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Synthesis of Reduced-Order Modeling, Global Sensitivity Analysis, and Uncertainty Quantification for Robust
Control Design of Nonlinear Smart Composite Systems

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This program focused on the synthesis of physical and reduced-order model development, quantification of model discrepancies, global sensitivity analysis (SA) and uncertainty quantification (UQ) to improve robust control designs for AFOSR applications utilizing advanced transductive materials. These actuators provide broadband drive capabilities, solid state nature, and potential for miniaturization. However, these advantages come at the cost of hysteretic, rate-dependent, and nonlinear dynamics. We addressed this via model-based uncertainty analysis and robust control design and verified these results for materials under consideration for present micro-air vehicles and energy-harvesting systems.

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Synthesis of Reduced-Order Modeling, Global Sensitivity Analysis, and Uncertainty Quantification for Robust Control Design of Nonlinear Smart Composite Systems

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Objectives

This program focused on the synthesis of physical and reduced-order model development, quantification of model discrepancies, global sensitivity analysis (SA) and uncertainty quantification (UQ) to improve robust control designs for AFOSR applications utilizing advanced transductive materials. We initially considered micro-air vehicles, such as Robobee shown in Figure 1, and macro-fiber composite (MFC) devices that serve as prototypes for aeroelastic systems. Both the Robobee and MFC-driven devices employ piezoelectric actuators due to their broadband drive capabilities, solid state nature, and potential for miniaturization. However, these advantages come at the cost of hysteretic, rate-dependent, and nonlinear dynamics. Furthermore, the unsteady aerodynamics inherent to the micro-air vehicles (MAV) must be quantified through full and reduced-order models that are formulated in terms of influential parameters – determined through global sensitivity and active subspace analysis – and accommodate uncertainties due to parameters, model discrepancies, and measurement errors. This yielded response uncertainties that we used to improve robust control designs.

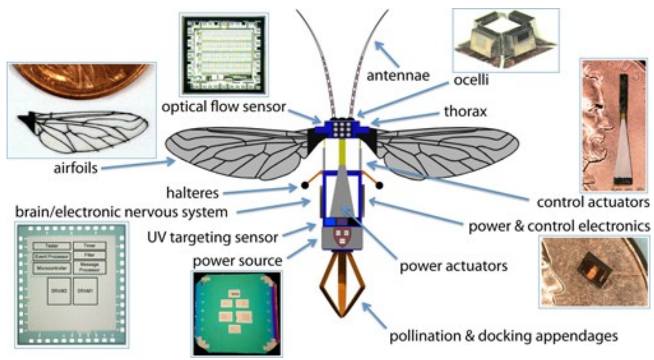


Figure 1: Schematic of Robobee.

Accomplishments

During the program, we focused on six broad topics: (i) development of a physics-based model for the lead-zirconate-titanate (PZT)-based drive mechanisms for Robobee, (ii) sensitivity analysis, parameter subset selection, and active subspace techniques to isolate subsets and subspaces of influential parameters, (iii) Bayesian inference and uncertainty quantification, (iv) implementation of a data-driven reduced-order model framework based on dynamic mode decomposions (DMD) and feedback control design, (v) development of fractional-order models to improve predictive capabilities for systems with rate-dependent dissipation properties, and (vi) implementation of a Python package for robust Bayesian inference to compute posterior distributions for model parameters.

1. Homogenized Energy model (HEM) for MAV Actuators

We summarize here the dynamic PDE model, developed during the program, which we use to quantify the nonlinear, rate-dependent, and hysteretic dynamics of PZT actuators employed as drive mechanisms for MAV such as Robobee; see Figure 2. As detailed in [9], the actuators are modeled by the Euler-Bernoulli beam relation

$$\rho(x)\frac{\partial^2 w(x,t)}{\partial t^2} + \gamma\frac{\partial w(x,t)}{\partial t} - \frac{\partial^2 M(x,t)}{\partial x^2} = 0, \tag{1}$$

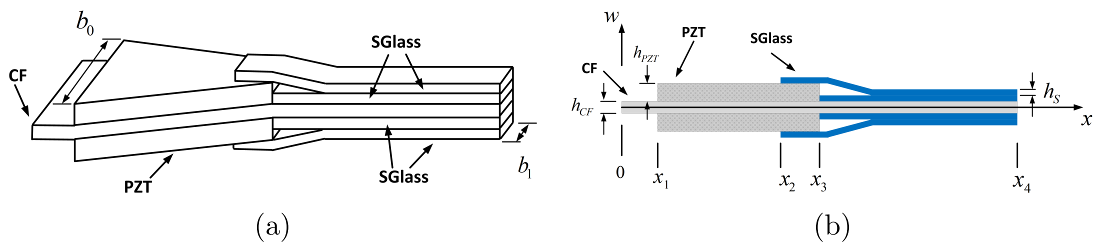


Figure 2: (a) Bimorph schematic and (b) cross-sectional view of the bimorph.

where $\rho(x)$ is a piecewise constant and the moment is

$$M(x,t) = -YI(x)\frac{\partial^2 w(x,t)}{\partial x^2} - cI(x)\frac{\partial^3 w(x,t)}{\partial t \partial x^2} + f(x,t).$$

The piecewise constant terms

$$YI(x) = Y_{cf} I_{cf}(x) + Y_s I_s(x) + \tfrac{1}{s^E} I_{pzt}(x),$$

$$cI(x) = c_{cf} I_{cf}(x) + c_s I_s(x) + c_{pzt} I_{pzt}(x),$$

$$I_{cf}(x) = \tfrac{1}{12} b_{cf}(x) h_{cf}^3,$$

$$I_s(x) = \tfrac{2}{3} b_1 \left[\chi_{s1}\left((h_{pzt}+h_s)^3 - h_{pzt}^3\right) + \chi_{s2}\left((h_{cf}+2h_s)^3 - h_{cf}^3\right)\right],$$

$$I_{pzt}(x) = \tfrac{2}{3} b_{pzt}(x)\chi_{pzt}(x)\left((h_{pzt}+\tfrac{1}{2}h_{cf})^3 - \tfrac{1}{8}h_{cf}^3\right)$$

and

$$f(x,t) = \tfrac{c_1}{s^E}\chi_{pzt}(x)\left[-d(E_1(t),\sigma)E_1(t) + d(E_2(t),\sigma)E_2(t) - \varepsilon_{irr}(E_1(t),\sigma) + \varepsilon_{irr}(E_2(t),\sigma)\right],$$

$$c_1 = \tfrac{1}{2} b_{pzt}(x)\left(\left(h_{pzt}+\tfrac{1}{2}h_{cf}\right)^2 - \tfrac{1}{4}h_{cf}^2\right)$$

contain material and geometric constants along with the nonlinear and hysteretic PZT inputs. Following theory in [19], the hysteretic and nonlinear PZT inputs are quantified by the relations

$$d(E,\sigma) = \int_0^\infty \int_{-\infty}^\infty \overline{d}\left(E(t)+E_I,\sigma;f_c\right)\nu_c(f_c)\nu_I(E_I)dE_I df_c$$

$$\varepsilon_{irr}(E,\sigma) = \int_0^\infty \int_{-\infty}^\infty \overline{\varepsilon}_{irr}\left(E(t)+E_I,\sigma;f_c\right)\nu_c(f_c)\nu_I(E_I)dE_I df_c$$

for the piezoelectric coupling coefficient and irreversible strain. Here $\bar{d}$ and $\bar{\varepsilon}_{irr}$ are energy-based kernels at the grain level and $\nu_c(f_c)$ and $\nu_I(E_I)$ are densities, which are estimated for a given dataset.

Based on physical constraints, we obtained 18 model parameters that need to be specified or calibrated through fits to data. Based on data from [20], we employed a simplex-based optimization method to obtain initial parameter estimates followed by a gradient-based technique to obtain the point estimates summarized in Table 1. The resulting hysteretic model fit to the data is illustrated in Figure 3.

Whereas a number of the parameters are physically reasonable, values such as the relaxation time $\tau_{180} = 1.02 \times 10^{-19}$ are not physically feasible. This indicates that the complete parameter set is not identifiable in the sense that not all the parameters are uniquely determined by the data.

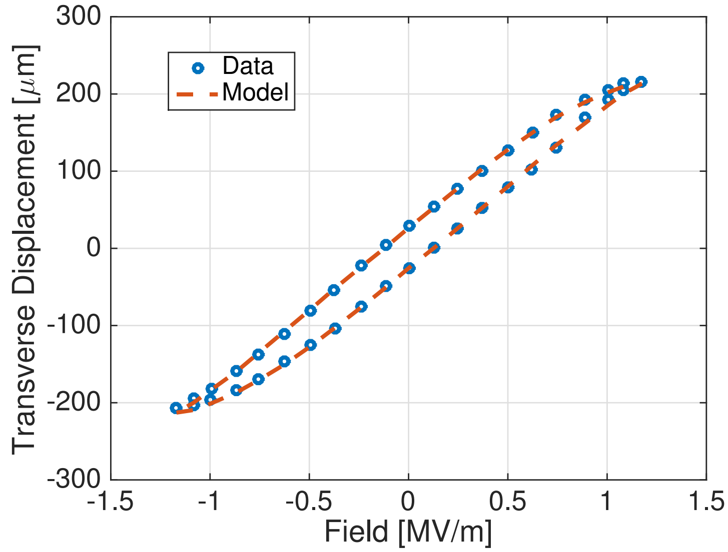


Figure 3: Optimized fit of the model (1) to data from [20].

Table 1: Model parameters and point estimates obtained using data from [20].

γ	Air damping coefficient	0.097
ρ_{cf}	Density of the CF layer (kg/m^3)	1.7×10^4
ρ_s	Density of the S2 Glass (kg/m^3)	2.27×10^4
ρ_{pzt}	Density of the PZT actuators (kg/m^3)	1.22×10^3
Y_{cf}	Elastic modulus of CF (Pa)	5.00×10^{11}
Y_s	Elastic modulus of S2 Glass (Pa)	8.04×10^{11}
c_{cf}	Damping coefficient for CF	1.12×10^4
c_s	Damping coefficient for S2 Glass	2.98×10^3
c_{pzt}	Damping coefficient for PZT	1.37×10^3
s^E	Elastic compliance (1/Pa)	$1.14 \times 10{-}11$
$d_\pm$	Piezoelectric coupling coefficient for $\alpha = \pm$ (m/V)	1.07×10^{-9}
$\varepsilon_R^\pm$	Remanent strain for $\alpha = \pm$ (%)	0.17
ε_R^{90}	Remanent strain for $\alpha = 90$ (%)	-8.79×10^{-13}
$P_R^\pm$	Remanent polarization for $\alpha = \pm$ (C/m^2)	0.13
χ^σ	Ferroelectric susceptibility (F/m)	1.66×10^{-6}
τ_{90}	Relaxation time for 90° switching (s)	7.11×10^{-6}
τ_{180}	Relaxation time for 180° switching (s)	1.02×10^{-19}
γ_{pzt}	Inverse of relative thermal energy (m^3/J)	0.08

2. Parameter Identifiability Analysis

To isolate identifiable subsets of parameters, which can be employed for subsequent Bayesian inference, we employed the following parameter subset selection (PSS) technique. We formulate the input-output relation as

$$Y = f(\Theta), \tag{2}$$

where Θ is a random vector corresponding to the $p = 18$ parameters in Table 1. Minimization of the functional

$$J(\theta) = \frac{1}{N}\sum_{n=1}^{N}\left[Y_n - f(\theta)\right]^2,$$

for realizations θ of the random vector Θ, yields an optimal parameter vector θ^*.

To relate this to the local sensitivity, we consider the multivariate Taylor expansion

$$f(\theta) \approx f(\theta^*) + \nabla_\theta f(\theta^*)\cdot\Delta\theta,$$

where

$$\nabla_\theta f(\theta^*) = \left[\frac{\partial f}{\partial\theta_1}(\theta^*),\ldots,\frac{\partial f}{\partial\theta_p}(\theta^*)\right]^T \tag{3}$$

and $\Delta\theta = \theta - \theta^*$. Based on the assumption that $Y_n \approx f(\theta^*)$ at the minimum θ^*, the cost functional can be approximated by

$$J(\theta) \approx \frac{1}{N}\sum_{n=1}^{N}\left[\nabla_\theta f(\theta^*)\cdot\Delta\theta\right]^2.$$

If we define the $1\times p$ sensitivity matrix S by

$$S = \left[\nabla_\theta f(\theta^*)\right]^T = \begin{bmatrix} \frac{\partial f}{\partial\theta_1}(\theta^*) & \cdots & \frac{\partial f}{\partial\theta_p}(\theta^*)\end{bmatrix}, \tag{4}$$

we can approximate the cost functional by

$$J(\theta) \approx \frac{1}{N}(S\Delta\theta)^T(S\Delta\theta),$$

or, equivalently,

$$J(\theta^* + \Delta\theta) \approx \frac{1}{N}\Delta\theta^T S^T S\Delta\theta.$$

If we take $\Delta\theta$ to be an eigenvector of S^TS, so that $S^TS\Delta\theta = \lambda\Delta\theta$, then

$$J(\theta^* + \Delta\theta) \approx \frac{1}{N}\lambda\|\Delta\theta\|_2^2.$$

We note that if $\lambda \approx 0$, the cost functional perturbations $J(\theta^* + \Delta\theta)$ are also approximately 0 and hence the corresponding parameters are locally nonidentifiable. This forms the basis for the algorithms in [6–8, 17].

Using this algorithm we obtained the results in Table 2. We observe that the eigenvalues corresponding to $d_\pm, \gamma_{pzt}, P_R^\pm$, and $\epsilon_R^\pm$ are significantly larger than those associated with the other parameters. This determines that these four parameters are identifiable in the sense that they are uniquely determined by data. We then performed Bayesian inference and forward uncertainty propagation using these four parameters.

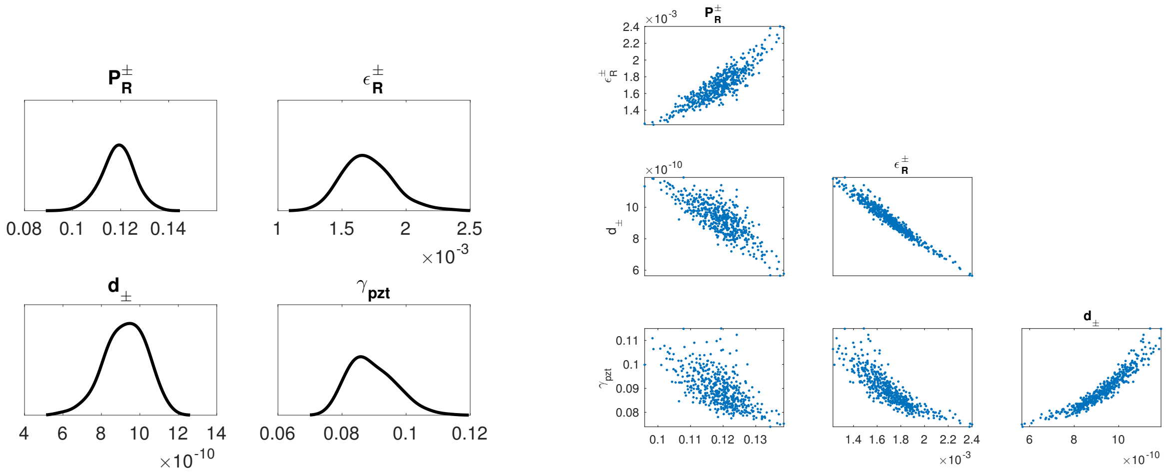
Table 2: Normalized eigenvalues associated with each parameter. Bold parameters are identifiable in the sense that they are uniquely determined by the data.

γ	Air damping coefficient	6.24e-23
ρ_{cf}	Density of the CF layer (kg/m^3)	-4.31e-18
ρ_s	Density of the S2 Glass (kg/m^3)	1.35e-20
ρ_{pzt}	Density of the PZT actuators (kg/m^3)	3.82e-13
Y_{cf}	Elastic modulus of CF (Pa)	7.75e-08
Y_s	Elastic modulus of S2 Glass (Pa)	2.26e-13
c_{cf}	Damping coefficient for CF	3.43e-18
c_s	Damping coefficient for S2 Glass	2.32e-23
c_{pzt}	Damping coefficient for PZT	2.09e-14
s^E	Elastic compliance (1/Pa)	3.14e-06
$d_\pm$	**Piezoelectric coupling coefficient for $\alpha = \pm$ (m/V)**	**1**
$\varepsilon_R^\pm$	**Remanent strain for $\alpha = \pm$ (%)**	**4.82e-4**
ε_R^{90}	Remanent strain for $\alpha = 90$ (%)	5.94e-23
$P_R^\pm$	**Remanent polarization for $\alpha = \pm$ (C/m^2)**	**0.011**
χ^σ	Ferroelectric susceptibility (F/m)	-1.62e-22
τ_{90}	Relaxation time for 90° switching (s)	8.88e-08
τ_{180}	Relaxation time for 180° switching (s)	6.88e-10
γ_{pzt}	**Inverse of relative thermal energy (m^3/J)**	**0.063**

3. Bayesian Inference and Uncertainty Quantification

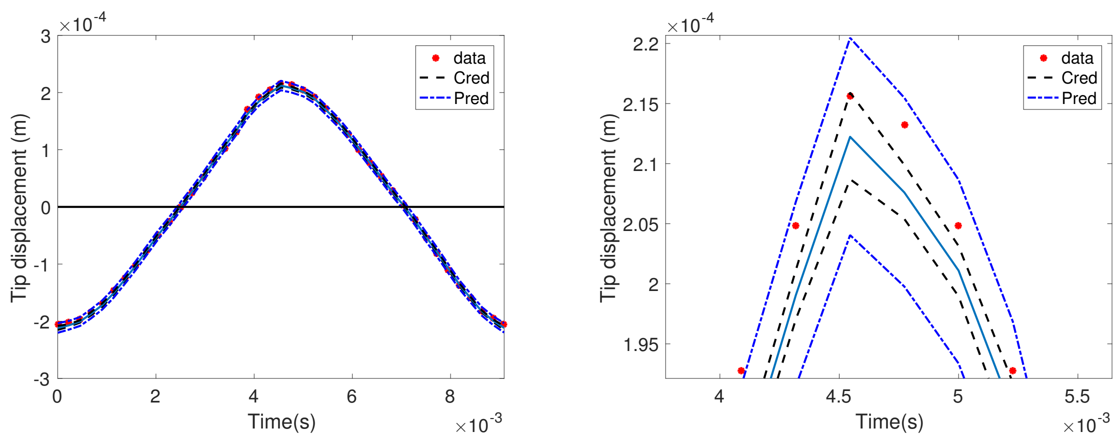
We performed Bayesian inference using the Delayed Rejection Adaptive Metropolis (DRAM) algorithm [10] to compute posterior distributions for these four parameters. We employed the nominal values in Table 1 as initial values in the algorithm and fixed non-identifiable parameters at those values for subsequent analysis. We plot the marginal posterior distributions plotted in Figure 4. We observe from the pairwise plots in Figure 4(b) that the parameters are correlated but identifiable.

To quantify resulting uncertainty in the predicted tip displacement, we randomly sampled 10,000 values for the posterior parameter distributions for $d_\pm, \gamma_{pzt}, P_R^\pm$, and $\epsilon_R^\pm$, along with the inferred observation error, and propagated these values through the Homogenized Energy Model to compute 95% credible and prediction intervals for the tip displacement. We plot in Figure 5 the predicted tip displacement, experimental data and intervals over a full cycle and at the peak of the cycle. We observe that for this operating regime, the intervals are very tight and are consistent with the data. Additional results are reported in [2,5].



(a) (b)

Figure 4: (a) Marginal posterior densities for identifiable parameters and (b) pairwise plots quantifying correlation.



(a) (b)

Figure 5: 95% credible intervals and prediction intervals for the actuator tip displacement.

4. Surrogate Model Development and Feedback Control Implementation

The HEM model is computationally intensive, which motivates the construction of a surrogate model for control design. We selected a dynamic mode decomposition (DMD) as a surrogate model.

We developed the DMD in the manner detailed in [11, 12] . The dynamics of a nonlinear system can be represented as a infinite-dimensional linear map,

$$x_{k+1} = Ax_k,$$

where A is a linear map termed the Koopman Operator. DMD approximates the Koopman Operator A to approximate the dynamical system. We combine periodic observations in a matrix as columns,

$$X_1^{M-1} = [x_1, x_2, ..., x_{M-1}].$$

The columns of X_1^{M-1} are elements of a Krylov subspace

$$X_1^{M-1} = [x_1, Ax_1, ..., A^{M-2}x_1].$$

We can express the final observation, x_M, as a linear combination of the Kyrlov basis and a residual term, r, that is orthogonal to the Krylov space:

$$x_M = \sum_{k=1}^{M-1} s_k x_k + r.$$

Note that $X_2^M = AX_1^{M-1}$. Using the expression for x_M, we obtain

$$X_2^M = X_1^{M-1}\tilde{A} + re_{(M-1)},$$

where $e_{(M-1)}$ is the $(M-1)^{th}$ unit vector and $\tilde{A}$ is the companion matrix with the s_i terms in the final column. We note that $\tilde{A}$, is the finite-dimensional tangential approximation of the Koopman Operator; i.e., the eigenvectors and eigenvalues of $\tilde{A}$ approximate the Koopman modes and eigenvalues. The approximate Koopman operator can be expressed as

$$A = X_2^M (X_1^{M-1})^\dagger,$$

where † denotes the Moore-Penrose inverse. We utilize the low dimensionality of the data matrix by taking an SVD of X_1^{M-1} when constructing the Moore-Penrose Inverse. We truncate X_1^{M-1} by retaining the r largest singular values and their corresponding singular vectors. This prohibits the problem from being ill-conditioned.

As detailed in [18], dynamic mode decomposition with control (DMDc) take similar ideas from DMD and applies it to a control system. We can express

$$X_2^M \approx [A \;\; B] \begin{bmatrix} X_1^{M-1} \\ U \end{bmatrix},$$

where $U = [u_1, u_2, \ldots, u_{m-1}]$ and u_i is the control vector at the i^{th} observation. We then observe that the tangential approximation for the A and B matrix in a control system is

$$[A \;\; B] = X_2^M \begin{bmatrix} X_1^{M-1} \\ U \end{bmatrix}^\dagger.$$

Once A and B are determined, we can approximate the nonlinear system by

$$x_{k+1} = Ax_k + Bu_k,$$

and use the solution for control design.

We employed the output from the tip of the PZT bimorph actuator, computed using the high-fidelity homogenized energy model (HEM), as synthetic data for the dynamic mode decomposition with control (DMDc) algorithm. In Figure 6, we display the data used to create the approximate A and B matrices used to develop the control algorithm. We note that DMDc accurately approximates the data. The data was sampled at a rate of 500 equally-spaced samples per second. The control data matrix is constructed from the voltages that are applied to both patches. When we attempted to use only one voltage to construct the control matrix, the algorithm failed to produce an accurate surrogate. This is due to the relation between the two voltages. We truncated the data and control matrix at $r = 3$ singular values since there is a drop-off on the order of 10^{-6} in magnitude of the singular values. This allows us to have at least two singular values for the control and at least one for the system.

As detailed in [4], we implemented feedback control based on the voltage $v_b(t)$. Figure 7 illustrates the tracking capabilities of a feedback control design from [1] applied to the surrogate model. Here $y(t)$ is the tip displacement of the PZT actuator characterized by the surrogate model. We added 5% noise to the control to display the stability of the controller and to analyze the controller's ability to acoomodate noise in the system. We observe that the system is able to track the target despite this added noise.

We then applied the feedback control, designed with the surrogate model, as a closed-loop controller to the high-fidelity homogenized energy model (HEM). Figure 8 displays the results of feedback control on the high-fidelity model with 5% noise added to the control. We note the capability of the feedback control to track the discontinuous target function. This demonstrates the robustness of the model-based design.

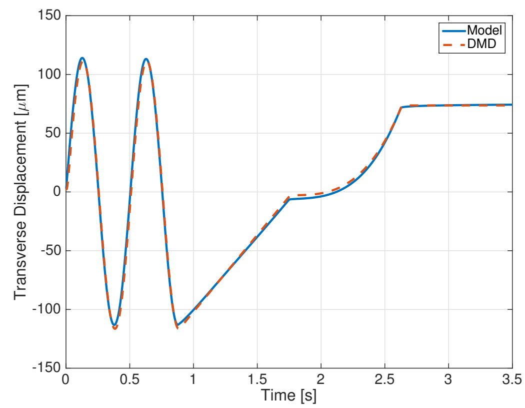


Figure 6: HEM-generated data and DMD approximation used to create the approximate A and B matrices.

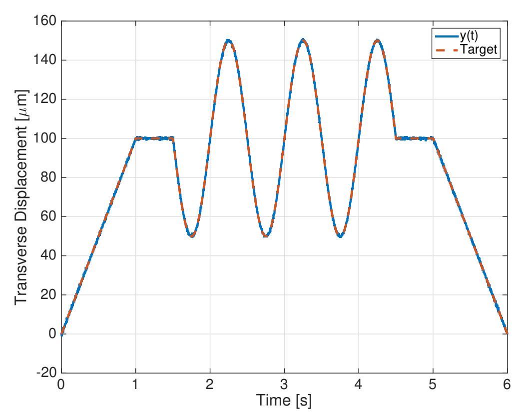


Figure 7: Performance of the feedback control applied to the DMD surrogate model.

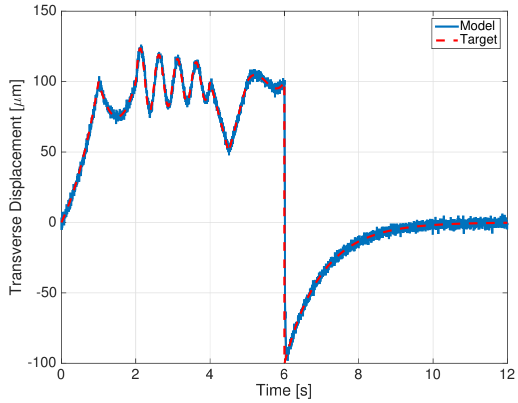


Figure 8: Performance of the DMDc-computed feedback control when applied to the high-fidelity homogenized energy model (HEM).

5. *Fractional-Order Model Development for Viscoelastic Materials*

Many smart materials being considered for AFOSR applications, including dielectric elastomers, exhibit complex rate-dependent viscoelastic behavior. The development of models that predict material behavior across a broad range of operating conditions is critical to achieve the unique actuator and sensor capabilities of the materials. Whereas the use of a nonlinear viscoelastic framework improves predictive capabilities, it does not provide the comprehensive predictive capabilities required to significantly improve model-based control design.

To address this, we investigated the use of fractional-order differential operators employed in conjunction with linear and nonlinear viscoelastic frameworks. Model calibration and validation using Very High Bond (VHB) 4910 experimental data demonstrated significantly improved predictive capabilities when using linear and nonlinear fractional order derivatives [13]. We illustrate 95% prediction intervals computed using Bayesian inference followed by uncertainty propagation techniques in Figure 9. These results demonstrate that by employing the fractional-order models, we are able to obtain very accurate predictions at both fast and slow stretch rates using a single set of parameters. This significantly improves the predictive capabilities of models and is expected to provide enhanced model-based control capabilities for systems exploiting these actuators. We present additional details about the model development, Bayesian inference, and uncertainty propagation in [15].

Evaluation of the fractional-order calculus operators is typically computationally expensive, which significantly complicates parameter inference using Bayesian algorithms. To improve efficiency, we

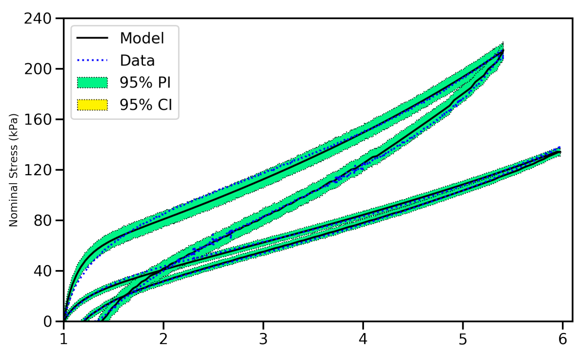


Figure 9: Uncertainty propagation and 95% prediction intervals at slow and fast stretch rates.

also investigated the development of novel quadrature methods to more accurately and efficiently approximate the Riemann–Liouville fractional derivatives. We presented results from this research in [14].

6. Development of a Python Package for Bayesian Inference

The accurate and robust inference of model parameters constitutes a critical first step when quantifying the uncertainty of model responses and computing robust feedback controls. Bayesian analysis is natural since it directly provides distributions for model parameters and initial conditions but it comes at the cost of significant computation overhead associated with sampling the posterior distribution thousands to millions of times. We employ a Delayed Rejection Adaptive Metropolis (DRAM) algorithm due to its robustness when inferring highly correlated parameters in nonlinearly parameterized models [10]. Specifically, delayed rejection (DR) enhances mixing of the chain, which mitigates stagnation regions. Furthermore, the DR method can be applied recursively for n-stages, although in practice a single stage of DR is often sufficient. For many problems DRAM decreases the number of simulations required to reach convergence and overcome bias from initialization.

Whereas there exists a robust MATLAB implementation of the algorithm, we increasingly employ Python due to its open source nature and facility when coupling production codes in various languages. This component of the program focused on the development and documentation of the Python package `pymcmcstat`, which provides a robust, user-friendly interface for running MCMC simulations using the DRAM algorithm.

We maintained project updates at project homepage and on the GitHub repository site. Code is written in a unit-testable manner, with coverage being checked by Coveralls – currently at 98%. Pipeline testing is done using Travis-CI, and the package is currently supported to run using Python 3.6. Documentation for the code was built using `Sphinx` and is hosted by ReadTheDocs. Tutorial notebooks provide the user with a wide variety of example implementations, and help demonstrate the various options available within the package.

To provide an example, we included in the tutorial a demonstration of the coupling of the `pymcmcstat` package when estimating the parameters for an integer-order linear viscoelastic model. Details regarding the package are reported in [16].

Project Links:

- Project Homepage: `https://prmiles.wordpress.ncsu.edu/codes/python-packages/pymcmcstat/`
- GitHub Repository: `https://github.com/prmiles/pymcmcstat`
- Build: `https://travis-ci.org/prmiles/pymcmcstat`
- Online Tutorials: `https://nbviewer.jupyter.org/github/prmiles/notebooks/blob/master/pymcmcstat/index.ipynb`
- Code Documentation: `http://pymcmcstat.readthedocs.io/`

Supported Personnel

Nikolas Bravo	PhD Student, NCSU, Raleigh, NC. PhD Defense 6/28/19, Dissertation: *Synthesis of Uncertainty Quantification, Surrogate Modeling, and Robust Control Design for PZT Birmorph Actuators*, Employment: Raytheon, Tuscon, AZ
Paul Miles	Postdoc, NCSU, Raleigh, NC. Employment: Staff Scientist at Sandia National Laboratories Albuquerque.
Lider Leon	Postdoc, NCSU, Raleigh, NC. Employment: Staff Scientist at the Johns Hopkins Applied Physics Lab.
Amanda Bernstein	Postdoc, NCSU, Raleigh, NC. Employment: ORISE Postdoc at the EPA.
Ralph Smith	Distinguished University Professor, NCSU, Raleigh, NC

Publications

1. A. Alexanderian, P. Gremaud and R.C. Smith, "Variance-based sensitivity analysis for time-dependent processes," *Reliability Engineering and System Safety*, under review.

2. N. Bravo, *Synthesis of Uncertainty Quantification, Surrogate Modeling, and Robust Control Design for PZT Birmorph Actuators*, PhD Dissertation, North Carolina State University, Raleigh, NC 27695.

3. N. Bravo, J. Crews and R.C. Smith, "Data-driven model development and feedback control design for PZT bimorph actuators," Proceedings of the ASME Conference on Smart Materials, Adaptive Structures and Intelligent Systems (SMASIS), Snowbird, UT, 2017.

4. N. Bravo and R.C. Smith, "Parameter dependent surrogate model development for PZT biomorph actuators employed for robobee," Proceedings of the SPIE, Smart Structures and Materials 2019, Denver, CO.

5. N. Bravo, R.C. Smith and J. Crews, "Surrogate model development and feedforward control implementation of PZT bimorph actuators employed for robobee," Proceedings of the SPIE, Smart Structures and Materials 2017, Portland,OR, doi:10.1117/12.2259948.

6. N. Bravo, R.C. Smith and J. Crews,"Uncertainty quantification for PZT bimorph actuators, Proceedings of the SPIE, Smart Structures and Materials 2018, Denver, CO, March 2018.

7. H.L. Cleaves, A. Alexanderian, H. Guy, R.C. Smith and M. Yu, "Derivative-based global sensitivity analysis for models with high-dimensional inputs and functional outputs," *SIAM Journal on Scientific Computing*, to appear.

8. J.H. Crews, N. Bravo and R.C. Smith, "Model development for a PZT bimorph actuator employed for micro-air vehicles," Proceedings of the ASME 2016 Conference on Smart Materials, Adaptive Structures and Intelligent Systems (SMASIS), September 28–30, 2016.

9. P. Miles, G. Pash, W.S. Oates and R.C. Smith, "Numerical techniques to model fractional-order nonlinear viscoelasticity in soft elastomers," Proceedings of the 2018 ASME Conference on Smart Materials, Adaptive Structures, and Intelligent Systems (SMASIS), San Antonio, TX, September 2018.

10. P.R. Miles, G.T. Pash, R.C. Smith and W. Oates, "Global sensitivity analysis of fractional-order viscoelasticity models," Proceedings of the SPIE, Smart Structures and Materials 2019, Denver, CO.

11. P. Miles, G.T. Pash, R.C. Smith and W.S. Oates, "Parameter estimation using efficient fractional-order viscoelastic models for dielectric elastomers," *Journal of Intelligent Material Systems and Structures*, submitted.

12. P.R. Miles and R.C. Smith, "Parameter estimation using the Python package pymcmcstat," Proceedings of the 18th Python in Science Conference (SCIPY), 2019.

AFRL Point of Contact

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Transitions/Interactions

Transitions: Bayesian Analysis for Parameter and Model Uncertainty – DOE CASL and NNSA CNEC: We transitioned aspects of the Bayesian analysis and sampling-based techniques to construct prediction intervals to the DOE Consortium for Advanced Simulation of Light Water Reactors (CASL) Energy Innovation Hub and the NNSA Consortium for Nonproliferation Enabling Capabilities (CNEC).

Conference, Colloquia and Workshop Presentations and Short Courses

1. Invited Presentation: INFORMS Summer Roundtable on Uncertainty Quantification, Jackson Lake Lodge, WY, July 20, 2015.

2. Colloquium: Applied Mathematics and Statistics, Colorado School of Mines, Golden, CO, Oct 4, 2015.

3. CSRI Seminar: CSRI Seminar, Sandia National Laboratories, Albuquerque, NM, Nov 20, 2015.

4. Invited Presentation: NIST Workshop on Uncertainty Quantification in Materials Science, Gaithersburg, MD, January 14, 2016.

5. Presentation: SPIE Symposium on Smart Structures and Materials, Las Vegas, NV, March 21, 2016.

6. Colloquium: NASA Langley Research Center, June 27, 2016.

7. Invited Presentation: IFAC Symposium on Nonlinear Control Systems, Monterey, CA, August 24, 2016.

8. Short Course on Uncertainty Quantification with W. Oates: ASME 2016 Conference on Smart Materials, Adaptive Structures and Intelligent Systems (SMASIS), Stowe, VT, September 27, 2016.

9. Keynote Presentation: ASME 2016 Conference on Smart Materials, Adaptive Structures and Intelligent Systems (SMASIS), Stowe, VT, September 28, 2016.

10. Presentation by N. Bravo: ASME 2016 Conference on Smart Materials, Adaptive Structures and Intelligent Systems (SMASIS), Stowe, VT, September 29, 2016.

11. Colloquium: Department of Computer Science and Engineering, University of South Carolina, October 21, 2016.
12. Keynote Presentation: 36th Annual Mathematics Symposium at Western Kentucky University, November 11, 2016.
13. Short Course on Uncertainty Quantification with W. Oates: SPIE Symposium on Smart Structures and Materials, Portland, OR, March 27, 2017.
14. Presentation by N. Bravo, SPIE Symposium on Smart Structures and Materials, Portland, OR, March 27, 2017.
15. Presentation: SIAM Conference on Computational Science and Engineering, Atlanta, GA, February 28, 2017.
16. Plenary Presentation: Workshop on Parameter Estimation and Uncertainty Quantification for Dynamical Systems, University of Pittsburgh, Pittsburgh, PA, March 5, 2017.
17. Invited Presentation: American Control Conference (ACC), Seattle, WA, May 25, 2017.
18. Short Course on Uncertainty Quantification with W. Oates: College of Engineering, Florida State University, June 23, 2017.
19. Short Course on Sensitivity Analysis and Uncertainty Quantification with W. Oates, Florida State University, ASME 2017 Conference on Smart Materials, Adaptive Structures and Intelligent Systems (SMASIS), Snowbird, UT, September 17, 2017.
20. Presentation by N. Bravo, ASME 2017 Conference on Smart Materials, Adaptive Structures and Intelligent Systems (SMASIS), Snowbird, UT, September 18, 2017.
21. Colloquium: Department of Aerospace Engineering Colloquium, Texas A&M University, College Station TX, November 2, 2017.
22. Invited Presentation: Workshop on Key UQ Methodologies and Motivating Applications, Newton Institute, Cambridge University, UK, January 8, 2018.
23. Invited Presentation: Workshop on Surrogate Models for UQ in Complex Systems, Isaac Newton Institute for Mathematical Sciences, Cambridge University, UK, February 5, 2018.
24. Colloquium: Applied Statistics Group, Lawrence Livermore National Laboratory, Livermore, CA, February 22, 2018.
25. Plenary Presentation: SPIE Smart Structures and Nondestructive Evaluation, Denver, CO, March 5, 2018.
26. Short Course on Sensitivity Analysis and Uncertainty Quantification with William Oates, Florida State University, SPIE Symposium on Smart Structures and Materials, Denver, CO, March 5, 2018.
27. Presentation by P. Miles, SPIE Symposium on Smart Structures and Materials, Denver, CO, March 5, 2018.
28. Presentation by N. Bravo, SPIE Symposium on Smart Structures and Materials, Denver, CO, March 6, 2018.

29. “Uncertainty Quantification,” Short Course given at DATAWorks 2018, Defense and Aerospace Test and Analysis (DATA) Workshop, Springfield, VA, March 20, 2018.

30. Keynote Presentation: Workshop on Dynamics, Control and Numerics for Fractional PDEs, Hotel Embassy Suites, Isla Verde, Carolina, Puerto Rico, December 5, 2018.

31. Colloquium: Department of Mechanical and Nuclear Engineering, Penn State University, State College, PA, February 4, 2019.

32. Keynote Presentation: SCALA 2019: Scientific Computing around Louisiana, Tulane University, New Orleans, LA, Feb 15, 2019.

33. “Uncertainty Quantification,” Short Course given at DATAWorks 2018, Defense and Aerospace Test and Analysis (DATA) Workshop, Springfield, VA, April 9, 2019.

34. Colloquium: NASA Jet Propulsion Laboratory, Pasadena, CA, June 24, 2019.

Program Websites: To facilitate the dissemination of models, codes, and data to the general community, we developed the website `https://rsmith.math.ncsu.edu/Smart_Materials_Database.html`. The goal is to establish a repository for students and researchers investigating a broad range of transductive materials for advanced applications.

To facilitate Bayesian inference using the Delayed Rejection Adaptive Metropolis (DRAM) algorithm, we implemented the Python package pymcmcstat. The package and detailed examples are maintained at the website `https://prmiles.wordpress.ncsu.edu/codes/python-packages/pymcmcstat/`.

Honors and Awards

- Smith was recipient of the 2016 ASME *Adaptive Structures and Material Systems Award* “for extraordinary contributions in the development of smart materials and adaptive structures through constitutive model development, modeling and nonlinear control, and uncertainty analysis; and for modeling research that has been validated across a broad range of smart materials.”

- Smith was recipient of the SPIE *2017 Smart Structures and Materials Lifetime Achievement Award* “In recognition of his sustained contributions to the advancement of Smart Structures and Materials Technologies.”

- Smith was elected a *SIAM Fellow* for the class of 2018 for contributions in uncertainty quantification and materials science.

- Smith was named a Distinguished University Professor of Mathematics for his work on modeling, control and uncertainty quantification for smart material systems and for improving the quality of NC State and serving its mission through service and involvement in the campus community.

Patents

None

Acknowledgement/Disclaimer
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[2] N. Bravo, *Synthesis of Uncertainty Quantification, Surrogate Modeling, and Robust Control Design for PZT Birmorph Actuators*, PhD Dissertation, North Carolina State University, Raleigh, NC 27695.

[3] N. Bravo, J. Crews and R.C. Smith, "Data-driven model development and feedback control design for PZT bimorph actuators," Proceedings of the ASME Conference on Smart Materials, Adaptive Structures and Intelligent Systems (SMASIS), Snowbird, UT, 2017.

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