

Network Realizable Controllers for Distributed Computational Systems

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Final Technical Report

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Optimal distributed controller design methods with network realizable controller.

One of the main objectives of this project was to develop networked control design methodologies that produce optimal network controllers which are guaranteed to be implementable over the given network and to provide the network implementation. The available design methods, based on optimal control, can be adapted to find efficiently optimal structured controllers, if the system and the desired structure satisfied the Quadratic Invariance (QI) property. In [1] we have shown that a large class of networked systems satisfies the QI when the controller is distributed on the same network of the plant. Such networked systems need be interconnected on a "strictly causal network", where neighbor nodes interact with at least one delay (accounting for non-instantaneous communication). Once the structured controller is obtained, it was not understood how such controller could be built/realized on the network in a way that did not introduce internal instability. The methodology presented in [1] completely solve the problem in the special case where an initial stabilizing networked controller could be found. With such initial controller could find an optimal networked implementable one. Optimal network implementable controllers can then be found for any stable networked plant, as the zero initial controller works.

In subsequent work [2-7] we have solved the general problem where the networked plant is unstable. We have developed two new methodologies. The first, [2,3] in collaboration with Prof. Naghnaeian and Prof. Voulgaris, is based on operator theory and derives all the networked implementable controllers based on a novel separation of structured dynamic state feedback and structured dynamic state estimation. Each part of the controller is networked implementable and thus the overall controller is too.

The second method, [6], is based on a new derivation of the parametrization of all stabilizing controllers. One important advantage of the derivation is that it is simple and far easier to teach and learn, as it does not require explicit doubly coprime factorization of the plant transfer function matrix. In addition, it resolves the problem of finding an initial stabilizing when the controller is QI structured. In fact, the approach directly searches over affine closed loop maps

with an extra algebraic constraint that are equivalent to closed loop stability. The stability constraint can be imposed through an equivalent norm minimization constraint, leading to a design methodology that directly provides the controller building blocks for network realization. Moreover, in [6] we have derived a realization procedure for a structured system in transfer function matrix form and not necessarily an optimal controller.

Some open problems remain. While the results provide an implementable controller, the order of the network implantation can be large. This is due mainly to two factors. We do not yet know how to obtain a minimal network realization, and typically optimal control methods tend to provide controllers of order similar to the networked plant, which when implemented on the network tend to be quite large. Finally, although the controller is network distributed, its design is centralized. The design scalability can become an issue for large networked systems. These problems are mostly left for future research. To start addressing some of the above issues, we began investigating two different and opposite cases discussed next.

Decentralized solutions that are Socially Optimal

In collaboration with Prof. Voulgaris, we consider multi-agent problems, where the agents are not necessarily connected over a given network, but they need to cooperate to minimize their own deviations from the collective. This problem would in general require either that each agent measures all the others, or that there is a collector in charge of measuring the mass average and reporting to all the agents. Another approach is based on stochastic Mean Field Theory (MFT). Each agent predicts the mass behavior and to behave consistently so that the actual mass behavior is consistent with the predicted one. This approach leads to a decentralized control strategy, however the calculation of the MFT controller is not trivial and requires the solutions of certain partial differential equations. We consider infinite horizon problems with different norm costs, e.g. H₂, H₀₀, l₁, measuring the deviation from average behavior [7-10]. It turns out that the optimal controllers are completely decentralized and do not explicitly care about tracking the average. The results are slightly different for different norms, and details are in the papers. The more interesting point is that these classes of minimum norm problems are naturally cooperating and do not require explicit information exchange among agents or controllers, moreover the controller design problem is decentralized too. We see that there are classes of problems where the agents can cooperate without coordination or awareness of being part of a collective. In these cases, the optimal controllers are decentralized and can be designed and realized in a decentralized way. These results may suggest that cost functions and network structures may need to be matched somehow, in a way that the cost function should not require high order controllers so to be achieved. While in search of general results we have focused on heterogeneous agents and arbitrary networks, we may need to restrict our attention to similar agents and symmetric topologies, to discover scalable solutions.

Design of new distributed optimization algorithms using control ideas

To this end, we have considered problems on the other end of the spectrum. These are distributed optimization systems, where the agents are quite simple (often simple integrator dynamics) and they need to cooperate to collectively solve a convex optimization problem. It turns out that natural dynamic interactions emerge with the dual variable being the distributed

controller states which steer the together with the local gradients each agent to the optimal solution. In this case, the distributed controllers are simple, low order, and naturally or easily implementable over the network. We do no longer know however, in what sense they are optimal, although they stabilize the networked system and contribute to optimize the cost.

These types of problems are interesting and useful in their own right and have generated a lot of interest in different areas, from cooperative robotics, to power systems, to AI training algorithms. We have developed some fast-distributed solver of system of equations based on passivity theory [11-13]. Since passivity and convexity are tightly connected, and passivity is connected to Positive Real transfer functions. It turns out that positive real controller/systems are quite constrained, not just in terms of relative order. It appears there is not much benefit from high order controllers in these settings.

We have further derived dynamical systems which solve certain class of Robust Optimization (RO) problems [13-15]. These systems can solve RO problems that are not easily solved by existing methods and can be also distributed. In these problems we see not just the controller structure of the minimizing agents, but also the controller structure of the attacking agents that want to make the constraints infeasible and the cost infinity. Again, both minimizers and maximizer strategies are not very complex. This is another indication that large order controllers could be due to mismatched problem formulations.

However, a key simplification of optimization systems is due the simpler objective of these networked systems, which is an asymptotic minimization of a final objective rather than classical performance objective in control applications. Distributed optimization systems provide a significant yet simpler to analyze and understand class of distributed systems. Following this line of thoughts, we have revisited classical distributed gradient systems with the scope of making them asynchronous. Using fixed point theorems for non-expansive maps, we derived new distributed algorithms where the agents cooperatively minimize the sum of their private convex costs in a completely asynchronous fashion [16-20]. While most results require a bounded time within which connectivity is guaranteed (a centralized assumption) our results do not require such condition in order to prove their convergence. Even in the presence of asynchronous communications and update the control actions are relatively simple.

Summary:

In this project we have solved important classes of distributed control problem with heterogenous agents and general network configurations. The complexity of the optimal controllers when implemented over the network tends to be large. This may be an indication that further network model reduction needs to be performed (an open problem) or that the nature of the cost and the networked structure are not "well matched". To start understanding this issue, we looked at two extreme cases. The first is about cooperative norm minimization problems, which do not require any communication network to be solved and lead to completely decentralized controllers. The second class of problems we studied is that of distributed convex optimization systems. In this case the distributed controllers tend to be simple albeit connected, but the cost is simpler than typical performance measure for control systems. We were able to obtain significant results, 1) resolved the network realizability problem, 2) provided new

methodologies for networked control design. 3) Identified setting where selfish agents are socially optimal. 4) Provided new algorithms for distributed optimization based on control systems, including fast distributed solvers of systems of linear equations, asynchronous distributed gradient systems, and a continuous-time system that solves robust distributed optimization problems. These results show that the field of networked distributed systems is very fertile and point to relevant directions of future research.

Personnel

Nicola Elia, Professor, Principal Investigator

Gulnihal Kucuksayacigil, Ph.D 2018, "Optimal Network Implementable Controllers for networked Systems".

Sayyed Shaho Alaviani, Ph.D 2019, "Application of Fixed Point Theory to Distributed Optimization, Robust Convex Optimization, and Stability of Stochastic Systems". (Currently Postdoctoral fellow Clemson University).

Ian McInerney, MS. 2017, "Development of a multi-agent quadrotor research platform with distributed computational capabilities. (currently Ph.D. candidate Imperial College UK) Abhishek Rawat, Ph.d. Student, (UMN).

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- 2. M. Naghnaeian, P. G. Voulgaris, N. Elia, "A unified framework for decentralized control synthesis", 2018 European Control Conference (ECC), pp. 2482-2487, 2018.
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- 9. P. G. Voulgaris and N. Elia, "Social optimization problems with decentralized and selfish optimal strategies", in Proc. IEEE 56th Conference on Decision and Control 2017.
- 10. P. G. Voulgaris and N. Elia, "When Selfish is Socially Optimal" Accepted in IEEE Transaction on Automatic Control 2020.
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- 15. K. Ebrahimi, N. Elia and U. G. Vaidya "A continuous time dynamical system approach for solving robust optimization", 2019 18th IEEE European Control Conference (ECC).
- 16. S. Sh. Alaviani and N. Elia, "A Distributed Algorithm for Solving Linear Algebraic Equations Over Random Networks," IEEE Transactions on Automatic Control (accepted).
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