Evaluation of the computer simulation model NTVPM for assessing military tracked vehicle cross-country mobility

J.Y. Wong[a,*], P. Jayakumar[b], and J. Preston-Thomas[c]

[a] *Vehicle Systems Development Corporation, Toronto, Ontario, Canada*
[b] *U.S. Army Tank Automotive Research, Development and Engineering Center, Warren, MI, USA*
[c] *National Research Council of Canada, Ottawa, Ontario, Canada*

Abstract: Ground vehicle mobility is an important issue for defence operations. Currently, in the United States and some other NATO countries, the NATO Reference Mobility Model (NRMM) is used to evaluate military ground vehicle mobility. The cross-country performance prediction module of NRMM is based on empirical relations established using test data collected decades ago. It has inherent limitations, such as the uncertainty whether the empirical relations can be extrapolated beyond the test conditions upon which they are derived. This suggests that there is a need for the development of a physics-based model that takes into account the advancements in terramechanics and in modelling/simulation techniques. This paper describes the results of a detailed evaluation of the physics-based model — the Nepean Tracked Vehicle Performance Model (NTVPM) — for assessing military tracked vehicle cross-country mobility. The performance of a notional tracked vehicle (an armoured personnel carrier) predicted by the latest version of NTVPM is compared with test data obtained on sandy terrain, muskeg, and snow-covered terrain. The correlations between the predicted and measured performance are evaluated using the coefficient of correlation, coefficient of

* Corresponding author.
E-mail addresses: vsdccanada@yahoo.ca (J.Y. Wong), paramsothy.jayakumar.civ@mail.mil (P. Jayakumar), jon.preston-thomas@nrc-cnrc.gc.ca (J. Preston-Thomas).

determination, root mean squared deviation, and coefficient of variation. The applications of NTVPM to predicting the maximum possible speed (speed-made-good) on a given terrain, the sensitivity of vehicle performance to terrain parameters, and the mean maximum pressure (MMP) are demonstrated. The results of this study indicate that NTVPM can form the basis for the development of the next generation cross-country performance assessment methodology for military tracked vehicles.

Keywords*:* Coefficient of correlation; Coefficient of variation; Computer simulation models; Cross-country performance; Experimental study; Mean maximum pressure; Physics-based models; Tracked vehicle mobility.

Nomenclature

A_l	rigid area of track link as a portion of the product of track pitch and width
A_u	parameter characterizing terrain response to repetitive normal loading
b	width; track contact width
b_{be}	width of vehicle belly (hull)
c	cohesion
c_{ru}	adhesion
D	diameter; road wheel diameter
F	tractive effort, thrust or gross traction
F_d	drawbar pull or net traction
f_t	radial deflection of pneumatic tire
i	vehicle or track slip; road wheel index
j	shear displacement
K	shear deformation parameter
K_r	ratio of residual shear stress to maximum shear stress
K_{ru}	shear deformation parameter of rubber-terrain shearing or belly-terrain shearing
K_w	shear displacement where the maximum shear stress occurs for a shear curve exhibiting a “hump”
k_c, k_ϕ	parameters of the Bekker pressure-sinkage equation
k_m	pressure-sinkage parameter for the underlying peat of muskeg
k_{p1}, k_{p2}	pressure-sinkage parameters for snow covers with a crust
k_{z1}, k_{z2}	pressure-sinkage parameters for snow covers with a crust
k_0	parameter characterizing terrain response to repetitive normal loading
L_{be}	length of vehicle belly (hull)
L_{cr}	crust (ice layer) bearing capacity factor
L_t	total length of track in contact with ground
l	length
M_{cr}	crust (ice layer) bearing capacity factor

m_m	parameter characterizing the strength of the surface mat of muskeg
n	exponent of the Bekker pressure-sinkage equation
n_r	number of road wheel stations on one track
p	pressure
p_{be}	normal pressure on the vehicle belly (hull)
R	coefficient of correlation
R^2	coefficient of determination
R_{be}	belly drag
R_t	motion resistance
R_{tex}	external motion resistance of track system
R_{tin}	internal motion resistance of track system
r	radius
s	shear stress
s_{be}	shear stress on vehicle belly (hull)
t_t	track pitch
V	actual vehicle forward speed
V_a	absolute velocity of a point on the track
V_j	slip velocity of a point on the track in contact with the ground
V_t	vehicle theoretical forward speed
W	vehicle weight
z	sinkage
α	angle
α_b	angle of vehicle belly (hull) with respect to horizontal
δ	inclination angle of track frame or vehicle body
ϕ	angle of internal shearing resistance
ϕ_{ru}	angle of rubber-terrain shearing resistance or belly-terrain shearing resistance
ω	angular speed

1. INTRODUCTION

Ground vehicle mobility on unprepared terrain is an important issue for defence operations, for resource exploration and exploitation industries, and for extraterrestrial surface exploration [1].

In the United States and some other NATO countries, military ground vehicle mobility is currently evaluated using the NATO Reference Mobility Model (NRMM) [2]. The NRMM methodology for predicting the cross-country performance of military vehicles is empirically based. It takes into account a limited number of vehicle design features and essentially a single terrain parameter measured by the cone penetrometer.

While NRMM has been used for evaluating military vehicle candidates in the procurement process or in the operational planning for deployment of military vehicles in the field, it has inherent limitations. These include the uncertainty whether empirical relations established with test data collected decades ago can be applied to evaluating current or future generation military vehicles with advanced design features, as well as whether they can be extrapolated beyond the operating environments upon which the empirical relations were derived. This suggests that there is a need for the development of a physics-based, next generation methodology for predicting military ground vehicle mobility that takes into account the advancements in terramechanics and in modelling/simulation techniques.

With the progress made in terramechanics over the years, computer simulation models for evaluating cross-country performance of off-road vehicles have emerged [3]. These include the computer simulation model - the Nepean Tracked Vehicle Performance Model (NTVPM) for performance and design evaluation of tracked vehicles, developed by

Vehicle Systems Development Corporation (VSDC), Toronto, Ontario, Canada. NTVPM is physics-based and is developed on the understanding of the physical nature and the mechanics of track-terrain interaction [3].

The objective of the study described in this paper is to evaluate whether the computer simulation model NTVPM could form the basis for the development of the next generation cross-country performance assessment methodology for military tracked vehicles.

2. FEATURES OF NTVPM

2.1 Basic features

NTVPM is for evaluating the steady-state cross-country performance of vehicles with segmented metal tracks with relatively short track pitch, commonly used in current generation military tracked vehicles; or rubber band tracks, proposed for use in future generation military tracked vehicles. These two types of track are hereinafter referred to as the flexible track.

NTVPM takes into account all major vehicle design parameters that affect its cross-country performance and all pertinent terrain characteristics.

2.2 Capabilities of NTVPM

NTVPM is intended to provide an effective and efficient engineering tool for guiding:

(a) the evaluation of cross-country performance of tracked vehicle candidates in the procurement process;

(b) the operational planning for deployment of tracked vehicles in a given operating environment;

(c) the development of future generation tracked vehicles from the cross-country performance perspective.

The capabilities of NTVPM include the evaluation of:

(a) the steady-state cross-country performance of single-unit or two-unit articulated tracked vehicles, expressed in terms of vehicle sinkage, tractive effort (thrust, gross traction), external motion resistance due to vehicle-terrain interaction, drawbar pull (net traction), and tractive (drawbar) efficiency as functions of slip on deformable terrains;

(b) the gradeability of single-unit or two-unit articulated tracked vehicles on deformable terrains (i.e., the maximum slope that a single-unit or two-unit articulated tracked vehicle can negotiate under steady-state operating conditions);

(c) the normal and shear stress distributions on the track-terrain interface of single-unit or two-unit articulated tracked vehicles under steady-state operating conditions, and the corresponding mean maximum pressure (MMP) on the track-terrain interface. MMP is used as an indicator for cross-country mobility of military vehicles in some NATO countries [4].

It should be noted that the steady-state cross-country performance is widely accepted as a basis for comparing the mobility of off-road vehicles. For instance, the cross-country prediction module of the current version of NRMM predicts the maximum possible speed (speed-made-good) of vehicles in straight-line motion, under steady-state operating conditions.

2.3 Vehicle input parameters

The vehicle design parameters that are taken into account in NTVPM include: vehicle sprung and unsprung weight; location of the center of gravity; number of road wheels; road wheel diameter and spacing; sprocket and idler (tensioning wheel) diameters and their locations (at the front or rear of the vehicle); supporting roller diameters and their locations; track dimensions and contact geometry with the terrain; track weight per unit length (for predicting the shape of the upper run of the track between supporting rollers); track longitudinal stiffness (for predicting the elongation of the track under tension); initial track tension (i.e., the tension in the track system when the vehicle is stationary on a level, hard ground, which is an indication of the initial tightness of the track); road wheel suspension characteristics (such as independent, pivot-arm or translational spring suspensions, with linear or nonlinear load-deflection characteristics); ground clearance; vehicle belly (hull) longitudinal profile and width (for evaluating vehicle belly-terrain interaction, when track sinkage being greater than vehicle ground clearance); and drawbar hitch location (for predicting the effect of drawbar pull on load transfer among road wheels).

The unique features of NTVPM include taking into account the effects of initial track tension, suspension characteristics, vehicle belly-terrain interaction, and track longitudinal stiffness on the cross-country performance of tracked vehicles. These factors have been shown to have significant effects on the cross-country mobility of tracked vehicles under certain circumstances.

The initial track tension has been found to have noticeable effects on cross-country performance, particularly on highly deformable terrain [5-8]. If the initial track tension is low, the track will be loose and track segments between road wheels will not support much load. Under theses circumstances, a tracked vehicle may essentially behave like a multi-wheeled vehicle, without the benefits of the track to provide a much larger ground contact area than the wheels. On the other hand, if the initial track tension is high, the track will be tight and track segments between road wheels will support substantial load. This reduces the peak normal pressures under the track, hence track sinkage and external motion resistance due to track-terrain interaction. On highly deformable terrain, such as deep snow, track sinkage may exceed vehicle ground clearance and the vehicle belly may come into contact with the terrain surface. Under these circumstances, increasing the initial track tension would have two significant effects. Firstly, it will reduce the sinkages of both the track and the belly, hence lowering the external track motion resistance and the belly drag caused by vehicle belly-terrain interaction. Secondly, it will reduce the load supported by the belly, hence increasing the proportion of the load exerted on the track. On terrains with a significant frictional component in their shear strength, this will enable the track to develop higher tractive effort, hence improving its mobility. These findings lead to the concept of an initial track tension regulating system, with which the driver may increase the initial track tension prior to crossing a soft terrain patch. This system is analogous to the central tire inflation system for improving wheeled vehicle mobility.

The road wheel suspension characteristics affect the load distribution among road wheels, track sinkage and associated external motion resistance [5, 7, 8]. It would also affect the attitude of the vehicle body (i.e., nose-up or nose-down). When operating on highly deformable terrain, such as deep snow, where vehicle belly is in contact with the terrain surface, vehicle attitude affects vehicle belly-terrain interaction. If the vehicle belly takes a nose-up attitude, then the vehicle belly behaves like a bulldozer and induces significant belly drag. On the other hand, if the vehicle belly takes a nose-down attitude, then the major part of the belly will not be in contact with the terrain surface resulting in a lower belly drag and higher tractive effort.

The track longitudinal stiffness characterizes track tension-elongation relationship. If the track longitudinal stiffness is low, its elongation under tension will be high. The portion of the track in contact with the terrain will become loose, similar to the effects of low initial track tensions. This will adversely affect the cross-country performance of tracked vehicles, especially on highly deformable terrain [7, 8].

As an example, the input parameters of the notional tracked vehicle (an armoured personnel carrier) used in this study are presented in Appendix A.

2.4 Terrain input parameters

The terrain input parameters that NTVPM takes into account include the following, obtained using the bevameter [3, 9, 10, 11]:

(a) pressure-sinkage parameters;

(b) shear strength parameters (cohesion and angle of internal shearing resistance);

(c) shear deformation parameters for characterizing the shear stress-shear displacement relationship;

(d) rubber-terrain shear parameters for tracks with rubber pads or for rubber band tracks;

(e) belly-terrain shear parameters for evaluating belly-terrain interaction when the vehicle belly contacts the terrain surface.

The parameters of various types of terrain used in this study are given in Appendix B.

It should be pointed out that a framework for the development of an automated bevameter terrain data acquisition and processing system for field use has been established [12]. It can provide an efficient tool for the rapid collection of terrain data in the field.

3. APPROACH TO THE DEVELOPMENT OF NTVPM

The development of NTVPM is based on the understanding of the physical nature of vehicle-terrain interaction and on the analysis of the mechanics of track-terrain interaction. In the analysis, the segmented metal track with relatively short track pitch or the rubber band track is idealized as a flexible and extensible belt. The focus is on the prediction of the normal and shear stress distributions on the track-terrain interface, from which the tractive performance parameters of a tracked vehicle can be derived. The detailed analysis of the mechanics of track-terrain interaction and the procedures for predicting tracked vehicle tractive performance are given in the reference [3]. A brief description of the general approach is outlined below.

3.1 Prediction of normal pressure distribution under the track

A schematic diagram of a flexible track on a deformable terrain in steady-state, straight-line motion is shown in Figure 1. When a tracked vehicle rests on a firm ground, the track segments between road wheels generally lie flat on the surface. On the other hand, when the vehicle travels on a deformable terrain, the normal load applied through the road wheel-track system causes the terrain to deform, which results in track sinkage. The track segments between road wheels take up load and as a consequence they deflect and have a form of a curve. In the analysis, the elongation of the track caused by tension is taken into account. Furthermore, the terrain under the track is subject to repetitive loading of consecutive road wheels, and the vehicle may take a nose-up attitude as illustrated in Figure 1.

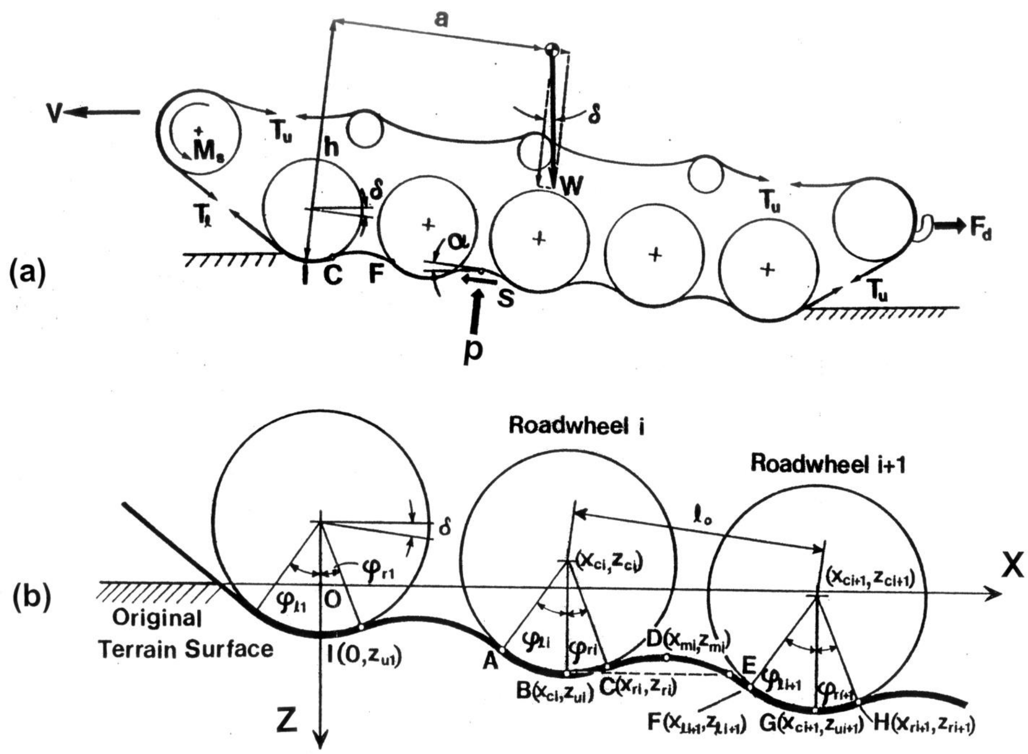


Figure 1 Interaction between a flexible track and deformable terrain.

The deflected track in contact with the terrain may be divided into two sections: one in contact with both the road wheel and the terrain, such as segments *AC* and *FH* shown in Figure 1 (b); the other in contact with the terrain only, such as segment *CF* shown in the figure. The shape of the track segment in contact with the road wheel, such as *AC*, is defined by the profile of the road wheel. The shape of the track segment in contact with the terrain only, such as *CF*, is determined by the track tension in the segment, spacing between two adjacent road wheels, and the pressure-sinkage relationship and response to repetitive loading of the terrain. With the shape of the track in contact with the terrain determined and with the pressure-sinkage relationship and response to repetitive loading of the terrain known, the normal pressure distribution under a flexible and extensible track is predicted. The details are given in the reference [3].

3.2 Prediction of shear stress distribution under the track

To predict the shear stress distribution, the shearing action of a flexible track is analysed and the approach to predicting the shear stress distribution is outlined below.

The shear stress at a given point on the track–terrain interface is a function of shear displacement, measured from the point where shearing (or reshearing) begins, and the normal pressure at that point. The shear displacement developed under a flexible track may be determined from an analysis of the slip velocity V_j, similar to that for a rigid track or a rigid wheel [3, 9, 11]. The slip velocity V_j of a point P on a flexible track relative to the terrain surface is the tangential component of the absolute velocity V_a shown in Figure 2. The magnitude of the slip velocity V_j is expressed by

$$V_j = V_t - V\cos\alpha = r\omega - r\omega(1-i)\cos\alpha = r\omega\left[1-(1-i)\cos\alpha\right] \quad (1)$$

where r and ω are the pitch radius and angular speed of the sprocket, respectively; i is the slip of the track; α is the angle between the tangent to the track at point P and the horizontal; V_t is the theoretical speed of the track (i.e., $V_t = r\omega$); V is the actual forward speed of the track (vehicle).

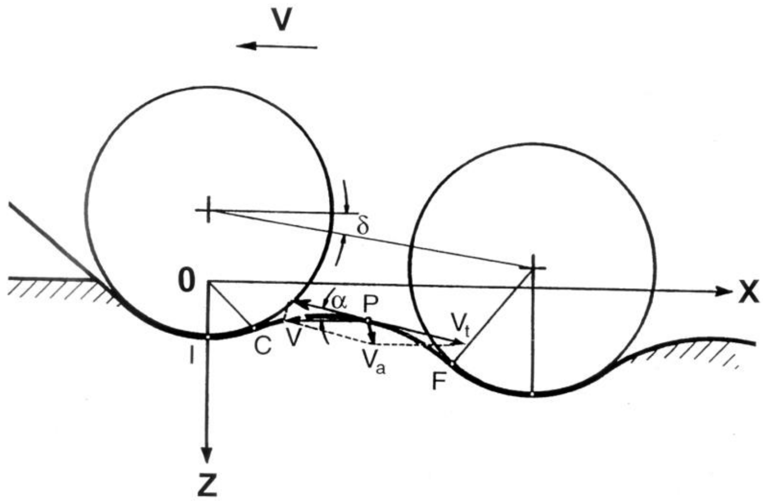


Figure 2 Slip velocity of a point on a flexible track in contact with deformable terrain.

The shear displacement *j* along the track-terrain interface is given by

$$j = \int_0^t r\,\omega[1-(1-i)\cos\alpha]\,dt = \int_0^l r\,\omega[1-(1-i)\cos\alpha]\frac{d\,l}{r\omega} = \int_0^l \left[1-(1-i)\frac{d\,x}{d\,l}\right]d\,l = l-(1-i)\,x \quad (2)$$

where *l* is the distance along the track between point *P* and the point where shearing (or reshearing) begins and *x* is the corresponding horizontal distance between point *P* and the initial shearing (or reshearing) point.

With the shear displacement *j* along the track-terrain interface determined and with the shear stress-shear displacement relationship and the response to repetitive shear loading of the terrain known, together with the normal pressure distribution determined previously, the shear stress distribution is predicted. The details are given in the reference [3]

.

3.3 Prediction of external motion resistance, tractive effort and drawbar pull

When the normal pressure and shear stress distributions under a tracked vehicle at a given slip have been determined, the tractive performance of the vehicle can readily be predicted [3]. The tractive performance of an off-road vehicle is usually characterized by its motion resistance, tractive effort, and drawbar pull (the difference between the tractive effort and motion resistance) as functions of slip.

(a) The external motion resistance R_{tex} of the track due to track-terrain interaction can be determined from the horizontal component of the normal pressure acting on the track in contact with the ground. For a vehicle with two tracks,

$$R_{tex} = 2b\int_0^{L_t} p\sin\alpha\, dl \qquad (3)$$

where b is the contact width of the track; L_t is the length of track in contact with the terrain; p is normal pressure; and α is the angle of the track element with respect to the horizontal.

(b) If the track sinkage is greater than the ground clearance of the vehicle, the belly (hull) will be in contact with the terrain, giving rise to an additional belly drag R_{be}. It can be determined from the horizontal components of the normal and shear stresses acting on the belly–terrain interface and is described by

$$R_{be} = b_{be}\left[\int_0^{L_{be}} p_{be}\sin\alpha_b\, dl + \int_0^{L_{be}} s_{be}\cos\alpha_b\, dl\right] \qquad (4)$$

where b_{be} is the width of the belly; L_{be} is the length of the belly; α_b is the angle of the belly with respect to the horizontal; p_{be} and s_{be} are the normal and shear stresses on the belly–terrain interface, respectively.

(c) The tractive effort F of the vehicle can be calculated from the horizontal component of the shear stress acting on the track in contact with the terrain. For a vehicle with two tracks, F is given by

$$F = 2b\int_0^{L_t} s\cos\alpha\, dl \tag{5}$$

where *s* is the shear stress on the track-terrain interface.

Since both the normal pressure *p* and shear stress *s* are functions of track slip, the external track motion resistance R_{tex}, belly drag R_{be} (if any), and tractive effort *F* vary with slip.

For a track with rubber pads, part of the total tractive effort is generated by the rubber–terrain shearing. To predict the tractive effort developed by the rubber pads, the portion of the vehicle weight supported by the rubber pads should be estimated and the characteristics of rubber – terrain shearing should be taken into account.

(d) The drawbar pull F_d of the vehicle can be considered as the difference between the tractive effort and the total external motion resistance (including the belly drag, if any), and can be expressed by

$$F_d = F - R_{tex} - R_{be} \tag{6}$$

From this equation, the relationship between drawbar pull F_d and track slip *i* can be predicted. The detailed procedures for predicting tracked vehicle tractive performance are given in the reference [3].

It should be pointed out that predictions by NTVPM of the normal and shear stress distributions on the track-terrain interface and of the vehicle tractive performance under steady-state operating conditions are based on solving a set of dynamic equilibrium equations of the vehicle and its sub-systems, such as the road wheel suspension system. For predicting steady-state vehicle performance, this is inherently more efficient than the time integration of a large set of equations of motion with small time steps, commonly used in multibody vehicle dynamics models. Simulating tracked vehicle tractive performance over a range of slips using NTVPM usually takes only a few seconds with personal computers. Thus, together with an automated terrain data acquisition and processing system noted in Section 2.4, NTVPM provides an effective tool for the speedy assessment of vehicle mobility for the deployment of military tracked vehicles in the field.

3.4 Operation of NTVPM

The procedures for predicting the tractive performance of tracked vehicles described above are implemented in the simulation model NTVPM in a user-friendly manner. All vehicle and terrain data are input to NTVPM using the dialog (edit) box format, for the convenience of the user. It runs on Microsoft Windows operating systems, including XP, 7, 8, and 10. Figure 3 shows the control centre, as displayed on the computer monitor screen, for the operation of the latest version of NTVPM.

NTVPM has been successfully employed to assist vehicle manufacturers in the development of high-mobility military tracked vehicles [13, 14], and governmental agencies in the evaluation of the cross-country mobility of military vehicles in North America, Europe, and Asia.

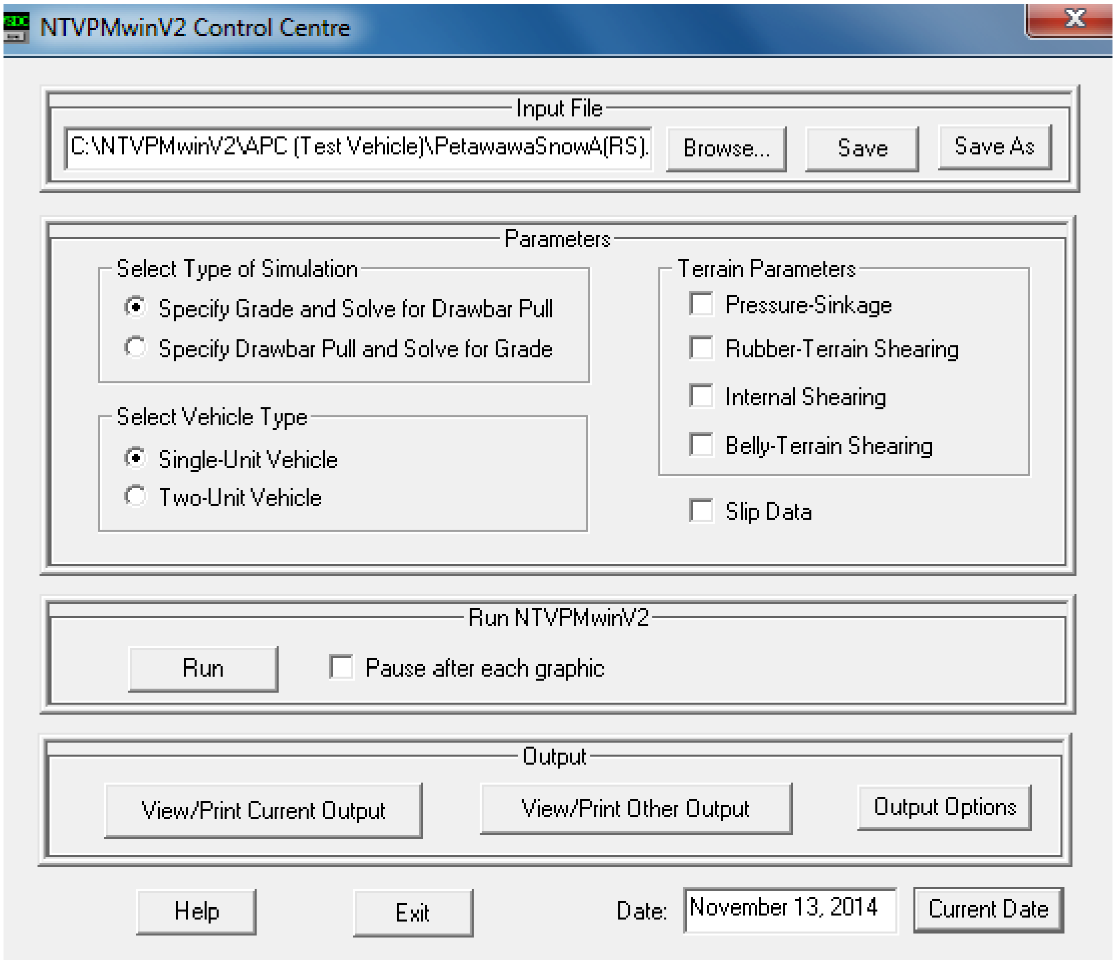


Figure 3 Control centre for the operation of NTVPM, as shown on the monitor screen.

4. FIELD TESTS FOR EVALUATION OF NTVPM

To evaluate the predictive capabilities of NTVPM, a series of field tests was conducted. The first set of tests was performed using a notional tracked vehicle (an armoured personnel carrier) and a two-unit articulated tracked vehicle on sandy terrain (LETE Sand), two types of muskeg (Petawawa Muskeg A and B) and two types of snow-covered terrain (Petawawa Snow A and B) [3, 15]. The second set of tests was performed using another two-unit articulated tracked vehicle on snow [13].

In this paper, the field test data for the notional tracked vehicle are used to evaluate the predictive capabilities of NTVPM. The instrumented notional tracked vehicle used in the tests is shown in Figure 4.

Figure 4 The instrumented notional tracked vehicle used in field tests.

To measure the normal pressure on the track-terrain interface of the test vehicle, four Kulite IPT-750 flush stainless steel diaphragm pressure transducers were used. These transducers employ semiconductor strain gauge elements, bonded directly to the inner surface of the diaphragm. The diameter of the diaphragm in contact with the terrain is 1.9 cm (0.75 in.). Four pressure transducers were mounted on a track link of the test vehicle: two on the rubber pad and the other two on the metal part of the track link. The signals from the transducers were transmitted through an adapter and a cable to a multi-channel signal conditioner and a recorder installed inside the test vehicle.

In addition to the normal pressure on the track-terrain interface, a number of other performance parameters of the test vehicle were monitored during tests. These include the dynamic sinkage of the front and rear road wheels with respect to the terrain surface; the trim angle of the vehicle body (hull); the distance that the vehicle travelled during tests; the revolutions of the sprockets of the tracks during tests; time signal; drawbar pull (monitored by a strain-gauge type load cell installed between the test vehicle and the towed vehicle), etc. The data recorded were later digitized and processed. The details of the vehicle tractive performance testing are given in the reference [3].

5. EVALUATION OF NTVPM FOR ASSESSING TRACKED VEHICLE CROSS-COUNTRY MOBILITY

A general comparison of the tractive performance of the notional tracked vehicle predicted by an earlier version of NTVPM and test data on sandy terrain (LETE Sand), muskeg (Petawawa Muskeg B) and snow-covered terrain (Petawawa Snow A) has been presented previously [3]. A detailed evaluation of the correlations between the tractive performance of the notional tracked vehicle predicted by the latest version of NTVPM and test data is given in this section. It includes the examination of the coefficient of correlation *R,* the coefficient of determination R^2, the root mean squared deviation *RMSD,* and the coefficient of variation *CV.* In the evaluation, the test data of the vehicle tractive performance are taken from the reference [3, 15]. As noted previously, the vehicle and terrain input data used in this study are given in Appendix A and B, respectively.

5.1 Correlations between the measured tractive performance and that predicted by NTVPM

(A) LETE Sand

(a) The measured and predicted tractive performance

Figure 5 shows the measured drawbar pull vs. slip data of the notional tracked vehicle on LETE Sand, together with the drawbar pull - slip curve predicted by NTVPM. The predicted curve shown in the figure is based on the mean values of terrain parameters for LETE Sand presented in Tables B.1 and B.2 of Appendix B.

(b) Correlation between the measured and predicted tractive performance

To quantitatively evaluate the overall correlations between the predictions by NTVPM and experimental data is complex. It appears that so far there are no formal standards [16]. Based on a review of the literature, to quantitatively evaluate the overall correlations between the tractive performance predicted by NTVPM and the measured data, two criteria are adopted in this study. Firstly, the coefficient of correlation *R* and the associated coefficient of determination R^2 are used to evaluate the correlations between the trends of the drawbar pull - slip relationships predicted by NTVPM and those measured. Secondly, the root mean squared deviation *RMSD* and the coefficient of variation *CV* (i.e., the ratio of *RMSD* to the mean of measured values) are used to quantitatively evaluate the deviations between the drawbar pulls predicted by NTVPM and those measured at corresponding slips.

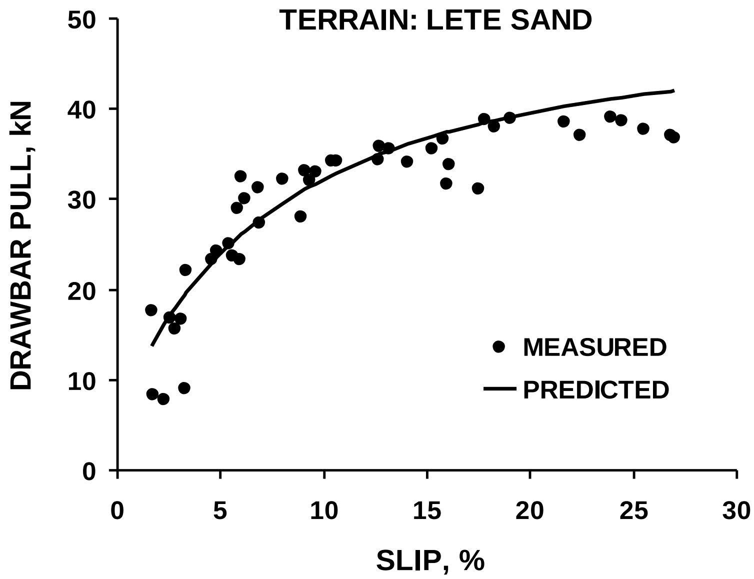


Figure 5 Correlation between the measured drawbar pull vs. slip of the notional tracked vehicle and that predicted by NTVPM on LETE Sand.

(i) The coefficient of correlation *R* and the coefficient of determination R^2

The coefficient of correlation *R* is defined as

$$R = \frac{n\sum xy - \left(\sum x\right)\left(\sum y\right)}{[\sqrt{n\left(\sum x^2\right) - \left(\sum x\right)^2}\,][\sqrt{n\left(\sum y^2\right) - \left(\sum y\right)^2}\,]} \tag{7}$$

where *x* and *y* represent the predicted and measured drawbar pulls at the corresponding slips, respectively; and *n* is the number of data points used in the evaluation.

A value of one (1) for the coefficient of correlation *R* indicates a perfect correlation between the trends of the predicted and measured data. The correlation will generally be regarded as strong, if the value of *R* is greater than 0.8. With a value of *R* less than 0.5, the correlation is usually regarded as weak. The coefficient of determination R^2 (the square of the coefficient of correlation *R*) gives the proportion of the variance of one variable that is predictable from the other variable. For example, if $R^2 = 0.85$, it will indicate that 85% of the variation in one variable is predictable from the other. In other words, R^2 is a measure that determines how certain one can be in making predictions from a particular model.

(ii) The root mean squared deviation *RMSD* and the coefficient of variation *CV*

The root mean squared deviation *RMSD* is defined as

$$RMSD = \sqrt{\frac{\sum (x-y)^2}{n}} \tag{8}$$

where *x, y* and *n* are the same as those in Equation (7).

If the value of *RMSD* is zero, the predicted and measured data will have a perfect match, with zero deviations between them. It should be noted that *RMSD* is dimensional. For this case, it is in the same units as that of drawbar pull in kN. To provide a non-dimensional indicator for the degree of match between the predicted and measured data, the coefficient of variation *CV* is introduced, as mentioned previously.

$$CV = RMSD / Y \tag{9}$$

where *Y* is the mean value of the measured drawbar pulls.

Table 1 shows the values of the coefficient of correlation *R*, the coefficient of determination R^2, the root mean squared deviation *RMSD*, and the coefficient of variation *CV*, for the predicted and measured data of the notional tracked vehicle on LETE Sand. As can be seen from the table, the values of *R* and R^2 are 0.922 and 0.850, respectively. Thus, the correlation between the trends of the predicted drawbar pull - slip relationship of the notional tracked vehicle by NTVPM and those of the measured data on LETE Sand can be regarded as strong. The values of *RMSD* and *CV* are 3.55 kN and 0.120, respectively.

Table 1 Correlation between the measured tractive performance of the notional tracked vehicle and that predicted by NTVPM on LETE Sand.

R	R^2	*RMSD*	*CV*
0.922	0.850	3.55 kN	0.120

(B) Petawawa Muskeg B

(a) The measured and predicted tractive performance

Figure 6 shows the measured drawbar pull vs. slip data of the notional tracked vehicle on Petawawa Muskeg B, together with the drawbar pull - slip curve predicted by NTVPM.

Based on field observations of the notional tracked vehicle operating on muskeg, the surface mat was broken due to heavy vehicle load. Consequently, vehicle weight is essentially supported by the underlying peat and vehicle traction is primarily developed through shearing of the peat. The predicted drawbar pull - slip curve by NTVPM shown in Figure 6 is based on the mean value of the terrain parameter of the underlying peat k_m presented in Table B.3, and on the mean values of the shear parameters of the peat c, ϕ and K given in Table B.4 of Appendix B. It should be mentioned that the values of k_o and A_u given in Table B.3 represent the repetitive loading behaviour of the surface mat and the underlying peat combined. The values of the repetitive loading parameters for the underlying peat alone are not available. As an expedient, the values of k_o and A_u given in Table B.3 are used in predictions of vehicle performance on Petawawa Muskeg B. It will be shown later that the values of k_o and A_u have only minor effects on vehicle performance.

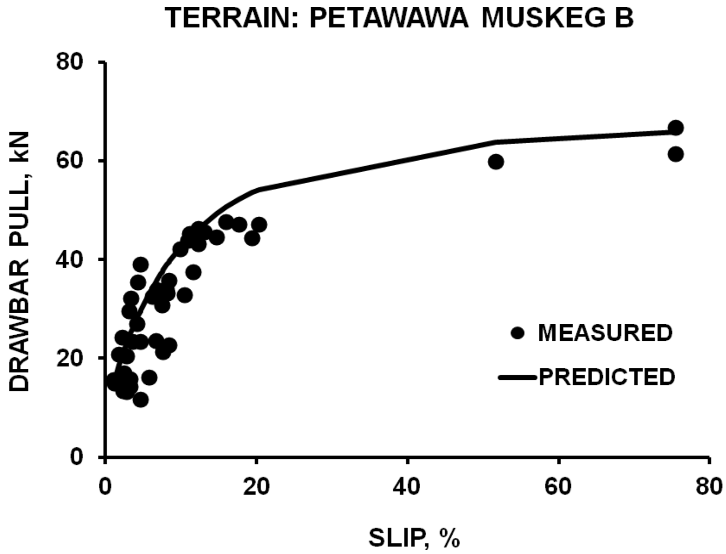


Figure 6 Correlation between the measured drawbar pull of the notional tracked vehicle and that predicted by NTVPM on Petawawa Muskeg B.

(b) Correlation between the measured and predicted tractive performance

Table 2 shows the values of the coefficient of correlation *R*, the coefficient of determination R^2, the root mean squared deviation *RMSD*, and the coefficient of variation *CV*, for the predicted and measured data of the notional tracked vehicle on Petawawa Muskeg B. As can be seen from the table, the values of *R* and R^2 are 0.903 and 0.815, respectively. Thus, the correlation between the trends of the predicted drawbar pull - slip relationship of the notional tracked vehicle by NTVPM and those of the measured data on Petawawa Muskeg B can be regarded as strong. The values of *RMSD* and *CV* are 7.25 kN and 0.225, respectively.

Table 2 Correlation between the measured tractive performance of the notional tracked vehicle and that predicted by NTVPM on Petawawa Muskeg B.

R	R^2	*RMSD*	*CV*
0.903	0.815	7.25 kN	0.225

(C) Petawawa Snow A

(a) The measured and predicted tractive performance

Figure 7 shows the measured drawbar pull vs. slip data of the notional tracked vehicle on Petawawa Snow A, together with the drawbar pull - slip curve predicted by NTVPM. The predicted curve shown in the figure is based on the mean values of terrain parameters for Petawawa Snow A presented in Tables B.5, B.6 and B.7 of Appendix B.

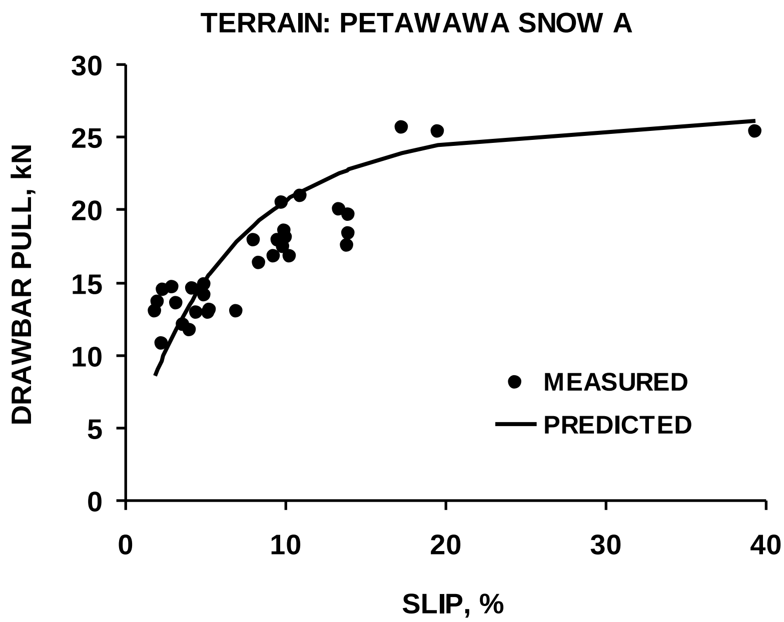


Figure 7 Correlation between the measured drawbar pull of the notional tracked vehicle and that predicted by NTVPM on Petawawa Snow A.

(b) Correlation between the measured and predicted tractive performance

Table 3 shows the values of the coefficient of correlation *R*, the coefficient of determination R^2, the root mean squared deviation *RMSD*, and the coefficient of variation *CV*, for the predicted and measured data of the notional tracked vehicle on Petawawa Snow A. As can be seen from the table, the values of *R* and R^2 are 0.845 and 0.714, respectively. Thus, the correlation between the trends of the predicted drawbar pull - slip relationship of the notional tracked vehicle by NTVPM and those of the measured data on Petawawa Snow A can be regarded as strong. The values of *RMSD* and *CV* are 2.79 kN and 0.168, respectively.

Table 3 Correlation between the measured tractive performance of the notional tracked vehicle and that predicted using NTVPM on Petawawa Snow A.

R	R^2	*RMSD*	*CV*
0.845	0.714	2.79 kN	0.168

In addition to using field test data described above to evaluate the predictive capability of NTVPM, an independent study was carried out in Sweden, using field test data obtained with a specially designed experimental vehicle on deep snow [17]. The general conclusion of the study is that "the simulation model NTVPM predicted the behaviour of the test vehicle reasonably well when compared to the experimental results".

A study of the application of NTVPM to the prediction of the tractive performance of a small, lightweight robotic track system was also conducted [18]. It is shown that the correlation between the performance predicted by NTVPM and that measured on a sandy soil is reasonably close. The results of the study provide evidence to support the view that physics-based models, such as NTVPM, are applicable to large, heavy, as well as small, lightweight tracked vehicles, provided that appropriate input terrain data are used.

5.2 The Maximum Possible Speed (Speed-Made-Good) Predicted by NTVPM

The physics-based procedures of NTVPM for predicting the maximum possible speed of a tracked vehicle in straight-line motion, under steady-state operating conditions on a level terrain are outlined below.

(a) The external motion resistance of the tracked vehicle R_{tex} due to vehicle-terrain interaction on a given terrain is predicted using NTVPM.

(b) The internal motion resistance of the track system R_{tin} (i.e., the sum of the resistances due to road wheels rolling on the track inner surface, mechanical losses in the pins or bushings caused by the relative movement between track links, friction between sprocket teeth and track links during engagement, etc.) is usually determined experimentally. In this study, the internal motion resistance coefficient (i.e., the ratio of the internal motion resistance to vehicle weight) of 0.0525 is used. It is from the NATO Reference Mobility Model Edition II, NRMM II Users Guide [2] and the U.S. Army Corps of Engineers Cold Regions Research and Engineering Laboratory Report 95-1 [19].

(c) For steady-state straight-line motion on a level terrain, the vehicle must develop the required tractive effort (thrust) F to overcome the sum of the internal and external motion resistances of the vehicle, that is, $F = R_{tex} + R_{tin}$.

(d) For a given tractive effort F, the corresponding vehicle theoretical speed V_t can be determined from the tractive effort-theoretical speed relationship of the vehicle. It is determined from the engine torque-speed curve (with throttle fully open), characteristics of the transmission (including the torque converter and gear box), final drive gear ratio, and sprocket pitch radius. The tractive effort-theoretical speed relationship for the notional tracked vehicle is shown in Figure 8.

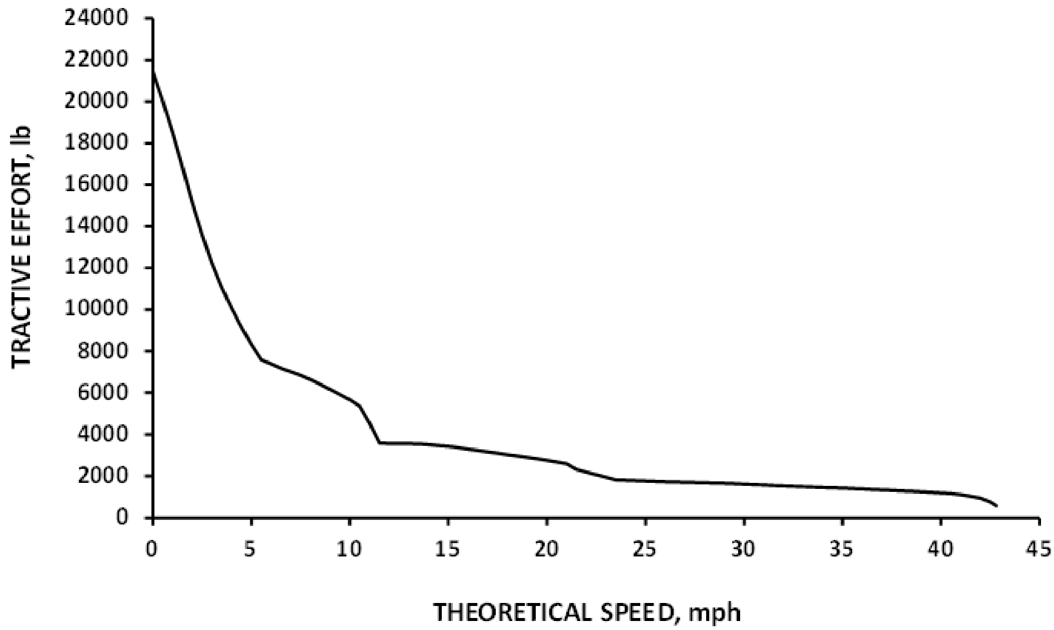


Figure 8 Tractive effort-theoretical speed relationship of the notional tracked vehicle.

(e) Based on the tractive effort-slip relationship of the vehicle predicted by NTVPM, the vehicle slip i for a given tractive effort can be defined. As an example, the tractive effort-slip relationships of the notional tracked vehicle on LETE Sand, Petawawa Muskeg B and Petawawa Snow A are shown in Figure 9.

(f) From the theoretical speed V_t determined in (d) and the slip i predicted in (e), for a given tractive effort required to maintain steady-state motion, the actual vehicle speed can finally be predicted. The steady-state maximum possible speed V_{max} of the vehicle in straight-line motion on a level terrain can be determined by

$$V_{max} = V_t \ (1- i) \qquad (10)$$

where V_t is theoretical speed of the vehicle for the required tractive effort F from (c), and is determined from the tractive effort-theoretical speed curve shown in Figure 8; i is the slip of the vehicle corresponding to the required vehicle tractive effort and can be determined using the tractive effort-slip relationship predicted by NTVPM shown in Figure 9.

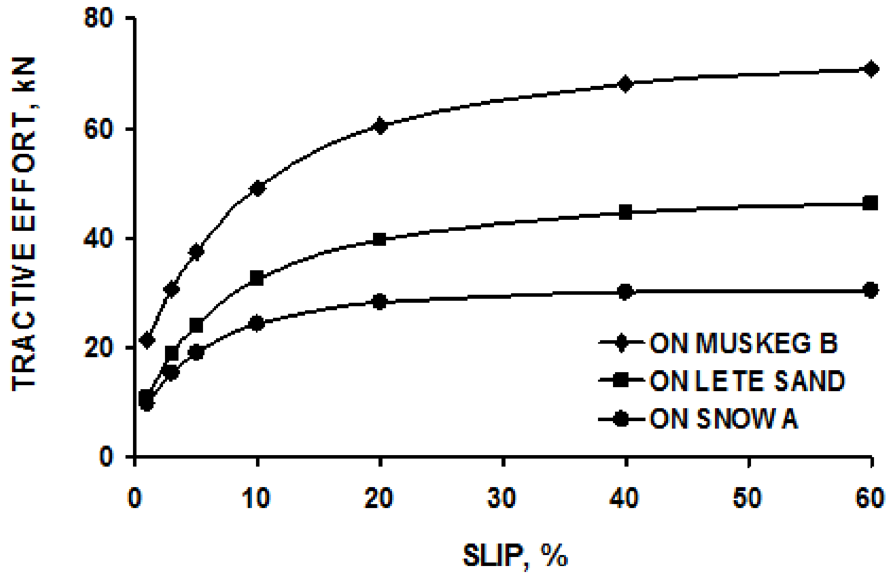


Figure 9 Tractive effort-slip relationships of the notional tracked vehicle on LETE Sand, Petawawa Muskeg B, and Petawawa Snow A predicted by NTVPM.

Implementing the above-noted procedures for the prediction of the maximum possible speeds of the notional tracked vehicle on LETE Sand, Petawawa Muskeg B and Petawawa Snow A yields the results shown in Table 4. This demonstrates the physics-based approach to predicting the maximum possible speed, in contrast to the empirically-based approach used in the current version of NRMM.

Table 4 Maximum possible speeds of the notional tracked vehicle predicted using NTVPM on LETE Sand, Petawawa Muskeg B, and Petawawa Snow A.

Terrain	Maximum possible speed (speed-made-good) predicted using NTVPM	
	mph	km/h
LETE Sand	41	66
Petawawa Muskeg B	21	33.8
Petawawa Snow A	23	37

6. SENSITIVITY OF VEHICLE PERFORMANCE TO TERRAIN VALUES

The study of the sensitivity of vehicle tractive performance to the values of terrain parameters would illustrate the effects of variability of terrain data on vehicle performance. The variability of terrain data is generally unavoidable, particularly in the field and even in controlled laboratory environment. The results of the study would also provide guidance for terrain data collection, with respect to the relative significance of various terrain parameters to vehicle performance.

The drawbar pull coefficient (i.e., the ratio of the drawbar pull to vehicle weight) at 20% slip is widely used as a performance indicator of an off-road vehicle. Accordingly, the effects on the drawbar pull coefficient at 20% slip of varying the values of major input terrain parameters from -30% to +30% of their baseline values are examined using NTVPM. The baseline values of the parameters for LETE Sand, Petawawa Muskeg A and B, and Petawawa Snow A and B are given in Appendix B

(A) LETE Sand

The sensitivity is examined of the predicted drawbar pull coefficient at 20% slip of the notional tracked vehicle to variations in the range of -30% to +30% of the baseline values of the following terrain parameters of LETE Sand:

(a) pressure-sinkage parameters k_c and k_ϕ;

(b) exponent *n* of the pressure-sinkage relationship;

(c) repetitive loading parameter A_u;

(d) terrain internal shear strength parameters *c* and ϕ, and rubber-terrain shear strength parameters c_{ru} and ϕ_{ru};

(e) terrain internal shear deformation parameter *K*, and rubber-terrain shear deformation parameter K_{ru}.

The sensitivity of the drawbar pull coefficient at 20% slip of the notional tracked vehicle to the values of terrain parameters on LETE Sand are summarized in Figure 10. It is shown that vehicle tractive performance is most sensitive to the values of shear strength parameters, among all the terrain parameters examined. It should be noted that as shown in Table B.2 of Appendix B, the angle of internal shearing resistance ϕ and the angle of rubber-terrain shearing resistance ϕ_{ru} are the dominant shear strength components, in comparison with the cohesion of the terrain *c* and the rubber-terrain adhesion c_{ru}. This indicates that vehicle tractive performance is most sensitive to the variations of the values of the angle of internal shearing resistance ϕ and the angle of rubber-terrain shearing resistance ϕ_{ru}, among the terrain parameters examined. It is shown that variations of other terrain parameters, such as pressure-sinkage parameters and repetitive loading parameters, have only relatively minor effects on vehicle performance on LETE Sand, in the range examined.

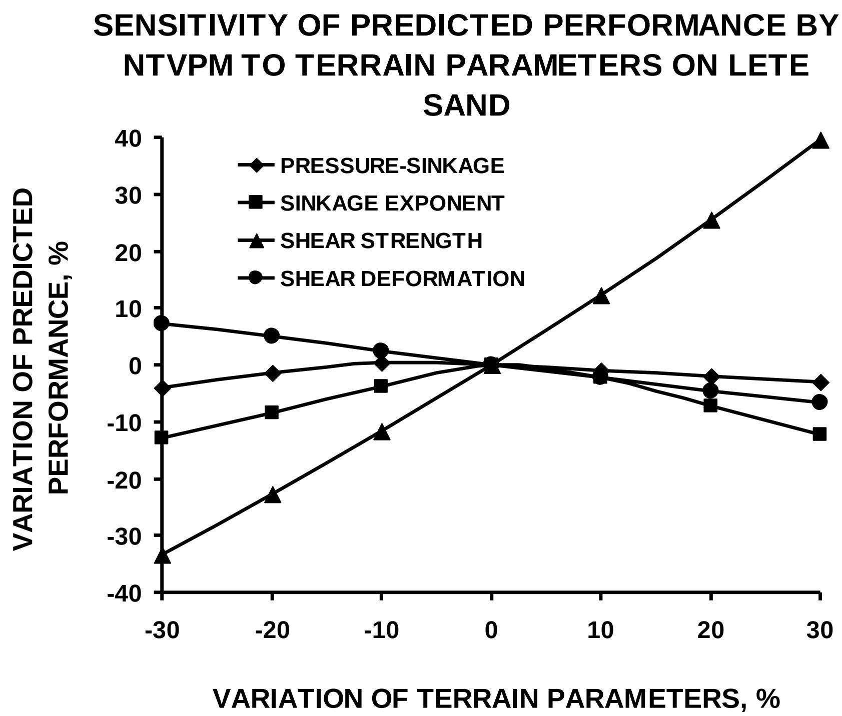


Figure 10 Sensitivity of the drawbar pull coefficient at 20% slip of the notional tracked vehicle predicted by NTVPM to variations of terrain parameters of LETE Sand: pressure-sinkage parameters k_c and k_ϕ; exponent n; terrain internal shear strength parameters c and ϕ and rubber-terrain shear strength parameters c_{ru} and ϕ_{ru}; terrain internal shear deformation parameter K and rubber-terrain shear deformation parameter K_{ru}. Variations are with respect to the baseline values of terrain parameters.

(B) Petawawa Muskeg B

The sensitivity is examined of the predicted drawbar pull coefficient at 20% slip of the notional tracked vehicle to variations in the range of -30% to +30% of the baseline values of the following terrain parameters of Petawawa Muskeg B:

(a) pressure-sinkage parameter of the underlying peat k_m;

(b) repetitive loading parameters k_o and A_u;

(c) shear strength parameters of the underlying peat c and ϕ;

(d) shear deformation parameter of the underlying peat K.

The sensitivity of the drawbar pull coefficient at 20% slip of the notional tracked vehicle to the values of terrain parameters on Petawawa Muskeg B are summarized in Figure 11. It is shown that vehicle tractive performance is most sensitive to the values of shear strength parameters of the underlying peat *c* and ϕ, among all the terrain parameters examined. It should be noted that as shown in Table B.4 of Appendix B, the angle of internal shearing resistance ϕ of the underlying peat is the dominant shear strength component, in comparison with the cohesion *c*. This indicates that vehicle tractive performance is most sensitive to the variation of the value of the angle of internal shearing resistance of the peat ϕ, among the terrain parameters examined.

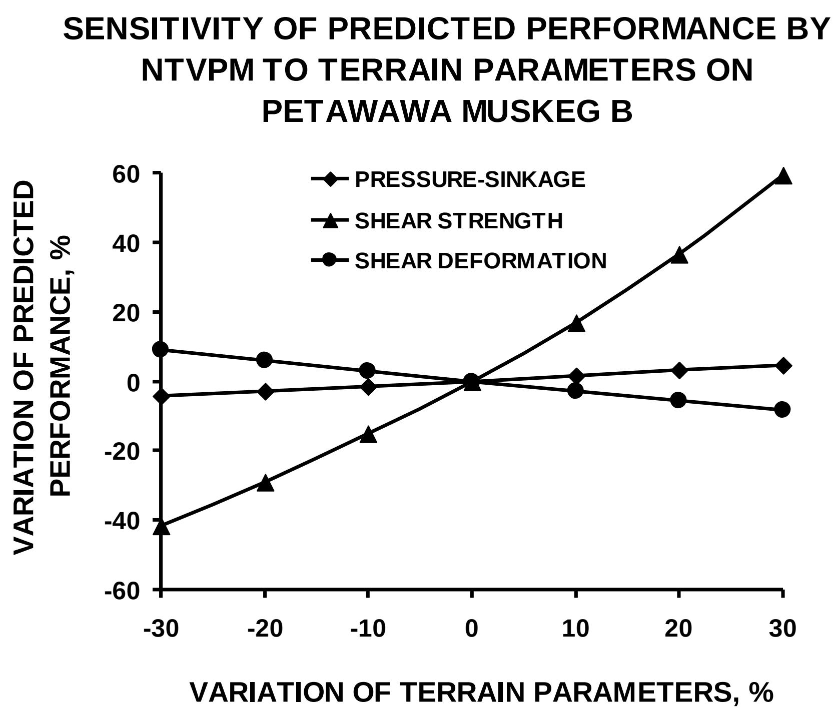


Figure 11 Sensitivity of the drawbar pull coefficient at 20% slip of the notional tracked vehicle predicted by NTVPM to variations of terrain parameters of Petawawa Muskeg B: pressure-sinkage parameter k_m; terrain internal shear strength parameters *c* and ϕ; terrain internal shear deformation parameter *K*. Variations are with respect to the baseline values of terrain parameters.

(C) Petawawa Snow A

The sensitivity is examined of the predicted drawbar pull coefficient at 20% slip of the notional tracked vehicle to the variations in the range of -30% to +30% of the baseline values of the following terrain parameters of Petawawa Snow A:

(a) pressure-sinkage parameters k_{p1} and k_{p2} and k_{z1} and k_{z2} (for detailed descriptions of these parameters, please refer to the reference [3]);

(b) terrain internal shear strength parameters c and ϕ and rubber-terrain shear strength parameters c_{ru} and ϕ_{ru};

(c) terrain internal shear deformation parameters K_r and K_w, and rubber-terrain shear deformation parameter K_{ru}.

The sensitivity of the drawbar pull coefficient at 20% slip of the notional tracked vehicle to the values of terrain parameters on Petawawa Snow A are summarized in Figure 12. It is shown that vehicle tractive performance is most sensitive to the values of shear strength parameters, among all the terrain parameters examined. It should be noted that as shown in Tables B.6 and B.7 of Appendix B, the angle of internal shearing resistance ϕ and the angle of rubber-terrain shearing resistance ϕ_{ru} are the dominant shear strength components, in comparison with the cohesion of the terrain c and the rubber-terrain adhesion c_{ru}. This indicates that vehicle tractive performance is most sensitive to the variations of the values of the angle of internal shearing resistance ϕ and the angle of rubber-terrain shearing resistance ϕ_{ru}, among the terrain parameters examined. It is shown that variations of other terrain parameters, such as pressure-sinkage parameters and repetitive loading parameters, have only relatively minor effects on vehicle performance on Petawawa Snow A, in the range examined.

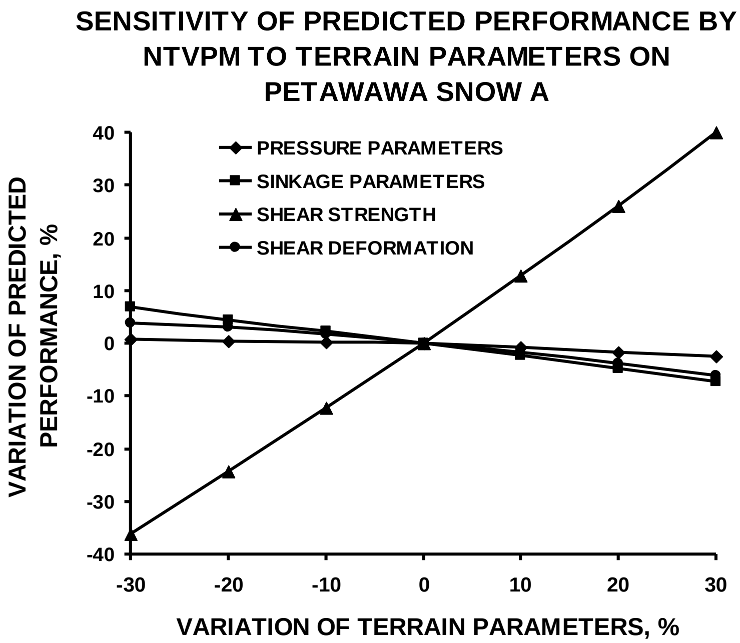


Figure 12 Sensitivity of the drawbar pull coefficient at 20% slip of the notional tracked vehicle predicted by NTVPM to variations of terrain parameters of Petawawa Snow A: pressure parameters k_{p1} and k_{p2} before and after breaking the crust; sinkage parameters k_{z1} and k_{z2} before and after breaking the crust; terrain internal shear strength parameters c and ϕ and rubber-terrain shear strength parameters c_{ru} and ϕ_{ru}; terrain internal shear deformation parameters K_w and K_r and rubber-terrain shear deformation parameter K_{ru}. Variations are with respect to the baseline values of terrain parameters.

In summary, for the three types of terrain examined in the study, vehicle tractive performance is most sensitive to the variations of the values of shear strength parameters of the internal and rubber-terrain shearing for LETE Sand and Petawawa Snow A, and to the variation of the internal shear strength of the underlying peat for Petawawa Muskeg B, among the terrain parameters examined. For instance, on LETE Sand, varying the values of shear strength parameters of the internal and rubber-terrain shearing within the range of -30% to +30% of their baseline values causes changes in the drawbar pull coefficient at 20% slip of the notional tracked vehicle in the range from -33.4% to +39.7%. On Petawawa Muskeg B, varying the values of shear strength parameters of internal shearing of the underlying peat within the range of -30% to +30% of their baseline values causes changes in the drawbar pull coefficient at 20% slip of the notional tracked vehicle in the range from -41.6% to +59.3%. On Petawawa Snow A, varying the values of shear strength parameters of the internal and rubber-terrain shearing within the range of -30% to +30% of their baseline values causes changes in the drawbar pull coefficient at 20% slip of the notional tracked vehicle in the range from -36.2% to +40%. It is shown that the sensitivity of vehicle performance to variations of pressure-sinkage parameters, repetitive loading parameters, and shear deformation parameters is much less than those of the shear strength parameters for the three types of terrain examined.

7. CORRELATIONS BETWEEN THE MEASURED AND CALCULATED MEAN MAXIMUM PRESSURE (MMP) AND THAT PREDICTED BY NTVPM

The mean maximum pressure (MMP), which is defined as the mean value of the maxima occurring under all road wheel stations of a track, was first proposed by Rowland as an indicator of cross-country mobility of tracked vehicles to replace the nominal ground pressure (NGP) [20, 21]. NGP is defined as the ratio of the vehicle weight to the total gross contact area of the tracks. NGP had earlier been regarded as a parameter of relevance to vehicle mobility. However, it was later found that in many cases, vehicles having the same value of NGP exhibit significantly different cross-country mobility. This indicates that NGP may not be an appropriate indicator for vehicle mobility. MMP is used as a parameter for evaluating cross-country mobility of military vehicles in some NATO countries, as mentioned previously [4].

Based on test data, Rowland developed the following empirical formulas for predicting MMP of vehicles with different road wheel-track system designs [20, 21]:

for link and belt tracks on rigid road wheels,

$$MMP = \frac{1.26\,W}{2n_r\,A_l\,b\sqrt{t_t\,D}} \quad kPa \tag{11}$$

and for belt tracks on pneumatic tired road wheels,

$$MMP = \frac{0.5\,W}{2n_r\,b\sqrt{D\,f_t}} \quad kPa \tag{12}$$

where A_l is the rigid area of link (or belt track cleat) as a proportion of b x t_t; b is the track (or pneumatic tire) width in m; t_t is track pitch in m; D is the outer diameter of the road wheel or pneumatic tire in m; f_t is the radial deflection of pneumatic tires under load in m; n_r is the number of road wheel stations on one track; and W is the weight of the vehicle with two tracks in kN.

To evaluate whether a particular vehicle with a specific value of MMP will have adequate mobility over a given terrain, Rowland suggested a set of desired values of MMP for different types of terrain [20].

It should be pointed out that in the empirical formulas proposed by Rowland, terrain characteristics are not taken into account in the calculation of MMP. Thus, the value of MMP calculated using either Equation (11) or (12) is independent of terrain conditions. In reality, the normal pressure distribution under a tracked vehicle, hence the actual value of MMP is strongly influenced by terrain characteristics [22]. As examples, Figure 13 shows the measured normal pressure distribution under the track of the notional tracked vehicle on LETE Sand at slip of 10.8% [15]. Figure 14 shows the measured normal pressure distribution under the track on Petawawa Muskeg A at slip of 8.5%. Figure 15 shows the measured normal pressure distribution under the track on Petawawa Snow A at slip of 5.6%, whereas Figure 16 shows the measured normal pressure distribution under the track on Petawawa Snow B at slip 7.9%. For comparison, the normal pressure distributions predicted by NTVPM are also shown in the figures.

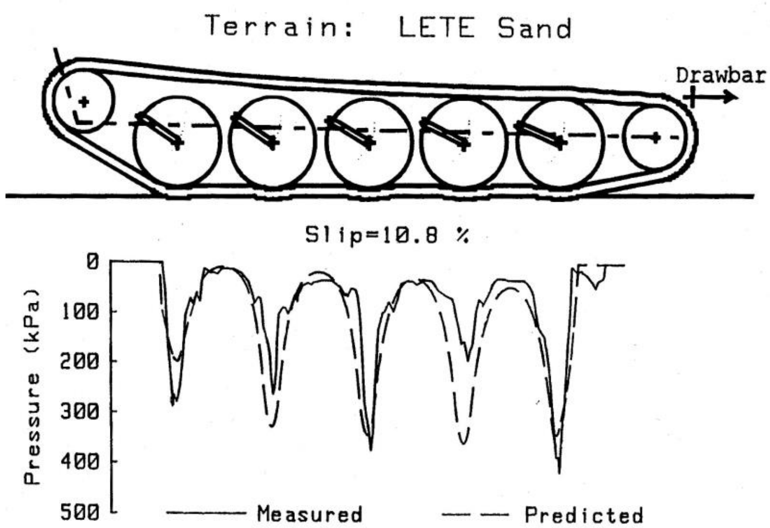


Figure 13 Comparison between the measured normal pressure distribution under the track of the notional tracked vehicle at 10.8% slip and that predicted by NTVPM on LETE Sand.

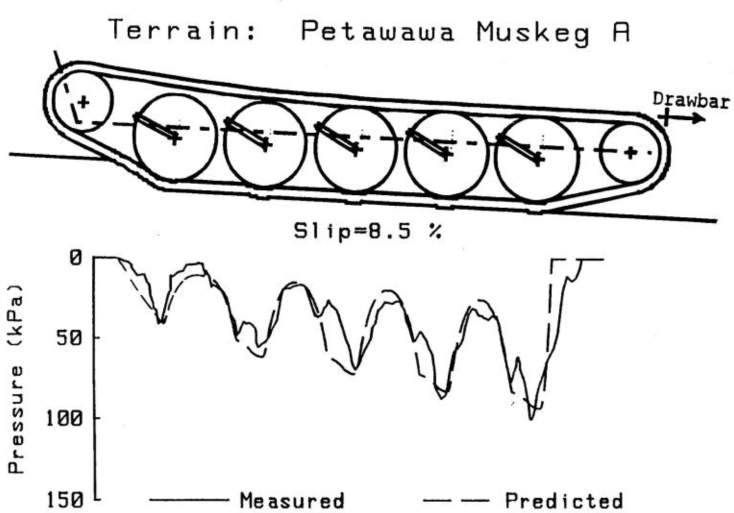


Figure 14 Comparison between the measured normal pressure distribution under the track of the notional tracked vehicle at 8.5% slip and that predicted by NTVPM on Petawawa Muskeg A.

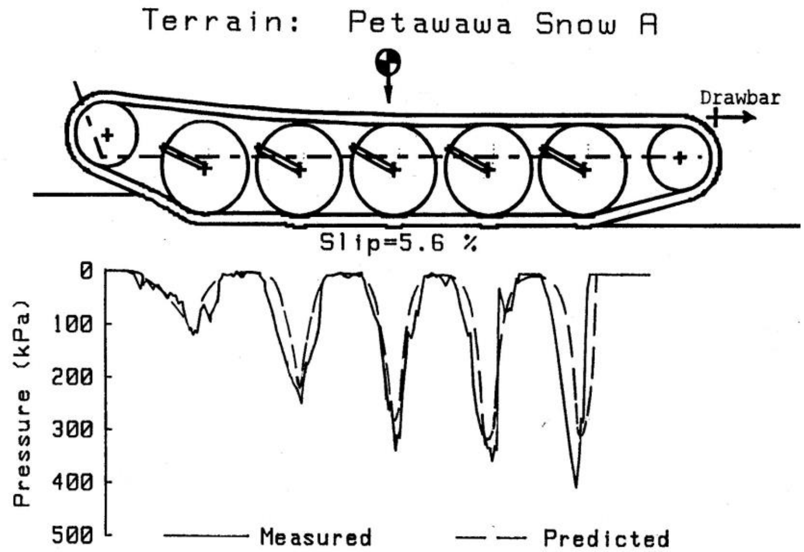


Figure 15 Comparison between the measured normal pressure distribution under the track of the notional tracked vehicle at 5.6% slip and that predicted by NTVPM on Petawawa Snow A.

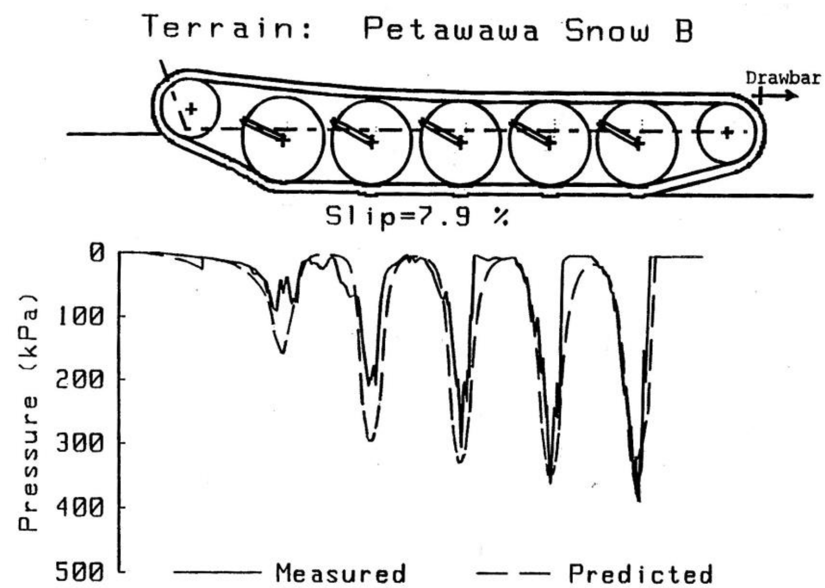


Figure 16 Comparison between the measured normal pressure distribution under the track of the notional tracked vehicle at 7.9% slip and that predicted by NTVPM on Petawawa Snow B.

As shown in the figures, the peak pressure under the track varies significantly with terrain conditions. This can be demonstrated by comparing the measured and predicted sinkages and pressure distributions on LETE Sand (a firm terrain) and on Petawawa Muskeg A (a much softer terrain). On LETE Sand vehicle sinkage is shallow, as shown in Figure 13. Vehicle weight is primarily supported by the track links immediately beneath the road wheels, and the maximum pressure under the rear road wheel at 10.8% slip is approximately 427 kPa. On Petawawa Muskeg A vehicle sinkage is much deeper, as shown in Figure 14. Vehicle weight is supported not only by the track links immediately beneath the road wheels but also by the track links between road wheels, and the maximum pressure under the rear road wheel at 8.5% slip is approximately 100 kPa, which is only 23.4% of that at 10.8% slip on LETE Sand. It is noted that track slip has an influence on the normal pressure distributions under the track as well.

As MMP has been used by some NATO countries as a parameter for evaluating the cross-country mobility of military vehicles, a study is carried out to compare the measured values of MMP of the notional tracked vehicle with that predicted by NTVPM, as well as that calculated by Rowland's empirical formula, on four types of terrain: LETE Sand, Petawawa Muskeg A, Petawawa Snow A and Snow B. The measured normal pressure distributions on Petawawa Muskeg B are not available and are not included in this study.

Table 5 shows the measured values of MMP, the predicted values by NTVPM and the calculated value using Rowland's empirical formula on the four types of terrain. It should be mentioned that since the notional tracked vehicle has segmented metal tracks with rigid road wheels (with rubber rims), Equation (11) is used in the calculation.

As noted previously, the measured values of MMP and that predicted values of MMP by NTVPM vary with slip. Therefore, the average measured values of MMP at all slips and the average predicted values of MMP at all slips by NTVPM on various types of terrain are also shown in the table. As pointed out previously, for a given vehicle the calculated value of MMP by Rowland's empirical formula is independent of terrain characteristics and slip. The value of MMP for the notional tracked vehicle calculated by Rowland's empirical formula is, therefore, a constant on all four types of terrain, as shown in Table 5.

Table 5 Measured MMP, predicted MMP by NTVPM, and calculated MMP by Rowland's empirical formula on various types of terrain.

Terrain type	Slip %	Measured MMP kPa	Predicted MMP by NTVPM kPa	Calculated MMP by Rowland's formula kPa
LETE Sand	1.8	317.6	312.2	100.3
	2.8	448.8	312.1	100.3
	5.9	522	311.1	100.3
	6.1	377.8	311.0	100.3
	7.1	432.6	310.6	100.3
	8.2	406	310.1	100.3
	10.8	311	309.1	100.3
	15.5	317.4	307.6	100.3
	Average	***391.7***	***310.5***	***100.3***
Petawawa Muskeg A	0.5	100.6	83.5	100.3
	2.7	91.1	80.3	100.3
	3.0	101.3	79.9	100.3
	6.6	111.0	76.0	100.3
	8.1	90.3	74.7	100.3
	8.5	70.4	74.5	100.3
	Average	***94.1***	***78.2***	***100.3***
	0.0	225.6	250.6	100.3
	2.2	257.5	250.2	100.3

Petawawa Snow A	2.4	246.5	250.1	100.3
	3.3	237.4	249.8	100.3
	4.8	287.5	249.3	100.3
	5.6	296.0	249.1	100.3
	8.3	246.4	245.7	100.3
	8.6	289.1	245.6	100.3
	Average	***260.8***	***248.8***	***100.3***
Petawawa Snow B	4.5	285.2	287.1	100.3
	5.8	280.4	286.9	100.3
	5.9	280.5	286.9	100.3
	6.5	309.7	286.8	100.3
	6.6	261.5	286.8	100.3
	6.9	307.7	286.7	100.3
	7.2	242.7	286.6	100.3
	7.9	269.2	286.5	100.3
	9.6	300.7	286.1	100.3
	10.2	244.1	286.0	100.3
	Average	***278.2***	***286.6***	***100.3***

Table 6 shows a comparison of the average measured values of MMP, the average predicted values of MMP by NTVPM, and the calculated value of MMP using Rowland's empirical formula, which is a constant on various types of terrain.

Table 6 Comparison of the average measured MMP, the average predicted MMP by NTVPM, and the calculated MMP by Rowland's empirical formula on various types of terrain.

Terrain type	Average measured MMP kPa	Average predicted MMP by NTVPM kPa	Calculated MMP by Rowland's formula* kPa	Average measured MMP/ average predicted MMP by NTVPM	Average measured MMP/ calculated MMP by Rowland's formula
LETE Sand	391.7	310.5	100.3	1.26	3.91
Petawawa Muskeg A	94.1	78.2	100.3	1.20	0.94
Petawawa	260.8	248.8	100.3	1.05	2.60

Snow A					
Petawawa Snow B	278.2	286.6	100.3	0.97	2.77

*For a given vehicle, the value of MMP calculated using Rowland's empirical formula is only a function of a limited number of design parameters of the vehicle, and is independent of terrain characteristics and slip.

It is shown that the ratios of the average measured values of MMP to the average values of MMP predicted by NTVPM on LETE Sand, Petawawa Muskeg A, Petawawa Snow A, and Petawawa Snow B are 1.26, 1.20, 1.05, and 0.97 respectively. The ratios of the average measured values of MMP to the calculated value of MMP by Rowland's empirical formula on LETE Sand, Petawawa Muskeg A, Petawawa Snow A, and Petawawa Snow B are 3.91, 0.94, 2.60, and 2.77 respectively. It can be concluded that the average values of MMP predicted by NTVPM are in much closer agreement with the measured values than the calculated value using Rowland's empirical formula.

8. CONCLUDING REMARKS

(A) Soundness of the approach of NTVPM

NTVPM is a physics-based, computer simulation model for predicting the cross-country performance of tracked vehicles under steady-state operating conditions on even terrain. It is based on the understanding of the physical nature of vehicle-terrain interaction, and on a detailed analysis of the mechanics of track-terrain interaction. Taking into account terrain characteristics measured using the bevameter, the normal and shear stress distributions on the track-terrain interface are predicted. The motion resistance, tractive effort, drawbar pull, and tractive efficiency of the vehicle at various slips are then derived from the normal and shear stress distributions under the track.

The procedures of NTVPM for predicting steady-state cross-country performance are based on solving a set of non-linear dynamic equilibrium equations of the tracked vehicle and its subsystems. For predicting steady-state performance, this method of approach is inherently much more efficient and effective than the time integration of a set of equations of motion that is used in multibody vehicle dynamics models. The computation time required for predicting steady-state cross-country performance of tracked vehicles using NTVPM is many orders of magnitude faster than the time integration of equations of motion, using the same type of computing facility. Thus, together with an automated terrain data acquisition and processing system to provide terrain input data, NTVPM will be an efficient tool for the rapid assessment of vehicle mobility for the deployment of military tracked vehicles in the field.

(B) Adequacy of the vehicle and terrain input parameters

NTVPM takes into account all major vehicle design features that affect cross-country performance of tracked vehicles. These include the road wheel suspension system, the initial track tension, and the longitudinal stiffness of the track.

NTVPM takes into consideration all pertinent terrain characteristics. These include: the pressure-sinkage relationship, shear strength, and shear stress-shear displacement relationship of the terrain; rubber-terrain shearing characteristics (for tracks with rubber pads or for rubber band tracks); terrain response to repetitive normal and shear loading; vehicle belly-terrain shearing characteristics (for analyzing belly-terrain interaction, in the event that the vehicle belly is in contact with the terrain surface).

(C) Correlations between test data and vehicle performance predicted by NTVPM

Results indicate that there are reasonably close correlations between the cross-country performance of the notional tracked vehicle predicted by NTVPM and test data on sandy terrain, muskeg (organic terrain) and snow-covered terrain. For instance, on sandy terrain

(LETE Sand), the coefficient of correlation *R* is 0.922, which indicates that the correlation between the trends of drawbar pull - slip relationship predicted by NTVPM and those of the measured one is strong. The coefficient of variation *CV* (i.e., the ratio of the root mean squared deviation to the mean of measured values) is 0.120, which indicates that the deviation of the predicted vehicle performance from the measured one is reasonable. Similar results are obtained on muskeg (Petawawa Muskeg B) and on snow-covered terrain (Petawawa Snow A). The reasonable agreement between the test data and the vehicle performance predicted by NTVPM provides evidence to support the soundness of the approach of NTVPM and the adequacy of the vehicle design parameters and terrain characteristics that are taken into account in NTVPM.

It is shown that NTVPM can be used to predict the maximum possible speed (speed-made-good) of tracked vehicles on a variety of terrains. In comparison with empirically-based NRMM, the physics-based approach of NTVPM to predicting vehicle maximum possible speed is universally applicable.

The mean maximum pressure (MMP) has been used as an indicator for military vehicle mobility in some NATO countries. It is demonstrated that the values of MMP predicted by NTVPM are much closer to the measured data than those calculated using Rowland’s empirical formula.

(D) User-friendliness of the operation of NTVPM

In the development of NTVPM, particular attention has been paid to its user-friendliness. Vehicle and terrain data are input using the dialog (edit) box format for the convenience of the user.

The output of NTVPM provides all major vehicle performance metrics, such as track sinkage, belly sinkage, belly load, belly drag, external track motion resistance, total external motion resistance, tractive effort, drawbar pull, tractive efficiency, mean maximum pressure, mean maximum shear stress, etc. They enable the procurement manager to evaluate, in detail, vehicle candidates in the acquisition process, or the vehicle designer to optimize the design of tracked vehicles in the product development process. This represents another dimension of the user-friendliness of NTVPM. Thus, it can be concluded that NTVPM is user-friendly.

(E) Suitability of NTVPM as the basis for the development of the next generation cross-country performance assessment methodology for military tracked vehicles

Based on the evaluation of the soundness of its approach, adequacy of the vehicle design parameters and terrain characteristics that are taken into account, correlations between test data and predicted performance on various types of terrain, and user-friendliness of the operation, it can be concluded that NTVPM provides a sound basis for the development of the next generation cross-country performance assessment methodology for military tracked vehicles.

(F) Future perspectives

(a) It would be beneficial to collect additional test data of a wider spectrum of military tracked vehicles on a broader range of operating environments to further substantiate the predictive capabilities of NTVPM.

(b) The physics-based procedures for predicting vehicle maximum possible speed (speed-made-good) outlined in this study can readily be incorporated into the current version of NTVPM to extend its capabilities. This additional capability can be applied to predicting maximum possible speeds on various terrain patches in a given region to produce a mobility map and mobility profile.

(c) The capability for evaluating vehicle operating fuel economy can be incorporated into the current version of NTVPM, through integrating the vehicle power plant fuel consumption characteristics and the power required to maintain a given vehicle speed on terrain patches with the mobility map for a given region.

(d) The development of an integrated system for speedy evaluation of military tracked vehicle mobility in the field should be considered. It will incorporate an automated terrain data acquisition and processing system to provide terrain input data with the extended-capability NTVPM. This system can be an efficient, physics-based tool for the rapid assessment of vehicle mobility for deployment of military tracked vehicles in any particular region.

ACKNOWLEDGEMENTS

The work described in this paper was performed by Vehicle Systems Development Corporation (VSDC), Toronto, Ontario, Canada, under Contract No. W911NF-11-D-0001, administered by Battelle Memorial Institute, Columbus, Ohio for the U.S. Army Tank Automotive Research, Development and Engineering Center (TARDEC), Warren, Michigan, U.S.A.

DISCLAIMER

Reference herein to any specific commercial company, product, process or service by trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favouring by the United States Government or the Department of the Army (DoA). The opinions of the authors expressed herein do not necessarily state or reflect those of the United States Government or the DoA, and shall not be used for advertising or product endorsement purposes.

APPENDIX A

Vehicle input parameters used in the study

PREDICTION OF TRACKED VEHICLE PERFORMANCE
(MODEL: NTVPMwinV2)
VEHICLE SYSTEMS DEVELOPMENT CORPORATION
OTTAWA, ONTARIO, CANADA

December 01, 2014

VEHICLE TYPE — APC (Test Vehicle)

VEHICLE PARAMETERS:

SPRUNG WEIGHT	78.57	kN
UNSPRUNG WEIGHT	10.14	kN
SPRUNG WEIGHT CENTRE OF GRAVITY X-COORDINATE	198.00	cm
SPRUNG WEIGHT CENTRE OF GRAVITY Y-COORDINATE	-48.10	cm
INITIAL TRACK TENSION	10.00	kN
DRAWBAR HITCH X-COORDINATE	427.50	cm
DRAWBAR HITCH Y-COORDINATE	-12.70	cm

FIXED WHEELS

WHEEL RADIUS (cm)	X-COORDINATE OF WHEEL CENTRE (cm)	Y-COORDINATE OF WHEEL CENTRE (cm)	NOTES
21.40	0.00	0.00	SPROCKET
21.90	402.30	15.10	

TORSION BAR SUSPENSION WHEELS

	TORSION ARM PIVOTS			TORSION ARM ANGLES (+ IS CW FROM HORIZONTAL)				
WHEEL RADIUS (cm)	X-COORD. (+ IS TO THE REAR) (cm)	Y-COORD. (+ IS DOWN) (cm)	TORSION BAR STIFFNESS (kN-m/deg)	REBOUND LIMIT (deg)	FREE POSITION (deg)	JOUNCE LIMIT (deg)	TORSION ARM LENGTH (cm)	NOTES
30.50	39.69	8.73	0.1668	50.00	43.00	-4.59	31.75	T
30.50	106.36	8.73	0.1668	50.00	43.00	-4.59	31.75	T
30.50	173.04	8.73	0.1668	50.00	43.00	-4.59	31.75	T
30.50	239.71	8.73	0.1668	50.00	43.00	-4.59	31.75	T
30.50	306.39	8.73	0.1668	50.00	43.00	-4.59	31.75	T

NOTE: T = TRAILING ARM

NOTE: COORDINATE ORIGIN IS AT THE CENTRE OF THE SPROCKET. POSITIVE X- AND Y-COORDINATES ARE TO THE REAR AND DOWN, RESPECTIVELY.

BELLY SHAPE	
WIDTH: 170.0 cm	
COORDINATES (cm)	
X	Y
-21.4	-34.3
-4.1	14.3
417.0	14.3

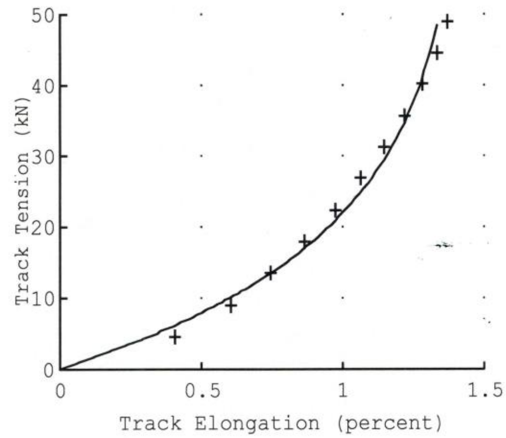
TRACK LINK CONTACT AREA			
SINKAGE (cm)		INCRE-MENTAL AREA	PERCENTAGE CAUSING EXTERNAL SHEARING
FROM	TO	(cm^2)	
0.00	0.00	77.43	100.0
0.00	0.51	80.65	100.0
0.51	1.46	54.84	100.0
1.46	2.22	40.65	0.0
2.22	3.05	98.06	0.0
3.05	4.06	41.94	0.0
4.06	5.08	45.16	0.0
5.08	5.97	83.87	0.0

TRACK PARAMETERS:

WEIGHT PER UNIT LENGTH	0.560 kN/m
WIDTH	38.0 cm
PITCH	15.0 cm
HEIGHT OF THE GROUSERS	4.7 cm
THICKNESS	6.7 cm
PERCENT EXTERNAL SHEAR AREA FOR COHESIVE SHEARING	41.0 %
LONGITUDINAL ELASTICITY CONST. Te (FROM T=-Te*ln(1-E/Emax))	18.208 kN
LONGITUDINAL ELASTICITY CONST. Emax (FROM T=-Te*ln(1-E/Emax))	1.434 %

TRACK ELASTICITY	
ELONGATION (%)	TENSION (kN)
0.000	0.00
0.408	4.45
0.605	8.90
0.745	13.34
0.867	17.79
0.975	22.24
1.063	26.69
1.151	31.14
1.219	35.59
1.282	40.03
1.334	44.48
1.373	48.93

NOTE: COORDINATE ORIGIN IS AT THE CENTRE OF THE SPROCKET. POSITIVE X- AND Y-COORDINATES ARE TO THE REAR AND DOWN, RESPECTIVELY.



50
40
30
20
10
0
Track Tension (kN)
0
0.5
1
1.5
Track Elongation (percent)

APPENDIX B

Terrain input parameters used in the study

The detailed descriptions of the terrain parameters used in the study are given in the reference [3].

(a) LETE Sand

The pressure-sinkage parameters k_c , k_ϕ , and n, and the parameters for characterizing the response to repetitive loading k_o and A_u, obtained using the bevameter, are given in Table B.1. The parameters for characterizing internal shear strength c and ϕ, and the shear deformation parameter (for characterizing the shear stress-shear displacement relationship) K, together with those corresponding to rubber-terrain shearing c_{ru}, ϕ_{ru}, and K_{ru}, are presented in Table B.2.

Table B.1 Pressure-sinkage and repetitive loading parameters for LETE Sand.

k_c , kN/m^{n+1}		k_ϕ , kN/m^{n+2}		n		k_o , kN/m^3	A_u, kN/m^4
Mean value	Standard deviation	Mean value	Standard deviation	Mean value	Standard deviation	Mean value	Mean value
102	54	5301	775	0.793	0.012	0	503,000

Table B.2 Parameters for internal and rubber-terrain shearing for LETE Sand.

Type of shearing	Cohesion or adhesion c or c_{ru} kPa		Angle of shearing resistance ϕ or ϕ_{ru} degrees		Shear deformation parameter K or K_{ru} cm	
	Mean value	Standard deviation	Mean value	Standard deviation	Mean value	Standard deviation
Internal	1.36	0.09	31.56	0.38	1.60	0.61
Rubber-terrain	0.65	0.23	27.51	0.05	1.14	0.34

(b) Petawawa Muskeg A and B

The pressure-sinkage parameter k_m and the parameters for characterizing the response to repetitive loading k_o and A_u, obtained using the bevameter, are given in Table B.3. The parameters for characterizing internal shear strength c and ϕ, and the shear deformation parameter K are presented in Table B.4.

Table B.3 Pressure-sinkage and repetitive loading parameters for Petawawa Muskeg A and B.

Terrain	k_m , kN/m^3		k_o , kN/m^3	A_u , kN/m^4
	Mean value	Standard deviation	Mean value	Mean value
Petawawa Muskeg A	424	95	123	23,540
Petawawa Muskeg B	555	105	147	29,700

Table B.4 Shear parameters for the peat of Petawawa Muskeg A and B.

Terrain type	Type of shearing	Cohesion or adhesion c kPa		Angle of shearing resistance ϕ degrees		Shear deformation parameter K cm	
		Mean value	Standard deviation	Mean value	Standard deviation	Mean value	Standard deviation
Peat - Muskeg A	Internal	5.19	1.54	37.73	3.7	3.29	1.16
Peat - Muskeg B	Internal	4.14	0.01	38.11	0.35	2.79	0.68

(c) Petawawa Snow A and B

Petawawa Snow A and B were two-layer snow covers with a crust (ice layer) in between [3]. The pressure-sinkage parameters k_{p1}, k_{p2}, k_{z1}, and k_{z2}, the parameters for characterizing the response to repetitive loading k_o and A_u, and the strength parameters of the crust L_{cr} and M_{cr}, obtained using the bevameter, are given in Table B.5. The parameters for characterizing internal shear strength c and ϕ, and the shear deformation parameters K_w and K_r are given in Table B.6, and those corresponding to rubber-terrain shearing c_{ru}, ϕ_{ru}, and K_{ru} are presented in Table B.7.

Table B.5 Pressure-sinkage and repetitive loading parameters for Petawawa Snow A and B.

Terrain	Petawawa Snow A		Petawawa Snow B	
Parameters	Before failure of the crust	After failure of the crust	Before failure of the crust	After failure of the crust
k_{p1}, kN/m^2	3.2	52.7	16.3	10.8
k_{p2}, kN/m^3	234	-48	0	0
k_{z1}, cm	0.9	14.2	24.8	41.0
k_{z2}, cm^2	39.7	67.3	0	0
L_{cr}, cm	16.7		26.1	
M_{cr}, kN	0.0402		0.0412	
k_o, kN/m^3	0		0	
A_u, kN/m^4	109,600		25,923	

Table B.6 Internal shear parameters for Petawawa Snow A and B.

Terrain type	Cohesion c, kPa		Angle of shearing resistance ϕ, degrees		Shear deformation parameter K_w, cm		Shear deformation parameter K_r	
	Mean value	Standard deviation	Mean value	Standard deviation	Mean value	Standard deviation	Mean value	Standard deviation
Petawawa Snow A & B	0.4	0.4	23.98	4.02	2.18	0.76	0.654	0.12

Table B.7 Parameters for rubber-snow shearing for Petawawa Snow A and B.

Type of shearing	Adhesion c_{ru}, kPa		Angle of shearing resistance ϕ_{ru}, degrees		Shear deformation parameter K_{ru}, cm	
	Mean value	Standard deviation	Mean value	Standard deviation	Mean value	Standard deviation
Rubber-snow	0.14	0.14	17	1.80	0.61	0.33

REFERENCES

1. Wong, J.Y. Terramechanics and its applications to the evaluation of terrestrial and extraterrestrial vehicle mobility: theory into practice. *Int. J. Vehicle Design,* 2014; 65(4): 384-410.

2. Ahlvin, R.B. and Haley, P.W. NATO Reference Mobility Model Edition II, NRMM II users Guide. Technical Report GL-92-19, U.S. Army Engineer Waterways Experiment Station, Vicksburg, Mississippi, U.S.A., 1992.

3. Wong, J.Y. *Terramechanics and Off-Road Vehicle Engineering,* 2nd ed. Oxford, England: Elsevier, 2010.

4. British Ministry of Defence. Defence Standard 23-6:2005. Technology Guidance for Military Logistics Vehicles, Issue 4.

5. Wong, J.Y. and Preston-Thomas, J. Investigation into the effects of suspension characteristics and design parameters on the performance of tracked vehicles using an advanced computer simulation model. *Proc IMechE, Part D: Transport Engineering*, 1988; 202(D3): 143-161.

6. Wong, J.Y. and Huang, W. An investigation into the effects of initial track tension on soft ground mobility of tracked vehicles using an advanced computer simulation model. *Proc IMechE, Part D: Journal of Automobile Engineering*, 2006; 220(D): 695-711.

7. Wong, J.Y. and Huang, W. Approaches to improving the mobility of military tracked vehicles on soft terrain. *Int. J. Heavy Vehicle Systems*, 2008; 15(2/3/4): 127-151.

8. Wong, J.Y. Development of high-mobility tracked vehicles for over snow operation. *J. Terramechanics*, 2009; 46(4): 141-155.

9. Bekker, M.G. *Theory of Land Locomotion.* Ann Arbor, MI: University of Michigan Press, 1956.

10. Bekker, M.G. *Off-The-Road Locomotion.* Ann Arbor, MI: University of Michigan Press, 1960.

11. Wong, J.Y. *Theory of Ground Vehicles,* 4th ed. New Jersey: John Wiley, 2008.

12. Wong, J.Y. Data processing methodology in the characterization of the mechanical properties of terrain. *J. Terramechanics,* 1980; 17(1): 13-41.

13. Wong, J.Y. Optimization of the tractive performance of articulated tracked vehicles using an advanced computer simulation model. *Proc IMechE, Part D: Journal of Automobile Engineering*, 1992; 206(D1): 29-45.

14. Wong, J.Y. Application of the computer simulation model NTVPM-86 to the development of a new version of the infantry fighting vehicle ASCOD. *J. Terramechanics,* 1995; 32(1): 53-61.

15. Wong, J.Y., Garber, M., and Preston-Thomas, J. Theoretical prediction and experimental substantiation of the ground pressure distribution and tractive performance of tracked vehicles. *Proc IMechE, Part D: Transport Engineering,* 1984; 198(15): 265-285.

16. Schunn, C.D. and Wallach, D. Evaluating goodness-of-fit in comparison of models to data. In: Tack, W. (ed) *Psychologie der Kognition: Reden und Vortrage anlasslich der Emeritierung von Werner Tack*. Saarbrueken, Germany: University of Saarland Press, 2005, pp.115-154.

17. Bodin, A. *Improving the mobility performance of tracked vehicles in deep snow.* Doctoral thesis (ISSN: 1402-1544), Department of Applied Physics and Mechanical Engineering, Lulea University of Technology, Sweden, 2002.

18. Wong, J.Y., Senatore, C., Jayakumar, P., Iagnemma, K. Predicting mobility performance of a small, lightweight track system using the computer-aided method NTVPM. *J. Terramechanics*, 2015; 61: 23-32.

19. Richmond, P.W., Shoop, S.A., and Blaisdell, G.L. *Cold regions mobility models.* Report 95-1, U.S. Army Corps of Engineers Cold Regions Research and Engineering Laboratory, Hanover, New Hampshire, U.S.A., 1995.

20. Rowland, D. Tracked vehicle ground pressure and its effect on soft ground performance. In: Proceedings of the 4th International Conference of the International Society for Terrain-Vehicle Systems, Stockholm-Kiruna, Sweden, 1972, I, pp. 353-384.

21. Rowland, D. A review of vehicle design for soft ground operation. In: Proceedings of the 5th International Conference of the International Society for Terrain-Vehicle Systems, Detroit-Houghton, Michigan, U.S.A., 1975, I, pp.179-219.

22. Wong, J.Y. On the role of mean maximum pressure as an indicator of cross-country mobility of tracked vehicles. *J. Terramechanics*, 1994; 31(3): 197-213.

23. Wong, J.Y., Radforth, J.R., and Preston-Thomas, J. Some further studies of the mechanical properties of muskeg in relation to vehicle mobility. *J. Terramechanics,* 1982; 19(2): 107-121.