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A Systematic Review of Corrections for Range Restriction: Five Things Consultants Should Know

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14. ABSTRACT
Data are often available only from a preselected-restricted sample in many applied research settings. Despite this, consultants and researchers must try to estimate population parameters for the unrestricted population to more accurately determine the relations among the scores. Although methods for addressing the effects of range restriction have been available for more than a century, often they are not applied, or sometimes applied incorrectly. Failure to correct causes erroneous conclusions to be drawn, erroneous beliefs to be held, and resources to be wasted. Five issues in the use of correction methods are discussed. These are the consequences of (1) failing to correct, (2) ignoring the effect of one or more selection variables, (3) using the wrong correction formula, (4) correcting to an inappropriate unrestricted sample, and (5) incorrectly estimating the standard error of the corrected correlation. The purpose of this paper is to enhance understanding of the consequences of these issues and propose best practices in applied psychology.

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1.0 INTRODUCTION

The role of the consultant is to employ proper techniques so that the client can make appropriate decisions. In practice, correlational studies frequently are conducted with samples of those already selected for training or with job incumbents. Although we want the expected value of our estimated correlations to equal the population parameters, in these studies the expected value of the correlations do not necessarily estimate the parameter. The use of selected samples can result in substantial change in the apparent magnitude and sign of the correlations (Ree, Carretta, Earles, & Albert, 1994; Thorndike, 1949). These changes are the consequences of the methods of sample selection. The selection methods reduce variance to create differences between the standard deviations of the unrestricted normative or applicant populations and the standard deviations of prior-selected samples used in studies. The bias created by using selected samples can cause the estimated correlation to be either higher (range enhancement) or lower (range restriction) than if the correlation were estimated in an unrestricted group (Johnson, Deary, & Bouchard, 2018, Levin, 1972). Methods to correct correlations for range restriction have been available for more than a century (Aitken, 1934; Lawley, 1943; Pearson, 1903; Thorndike, 1949). An informal expository review of a sample[1] of journals was done to determine the frequency with which range correction procedures were applied when applicable. The selection of journals was based on diversity of content in the area of applied psychology. All journals selected were frequently cited in the applied literature and should be familiar to applied psychologists. In the sample several applied journals showed that corrections for range restriction were used about 28 percent (%) of the time when sufficient information was available. This is consistent with Dahlke and Wiernik (2019a) who noted that the application of corrections for range restriction is common in some areas of educational, organizational, and psychological research, but relatively uncommon in other areas, though they could be highly relevant.

1.1 Early Range Restriction Correction Methods

The best known methods of range restriction correction are the Pearson-Thorndike (Pearson, 1903; Thorndike, 1949) Cases 1, 2, and 3. Cases 1 and 2 are appropriate for direct selection, and Case 3 for indirect selection. See Table 1 for appropriate and inappropriate uses of the correction formulas and the Appendix for the formulas and how to use them. The Case 1, 2, and 3 equations can be computed by hand, programmed in MasthCad, Excel, or the programming language R (Dahlke & weirnikm 2018, 2019b) otr using SPSS syntax,.

[1] This sample is described as a purposive Typical Case Sampling. The goal of the sampling was to focus on particular characteristics that are of interest that will answer a research question. The characteristics were (1) was a correction for range restriction used and (2) was the study published in an applied psychology journal.

Table 1. *Procedures for Range Restriction Correction*

Procedure	References	Intended Use Correction for	Appropriate Situations	Inappropriate Situations
Case 1	Pearson (1903) Thorndike (1949)	Direct truncation due to selection on one variable	Used to correct the correlation between 2 variables, a and *b*, when range restTariction occurs on variable *b*, the observed correlation between *a* and *b* is known, and the standard deviation of *b* is known in the observed sample and population	Indirect selection occurred; multivariate selection, non-linear relationships
Case 2	Pearson (1903) Thorndike (1949)	Direct truncation due to selection on one variable	Estimates the correlation between 2 variables of interest, *a* and *y,* when the variances for variable *a* are known in the restricted sample and population, but the correlation between *a* and *y* is available only in the restricted sample that has been directly selected on *a*.	Indirect selection occurred; multivariate selection, non-linear relationships
Case 3	Pearson (1903) Thorndike (1949)	Indirect restriction on variable *b* is produced by direct restriction on a third known variable (*a*)	Assumes direct top-down selection has been based entirely on *a* and has not been affected by any other information and r_{xz}, r_{ya}, r_{xy}, and *Sa* are known	Direct selection, violation of assumptions
Case IV	Hunter & Schmidt (2004) Le et al. (2016) Schmidt, Oh, & Le (2006)	Indirect selection	Used as an alternative to Case 3 (relaxes conditions (a) and (b) required for Case 3).	Multivariate selection, non-linear relationships
Case V	Dahlke, & Wiernik (2018) Le et al. (2016)	Indirect selection	Used for correction for indirect range restriction between variables X and Y without knowledge about selection on a third variable, Z.	multivariate selection, non-linear relationships, when correlations are not substantially different than zero. In that case use Case 3.
Multivariate	Aitken (1934) Lawley (1943)	Selection on multiple variables	Selection on two or more variables; correction for both direct and indirect selection; know the inter-correlations of the independent variables in both the restricted sample and unrestricted population	Insufficient information available, non-linear relationships

The multivariate case can be computed by any program that supports matrix algebra. R also includes programs (e.g., Psychmeta, Lmvrrc) for the multivariate correction. Further Alliger and Alexander (1984) reported code for the multivariate correction in SAS and Fortran IV. Direct selection (Cases 1 and 2) occurs when scores are only available for people who were selected or when measures are used for selection before a study. Both Case 1 and Case 2 correct for direct selection and yield very similar results. Case 1 is rarely found in the literature. An example of Case 2 is when a university wants to know the correlation between a test of academic achievement, used to select students, and first semester grades. They have both test scores directly used for selection and grades for those selected for admission and enrolled, but not for those rejected. The observed correlation of the test scores and grades was r =.27. After computing the Case 2 correction for range restriction the more accurate statistical estimate was r = .50.

Indirect range restriction (Case 3) occures when an experimental measure is administered, but not used for selection. Applicants are selected on scores from measures already being used. An example of indirect selection is when a new but unused measure is being considered for inclusion in the selection process. For example, the sample consists of those selected on anoperational measure already in use. The correlation of the new measure and the criterion is .30, while the correlation of the operational measure with the criterion is .21 due to direct range restriction. After correction for indirect range restriction the correlation of the new measure is r = .35 and the correlation for the operational measure is .40.

When more than one variable is used for selection a multivariate solution (Lawley, 1943) is available. This multivariate solution corrects simultaneously for direct and indirect selection. See the Appendix for a detailed explanation of the multivariate correction procedure.

1.2 Beyond Earlier Range Restriction Correction Methods

Sackett and Yang (2000) contend there is value in going beyond categorizing range restruiction as direct (Thorndike Cases 1 and 2) or indirect (Thorndike Case 3). Their focus was on sample selection procedures that result in a selected sample that is not representative of the population of interest. Sackett and Yang described 11 different range restriction scenarios that take into account (1) the variable(s) on which range restriction occurred (x, y, and/or a 3rd variable z), (2) whether unrestricted variances are available for the relevant variables, and (3) if a 3rd variable is involved, whether or not it is measured or unmeasured. Approaches for some of these scenarios have been developed and have been shown to provide more accurtate estimates than the more traditional Case 2 and Case 3 formulas (e,g., Lee, Oh, Schmidt, & Wooldridge, 2016).

1.3 Additional Methods

Other correction methods that show great promise have been proposed and examined in the last several years (e.g., Le, Oh, Schmidt, & Wooldridge 2016; Olson, & Becker, 1983; Pfaffel, Schober, & Spiel, 2016; Sackett & Yang, 2000). These correction methods have the potential to provide better estimates of corrected correlations that can lead to better decision making and more precise knowledge of the relationships among constructs. Although some of these methods have been devoloped for use in meta-analysis they are also appropriate for individual studies.

Hunter, Schmidt, and Le, (2006) presented another method (Case IV) to correct for indirect range restriction as an alternative to Case 3. Consider a situation with a predictor, X, a criterion, Y, and a third variable, S, on which the selection was actually made. Unlike other methods, Case IV assumes that the effect of the third variable (S) was on the true score of variable *Y* and that the true score relationship between X and Y is fully mediated. Le and Schmidt (2006) demonstrated by simulations that Case IV[2] always yields substantially more accurate estimates than the frequently misapplied Case 2[3], even when the full mediation assumption is violated. However, there remain concerns about the degree of bias when the mediation assumption is false. Despite the improved accuracy of the Case IV method, there are concerns when the full mediation assumption cannot be made (Beatty, Barratt, Berry, & Sackett, 2014; Fife, Mendoza, & Terry, 2013). Case IV always provides more accurate, less biased estimates than Case 2. Estimates from Case IV can differ from the population parameters to the degree of violation of the full mediation assumption. From these concerns comes Case V.

Based on an expansion of an equation from Bryant and Gokhale (1972) Case V allows correction for indirect range restriction when the mediation assumption may be false. Case V was developed to be applicable to meta-analysis as it incorporates estimates of true scores (Le et al., 2016). In a simulation, Case V was found to be more accurate than Case 2 and Case IV when full mediation did not exist. Le et al. (2016) identified two reasons for the underuse of the Case V correction. First, researchers often mistakenly apply the Case 2 (direct) correction when indirect selection has occurred. Second, the Case V correction equation requires knowledge of range restriction on variable Y $(e.g. \frac{sd_Y}{SDy})$. This information may not be available in some situations such as when Y is job performance measured in a restricted sample and standard deviation (SD_y) is unknown.

If the goal is to estimate corrected observed score correlation as in a validation study the Bryant and Gokhale equation can be applied. As with Cases 1, 2, and 3, Case IV and Case V corrections can be computed by hand or by several software packages.

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[2] Hunter and associates have chosen to use Roman numerals for Case IV and Case V.

[3] Hunter et al. (2006) noted that Case 2 (direct correction) is frequently applied in meta-analyses when Case 3 (indirect correction) is appropriate.

1.4 Range Restriction and Reliability

Reliability is a measure dependent on variance (Gulliksen, 1950, 1987). Range restriction changes variances and produces changes in reliability. For example, suppose the reliability of a test is reported in the test manual as .8. Because range restriction has changed variances it falls to .6. Gulliksen (1950, 1987, p. 124, Eq. 5) provided the following formula to correct the reliability for changes in variance.

$$Rxx = 1 - (vx/VX)(1 - rxx) \tag{1}$$

where *Rxx* is the corrected reliability, *vx* is the variance in the selected group, *VX* is the variance in the total group and *rxx* is the reliability in the range restricted group. Table 2 offers suggestions for situations where reliability estimates are unavaliable.

Although there have been advances in understanding corrections for range restriction and new models proposed, there is no current review. This paper provides a review and raises awareness of the advances in theory and practice, consequences of not correcting for range restriction, ignoring variables used in selection, using improper procedures, correcting to an inappropriate unrestricted sample, and incorrectly estimating the standard error of the corrected correlation.

1.5 Real-World Validation

The method for predictive validation seems straightforward. One approach is to find an existing test thought to measure the appropriate construct. Alternatively, a new construct could be developed; a measure of it created and then administered to a sample of applicants, trainees, or job incumbents. Correlational analyses are conducted to determine if the test of the existing construct or new construct is statistically significant or has incremented the variance accounted for in the criterion. The presence of range restriction can make the results of these studies very misleading.

1.6 Importance of Correcting for Range Restriction/Enhancement

Recognizing and correcting for range restriction/enhancement is especially important if the goal is to estimate the relationships between scores or between constructs, for example in regression or factor analysis. Often only selected samples, such as accepted students or hired applicants are available. Correlations estimated in selected samples will give a biased value.

Sackett, Lievens, Berry, and Landers (2007) offered a caution on interpreting correlations of predictors in range-restricted samples. They demonstrated that the correlations among predictors in range-restricted samples "...can differ dramatically from population values…" (p. 538). Biased correlations will be found in statistical procedures based on range restricted samples.

The consequence of failing to correct for range restriction is incorrectly estimated correlations that will not represent those in the normative or applicant populations. Failing to

Table 2. ***Techniques to use when reliability information is fully or only partially unavailable***.

Reliability Missing Data Situation	Technique	Comments
Presented only in study	Estimate based on reliability in test manual	Use formula presented by Gulliksen (1950, 1987)†
Reliability not available in study or test manual	Use R^2 commonality measure from factor analysis Use reliability value from a large representative study	Commonality provides a lower bound estimate of reliability that may conservative See Rummel (1988) and Lord and Novick (1968).
Available for some studies, but not for all studies	Use sample weighted mean or median estimate Use pooled value based on aggregating reliability and variance from all samples Use multiple imputation based on available studies	Use median if distribution of reliabilities substantially deviates from normality All these techniques requires sample size from available studies
Not available in study, but variance available in study	Estimate based on reliability in test manual but using variance from study	Use formula presented by Gulliksen (1950, 1987)

† See Gulliksen (1950, 1987) page 124, equation 5 and Freeware is available from pbarrett.net.

correct for range restriction can, and will, lead to inappropriate conclusions (i.e., over or underestimating the relations between scores) and actions (Alexander, Bennett, Alliger, & Carson, 1986; Hunter, Schmidt, & Le, 2006; Linn, 1968; Ree et al., 1994; Sackett & Yang, 2000).

1.6.1. Range restriction or range enhancement?

It should be noted that prior selection will not always decrease the magnitude of the correlations. Under unusual circumstances when there is “range enhancement” meaning increased variance in either the independent or the dependent variable due to selection, increased correlations would be observed (Levin, 1972). Also, Zimmerman and Williams (2000) noted that distributions with extreme outliers can cause correlations to be downwardly biased or under some situations increase the variance of the variables and produce increased correlations. They provided a statistical method to correct correlations in these situations. Some studies compare extreme groups, eliminating the middle of the distribution. This may cause “range enhancement” leading to overestimation of population parameters. How well do correction formulas do under this circumstance? Preacher, Rucker, MacCallum, and Nicewander (2005) referencing Pearson (1903) and Wherry (1984) observed that the range restriction correction formulas can be used to correct for overestimation when only the extreme ranges are used. However, Preacher et al. stated that they have not found any examples of the correction being applied to correlational analyses of extreme groups.

2.0 Five Issues in Correcting Correlations for Restriction[4] of Range

Five issues in the use of correction for range restriction were identified. The first is failing to correct correlations from selected samples (Thorndike, 1949). The second is ignoring the effect of one or more selection variables in range correction (Jackson & Ree, 1992; Olson & Becker, 1983; Ree et al., 1994). The third is using the wrong correction formula (Alexander et al., 1986; Hunter et al., 2006; Linn, 1968; Ree et al., 1994; Sackett & Yang, 2000; Stauffer & Pai, 2016). The fourth issue is correcting to an inappropriate unrestricted sample (Dunbar & Linn, 1991). The fifth is incorrectly estimating the standard error of the corrected correlations. Each of these five issues may cause erroneous conclusions to be drawn.

2.1 Failure to Correct for Range Restriction

Although convincing evidence exists about the benefits of correcting for range restriction, corrections are used infrequently as shown by our review of several applied journals. Among other effects, not correcting for range restriction hinders the effort to find predictive validity and incremental validity (Alderton, Larson, & Held, 1995; Carretta & Ree, 2003; Hunter et al., 2006; Linn, 1968**)**. Frequently, applied studies suffer from the effects of uncorrected range restriction leading to inappropriate conclusions. Interpreting uncorrected correlation coefficients can lead to misunderstanding of the construct and its relationships (Ree et al., 1994; Sackett & Yang, 2000; Thorndike, 1949). The result is that conclusions are likely to be wrong. Not correcting for range restriction obstructs the effort to find predictive validity and incremental validity (Carretta & Ree, 2003; McHenry, Hough, Toquam, Hanson, & Ashworth, 1990; Wolfe, Alderton, Larson, & Held, 1995). Correcting for range restriction improves statistical accuracy and allows for an improved understanding of the relationships among variables and constructs. For example, Stauffer, Ree, and Carretta (1996) demonstrated that scores from cognitive components tests (i.e., working memory, processing speed, declarative knowledge, and procedural knowledge) correlated strongly with scores from existing cognitive ability tests and lacked unique variance beyond existing test scores. Although cognitive ability theory suggested that the cognitive components and traditional cognitive measures assessed different constructs, they both were indicators of general mental ability.

2.1.1. Critique of range restriction correction

Range restriction correction formulas have been known for more than a century (Lawley, 1943; Pearson, 1903; Thorndike, 1949) and their utility has been well documented. Nonetheless some question their utility when there is not complete truncation (i.e., complete truncation would be having no scores below a specified numeric value). Campbell (1976, p. 218) concluded about corrections for range restriction that "… the safest recourse is to not use them." Damos (1996) referred to range restriction as a "red herring" in explanation for low predictive validities observed in commercial and military pilot selection batteries. She argued that commercial air carriers are not going "to administer tests to a completely unrestricted population: some type of selection based on the candidate's background and experiences will occur before any testing is conducted" (p. 202). Damos concluded that the uncorrected correlations provide the most accurate estimates of the predictive validity of a test in most cases. She acknowledged that some

[4] Uncorrected correlations can show the effects of range restriction or range enhancement (Levin, 1972).

type of selection has occurred, but does not suggest correcting for that selection. Testing a random sample of the population is not advocated. The appropriate unrestricted sample for pilot selection is applicants who present themselves for testing.

The argument for the application of range-restriction-correction formulas is that they provide a more accurate estimate of correlations (Hunter & Schmidt, 2004; Linn, Harnish, & Dunbar, 1981; Martinussen, 1997; Ree et al., 1994). Linn et al. (1981) analyzed the results of more than 700 validity studies and concluded "Thus it seems desirable to routinely compute and report corrected correlations along with their uncorrected counterparts. Though still conservative, the corrected values will generally provide a better indication of predictive validity and be less misleading than uncorrected correlations alone" (p. 662).

Hunter and Schmidt (2004) concluded that in educational and employment selection, the predictive validity of cognitive ability has been underestimated considerably due to the failure to correct for range restriction. Held and Foley (1994) provided an empirical example. They calculated the predictive validity of aptitude scores while varying the selection ratio from 1.00 (unrestricted group) to .10 (highly restricted group). The predictive validity (r) of the aptitude scores steadily decreased as selection became more restrictive. In all instances the corrected values were closer to the unrestricted validities than were the uncorrected validities.

Thorndike (1949, pp. 170-171) provided a dramatic illustration of the detrimental effects of not correcting for range restriction in an applied setting. In 1944 during World War II, an experimental group of 1,036 United States (US) Army Air Corps pilot applicants was admitted to training without regard to their scores on five aptitude tests. Correlations were computed with the training criterion for all participants (unrestricted sample N = 1,036) and for those (restricted sample N = 136 out of 1,036) who would have been selected had the strict standards in effect at the end of World War II been used. The pilot selection composite derived from the five tests had a correlation of .64, r^2 = .41, with training outcome in the unrestricted sample. This correlation dropped to .18, r^2 = .03, in the range-restricted sample. The most dramatic change from the unrestricted to the range-restricted sample was for a psychomotor test where the correlation changed sign from +.40 (all participants) to -.03 ("qualified" participants only). The average decrease in the five validity coefficients in the range-restricted sample was .29. Incorrect decisions about the effectiveness of the tests would have been made if only the range-restricted correlations had been considered. It is likely that the psychomotor test would have been deemed ineffective and dropped from further consideration if only the restricted data had been available.

2.1.2. Changes in regression coefficients and reliability due to range restriction

Mendoza and Mumford (1987) noted that while direct range restriction has no effect on regression slopes, indirect restriction leads to a reduction in the regression slope. See also Hunter et al. (2006). If indirect range restriction has occurred, the equality of the regression slopes in the population and range-restricted sample would occur only if the independent variable were perfectly reliable.

Ree and Carretta (2006) noted that reduced variance due to range restriction causes the reliability of variables to decrease (Gulliksen, 1950, 1987). This reduction of reliability causes the regression coefficients not to estimate the population parameters but to estimate the population parameter times the reduced reliability. This reduction of regression coefficients due to restricted variance caused by reduced reliability can be especially problematic when comparisons among or between regression coefficients are being made across several groups. Carretta (1997) demonstrated that correcting regression coefficients for unreliability reduced differences in regression slopes for sex and racial/ethnic groups to a trivial amount. He found that uncorrected regression coefficients between groups were statistically significant. After correction, all the differences were .0004 or less.

Using uncorrected regression coefficients, Johnson and Zeidner (1991) suggested a linear programming (Dantzig, 1981) approach to optimally assign applicants to jobs. When regression coefficients were estimated in several range-restricted job-incumbent samples, the prior differential job requirements and differential selection causes the reliabilities of the test scores to vary across samples. Varying test score reliabilities across samples causes biases in the regression coefficients (Gulliksen, 1950, 1987, p. 124, Eq. 5). Separately correcting each of the job incumbent samples for range restriction, which restores unattenuated reliability to a common applicant population, corrects this problem. The order in which to apply the corrections is dependent on the situation that caused the range restriction. Stauffer and Mendoza (2001) offer informative guidance on this topic.

2.1.3. Failure to correct for range restriction when estimating incremental validity

As illustrated by Dunbar and Linn (1991), the predictive validity (correlation) of a selection test, cannot be estimated accurately unless the effects of range restriction are taken into account. Dunbar and Linn illustrated the effect of different amounts of range restriction due to direct selection or indirect selection on the correlations of two measures with a criterion (See their Table 1, p. 133). When no selection occurs on the direct variable, the validities for the direct selection (r = .50) and indirect selection (r = .45) variables are equal to the population values. As expected, the correlations between the predictors and criterion decreased steadily as the selection ratio decreased. The rate of decrease was greater for direct selection variable than for indirect selection variable. The direct selection variable had greater validity than the indirect selection variable in the unrestricted population (r = .50 versus r = .45). Once the selection ratio dropped below 60%, the observed validity of the direct selection variable was lower than that of the indirect selection variable (r = .35 versus r = .36). The seeming superiority of the indirect variable continued to increase as the selection ratio decreased. If the correction for range restriction had not been applied the wrong decision about the validity of the two predictors would have been made.

2.2 Ignoring the Effect of One or More Unaccounted-for-Selection Variables

Frequently a problem is the effect of variables that are not administered by the practitioner or researcher, but are influential in creating range restriction nonetheless (see Gross, 1990; Gross, & McGanney, 1987; Jackson & Ree, 1992; Olson & Becker, 1983). Gross and McGanney (1987) identified this failure to correct for all variables as "nonignorable." It would be desirable to correct for range restriction in the selection processes caused by variables both administered and not administered by the practitioner or researcher.

For example, a job knowledge test may be administered for selection, but the examinees must have prior qualifications such as high school completion and no record of felonies. These qualifications are examples of unaccounted-for variables administered before the job knowledge test and may have noticeable impact. Identifiable groups may differ on these unaccounted-for selection variables. The impact of not accounting for all selection variables may influence the overall correlation and correlations for identifiable groups leading to differential range restriction by group. This in turn can cause practitioners and researchers to infer the presence of real differences in correlations for the groups when it is unwarranted.

Self-selection is another source of range restriction. Not all applicants want every job or admission to every educational institution and there are personal preferences that guide individuals toward some options and away from others. Additionally, self-selection results in applications to differing sources of employment or education. For example, a non-selective university does not get many applicants with very high standardized scholastic aptitude test scores. Applicants with very low standardized test scores rarely apply to highly selective universities. This self-selection creates range restriction.

Yang, Sackett, and Nho (2004) noted that range restriction correction formulas typically assume that applicants do not reject training or job offers when they are presented, whereas in an applied setting some applicants who have received offers reject them. Yang et al. proposed a correction procedure that involves both applicant rejection of offers and institutional rejection of applicants. They demonstrated the utility of this procedure in a series of Monte Carlo simulations. Van Iddekinge and Ployhart (2008) questioned the utility of the Yang et al. approach because sufficient information may not be available to model the effects of self-selection (i.e., plausible reasons why an applicant may reject an offer).

Consider the effect of one or more explicit selection variables that were not included (i.e., hidden variables) in the calculation of corrected correlations using the multivariate method (Lawley, 1943). The reason for the exclusion of these variables may be that they were unknown to those doing the correction or that they were difficult to measure. Jackson and Ree (1992) showed that the conditions of Lawley's (1943) theorem would be met when all explicit selection variables were included. The conditions would not be satisfied if one or more of these variables were omitted from consideration when correcting for range restriction. Jackson and Ree mathematically demonstrated why the correction formula should be expected to fail and performed several simulations to illustrate the magnitude of the inaccuracy caused by exclusion of the unaccounted for variables. They noted that "It is difficult to imagine any statistic that attempts to correct for hidden variables that does not make substantial assumptions about the form of the selection criteria and the distribution of the error terms" (p. 12). Leaving hidden variables out of the correction for range restriction results in poorer estimates of the population correlations. Jackson and Ree concluded "However, Pearson correction appears to work very well and should be replaced with another statistic only in the presence of overwhelming evidence of superior performance. The effect of range restriction is almost always present and Pearson correction does a very good job of correcting for this effect." (p. 26)

2.3 Use of an Inappropriate Correction Formula

In all instances how the range restriction was created is the key to which correction formula to use. Hunter et al. (2006) noted that it is common in educational and employment selection situations to find that data are available for only a restricted population. If applicants were selected directly based on their aptitude test scores, direct range restriction has occurred. However, if selection has occurred on some other variable that is correlated with the aptitude test score (e.g., achievement, prior experience), indirect range restriction has occurred for the test score. Hunter et al. noted that range restriction in most data sets is indirect. In a review of meta-analytic studies, they found that it was common practice to apply the direct correction for range restriction formula, although indirect selection actually occurred. They illustrated that the misapplication of the correction formula resulted in an underestimation of the corrected correlations, which could be substantial. Hunter et al. illustrated that due to misapplication of the range correction procedures, previous large-scale meta-analyses of the relations between general mental ability and job performance underestimated the correlations by about 25%.

Two important issues are correcting for the appropriate type of range restriction (direct or indirect) and the accuracy of the estimates provided by the univariate and multivariate procedures. Held and Foley (1994) demonstrated that when sufficient data are available the multivariate correction is more accurate than the univariate corrections. They compared the accuracy of univariate and multivariate corrections for range restriction in a large sample (*N* = 147,288) of US Navy applicants. Two US Navy aptitude selector composites derived from the Armed Services Vocational Aptitude Battery (ASVAB) were used separately and together to simulate three selection situations for nine selection ratios. Another ASVAB aptitude test was used as the criterion. The selection situations consisted of; (1) explicit selection on composite 1 and implicit selection on composite 2, (2) implicit selection on composite 1 and explicit selection on composite 2, and (3) explicit selection on both composites. The selection ratio (SR) ranged from .10 to .90, with the unrestricted SR at 1.0. Observed (uncorrected) validities for the explicit and implicit selection composites were corrected for range restriction. This was done using both univariate and multivariate correction procedures. In the univariate case, the Case 2 – direct and Case 3 - indirect (Thorndike, 1949) correction formulas were used as appropriate. Results indicated that the multivariate-corrected correlations were generally more accurate than the univariate-corrected corrections, especially when the correlation assumptions of linearity and homoscedasticity were violated.

Case IV corrects for indirect range restriction. See the Appendix for a detailed description. Fife, Mendoza, and Terry (2013) suggested that meeting the assumptions for Case IV may be impossible in some studies, but Le et al. (2016) found that even when the assumptions were not met Case IV was superior to misapplication of Case 2.

The Case V formula allows for correction for indirect range restriction between X and Y without knowledge about a third variable on which selection occurred. Le et al. (2016) examined the utility of these methods using Monte Carlo simulation and concluded that the Case V correction was superior to that provided by both Case 2 and Case IV. It should be noted that Case IV and Case V have not been compared to the multivariate correction. See the Appendix for the Case V formula and more detailed description.

2.4 Use of an Inappropriate Unrestricted Population or Sample

Another problem is choice of which unrestricted population or sample to use in the corrections. This choice may affect not only the resulting values, but their interpretation (i.e., generalizability, relevance). For example, if the research question is to estimate the relation between personality and job satisfaction in the general workforce, the data for employees would need to be corrected for range restriction. However, if the research question is how accurately the job satisfaction of *current* employees can be predicted from a personality test, the population of interest and the available sample are the same. Under these circumstances, no correction for range restriction is appropriate even if it is clear that the distribution of personality scores for the employees is severely restricted.

2.4.1. Specific applicant pools

Sackett and Ostgaar (1994) tested the hypothesis that applicant samples were more homogeneous than national norms and evaluated the difference between the standard deviation (SDs) from national norms for the Wonderlic Personnel Test to SDs derived from applicant samples for 80 moderate complexity jobs. They found that on average applicant SDs in the job-specific samples were about 10% lower than for the national normative group. Additionally, Lang, Kersting, and Hulsheger (2010) found that SDs for job-specific applicant pools in Germany were 10% to 12% smaller on average than estimates from a normative population.

2.4.2. Use of a common unrestricted reference group

The sample is fleeting and ephemeral. It doesn't return. This is an argument for correcting data to a common normative reference group. In the case of large scale testing programs, normative groups are established to provide a common metric for longitudinal consistency in score meaning. Further, using a common reference group allows comparison of results across multiple organizations or jobs (see Carretta, Ree, & Teachout, 2016).

However, there are instances where specialized normative reference groups (as opposed to common normative reference norms) are needed when subgroup differs markedly from the population on one or more attributes (e.g., specialized norms on the Multidimensional Aptitude Battery and NEO-PI-R for US Air Force aircrew applicants, see Carretta, King, Ree, Teachout, & Barto, 2016). See also Berry, Sackett and Sund (2013).

Several studies have used group-specific norms to correct data for range restriction when examining differential validity for racial/ethnic groups. Berry, Sackett, and Sund, (2013) showed in a large sample (N > 140,000) of SAT scores that within group-specific norms for range restriction correction aided in the assessment of differential validity for racial/ethnic groups. Results indicated that uncorrected validities underestimated differential validity compared to when range restriction and criterion contamination were controlled.

Roth Le, Oh, Van Iddekinge, Buster, Robbins, and Campion (2014) used simulation to examine differential validity for racial/ethnic groups in employment and educational settings. When no differential validity existed in the simulated population, different levels of range restriction for groups explained the differential validity.

2.5 Incorrectly Estimating the Standard Errors of Corrected Correlations

The standard error of correlations corrected for range restriction is not necessarily the same as that of a correlation estimated in a full range sample. Millsap (1989) using Monte Carlo simulations demonstrated that range restricted correlations had a different standard error than correlations estimated in a full range sample. He concluded "The negative bias in the sampling variance estimator under range restriction has several consequences for statistical practice. … The error rate inflation would be most serious in small or moderate sample sizes" (p. 460).

Bobko and Reich (1980) provided an analytic proof for a standard error for correlations corrected for direct range restriction. Based on a Taylor series expansion and asymptotic covariance of two variables administered to a single sample, a large sample estimator of sampling variance of correlations corrected for range restriction, was derived.

More recently, Wolfe and Held (2010) conducted Monte Carlo studies to produce estimates of the standard errors of correlations corrected using the multivariate correction (Lawley, 1943). They also compared the Bobko and Reich (1980) standard error for univariate correction to the Monte Carlo produced standard errors. They affirmed Millsap (1989) showing that the range-restricted standard errors were larger than the standard errors using the usual formula for a correlation.

3.0 RESOURCES AND PAYBACK

3.1 Accurate Estimation of Utility

Why bother doing the corrections if the correlations are statistically significant in the restricted sample? A reason already noted is to properly estimate the values of the true correlations (population parameters). The utility of the predictor(s) is at least partially embodied in the correlation. Utility may be defined as the economic value or cost avoidance value in the use of the variable(s). Utility may be substantially underestimated when uncorrected correlations are used (Schmidt & Hunter, 1998).

Taylor and Russell (1939) and Naylor and Shine (1965) proposed approaches to utility analysis that depend on accurate estimates of the correlation between the selection score and job success. The utility analysis approaches proposed by both Taylor and Russel (1939) and Naylor and Shine (1965) depend on the numeric value of the correlation between the selection score and job success. The use of uncorrected correlations will frequently lead to underestimates, sometimes severely, of the utility of the predictor. Corrected correlations would give a more accurate estimate of utility.

3.2 More Accurate Meta-Analyses

Using appropriate range restriction corrections benefits the interpretation of meta-analytic results. Correcting for range restriction appropriately provides better estimates of the true relationship of the variables. Some developers of meta-analysis software have provided correction for range restriction beyond Case 2 or Case 3. General-purpose statistical software, such as R, usually allows procedures to be written for any type of correction for range restriction. Dahlke and Wiernik (2019a) using the term bivariate indirect range restriction (BVIRR) to describe Case V, noted that Le et al.'s (2016) method did not take into account important impacts of the BVIRR correction on the sampling error of corrected correlations and the implications of applying the BVIRR correction in primary research and meta-analyses. Dahlke and Wiernik provided a generalized Case V (BVIRR) correction formula that can correct for either range restriction or range enhancement and substantially improved parameter estimation by reducing bias. They also described new methods to adjust for its impact on the sampling variance of correlations. Dahlke and Wiernik noted that the Case V (BVIRR) correction will function very differently than univariate direct range restriction (UVDRR or Case 2) or univariate indirect range restriction (UVIRR or Case IV) corrections or correction for measurement error alone when used in psychometric meta-analyses.

Dahlke and Wiernik (2018, 2019b) provide R code for performing both univariate and multivariate range correction.

4.0 CONCLUSIONS AND RECOMMENDATIONS

It is better to correct for range restriction in almost all instances. The correction will adjust the correlations for bias and generally provide more accurate statistical estimates. This can lead to clearer interpretation of scores and tests of relationships. In keeping with Linn, Harnisch and Dunbar (1981) report both the uncorrected and corrected correlation.

Apply the range correction formula that fits the restriction situation. How the range restriction was created is the key to which correction formula to use. Application of an inappropriate formula leads to misinterpretation. Use the appropriate unrestricted population or sample. It should represent the population or sample about which inferences are to be made. If enough information is available, the Lawley (1943) multivariate procedure is more accurate than Cases 2 and 3, especially when the correlation assumptions of linearity and homoscedasticity have been violated. Additional studies are needed to compare the effectiveness of the multivariate method with Cases IV and V.

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APPENDIX

Range Restriction Equations

Please note that all correction formulas assume linearity and homoscedasticity. The equations have been presented in the notation of the cited articles for ease of reference by reader.

Case 1 Correction (direct range restriction)
The Case 1 correction applies when the correlation to be corrected is between two variables, a and b, selection occurred on variable a, and both restricted and unrestricted variances are known only for variable b. There is indirect selection on variable b. The correlation between a and b has been estimated in the range restricted sample of those selected on a. The Case 1 correction formula is:

$$r_c = \sqrt{1 - (sd_b^2)/(SD_b^2)\,(1 - {r_{ab}}^2)}$$

where r_c is the correlation between variable a and variable b in the unrestricted population, r_{ab} is the correlation between a and b in the restricted sample, sd_b is the standard deviation (SD) of b in the restricted sample, and SD_b is the SD of b in the unrestricted population.

Case 2 Correction (direct range restriction)
The Case 2 correction (direct selection) is well-known and frequently used. In Case 2, the variances for a are known in both the restricted sample and unrestricted population. However, the correlation between a and y is known only for the restricted sample that has been directly selected on a. The Case 2 (direct range restriction) correction equation is:

$$r_c = r_{aY}\left(\frac{SD_a}{sd_a}\right) / \sqrt{(1 - {r_{ay}}^2) + r_{ay}^2\left(\frac{{SD_a}^2}{sd_a^2}\right)}$$

where r_c is the correlation between variable a and variable y in the unrestricted population, r_{ay} is the observed correlation between a and y in the restricted sample, sd_a is the standard deviation (SD) of a in the restricted sample and SD_a is the SD of a in the unrestricted population),

Case 3 Correction (indirect range restriction)
Case 3 is sometimes also called correction for incidental selection. Suppose a test, a, were given to a group of applicants some of whom were selected based on their test (a) scores and the variance for test a is known for both the restricted sample and unrestricted population. The correlation for an experimental test, b, and the job performance criterion, y are available only for the selected (restricted) sample. When all three correlations can be calculated the Case 3 (indirect range restriction) correction equation is:

$$r_c = r_{ba} r_{ya}((S_a^2/s_a^2) - 1) / \sqrt{\left[1 + r_{ba}^2\left(\frac{S_a^2}{s_a^2}\right) - 1\right]\left[1 + r_{ya}^2\left(\frac{S_a^2}{s_a^2}\right) - 1\right]}$$

where S_a/s_a is the ratio of the unrestricted to the restricted standard deviation of test a.

Case IV Correction (indirect range restriction)
The Case IV correction equation is described below. As noted by Schmidt et al. (2006), an important difference between direct and indirect range restriction is that in the direct case, because observed scores are used to select, restriction occurs on the observed scores. Observed scores are not used in selection in the indirect case. In the indirect case range restriction occurs on *true* scores rather than on observed scores (Hunter & Schmidt, 2004, Ch. 5; Hunter et al., 2006). In the Case IV correction for indirect range restriction, the restriction on the true scores t_x for the observed score x is estimated from the amount of range restriction on the observed scores x. It is assumed that y is fully mediated by the true score (T) of variable X. As in Case 2, the ratio of the restricted to the unrestricted observed *SD*s of x is defined as sx/Sx. The range restriction on the true scores t is estimated from observed scores x and is defined as st/St. The equation that gives st/St (Hunter & Schmidt, 2004, Equations 3.16 and 5.31; Hunter et al., 2006, Equation 22) is:

$$\left(\frac{s_t}{S_t}\right) = \frac{\sqrt{\left(\frac{s_X^2}{S_X^2}\right) - (1 - r_{XXa})}}{r_{XXa}}$$

where r_{XXa} is the reliability of the predictor in the unrestricted group (i.e., applicant group). The Case IV range correction equation is:

$$r_{XtYt} = \left[\left(\frac{1}{\frac{s_t}{S_t}}\right) r_{XtYt}\right] / \sqrt{\left[\left(\frac{1}{\frac{s_t^2}{S_t^2}}\right) - 1\right] r_{XtYt}^2 + 1}$$

In their discussion of various scenarios that can produce range restriction, Sackett and Yang (2000) introduced the little-known Case V correction for range restriction (Bryant & Gokhale, 1972) which was derived from the Case 3 equation. Le et al. (2016) conducted studies to improve the accuracy and reliability of the Case V formula for use in meta-analyses.

Case V Correction (indirect range restriction)
The Case V equation enables correction for indirect range restriction without requiring the full mediation assumption of Case IV.
Bryant and Gokhale's (1972) correction equations is:

$$R_{XY} = r_{XY}\left(\frac{sd_X}{SD_X}\right)\left(\frac{sd_Y}{SD_Y}\right) + \sqrt{\left(1 - \frac{sd_X^2}{SD_X^2}\right)\left(1 - \frac{sd_Y^2}{SD_Y^2}\right)}$$

where R_{XY} is the correlation between variables X and Y in the unrestricted population, r_{XY} is the observed correlation between X and Y in the restricted sample, the range restriction ratio of X is sd_X/SD_X, where sd_X is the standard deviation (SD) of X in the restricted sample and SD_X is the SD of X in the unrestricted population, and the range restriction ratio of Y is sd_Y/SD_Y, where sd_Y is the SD of Y in the restricted sample and SD_Y is the SD of Y in the unrestricted population. The Bryant and Gokhale (1972) correction equation allows for correction of indirect range restriction on X and Y without knowledge about the third variable on which selection occurred. The Bryant and Gokhale equation can be used to correct individual study correlations. Case V is suggested for meta-analysis.

The Case V method, building on Bryant and Gokhale (1972), first requires the estimation of true score means and true score correlations. Le et al. (2016) modified the Bryant and Gokhale formula to also correct for measurement error and described a revised meta-analytic approach that incorporates the Case V correction. Le et al. provided the equations for the multi-step procedure for estimating true score elements and should be consulted for descriptions of the detailed steps.
Even though Case V has been applied in meta-analysis it is also appropriate for correction of a single correlation to estimate the true relationship of X and Y. As noted by Le et al. the Case V method does not require any further assumptions beyond those of linearity and homoscedasticity, which underlie all existing range correction formulas. The steps below describe the process to perform correction meta-analysis based on Case V.

(1) **Correct for measurement error in measure Y.** For each study, the observed correlation, r_{XYi}, provides the best estimate for the restricted correlation, ρ_{XYi}.

$$\rho_{XYi} = r_{XYi}$$

Then using the familiar equation we can correct for measurement error in *Y*:

$$\rho_{XPi} = \rho_{XYi} / \sqrt{\rho_{YYi}}$$

where ρ_{XPi} is the reliability corrected correlation of variable *X* and its true score, the underlying construct, *P*. ρ_{XYi} is the observed correlation between X and Y and ρ_{YYi} is the reliability of Y.

(2) **Correct for measurement error in measure X.** Nest, correct for measurement error in X using the reliability of measure X on the underlying construct T, in the restricted population, ρ_{XXi}.

$$\rho_{TPi} = \rho_{XPi} / \sqrt{\rho_{XXi}}$$

where ρ_{TP1} is the correlation between construct T (the reliability corrected construct underlying variable X) and construct P underlying variable Y in the restricted population. ρ_{xp1} is the observed correlation between X and Y and ρ_{xx} is the reliability of X. Next μ_t and μ_p

(3) **Correct for indirect range restriction.** Le et al. (2016) adapted the Bryant and Gokhale (1972)m equation shown above correcting the variables for measurement error. Le et al. replaced the range restriction ration on X with the range restriction ratio on T, and replaced the range restriction ration on Y with the range restriction ratio on P.

Estimate u_T from $u_X\left(\frac{sd_X}{SD_X}\right)$ and ρ_{XXi}:

$$u_T = \sqrt{\frac{\rho_{XXi}u_X^2}{1 + (\rho_{XXi}u_X^2 - u_X^2)}}$$

Likewise, estimate u_P from $u_Y\left(\frac{sd_Y}{SD_Y}\right)$ and ρ_{YYi}:

$$u_P = \sqrt{\frac{\rho_{YYi}u_Y^2}{1 + (\rho_{YYi}u_Y^2 - u_Y^2)}}$$

Finally, estimate the correlation between T and P in the unrestricted population using the following formula:

$$\rho_{TPa}u_T u_P + \sqrt{(1 - u_T^2)(1 - u_P^2)}$$

Multivariate Correction

Lawley (1943) developed the multivariate, or general case that allows for correction for range restriction resulting from selection on several variables and provided a theorem. The theorem is explained in terms of variance-covariance matrices that can be converted to correlation matrices. The discussion that follows uses the notation of Birnbaum, Paulson, and Andrews (1950). Suppose one has a current test battery of p variables for which population information is available. A sample is selected on the basis of test scores on the current battery and suppose that n-p additional variables (perhaps a combination of new tests and criteria variables) are collected on the selected (or restricted) sample.

The variance-covariance matrix from the restricted sample is:

$$v^* = \begin{bmatrix} v^*_{p,p} & v^*_{p.n-p} \\ v^*_{n-p,p} & v^*_{n-p,n-p} \end{bmatrix}.$$

All of $v^*$is known. The comparable variance-covariance matrix from an unrestricted (mostly unknown values) population is:

$$v = \begin{bmatrix} v_{p,p} & v_{p,n-p} \\ v_{n-p,p} & v_{n-p,n-p} \end{bmatrix},$$

where there is only knowledge of $v_{p,p}$. We want to estimate the other three parameters of v. $v_{n-p,n-p}$ gives estimates of unrestricted variances and covariances for variables of the new tests and criteria. $v_{p,n-p}$(or its transpose $v_{n-p,p}$)gives estimates of unrestricted covariances for variables of the current test battery with the new tests and criteria.

Lawley's (103) theorem follows where x_i and x_j are any pair of variables.

The multivariate correction is expressed in matrix algebra notation.

Assumption 1 – Linearity

Assumption 2 – Equality of Error Variance. (Homoscedasticity)

Then the following equations use the known variances and covariances plus the variances and covariances from the restricted sample to provide corrected variances and covariances for all variables. These are then converted to correlations. The matrix equations follow:

$$v_{p,n-p} = v_{p,p} v_{p,p}^{*-1} \left(v_{p,n-p}^{*}\right) \text{ and}$$

$$v_{n-p,n-p} = v_{n-p,n-p}^{*} + \sqrt{v_{n-p,p}^{*} v_{p,p}^{*}} \left(v_{p,n-p} - v_{p,n-p}^{*}\right)$$

LIST OF SYMBOLS, ABBREBIATIONS AND ACRONYMS

%	percent/percentage
ASVAB	Armed Services Vocational Aptitude Battery
BVIRR	bivariate indirect range restriction
SDs	standard deviation
SR	selection ratio
US	United States
UVDRR	univariate direct range restriction