among individual resting-state networks within SC and FC. In addition
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Article Title: Emergence of power laws in noncritical neuronal systems
Authors: Ali Faqeeh, Saeed Osat, Filippo Radicchi, James P. Gleeson
Keywords: complex networks, critical phenomena
Abstract: Experimental and computational studies provide compelling evidence that neuronal systems are characterized by power-law distributions of neuronal avalanche sizes. This fact is interpreted as an indication that these systems are operating near criticality, and, in turn, typical properties of critical dynamical processes, such as optimal information transmission and stability, are attributed to neuronal systems. The purpose of this Rapid Communication is to show that the presence of power-law distributions for the size of neuronal avalanches is not a sufficient condition for the system to operate near criticality. Specifically, we consider a simplistic model of neuronal dynamics on networks and show that the degree distribution of the underlying neuronal network may trigger power-law distributions for neuronal avalanches even when the system is not in its critical regime. To certify and explain our findings we develop an analytical approach based on percolation theory and branching proce
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Date Submitted: 8/13/19 12:00AM Date Published: 3/1/19 5:00AM
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Article Title: Error-correcting decoders for communities in networks
Authors: Krishna C. Bathina, Filippo Radicchi
Keywords: complex networks, community detection
Abstract: As recent work demonstrated, the task of identifying communities in networks can be considered analogous to the classical problem of decoding messages transmitted along a noisy channel. We leverage this analogy to develop a community detection method directly inspired by a standard and widely-used decoding technique. We further simplify the algorithm to reduce the time complexity from quadratic to linear. We test the performance of the original and reduced versions of the algorithm on artificial benchmarks with pre-imposed community structure, and on real networks with annotated community structure. Results of our systematic analysis indicate that the proposed techniques are able to provide satisfactory results.
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Article Title: Systematic comparison between methods for the detection of influential spreaders in complex networks
Authors: S. Erkol, C. Castellano, F. Radicchi
Keywords: complex networks, influence maximization
Abstract: Influence maximization is the problem of finding the set of nodes of a network that maximizes the size of the outbreak of a spreading process occurring on the network. Solutions to this problem are important for strategic decisions in marketing and political campaigns. The typical setting consists in the identification of small sets of initial spreaders in very large networks. This setting makes the optimization problem computationally infea- sible for standard greedy optimization algorithms that account simultaneously for information about network topology and spreading dynamics, leaving space only to heuristic methods based on the drastic approximation of relying on the geometry of the network alone. The literature on the subject is plenty of purely topological methods for the identification of influential spreaders in networks. However, it is unclear how far these meth- ods are from being optimal. Here, we perform a systematic test of the performance of a multitude of heuristic meth
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Article Title: Classical information theory of networks
Authors: F. Radicchi, D. Krioukov, H. Hartle and G. Bianconi
Keywords: complex networks, space embedding, information theory
Abstract: Heterogeneity is among the most important features characterizing real-world networks. Empirical evidence in support of this fact is unquestionable. Existing theoretical frameworks justify heterogeneity in networks as a convenient way to enhance desirable systemic features, such as robustness, synchronizability and navigability. However, a unifying information theory able to explain the natural emergence of heterogeneity in complex networks does not yet exist. Here, we fill this gap of knowledge by developing a classical information theoretical framework for networks. We show that among all degree distributions that can be used to generate random networks, the one emerging from the principle of maximum entropy is a power law. We also study spatially embedded networks finding that the interactions between nodes naturally lead to nonuniform distributions of points in the space. The pertinent features of real-world air transportation networks are well described by the proposed framework.
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Article Title: Classical information theory of networks
Authors: F. Radicchi, D. Krioukov, H. Hartle and G. Bianconi
Keywords: complex networks
Abstract: Heterogeneity is among the most important features characterizing real-world networks. Empirical evidence in support of this fact is unquestionable. Existing theoretical frameworks justify heterogeneity in networks as a convenient way to enhance desirable systemic features, such as robustness, synchronizability and navigability. Information theory is one of the most fundamental theoretical frameworks of network science and machine learning. However, the current information theory frameworks for understading networks, based on maximum entropy network ensembles, are not able to explain the emergence of heterogeneity in complex networks. Here, we fill this gap of knowledge by developing a classical information theoretical framework for networks based on finding a trade-off between the information content of a compressed representation of the ensemble and the information content of the actual network ensemble. We show that among all degree distributions that can be used to generate random
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Article Title: Classes of critical avalanche dynamics in complex networks
Authors: F. Radicchi, C. Castellano, A. Flammini, M.A. Muñoz and D. Notarmuzi
Keywords: avalanche dynamics, networks
Abstract: Dynamical processes exhibiting absorbing states are essential in the modeling of a large variety of situations from material science to epidemiology and social sciences. Such processes exhibit the possibility of avalanching behavior upon slow driving. Here, we study the distribution of sizes and durations of avalanches for well-known dynamical processes on complex networks. We find that all analyzed models display a similar critical behavior, characterized by the presence of two distinct regimes. At small scales, sizes and durations of avalanches exhibit distributions that are dependent on the network topology and the model dynamics. At asymptotically large scales instead --irrespective of the type of dynamics and of the topology of the underlying network-- sizes and durations of avalanches are characterized by power-law distributions with the exponents of the mean-field critical branching process.
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Article Title: Defining and identifying the optimal embedding dimension of networks
Authors: W. Gu, A. Tandon, Y.Y. Ahn and F. Radicchi
Keywords: neural embedding
Abstract: Network embedding is a general-purpose machine learning technique that encodes network structure in vector spaces with tunable dimension. Choosing an appropriate embedding dimension -- small enough to be efficient and large enough to be effective -- is challenging but necessary to generate embeddings applicable to a multitude of tasks. Unlike most existing strategies that rely on performance maximization in downstream tasks, here we propose a principled method for the identification of an optimal dimension such that all structural information of a network is parsimoniously encoded. The method is validated on various embedding algorithms and a large corpus of real-world networks. Estimated values of the optimal dimension in real-world networks suggest that efficient encoding in low-dimensional spaces is usually possible.
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Final Report

Structural and dynamical transitions in networks of networks

W911NF-16-1-0104

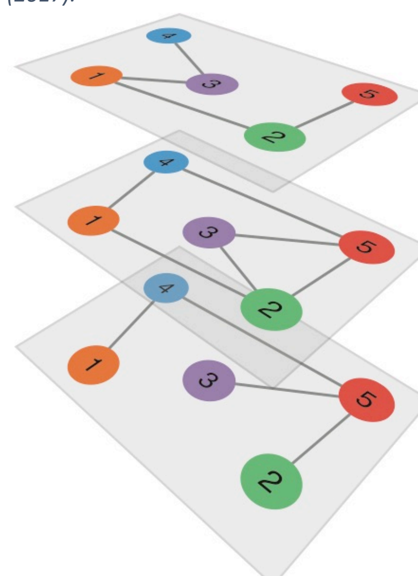
Filippo Radicchi

Center for Complex Networks and Systems, Luddy School of Informatics, Computing, and Engineering, Indiana University

Objective

Our world is linked by a complex mesh of networks where information, people and goods flow. These networks do not exist in isolation, but they depend one on the other (see Figure 1). Interdependencies are essential for the function of the individual networks, but they may also cause phenomena that would be absent if networks were in isolation. For example, the presence of interdependencies among power grids and communication networks dramatically augment their vulnerability, by boosting the potential for cascading failures due to the amplification of small-scale initial failures to catastrophic proportions. The urgency of this issue was stressed already in 1996 by the President's Commission on Critical Infrastructure Protection, but recent events, such as the 9/11 terrorist attack, the 2003 Northeast power blackout, and the 2005 Gulf Coast hurricane, indeed highlighted that the infrastructural system is constantly risk of catastrophic failures. Similarly, the 2008 crisis demonstrated how failures can propagate across interconnected financial sectors and disrupt the stability of global economy. Given the current inability to prevent such catastrophic events, there is a urgent need to better understand the role of interdependencies among real networks, and make them more secure, stable, robust and resilient. The objective of this project was to start building the fundamentals of a theory for real interdependent networks, including the generation of novel sets of analytic and computational tools to be used in the analysis of critical interdependent infrastructures.

Figure 1 Schematic representation of an interdependent network. Colors and labels identify interdependent nodes among the network layers that compose the system. Figure adapted from Phys. Rev. X 7, 011013 (2017).



Approach

The study of interdependencies in infrastructural networks has a relatively long tradition in civil engineering research. It has been mainly based on traditional approaches such as agent-based simulations, economic theoretical analyses, system dynamical equations, and reliability theory. Our goal was to contribute to the advancement of the field by studying interdependencies among networks from a different perspective by using tools and methods from statistical physics of complex networks. The approach focuses on understanding how microscopic properties of the system structure and dynamics lead to significant emergent phenomena at the global scale. The potential of such an approach is to provide guidance for the design of more robust and resilient systems, including the provision of ready-to-use formulas of intervention for fast recovery after catastrophic failures. We planned to apply theoretical and numerical methods to study of the real-world networks of networks.

Scientific Barriers

The development of mathematical frameworks able to describe real-world systems composed of multiple interacting networks is challenging. The task requires to overcome scientific obstacles that may be grouped in two main categories.

The first category regards theoretical challenges in the development of the mathematical framework. Most of the state-of-the-art results are valid for unrealistic scenarios, as they make use of random network models, and rely on the assumption that system size is infinite. Real networks do not satisfy these properties. For example, a real-world transportation network is composed of a certain number (generally small) of geographical locations that are connected (through modes of transportation, such as buses, trains, subways, etc.) in a nonrandom fashion. Also, in state-of-the-art approaches, interdependencies among networks are generally modeled in a way that simplifies their mathematical treatment, but poorly describes their true role in real-world systems. According to these models for example, the addition of a new mode of transportation in a city would counterintuitively have the detrimental effect of making the city transportation system less reliable. Finally, state-of-the-art approaches to study the robustness of interdependent networks focus their attention on the consequences of random and independent local failures. However, failures in real systems do not generally occur in this way. Further, in engineering applications, it is generally more interesting to consider the worst-case scenario, where the system is under maximal stress, rather than considering a scenario where elements fail at random.

The second category of scientific challenges regards the retrieval, collection and analysis of data describing real-world interconnected networks, and the use of these data to validate theoretical models. Only a few repositories of interdependent networks exist. They are mainly focusing on social and biological systems, and multimodal transportation graphs. Retrieving information about individual networks is generally a doable task, however, establishing how these systems interact one with the other is much more challenging. The task requires integration of data from different sources, and the development of suitable techniques for inferring interactions among networks.

Significance

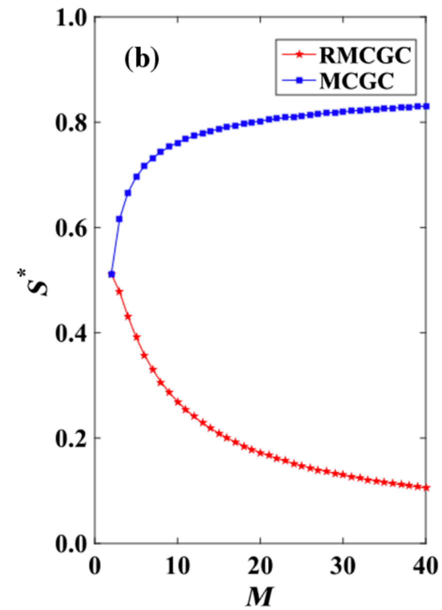
The project has been very successful in terms of number of papers produced and prestige of the journals where these papers have been published. For a complete list of publications funded by the project, see section "Accomplishments." We summarize here only a few main results. For sake of clarity, we divide them into three main areas.

Robustness

This is the area where we produced the highest number of results. For brevity, we describe here only the two most important ones.

First, we developed a new model to study the robustness of interdependent networks. The model, named redundant percolation model, describes a scenario where the addition of a new network layer in a preexisting interdependent system boosts its robustness (Figure 2). The model is much more realistic that the one generally used in this context, where the addition of a new network instead makes the system more fragile. Adding a new mode of transportation in a preexisting multimodal transportation system should make the system more reliable against eventual failures. Similarly, in a living organism, the development of new types of interactions among constituents should increase the stability of the same organism against possible mutations. The new model is mathematically much more challenging than its less realistic counterpart. In our paper, we developed theoretical methods to find solutions of the model for both artificial and real-world networks.

Figure 2 Robustness of an interdependent network as a function of the number of its layers M. The quantity S represents the proportion of the system that fails suddenly when the system collapses. The higher the value of S*, the less robust is the system. The two curves show the comparison between the standard model used in the literature (MCGCC, blue) and the new model we proposed (RMCGC, red). Figure adapted from Phys. Rev. X 7, 011013 (2017).*

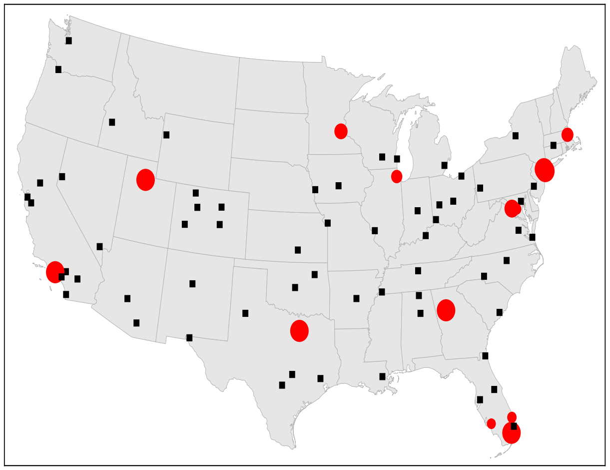


Second, we performed a study of the robustness of real-world interdependent networks under maximal stress. We performed the analysis by finding approximated solutions to the so-called optimal percolation problem. This is a NP problem that consists in identifying the minimal set of nodes whose failure leads to the collapse of the entire system. An example of the application of our results in a real system is reported in Figure 3. Our contribution consists in a complete characterization of the model, including the development of efficient yet effective algorithms to approximate the solution of the original NP problem.

Observability

We investigated theoretical methods to understand how to properly observe or monitor the dynamical state of networks. The problem is extremely important in any context where resource constraint allow to place sensors only on a small portion of the system. These observations are in turn used to infer the state of the unobserved part of the system. In this context, we considered two scenarios. The first applies to deterministic dynamical systems such as those used in modeling power grids. The framework used in this context is similar to the one considered in ordinary percolation. We leveraged our percolation framework to study observability in both isolated and interdependent networks. The second scenario we considered applies instead to stochastic processes on networks. This scenario is much more general, but mathematically much more challenging. We deployed a novel algorithm that allows to perform optimal decisions about the placement of sensors in a network.

Figure 3 Optimal percolation on the US air transportation network. The network has been created aggregating data about US domestic flights operated by American Airlines and Delta. Red circles represent airports that play a central role in the system, in the sense that their failure only is sufficient to cause system collapse. The size of the circles is proportional to the centrality of the airports. Black squares represent all other airports in the network. Connections among airports have been omitted from the visualization for clarity. Figure adapted from Nature Comm. 8, 1540 (2017).



Spreading and influential nodes

We leveraged our knowledge of percolation processes on networks to better understand the problem of identification of influential spreaders in opinion dynamical models. Solutions to this problem may play an important role in marketing or political campaigns, as they represent the best strategies to identify key actors in social networks when the goal is to maximize the diffusion of a given product or opinion. The problem of identification of influential spreader in a network is NP complete. Based on results reported in recent literature, it was believed that the solution to the problem could be well approximated by the one valid for optimal percolation. We demonstrated instead that the two problems are fundamentally different.

We further produced the first systematic study of the influence maximization problem in real-world networks. Specifically, we considered tens of approximated algorithms used for the solution of the influence maximization problem, applied them to hundreds of real-world networks, and measured their performance against the best achievable solution of the influence maximization problem obtained with greedy optimization. We found that no single method is able to outperform all others in all networks, but also that quasioptimal solutions can be obtained via the combination of various techniques in hybrid identification methods. This finding indicates potential of hybrid optimization techniques in network optimization problems other than influence maximization.

Geometric interpretation of physical properties of networks

Geometric embeddings aim at reducing the complexity of a network, exploiting the flexibility of a continuous distribution of points in space and providing a geometric interpretation of structural relationships. Their main application in network science is restricted to standard tasks such as link prediction and graph clustering. We started working on the possibility to leverage geometric descriptions to characterize and predict physical properties of multilayer networks. So far, our results are extremely encouraging, opening the way for a really promising research direction.

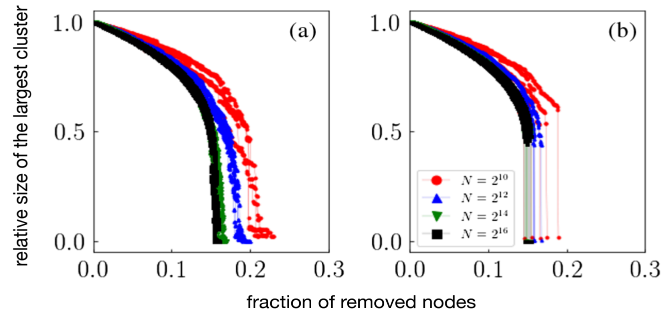


Figure 4 Robustness of multiplex networks with correlated community structure. We measure the relative size of the largest mutually connected cluster as a function of the fraction of nodes removed from the system. We considered synthetic multi-layer graphs where we could control for the level of correlation between the pre-imposed community structures. Panel a refers to a case with highly correlated community structure among the network layers. The collapse of the network is smooth, indicating robustness of the system against disruption. In panel b, we present results for the case of weak correlation between the community structure of the layers. The collapse is abrupt in this case. Figure adapted from Phys. Rev. Lett. 121, 098301 (2018).

We demonstrated that descriptions of networks in non-metric spaces, as for example those obtained using the detection of communities via the stochastic block model, are for many respects equivalent to those achievable in metric spaces. The equivalence is valid not only for the description of networks, but also for the prediction of their physical properties. For instance, the correlation between the community structure of the layers of a multi-layer network is a good predictor of the actual robustness of the network itself. High correlation, meaning that nodes in the various layers are clustered similarly, implies resilience of the multi-layer network against intentional attacks. On the contrary, low correlation leads to fragility of the network (see Figure 4).

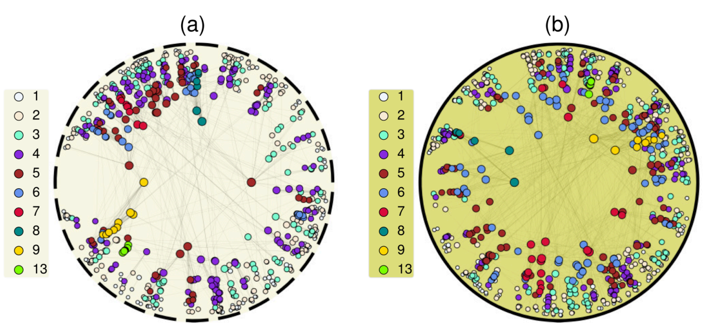


Figure 5 k-core structure of real-world multiplex networks. Visualization of the hyperbolic embedding for the arXiv multi-layer collaboration network. The position of the nodes in the disk is determined by their hyperbolic coordinates. Different colors serve to differentiate nodes depending on their k-shell index value. Figure adapted from Phys. Rev. Research 2, 023176 (2020).

We investigated also the ability of hyperbolic embedding to characterize the k-core structure of real-world multilayer networks (Figure 5). k-core structure is a technical term used to describe the organization in shells of nodes with an increasingly high degree. Understanding of k-core structures is important in the study of spreading processes taking place on networks, as for example in the identification of influential spreaders and the emergence of localization phenomena. Our results demonstrated that the relative position of the nodes in the embedding space can well predict the level of strength of the k-core structure of a multi-layer network. Specifically, networks with heterogenous degree distributions display strong k-structure only if the degree of the nodes in the various layers are significantly correlated. In network with homogenous degree distributions instead, strong k-core structure is visible only in presence of high correlation among the cluster structure of the layers.

Accomplishments

Published papers

1. S. Osat, F. Radicchi and F. Papadopoulos, *k-core structure of real multiplex networks*, Phys. Rev. Research 2, 023176 (2020).
https://journals.aps.org/prresearch/abstract/10.1103/PhysRevResearch.2.023176
2. S. Erkol, C. Castellano and F. Radicchi, *Systematic comparison between methods for the detection of influential spreaders in complex networks*. Sci. Rep. **9**, 15095 (2019).
https://www.nature.com/articles/s41598-019-51209-6
3. K. Bathina and F. Radicchi, *Error-correcting decoders for community detection*, Appl. Netw. Sci. **4**, 9 (2019).
https://appliednetsci.springeropen.com/articles/10.1007/s41109-019-0114-7
4. A. Faqeeh, S. Osat, F. Radicchi and J.P. Gleeson, *Emergence of power-laws in noncritical neuronal systems*, Phys. Rev. E **100**, 010401(R) (2019).
https://journals.aps.org/pre/abstract/10.1103/PhysRevE.100.010401
5. S. Lim, F. Radicchi, M. de Reus, M.P. van den Heuvel and O. Sporns, *Discordant attributes of structural and functional brain connectivity in two-layer multiplex network*, Sci. Rep. **9**, 2885 (2019).
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https://journals.aps.org/pre/abstract/10.1103/PhysRevE.98.062317
7. S. Erkol, A. Faqeeh and F. Radicchi, *Influence maximization in noisy networks*, EPL **123**, 58007 (2018).
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9. A. Faqeeh, S. Osat and F. Radicchi, *Characterizing the analogy between hyperbolic embedding and community structure of complex networks*, Phys. Rev. Lett. **121**, 098301 (2018).
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10. F. Radicchi and C. Castellano, *Uncertainty reduction for stochastic processes on complex networks,* Phys. Rev. Lett. **120**, 198301 (2018).
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11. S. Osat and F. Radicchi, *Observability transition in multiplex networks,* Physica A **503**, 745-761 (2018).
https://www.sciencedirect.com/science/article/pii/S0378437118302310
12. F. Radicchi, *Decoding communities in networks,* Phys. Rev. E **97**, 022316 (2018).
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13. S. Osat, A. Faqeeh and F. Radicchi, *Optimal percolation on multiplex networks,* Nature Comm. **8**, 1540 (2017).
https://www.nature.com/articles/s41467-017-01442-2
14. F. Radicchi and G. Bianconi, *Redundant interdependencies boost the robustness of multilayer networks*, Phys. Rev. X **7**, 011013 (2017).
http://journals.aps.org/prx/abstract/10.1103/PhysRevX.7.011013
15. F. Radicchi and C. Castellano, *Fundamental difference between superblockers and superspreaders in networks*, Phys. Rev. E **95**, 012318 (2017)
http://journals.aps.org/pre/abstract/10.1103/PhysRevE.95.012318
16. G. Bianconi and F. Radicchi, *Percolation in real multiplex networks*, Phys. Rev. E **94**, 060301(R) (2016).
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17. Y. Yang and F. Radicchi, *Observability transition in real networks,* Phys. Rev. E **94**, 030301(R) (2016).
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18. F. Radicchi and C. Castellano, *Leveraging percolation theory to identify influential spreaders in real networks,* Phys. Rev. E **93**, 030302(R) (2016).
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19. F. Radicchi and C. Castellano, *Beyond the locally tree-like approximation for percolation on real networks,* Phys. Rev. E **93**, 030302(R) (2016).
http://journals.aps.org/pre/abstract/10.1103/PhysRevE.93.030302

Manuscripts accepted/under review

20. F. Radicchi, D. Krioukov, H. Hartle and G. Bianconi, *Classical Information Theory of Networks*, accepted for publication in J. Physics Complexity (2020).
https://arxiv.org/abs/1908.03811
21. W. Gu, Y.Y. Ahn and F. Radicchi, *Defining and identifying the optimal embedding dimension of networks* (2020).
https://arxiv.org/abs/2004.09928
22. F. Radicchi, C. Castellano, A. Flammini, M.A. Munoz, D. Notarmuzi, *Classes of critical avalanche dynamics in complex networks* (2020).
https://arxiv.org/abs/2002.11831

Presentations

- Contributed talk, CCS 2019, Singapore, Singapore, September 2019.
- Contributed talk, NetSci 2019, Burlington, Vermont, May 2019.
- Invited talk, SIAM DS, Snow Bird, Utah, April 2019.

- Invited talk, DPG Spring Meeting, Regensburg, Germany, April 2019.
- Invited talk, Workshop "International Workshop on Theoretical perspectives in network science," Seoul, South Korea, December 2018.
- Invited talk, Workshop "DOOCN-XI: Dynamics On and Of Complex Networks," CCS 2018, Thessaloniki, Greece, September 2018.
- Contributed talk, CCS 2018, Thessaloniki, Greece, September 2018.
- Contributed talk, NetSci 2018, Paris, France, June 2018.
- Invited talk, NetSciX 2018, Hangzhou, China, January 2018.
- Invited lecture, CCS 2018, Cancun, Mexico, September 2018.
- Invited talk, Workshop "Social influence in networks," NetSci 2017, Indianapolis, USA, June 2017.
- Contributed talk, CCS 2016, Amsterdam, Netherland, September 2016.
- Contributed talk, Netsci 2016, Seoul, Korea, June 2016.

Synergistic activities

- Satellite chair, NetSci 2017, Indianapolis, USA, June 2017.
- Program committee member of various conferences, including NetSci 2016, WWW16, WWW17, NetSci 2018, NetSci 2019, and NetSciX 2018.

Collaborations and Leveraged Funding

Collaborators

- Claudio Castellano, University of Rome "La Sapienza", Italy
- Ginestra Bianconi, Queen Mary University, London, UK
- Francesco Coghi, Queen Mary University, London, UK
- Olaf Sporns, Indiana University, USA
- Krishna Bathina, Indiana University, USA
- Xiaoran Yan, Indiana University, USA
- Lucas Jeub, Indiana University, USA
- Santo Fortunato, Indiana University, USA
- Alessandro Flammini, Indiana University, USA

- Yang Yang, Northwestern University, USA
- Saeed Osat, Skolkovo Institute of Science and Technology, Moscow, Russia
- Ali Faqeeh, Indiana University, USA
- Sirag Erkol, Indiana University, USA
- Daniele Notarmuzi, Indiana University, USA
- Dario Mazzilli, Indiana University, USA
- Sol Lim, Indiana University, USA
- Miguel Angel Munoz, University of Granada, Spain
- James Gleeson, University of Limerick, Ireland
- Fragkiskos Papadopolous, University of Cyprus, Cyprus
- Dimitry Krioukov, Northeastern University, USA
- Harrison Hartle, Northeastern University, USA

Training opportunities

The project funded by the Army Research Office (ARO) provided partial support to the PI (1 summer salary). It further provided full support to the research activity of one postdoctoral researcher. In the course of the project, we supported two postdocs: Ali Faqeeh and Dario Mazzilli. Ali started his appointment in November 2016, and worked on the project till April 2018. He left the position for a prestigious 5-years fellowship that he received from the Science Foundation Ireland based at the University of Limerick. Although he worked only for a bit more than 1 year in this project, his performance can be considered good. He was able to publish in top journals in our field, and secure a long-term academic position. Dario started his appointment in April 2019. He has been funded by the ARO project till April 2020. Currently, he is still at Indiana University supported by other funds. His performance can be considered satisfactory. Some of the research that he performed while working on the ARO project is near to completion.

Related projects

The ARO project was perfectly integrated with another project funded by the National Science Foundation (NSF) titled “CAREER: Network Theory of Critical Interdependent Infrastructures” (Grant No.: CMMI-1552487). The two projects shared the same line of research. The projects were, however, complementary in terms of funded activities, and they differed much in terms of priorities. The ARO project focused mainly on theoretical aspects of the research. The NSF project gave instead high emphasis to the integration between research and education, focusing also on the development of undergraduate and graduate courses. Also, the NSF project provided 1 summer salary to the PI, and supported two graduate students.

Conclusions

Our research had the ambitious goal to build the fundamentals of a theory for real interdependent networks, and perform a systematic empirical validation of such a theory. This is a very general and broad research topic with numerous contexts of application. The research conducted during the ARO project was successful in many respects. We achieved many of the planned goals and opened novel research directions. The research activity has produced a good number of high-quality scientific publications, appeared mostly in physics and multidisciplinary journals. Research outcomes from the project have been presented in the top scientific conferences of network and complexity science.

The ARO project was successful also in the training of young researchers. One the postdoctoral researchers supported by the ARO project was able to move forward in his career securing a long-term position in an European university. The synergy of the ARO research activities with those supported by another project funded by the NSF was very helpful as it created the opportunity to have a critical mass of young researchers working together on similar research objectives. We remark that the ARO project focused entirely on research and supported postdoctoral researchers; the NSF project was instead dedicated to the integration between research and education, and supported graduate students.

Technology Transfer

The project supported basic research activities. Some of the algorithms deployed in the project may have potential for future technology transfer.

Future Plans

We have two main research plans for future research that followed up directly from the research activity supported by the ARO project.

First, while working on the project, we realized that there is a urgent need for a better understanding of optimization problems on networks. We identified large gaps in the state-of-the-art knowledge regarding similarity/differences across optimization problems. Further, we believe that better optimization techniques should be developed for several outstanding optimization problems on networks. Optimization problems that we want to study regard both structural and dynamical, either stochastic and deterministic, processes on networks. Specifically, we plan to address the practical, algorithmic and theoretical aspects of several optimization problems on networks, focusing on (i) the generalization of the problem settings to realistic scenarios, (ii) the development of numerical techniques for the solution of the optimization problems, and (iii) the establishment of analytical baselines for the objective assessment of the performance of the optimization algorithms. Given the ubiquity of networks in the real world and the generality of the type of optimization problems that we want to consider, we believe that developments of this research program may be useful in a multitude of real applications.

Second, we want to expand our understanding of physical properties of multi-layer networks via their geometric embedding. We believe that efforts in this direction are still in their infancy. However, several of the results from our ARO project indicate that this a very promising line of research. Specifically, we want to test whether embedding strategies provide better solutions to critical problems in network science, including (i) the identification of the features that make networks robust against failures and malicious attacks, (ii) the definition of centrality scores of nodes of social networks that are predictive of their role as influencers, (iii) the design of effective protocols to search and navigate networks. PI Radicchi submitted a research proposal on this topic to the ARO. The proposal is currently under consideration.