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14. ABSTRACT
Protection of humans from large, finite time perturbations is an important area of research. Hence it is of interest to develop systems that are thin, light-weight and capable of absorbing large accelerations. An area of special interest concerns highly nonlinear systems which possess a rich variety of dynamical responses. This study focused on whether it is possible to convert any incident perturbation into localized excitations. The study explored the dynamics of a weakly nonlinear system with quadratic and quartic on-site and inter-site potentials where the linear

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Report Title

Final Report: Stable Intrinsic Localized Modes in Microelectromechanical Cantilever Structures

ABSTRACT

Protection of humans from large, finite time perturbations is an important area of research. Hence it is of interest to develop systems that are thin, light-weight and capable of absorbing large accelerations. An area of special interest concerns highly nonlinear systems which possess a rich variety of dynamical responses. This study focused on whether it is possible to convert any incident perturbation into localized excitations. The study explored the dynamics of a weakly nonlinear system with quadratic and quartic on-site and inter-site potentials where the linear pieces are significantly stronger than the nonlinear pieces that has been introduced by Sievers and coworkers. The system is realized by a micromechanical cantilever structure. We showed that the Sievers system can trap excitations that are in a specific amplitude and frequency window. We have studied whether breathers form from noisy perturbations in the Fermi-Pasta-Ulam system which consists of masses interacting via a quadratic and a quartic potentials. Turns out that only weak breathers form from such perturbations. Our work suggests the emergence of highly localized excitations when synchronized or nearly synchronized perturbations from two ends meet in granular chains with soft centers. Similar physics is also seen in preliminary studies on Fermi-Pasta-Ulam chains.

Enter List of papers submitted or published that acknowledge ARO support from the start of the project to the date of this printing. List the papers, including journal references, in the following categories:

(a) Papers published in peer-reviewed journals (N/A for none)

Received		Paper
06/10/2015	7	Ding Han, Matthew Westley, Surajit Sen. Mechanical energy fluctuations in granular chains: The possibility of rogue fluctuations or waves, PHYSICAL REVIEW E, (09 2014): 32904. doi:
06/10/2015	8	Yannan Shen, Panos G Kevrekidis, Surajit Sen, Aaron Hoffman. Characterizing traveling-wave collisions in granular chains starting from integrable limits:The case of the Korteweg–de Vries equation and the Toda lattice, PHYSICAL REVIEW E, (08 2014): 22905. doi:
06/10/2015	9	Mukesh Tiwari, T R Krishna Mohan, Surajit Sen. Drag-force regimes in granular impact, PHYSICAL REVIEW E, (12 2014): 62202. doi:

TOTAL: **3**

Number of Papers published in peer-reviewed journals:

(b) Papers published in non-peer-reviewed journals (N/A for none)

Received Paper

TOTAL: **4**

Number of Papers published in non peer-reviewed journals:

(c) Presentations

Yoichi Takato and Surajit Sen, Nanoparticle collisions, PASI Conference, La Plata, Argentina, August 2014 (Takato was awarded a NSF Fellowship to present this work at La Plata).

Number of Presentations: 1.00

Non Peer-Reviewed Conference Proceeding publications (other than abstracts):

Received Paper

TOTAL:

Number of Non Peer-Reviewed Conference Proceeding publications (other than abstracts):

Peer-Reviewed Conference Proceeding publications (other than abstracts):

Received Paper

TOTAL:

Number of Peer-Reviewed Conference Proceeding publications (other than abstracts):

(d) Manuscripts

Received	Paper

TOTAL:

Number of Manuscripts:

Books

Received	Book

TOTAL:

Received	Book Chapter

TOTAL:

Patents Submitted

Patents Awarded

Awards

Graduate Students

NAME	PERCENT_SUPPORTED	Discipline
Yoichi Takato	0.47	
Matthew Westley	0.20	
Rahul Kashyap	0.28	
FTE Equivalent:	**0.95**	
Total Number:	**3**	

Names of Post Doctorates

NAME	PERCENT_SUPPORTED
FTE Equivalent:	
Total Number:	

Names of Faculty Supported

NAME	PERCENT_SUPPORTED	National Academy Member
Surajit Sen	0.25	
FTE Equivalent:	**0.25**	
Total Number:	**1**	

Names of Under Graduate students supported

NAME	PERCENT_SUPPORTED	Discipline
Michael E Benson	0.00	BS (Mathematical Physics)
FTE Equivalent:	**0.00**	
Total Number:	**1**	

Student Metrics

This section only applies to graduating undergraduates supported by this agreement in this reporting period

The number of undergraduates funded by this agreement who graduated during this period: 0.00

The number of undergraduates funded by this agreement who graduated during this period with a degree in science, mathematics, engineering, or technology fields:...... 0.00

The number of undergraduates funded by your agreement who graduated during this period and will continue to pursue a graduate or Ph.D. degree in science, mathematics, engineering, or technology fields:...... 0.00

Number of graduating undergraduates who achieved a 3.5 GPA to 4.0 (4.0 max scale):...... 0.00

Number of graduating undergraduates funded by a DoD funded Center of Excellence grant for Education, Research and Engineering:...... 0.00

The number of undergraduates funded by your agreement who graduated during this period and intend to work for the Department of Defense 0.00

The number of undergraduates funded by your agreement who graduated during this period and will receive scholarships or fellowships for further studies in science, mathematics, engineering or technology fields:...... 0.00

Names of Personnel receiving masters degrees

NAME
Total Number:

Names of personnel receiving PHDs

NAME	
William J Falls	
Yoichi Takato	
Total Number:	**2**

Names of other research staff

NAME	PERCENT_SUPPORTED
FTE Equivalent:	
Total Number:	

Sub Contractors (DD882)

Inventions (DD882)

Scientific Progress

Outcomes

William J Falls and Yoichi Takato defended their PhD theses during the period of the STIR Grant. Dr Falls is now an assistant professor at Erie Community College of the SUNY and Dr Takato is a Research Associate at the Okinawa Institute of Science and Technology in Okinawa, Japan. The papers published and under consideration during the grant period reflect a part of what was accomplished. The complete summary of the actual project work has been captured in this document. Published work included analyses of solitary waves and breathers in Fermi-Pasta-Ulam systems, of how solitary waves get temporarily localized near soft walls in Hertz systems, of possible interactions between nanoscale grains, of the nature of cratering in impacts on 3D beds with the most recent manuscript (under preparation) being on a pilot study of how granular chains with soft centers may help localize any incident energy. We have also carried out studies on how to localize energy in Fermi-Pasta-Ulam systems and a detailed paper is being completed now. The work has been geared to not just solve the problem at hand but do so in an encompassing way by exploring issues such as miniaturization, fundamentals such as the relationships between solitary waves and breathers and the formal theoretical underpinnings that tie the two, and even on how impacts damage a surface.

In the next stage, we are looking forward to solving this localization problem and that is precisely the focus of the newly submitted proposal.

Technology Transfer

Stable intrinsic localized modes in microelectromechanical cantilever structures and related studies:

Final Report to the Army Research Office on the STIR Grant (W911NF1410202)

Surajit Sen, PI

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Abstract

Protection of humans from large, finite time perturbations is an important area of research. Hence it is of interest to develop systems that are thin, light-weight and capable of absorbing large accelerations. An area of special interest concerns highly nonlinear systems which possess a rich variety of dynamical responses. This study focused on whether it is possible to convert any incident perturbation into localized excitations. The study explored the dynamics of a weakly nonlinear system with quadratic and quartic on-site and inter-site potentials where the linear pieces are significantly stronger than the nonlinear pieces that have been introduced by Sievers and coworkers. The system is realized by a micromechanical cantilever structure. We showed that the Sievers system can trap excitations that are in a specific amplitude and frequency window. We have studied whether breathers form from noisy perturbations in the Fermi-Pasta-Ulam system which consists of masses interacting via a quadratic and a quartic potentials. Turns out that only weak breathers form from such perturbations. Our work suggests the emergence of highly localized excitations when synchronized or nearly synchronized perturbations from two ends meet in granular chains with soft centers. Similar physics is also seen in preliminary studies on Fermi-Pasta-Ulam chains.

1. Introduction

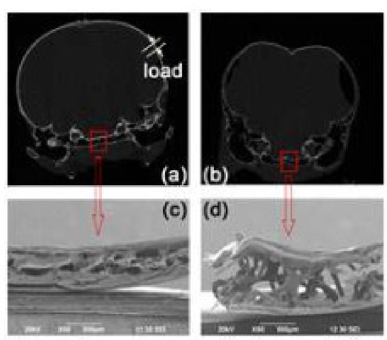

Fig. 1 (left): Great spotted woodpecker on the left and the Eurasian Hoopoe on the right; **Fig. 1 (right):** Woodpecker's head and skull bones on the left (a,c) and Hoopoe's head and skull bone on the right (b,d) [From L. Wang, *et al.* PLOS One 6(2011)e26490].

Protection of humans, equipment and machinery from high amplitude, short duration perturbations is an important area of basic and applied research. It is hence of significant interest to develop thin, strong and light mass materials for purposes of protection from impacts.

One can then ask whether it would be possible to make systems which would be very thin, very light, non-brittle and at the same time capable of damping sudden large changes in acceleration. While developing such materials has been a challenging journey, woodpeckers provide an inspiring example of what may someday be possible!

Typically, woodpeckers peck on wood about 20-22 times per second. Such rapid impacts can result in accelerations ~ 1200g [1]. However, woodpeckers don't get brain damage! An acceleration change of 1200g is a good order of magnitude larger than the protection that the best helmets can provide [2,3]. Studies show that the woodpecker skulls are thin and lightweight and have almost no bulk fluids (Fig. 1) [1,4,5]. Their skulls also have dense platelike structures and have low porosity [5]. How exactly do these skulls absorb such large acceleration changes is presently unclear.

While one way to build highly impact absorbing structures may be to mimic the woodpecker skull itself, we feel there may be other and more scalable ways to do so. One possibility is by invoking the very special properties of *strongly nonlinear systems*, which allow pulse transformation/disintegration on very short distances unlike linear or weakly nonlinear systems [6-9].

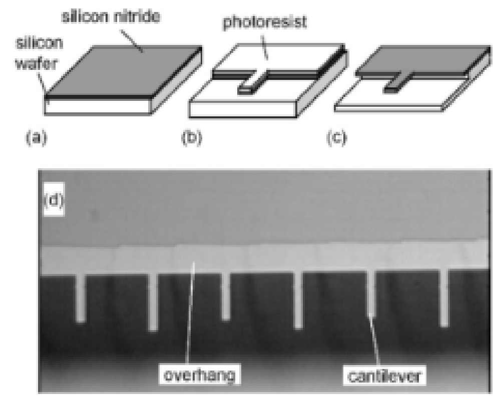

Fig. 2: The microelectromechanical (MEM) cantilever system in one of Sievers' studies with two different overhangs relating to a diatomic system (From [17-18]).

Special features of nonlinear many body systems: Many body systems with strongly nonlinear interactions are special because they exhibit a rich variety of ways to propagate energy through them [6-11]. These systems can admit the propagation of non-dispersive bundles of energy as compression pulses or solitary waves [12] and dilation pulses or anti-solitary waves, localized excitations or breathers [10-11], and localized or collective harmonic or acoustic oscillations or phonons. The existence of any or all of these dynamical quantities depends upon the system properties, the boundary conditions and the nature of the perturbations (e.g., amplitude, duration) effected onto the system [12,13]. The complex nature of the possible excitations also provides unique opportunities to control propagation and localization.

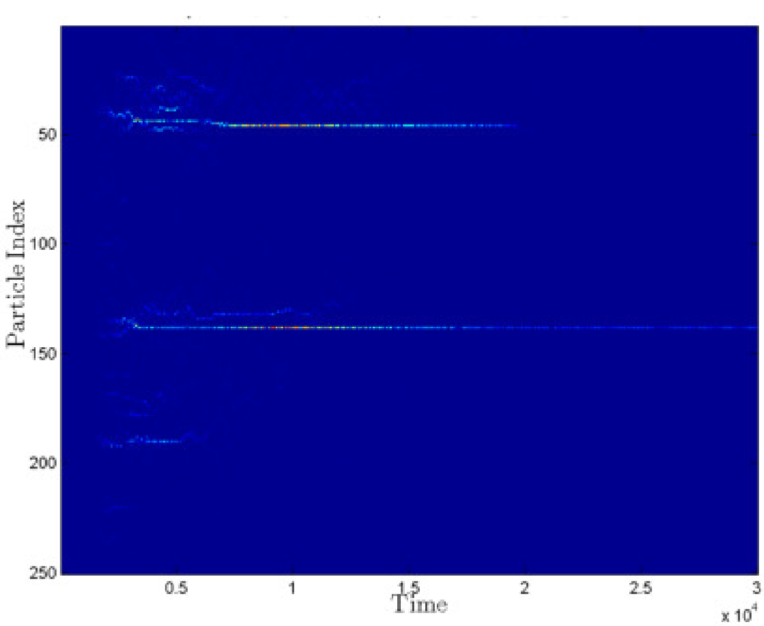

Fig. 3: Intrinsic localized modes precipitated in a driven-dissipative Sievers system (see Eq. 2 in Sec. 2), similar to what has been seen experimentally [24].

Why 1D matters: Ours and many other studies suggest that strongly nonlinear one dimensional systems are truly special [9,13]. Some of their behavior are translated into 2-D and 3-D systems [15,16], but many of the non-dispersive properties of the purely nonlinear entities such as solitary waves and breathers may not hold up very well in 2 or 3D systems except in special cases. It is hence desirable to place arrays of 1D chains to construct higher dimensional systems.

Using the geometry and the simplicity of the 1D systems would hence in principle be best to explore how to convert incident perturbations into energetic breathers. However, making breathers out of incident perturbations is a challenging task.

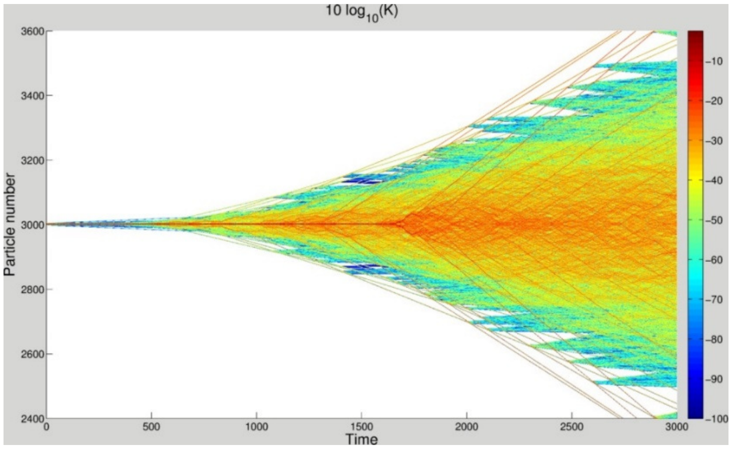

Fig. 4: A 6000 particle FPU system is studied where a breather has been seeded by stretching the center bond. We plot kinetic energy (in color) as functions of space and time. Kinetic energy is plotted on a log scale. Breathers decay here by simultaneously emitting solitary and anti-solitary waves, which in turn generate more such waves. In a smaller system wall effects lead to shorter breather lifetimes.

Much of Sievers' works have been done with small systems, typically the cantilever structures being ~ 50μm in size [17]. The ILMs can form when the system is driven at sufficiently high frequencies (how high depends of course on the dimensions of the system) (see Fig. 3).

Seeded and Unseeded Breathers: Breather formation can be extremely robust in many body systems with quadratic and quartic inter-site interaction potentials (i.e., a form of the Fermi-Pasta-Ulam systems) provided the breathers are initially seeded by stretching one or more bonds (Fig. 4). However, breather formation is weak when initiated by any form of random perturbations in such systems. No significant spontaneous breather formation exists in driven-dissipative FPU systems [25]. Whatever breather formation we have seen are for limited ranges of the harmonic and quartic spring couplings and the breathers are very low energy and short lived. In short, something special is needed to reliably make energetic breathers out of arbitrary excitations. In addition to the above studies in Fermi-Pasta-Ulam [6-8] and so called Sievers systems [17-23], precompressed diatomic granular chains, when sufficiently excited, can also precipitate ILM formation in much the same way as in the works of Sievers and coworkers [26].

Intrinsic Localized Modes/Unseeded Breathers with High Frequency Driving: Thus far, robust breather-like structures [10,11], referred to in the literature as intrinsic localized modes or ILMs, have been seen by Sievers and by several other groups following a recipe pioneered by Sievers [11,17-23]. Their driven-dissipative system is the one described by a potential energy function which consists of an inter-site coupling between the masses, which has both quadratic and quartic springs along with an on-site potential well which also exhibits both quadratic and quartic terms. In experiments, these systems can be realized by cantilever structures (Fig. 2). In all of Sievers' works, the magnitudes of the quartic interactions, inter-site and on-site, are two orders of magnitude less than those of the corresponding harmonic terms. These systems hence are weakly nonlinear systems.

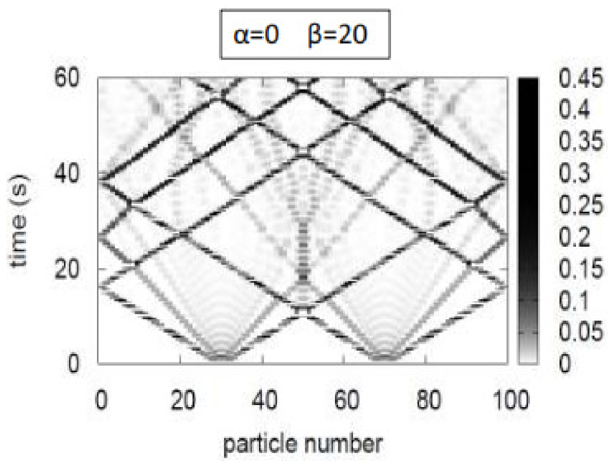

Fig. 5: A compression pulse moving right and a dilation pulse moving left collide at particle 50 in this space-time plot of energy propagation in an FPU chain to precipitate a metastable breather. Dark lines depict higher kinetic energy [27].

Breathers made to order?: Given the difficulty in spontaneously generating energetic breathers out of random perturbations in the various systems, it is important to see whether there are indirect ways of enhancing breather generation and thus increasing energy trapping. As we shall see below, our studies suggest that breathers tend to be initiated when a large amount of potential energy ends up at a site or a group of sites. A possible way to achieve this would be in regions where solitary waves or solitary and anti-solitary waves collide such that the region ends up getting squeezed. Specifically, we see that breathers form in the Fermi Pasta Ulam systems when two opposite propagating waves, one a solitary wave or a compression pulse and one an anti-solitary wave or a dilation pulse, both carrying equal energies, collide (Fig. 5) [27].

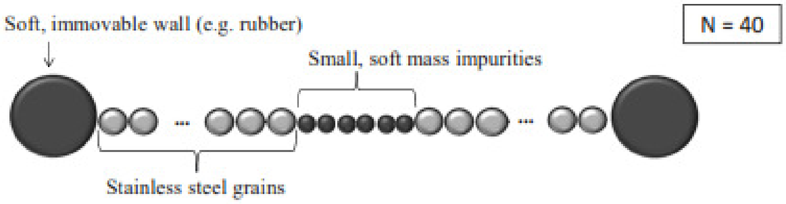

Fig. 6: Elastic grains placed between two soft end walls with 6 soft grains in the center.

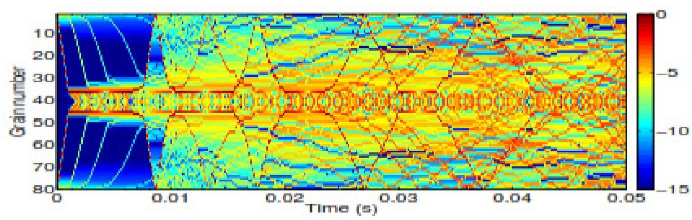

Fig. 7: Two colliding solitary waves make a central breather in a non-dissipative granular chain with soft central impurities. In a dissipative system, the center will act as an energy sink [28].

Breather formation is more enhanced when two solitary waves collide in granular chains where the collision region has a few soft grains [28]. Such granular chain systems can be best realized via use of elastic spherical grain alignments where the centers can be made quite deformable (Figs. 6 and 7) (in this context see Refs. [14] and [29]). Similar behavior is also seen in Fermi Pasta Ulam chains when two opposite propagating solitary waves collide in the middle and the middle region carries nonlinear springs that are stiffer than that elsewhere [25]. Our pilot studies hence suggest that it may be possible to precipitate breathers out of a variety of different forms of excitations provided that the right forms of interactions, the right interplay of time scales and the right geometric criteria can be realized [25].

2. Results from the STIR Study

It is meaningful to consider the problem of breather formation for the simplest possible nonlinear many body systems. The first one is the classic system introduced by Fermi, Pasta and Ulam (FPU) in 1955 given by the Hamiltonian below [6-8],

$$H_{FPU} = \sum_{i=1}^{N} \frac{p_i^2}{2m_i} + \sum_{i=1}^{N} [\frac{\alpha_{i,i+1}}{2}(x_i - x_{i+1})^2 + \frac{\beta_{i,i+1}}{4}(x_i - x_{i+1})^4], \quad (1)$$

where p_i, m_i, $\alpha_{i,i+1}$, $\beta_{i,i+1}$ represent the particle momenta, masses, and strengths of the quadratic and quartic inter-site couplings, respectively and N represents the number of particles in the system. Here and below x_i denotes the displacement from the equilibrium position. A version of the FPU-type system, but with added on-site potentials can be realized using cantilever structures and it was invoked by Sievers and coworkers, which we refer to as the Sievers system. The Sievers system is described as follows [17-18]

$$H_{Sievers} = \sum_{i=1}^{N} \frac{p_i^2}{2m_i} + \sum_{i=1}^{N} [\frac{k_{2,i}}{2}(x_i)^2 + \frac{k_{4i}}{4}(x_i)^4] + \sum_{i=1}^{N} [\frac{k_{2,i,i+1}}{2}(x_i - x_{i+1})^2 + \frac{k_{4i,i+1}}{4}(x_i - x_{i+1})^4], \quad (2)$$

where the on-site ($k_{2,i}, k_{4i}$) and inter-site ($k_{2,i,i+1}, k_{4,i,i+1}$) quadratic and quartic couplings are present. Thus, the Sievers system provides the capability to pin excitations at specific sites. Further, the parameter space available can be quite vast.

Both systems can be experimentally realized in precompressed granular metamaterials or laminate systems made from plates and o-rings. Additional modifications corresponding to changes in the onsite potential can be easily added in these systems, such as those introducing strong nonlinearity and effective dissipation with a unique capability for tuning by the initial precompression. These mechanical systems can be tuned into linear elastic, weakly nonlinear, strongly nonlinear or purely nonlinear responses depending on the initial precompression and the amplitude of the disturbance [6]. The simplest of these is the Hertz system discussed in Sec. 2, which consists of an alignment of elastic spheres and is described below. For simplicity we describe the Hertz system below [see also, 30-32],

$$H_{Hertz} = \sum_{i=1}^{N} \frac{p_i^2}{2m_i} + \sum_{i=1}^{N} [a_{i,i+1}(2R - (x_i - x_{i+1}))^{\frac{5}{2}}], \quad (3)$$

where $2R$ is the diameter of each spherical grain in the alignment and $a_{i,i+1}$ is a function of the Young's modulus and the Poisson ratio and refers to the prefactor associated with the Hertz potential which is the second term on the right hand side in the above equation [33]. Note that there is no interaction between the elastic spheres in the Hertz system when grain-grain contact is broken or ($x_i - x_{i+1} > 2R$) and the interaction between the adjacent grains is intrinsically nonlinear. When the grains are pre-compressed in the Hertz system, the grains there may no longer lose contact so easily and have oscillations about some mean positions. Such a system carries the presence of the harmonic component as well as nonlinear terms, both coming from the expansion of the Hertzian potential about strain caused by initial compression [6,13]. In this case the granular chains or the metamaterials assembled from plates and o-rings are essentially weakly nonlinear FPU systems for relatively small amplitudes of the perturbations [6,30-32].

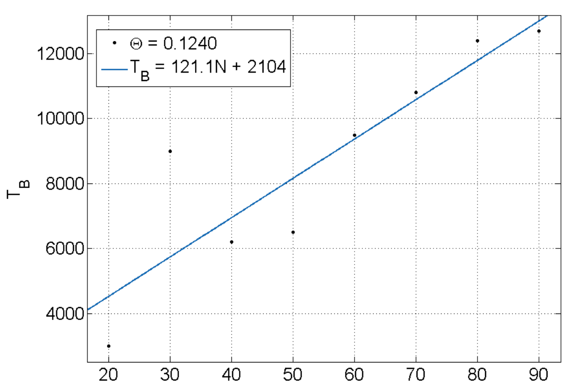

Fig. 8: The lifetime (T_B) of a seeded breather in the center of a FPU system is shown to grow as number of elements (N) increases.

In all dynamical many body systems at late enough times, on average, only a part of the total energy is kinetic and the rest is potential. Virial theorem dictates how much of the total energy is kinetic and how much is potential [13]. In systems with quadratic potentials, half the system is kinetic and half is potential. In nonlinear systems, the amount of kinetic energy exceeds the amount of potential energy. By how much kinetic energy exceeds the potential energy can depend on a number of parameters such as the strengths of the various terms in the potential energy function, the amplitude of the perturbation and the boundary conditions. If a lot of potential energy can somehow end up in a region of a strongly nonlinear system, it would be difficult for it to quickly disperse off that energy to satisfy the virial theorem globally. Our studies reveal that fairly long lived (metastable) breathers emerge only when a significant amount of potential energy somehow ends up in a specific site [34,35,36].

As alluded to in Sec. 1, nowhere is the issue of excess potential energy leading to stable breathers better seen than in the case of a seeded breather for the FPU system. To seed a breather, one can stretch a bond or two adjacent bonds at some location in a linear chain, for simplicity's sake say at the center. All the system's energy cannot rapidly leak out of the high energy bond(s). In fact, in many of these systems bonds can only leak energy via emission of solitary and anti-solitary wave pairs (see Fig. 4 above). These waves eventually bounce off the boundaries and perturb the breather itself, ultimately destabilizing the breather and leading to its demise into an equilibrium-like phase [36]. In the presence of strongly harmonic terms, phonons also appear as part of the leakage process [34,35]. However, it is somehow not easy for a breather to quickly convert its energy into phonons [34,35]. The upshot is that the breather lives for a span of time. For systems with just the right conditions, the breathers can live for very long times. In general, the larger a system, farther the walls and longer the breather may be able to survive (see Fig. 4). Such breathers can also be initiated in the Sievers system and possibly in the compressed Hertz system though the compressed Hertz system (grain or o-ring based) has not been exhaustively investigated. However, it is not necessarily of great interest to seed a breather if one is to make use of breathers to trap incident energy where the incident energy can be from arbitrary sources and across arbitrary times.

Breathers can be excited in the Sievers-type discrete systems by driving the system at some specific high optical band frequency window and allowing the additional energy to dissipate at an appropriate rate such that there is long term energy balance in the system alluded to in Sec. 1 [18]. In the system's natural units, the quadratic couplings are significantly stronger than the nonlinear couplings, i.e., when excited, these systems have weakly nonlinear interactions and an abundance of phonons [25]. Sievers and coworkers have worked mostly with diatomic systems though some monatomic system work has been done as well [18]. The existence of some high frequency response in these systems is needed. When these systems are excited by driving at high enough frequencies for long enough intervals, enough energy ends up in high frequency vibrations, which in turn can get localized in space while surrounded by a phonon bath. The phonon bath presumably suppresses the leakage of the ILMs thereby giving the structure some stability. In experiments, and simulations driven dissipative systems are typically needed to sustain ILMs. We will specifically differentiate ILMs and breathers because as we shall see in Sec 2 below, ILMs may be a specific type of breathers.

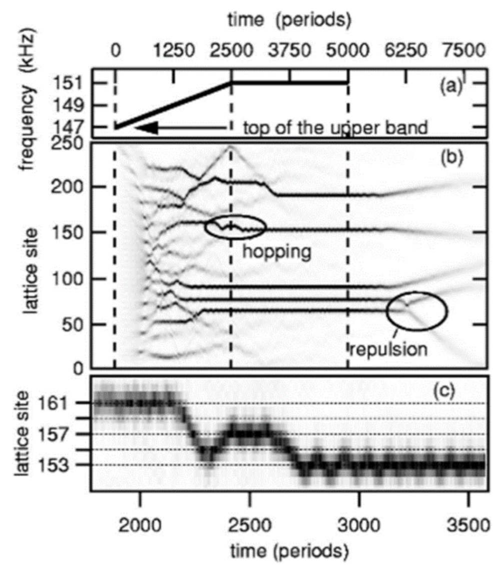

Fig. 9: The figure from Sievers' work shows the driving frequency behavior in time (upper panel), ILM (or breather) formation in real time and space (middle panel) ad a typical drift of an ILM (lower panel).

However, even this approach is not of much practical use when it comes to trapping just about any form of the frequency spectrum of the incident energy. Thus, we are left with limited choices on how to directly initiate breather formation from any form of incident perturbation.

One hope when it comes to the Sievers and FPU-type systems is whether it is possible to precipitate breathers out of the right range of system constants and the right perturbations which are not necessarily high frequency and within a well-defined window. There indeed are parameter regimes and perturbation types where one finds breathers forming in some situations. However, in no case that we have thus far identified are we able to find direct precipitation of *energetic* breathers from random perturbations [25]. Ultimately, these studies suggest that there may not be any direct way to generate energetic breathers from arbitrary perturbations. So we turned our attention toward using the help of solitary and anti-solitary waves as an intermediate step to generate breathers out of random perturbations. And this, as we shall see below, turns out to be a promising avenue to explore.

Although the FPU model was introduced in 1955 [6], there is only a relatively modest amount of understanding of how solitary and anti-solitary waves form, propagate and collide in these systems [27]. We have performed extensive analyses on the formation and propagation of solitary and anti-solitary waves in the FPU systems (which was originally introduced strictly to explain mechanism of thermlization/heat conduction using weakly nonlinear interaction). We find that in the strongly nonlinear regime, head on collisions between solitary and anti-solitary waves can lead to the formation of breathers in the vicinity of the collision regime (Fig. 5) [27]. Nevertheless, while these breathers can be stronger than those that are precipitated out of random perturbations in the FPU and Sievers systems, *it may still not be easy to make breathers with significant amounts of energy*. However, the study of the collision of solitary and anti-solitary waves suggested a new avenue to pursue – is it conceivable then that one can use solitary wave collisions to generate breathers? If so, can one make those breathers stick? And can one precipitate breathers with significant energies?

Hertz systems are simpler in the sense that the grains interact only when in contact. However, when the grains gently touch one another, the repulsive force is intrinsically nonlinear in nature, even nonlinearizable, which makes their behavior dramatically different from classical weakly nonlinear FPU system. If the granular chain is kept in a compressed state with some preloading, then a harmonic force is invoked in addition to the nonlinear force component, both due to the Hertz interaction, making its behavior similar to FPU system. At zero or very low preloadings, any delta function perturbation initiated at the edge of the chain becomes a solitary wave. Further, our earlier work has shown that these solitary waves can be rather sticky when they interact with very soft walls [14].

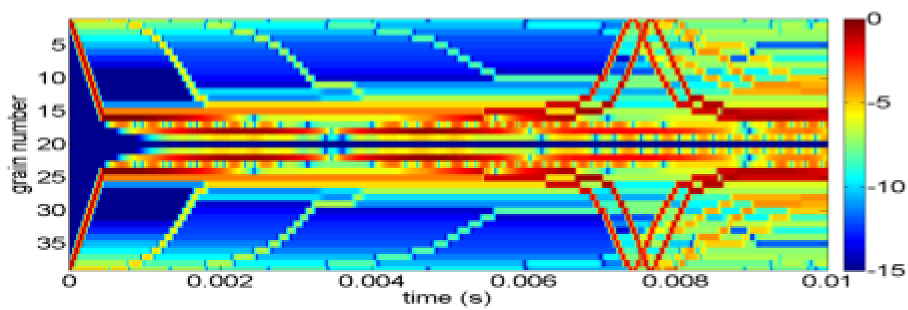

Fig. 10: A granular chain is shown where there are 3 central soft grains with a hard and a soft grain to each side of the central grains. Here we see the formation of two breathers in the center. These preliminary studies were not optimized to trap maximum energy. Also dissipative effects have been ignored. Observe the cold spot in the center (the blue line) [28].

These findings raise an intriguing possibility. What if we can initiate two solitary waves from the two ends of a granular chain and make them collide in a central region made up of soft grains and trap the energy for a while?

Our preliminary studies show that indeed such solitary wave collisions can precipitate breathers in granular chains [28]. We have done some pilot studies on collision between solitary waves in FPU chains where the central region contains springs with large spring constants. The experimental and theoretical investigations of the co-PI strongly suggest that similar results are to be anticipated for the more versatile and tunable discrete metamaterial assembled system of o-ring and plate periodic alignments, with sufficiently rigid o-rings [25,30-32,40]. Our successes hence suggest that there is indeed a way to make breathers at will and possibly out of arbitrary perturbations provided the system is designed right and this is what we will pursue in the proposed research section below. The simulational and phenomenological research will be largely pursed by the PI. In addition, experimental work on the highly tunable and strongly nonlinear o-ring based systems would be carried out by the co-PI.

3. Outcomes

William J Falls and Yoichi Takato defended their PhD theses during the period of the STIR Grant. Dr Falls is now an assistant professor at Erie Community College of the SUNY and Dr Takato is a Research Associate at the Okinawa Institute of Science and Technology in Okinawa, Japan. The papers published and under consideration during the grant period reflect a part of what was accomplished. The complete summary of the actual project work has been captured in this document. Published work included analyses of solitary waves and breathers in Fermi-Pasta-Ulam systems, of how solitary waves get temporarily localized near soft walls in Hertz systems, of possible interactions between nanoscale grains, of the nature of cratering in impacts on 3D beds with the most recent manuscript (under preparation) being on a pilot study of how granular chains with soft centers may help localize any incident energy. We have also carried out studies on how to localize energy in Fermi-Pasta-Ulam systems and a detailed paper is being completed now. The work has been geared to not just solve the problem at hand but do so in an encompassing way by exploring issues such as miniaturization, fundamentals such as the relationships between solitary waves and breathers and the formal theoretical underpinnings that tie the two, and even on how impacts damage a surface.

In the next stage, we are looking forward to solving this localization problem and that is precisely the focus of the newly submitted proposal.

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