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Ultra High Speed Communications System with Finite Rate of Innovation

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This paper presents a novel realization of a real-world ultra high speed communications system using FRI. This system consists of existing components and promises greater data throughput using lower bandwidth signals than conventional systems; therefore, FRI communications systems would provide better performance with lower SWAP and cost.

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Contents

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Ultra High Speed Communications System with Finite Rate of Innovation

Introduction

The modern world possesses a nearly insatiable thirst for data. Many user applications across defense, consumer, commercial, government, and academic sectors have demanding requirements for high data throughput and low latency. The burden of data transport falls on communications systems which in turn are constrained by regulation (spectrum management and human safety), engineering (size, weight, power and cost) and by physical realities (e.g. the finite speed of light). The maximum rate of data potentially transmitted across a channel depends on both the bandwidth and the signal to noise ratio (SNR) [1]. Hence for a fixed SNR, increasing the data throughput of a communications system involves increasing the bandwidth (number of degrees of freedom per unit time) of the information encoded onto the signal which has several consequences: higher performance transmitter modulators, wider bandwidth receiver components, faster digital to analog converters and reduced sensitivity, to name a few. Objectively speaking, an ideal solution for meeting high data rate requirements without increasing SNR is to increase the information content of a signal without increasing its bandwidth, a seemingly quixotic endeavor. This paper proposes a novel method for realizing this solution in practice.

Background

Band-limited Signals

Conventional communications signals are bandlimited, that is to say the baseband representation of a signal $x(t)$ with Fourier transform $X(\omega)$ obeys the following

$$X(\omega) = 0, |\omega| > \omega_m$$

The number of degrees of freedom per unit time of a bandlimited signal is equivalent to the bandwidth B [2]

$$B = \frac{\omega_m}{\pi}$$

In practice, communications systems obtain a discrete set of samples of a signal with an objective of minimizing the number of samples necessary to represent a signal. It is readily apparent that the high bandwidth signals required by ultra high speed communications require a greater number of samples than their lower speed counterparts as they contain more degrees of freedom per unit time.

Another impact of high bandwidths is on the receiver subsystem. High bandwidths reduce the sensitivity of receivers hence requiring greater link margin. For illustration, the equation below describes the sensitivity of a room temperature RF system as a function of bandwidth, BW, and noise figure, NF.

$$S_{RF} = -174 + 10\log_{10} BW + NF$$

Above, S_{RF} is in dBm with lower values representing better sensitivity.

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Pulse Position Modulation (PPM)

PPM is a widely used modulation scheme in communications systems. PPM encodes information using the position of a pulse in the time domain. [3] provides a review of PPM. The equation below illustrates PPM algebraically as a series of dirac delta functions

$$x(t) = \sum_{n \in Z} \delta(t - t_n)$$

Above t_n is the displacement of the dirac delta function in the time domain.

Finite Rate of Innovation (FRI)

In [4], the concept of Finite Rate of Innovation (FRI) was introduced. FRI signals differ from conventional communications systems as they are not bandlimited. Instead they posses a quality known as innovations; it is this quality that is finite per unit time. The equation below shows an illustrative form of FRI signals

$$x(t) = \sum_{n \in Z} \sum_{r=0}^{R} c_{nr} \delta\left(\frac{t - t_n}{T}\right)$$

Above c_{nr} are scalar coefficients, $\delta(t)$ is the dirac delta function and t_n are time instants. The degrees of freedom present in the signal are c_{nr} and t_n. Note that while $\delta(t)$ is highly localized in time, it is not bandlimited. Define a function $C_x(\tau_a, \tau_b)$ which counts the degrees of freedom on an interval from τ_a to τ_b. The rate of innovation of a signal is defined as

$$\rho = \lim_{\tau \to \infty} \frac{1}{\tau} C_x\left(-\frac{\tau}{2}, \frac{\tau}{2}\right)$$

As shown in [4], only ρ measurements per unit time are necessary to fully represent a signal with a finite rate of innovation. The implications of this statement are that relative to bandlimited signals, *FRI signals contain more information per measurement*; FRI signals are an attractive candidate

What has prevented the realization of real world FRI communications systems? The challenge behind measuring FRI signals is that they require non-bandlimited sampling kernels [4]; non-bandlimited sampling kernels are not practical to implement in real-world digital receivers. The figure below shows a representative sampling schema.

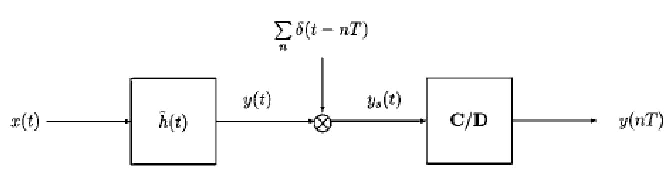

Fig. 1. Sampling setup: $x(t)$ is the continuous-time signal; $\tilde{h}(t) = h(-t)$ is the smoothing kernel; $y(t)$ is the filtered signal; T is the sampling interval; $y_s(t)$ is the sampled version of $y(t)$; and $y(nT)$, $n \in \mathbb{Z}$ are the sample values. The box C/D stands for continuous-to-discrete transformation and corresponds to reading out the sample values $y(nT)$ from $y_s(t)$.

Figure 1 - FRI Block Diagram from [4]

Method

This report now presents a novel method for implementing FRI signals in a real-world communications system. This section presents methods for both RF and Optical communications systems.

RF Implementation

The novel idea presented here to realize a real-world RF RFI communications system is modulation of the RF signal with either a sinc function kernel or a gaussian kernel. The equations below show archetypes for the sinc kernel and the gaussian kernel respectively.

$$\phi(t) = sinc\left(\frac{t}{T}\right)$$

$$\phi(t) = exp\left(\frac{-t^2}{2\sigma^2}\right)$$

The FRI modulator realizes the sinc kernel through amplitude and phase modulation. The FRI modulator realizes the gaussian kernel through amplitude modulation.

The remainder of the RF RFI system utilizes standard RF components. Recovery of the information is performed through standard methods, for example the annihilator method [4] or noisy spectral estimation techniques [5].

The figure below shows a block diagram of a RF RFI communications system. It highlights the novel FRI modulator in red.

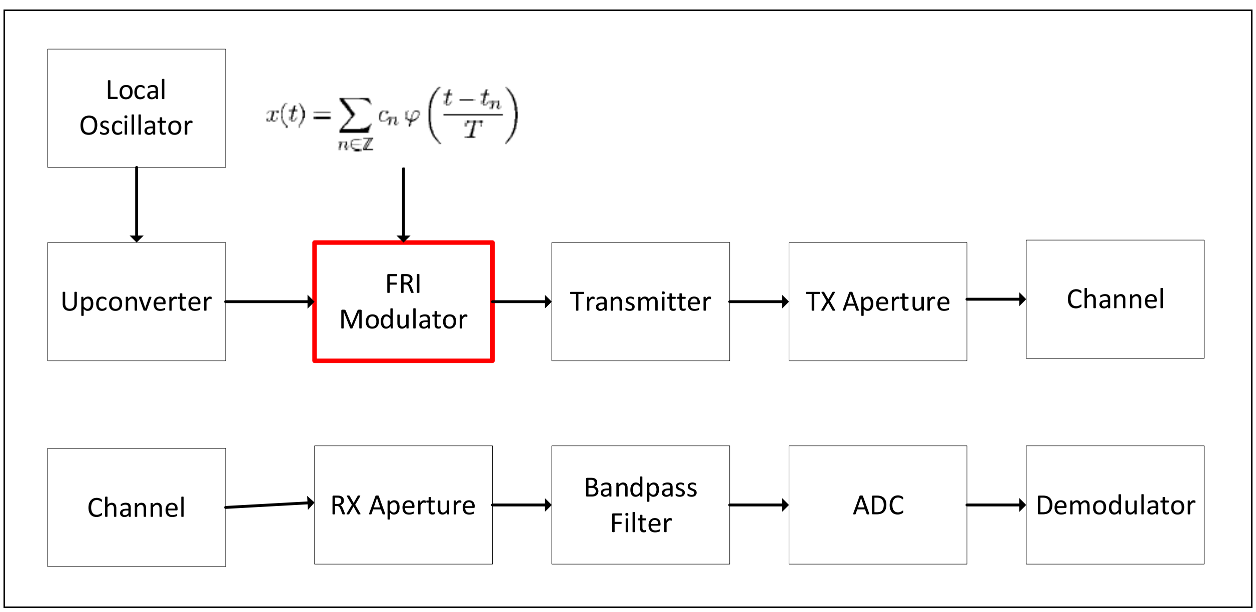

Figure 2 - RF FRI Communications System Block Diagram

The table below identifies and describes the individual components.

Component	Description
Local Oscillator	Reference clock source
Upconverter	Provides RF reference derived from clock source
FRI Modulator	Applies FRI kernel to PPM modulation, encodes information onto RF signal
Transmitter	Amplifies RF signal
TX Aperture	Couples transmitter to channel (antenna for OTA applications)
Channel	Media between TX and RX subsystems (atmosphere for OTA applications)
RX Aperture	Couples channel to receiver (antenna for OTA applications)
Bandpass Filter	Bandwidth limiting and anti-aliasing filter
ADC	Analog to digital conversion
Demodulator	Recovers information from signal

Table 1 - RF FRI Component Descriptions

Optical Implementation

The novel idea presented here to realize a real-world Optical RFI communications system is spatial modulation of the Optical signal by slewing the positioning optics, such as fast steering mirror (FSM), of the transmitter. As the gain of an optical beam is gaussian spatially, the consequence of slewing the FSM is imposing a gaussian kernel onto the time domain at the receiver.

The remainder of the Optical RFI system utilizes standard optical components. Recovery of the information is performed through standard methods, for example the annihilator method [4] or noisy spectral estimation techniques [5].

The figure below shows a block diagram of an Optical RFI communications system. It highlights the novelty in red.

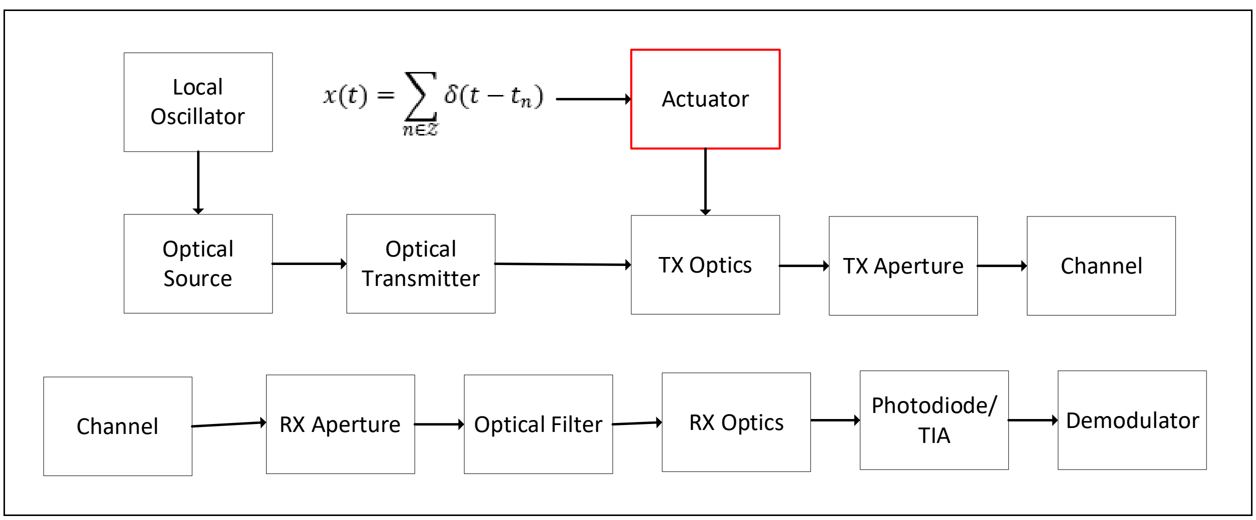

Figure 3 - Optical FRI Communications System Block Diagram

The table below identifies and describes the individual components.

Component	Description
Local Oscillator	Reference clock source
Optical Source	Generates CW tone at optical wavelength from clock source
Optical Transmitter	Amplifies optical source (commonly an EDFA)
TX Optics	Steers optical beam (commonly a FSM)
TX Aperture	Couples transmitter to channel (lenses for OTA applications)
Channel	Media between TX and RX subsystems (atmosphere for OTA applications)
RX Aperture	Couples channel to receiver (lenses for OTA applications)
Optical Filter	Spectral limiting filter
RX Optics	Steers optical beam (commonly a FSM)
Photodiode/TIA	Develops digital signal from optical signal (commonly APD and TIA)
Demodulator	Recovers information from signal

Table 2 - Optical FRI Component Descriptions

Summary

This paper presented a novel realization of a real-world ultra high speed communications system using FRI. This system consists of existing components and promises greater data throughput using lower bandwidth signals than conventional systems for a fixed SNR; therefore, FRI communications systems would provide better performance with lower SWAP and cost. Future work includes:

1. Modeling and simulation to determination nominal system performance
2. Paper studies on FRI kernels and optimization of their parameters
3. Development of a prototype system

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