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DATAWorks 2020:
A Notional Case Study of Uncertainty Analysis in Live Fire Modeling and Simulation

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Executive Summary

A vulnerability assessment, which evaluates the ability of an armored combat vehicle and its crew to withstand the damaging effects of an anti-armor weapon, presents a unique challenge. Since there are many areas that need to be assessed on the vehicle and testing is destructive, this limits the number of full-up, system-level tests to quantities that generally do not support meaningful statistical inference.

The prevailing solution to this problem is to obtain test data that is more affordable from sources that include component- and subsystem-level testing. This creates a new challenge that forms the premise of this paper: how can we connect lower-level data sources to provide a credible system-level prediction of vehicle vulnerability? This paper presents a notional case study of an approach to this problem that emphasizes the use of fundamental statistical techniques—design of experiments, statistical modeling, and propagation of uncertainty—in the context of a combat scenario that depicts a ground vehicle engaged by indirect artillery.

A Notional Case Study of Uncertainty Analysis in Live Fire Modeling and Simulation

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February 19, 2020

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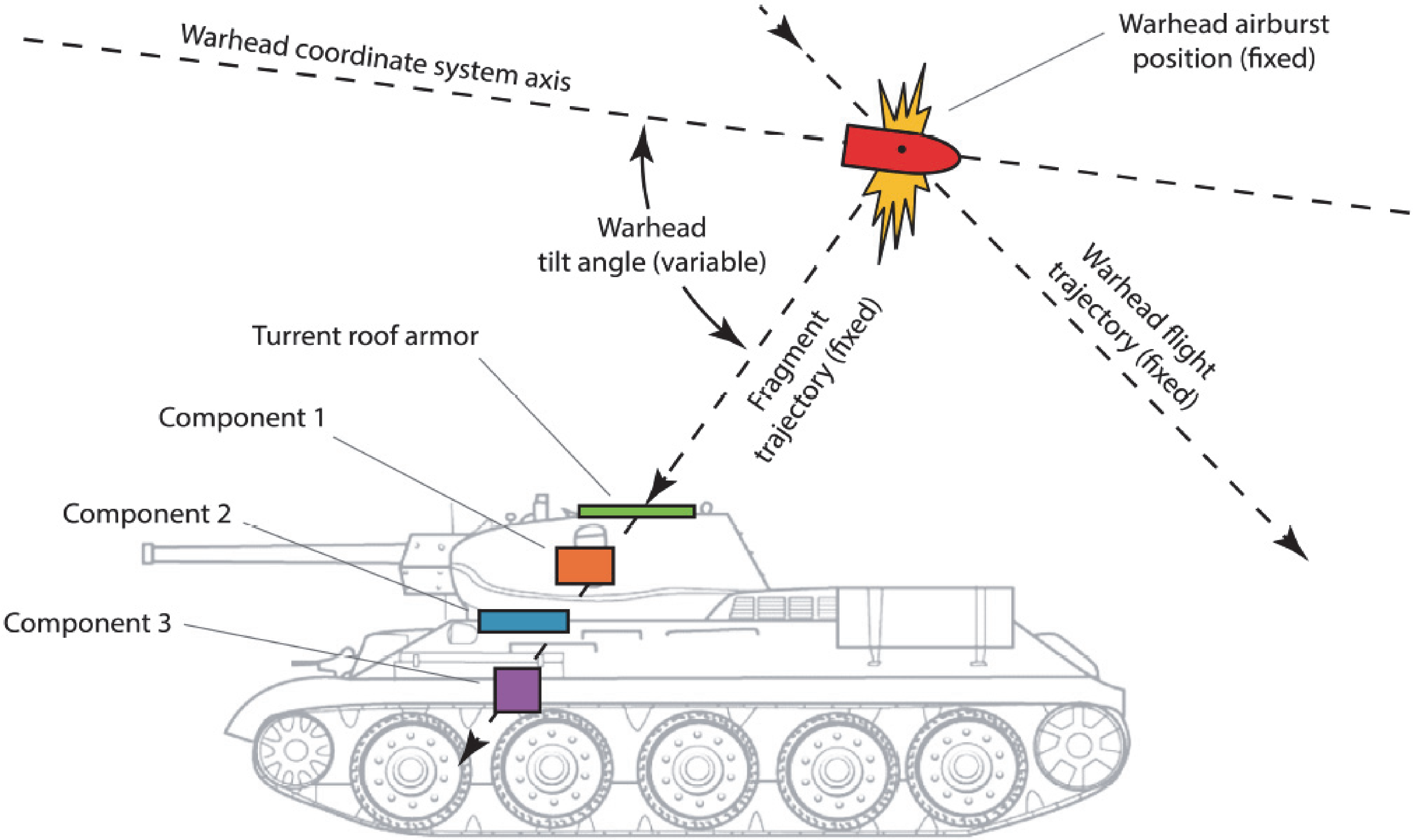
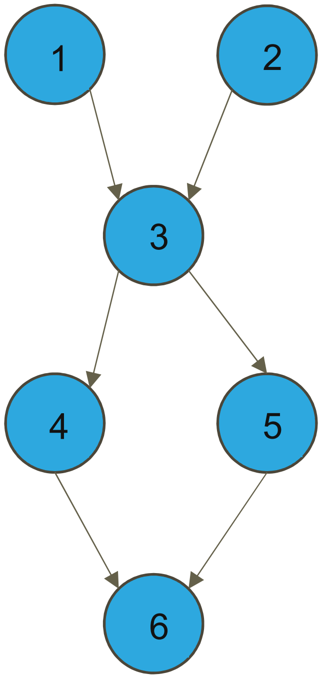
Case Study Engagement Scenario

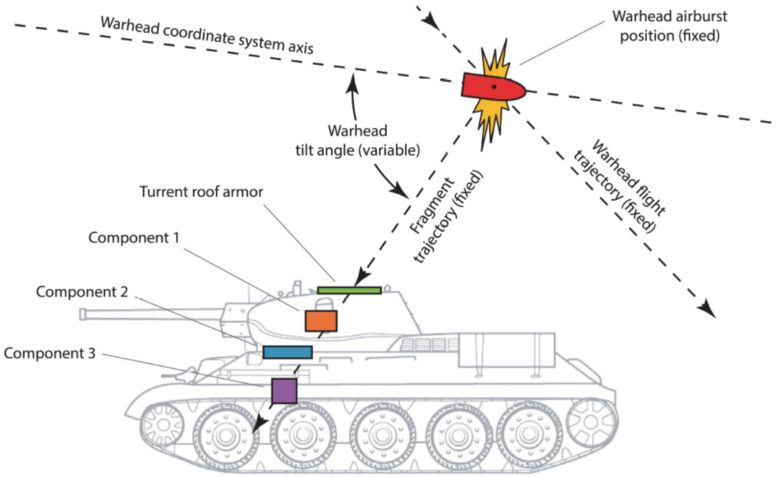


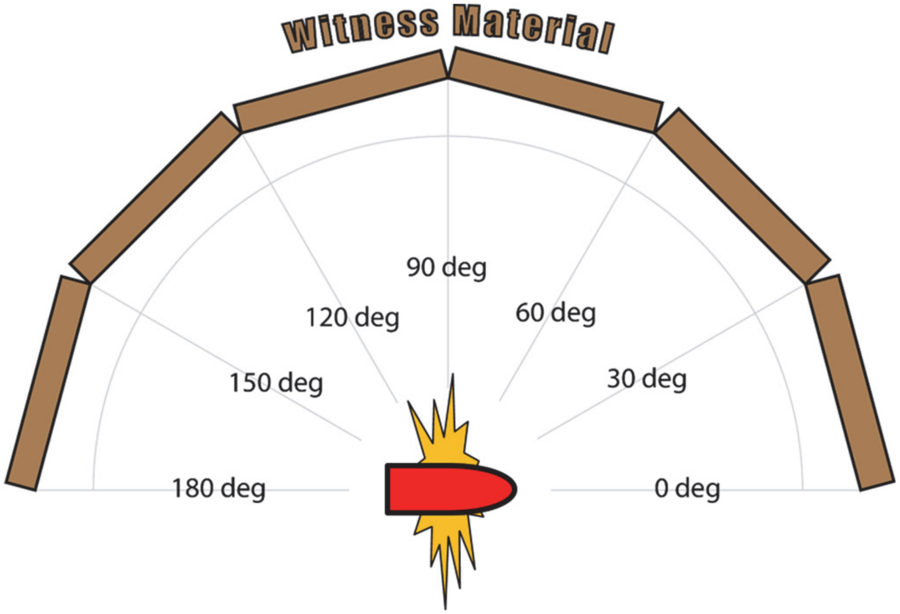
Three experiments, six models

- Warhead Airburst Experiment
  1. Fragment velocity model
  2. Fragment mass model
- Armor Coupon Experiment
  3. Probability of penetration model
  4. Residual velocity model
  5. Residual mass model
- Component Damage Experiment
  6. Probability of component damage model



Experiment 1: Warhead Airburst





Experiment 1: Warhead Airburst

Model 1: Fragment velocity model

Gaussian Process Model – typical formulation

$$
\begin{aligned}
y_{ij}^{\mathrm{I}} &\sim \text{normal}(f_i^{\mathrm{I}}, \sigma^{\mathrm{I}}) \quad \forall i \in \{1, 2, \ldots, 13\} \text{ and } j \in \{1, 2, \ldots, n_i^{\mathrm{I}}\} \\
f^{\mathrm{I}} &\sim \text{multivariate normal}\left(0, K(x \mid \alpha^{\mathrm{I}}, \rho^{\mathrm{I}})\right) \\
\alpha^{\mathrm{I}} &\sim \text{half normal}(0, .05) \\
\rho^{\mathrm{I}} &\sim \text{inv gamma}(.05, .05) \\
\sigma^{\mathrm{I}} &\sim \text{half normal}(0, 1)
\end{aligned}
$$

Experiment 1: Warhead Airburst

Model 1: Fragment velocity model

Gaussian Process Model – latent variable formulation

$$
\begin{aligned}
y_{ij}^{\mathrm{I}} &\sim \text{normal}(f_i^{\mathrm{I}}, \sigma^{\mathrm{I}}) \quad \forall i \in \{1, 2, \ldots, 13\} \text{ and } j \in \{1, 2, \ldots, n_i^{\mathrm{I}}\} \\
f^{\mathrm{I}} &= L^{\mathrm{I}} \eta^{\mathrm{I}} \\
L^{\mathrm{I}} &= \text{cholesky decompose}\left(K(x \mid \alpha^{\mathrm{I}}, \rho^{\mathrm{I}})\right) \\
\eta_i^{\mathrm{I}} &\sim \text{normal}(0, 1) \quad \forall i \in \{1, \ldots, 13\} \\
\alpha^{\mathrm{I}} &\sim \text{half normal}(0, 1) \\
\rho^{\mathrm{I}} &\sim \text{inv gamma}(.1, .1) \\
\sigma^{\mathrm{I}} &\sim \text{half normal}(0, 1)
\end{aligned}
$$

Experiment 1: Warhead Airburst

Model 2: Fragment mass model

Gaussian Process Model – latent Mott variable formulation

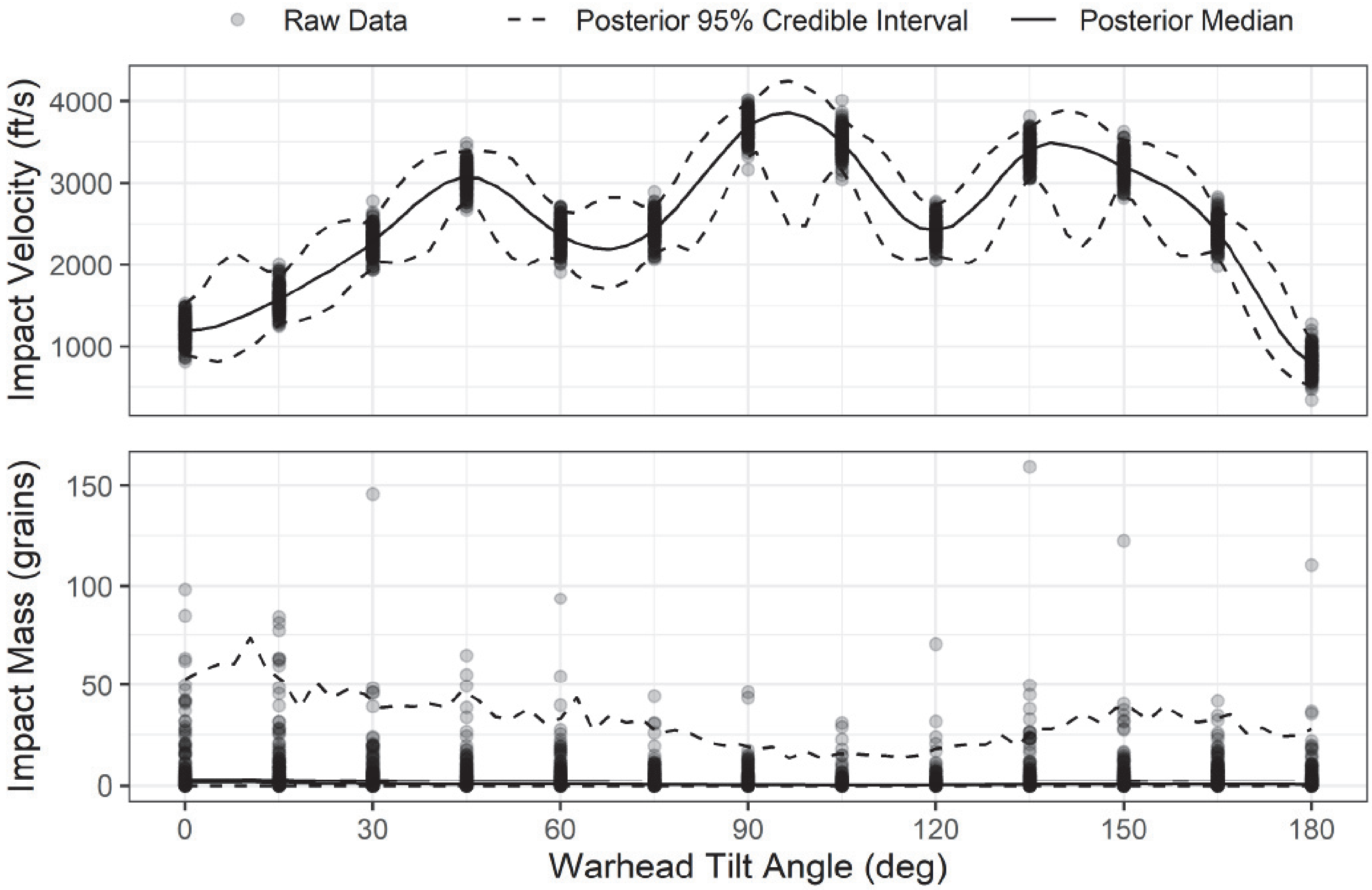
$$
\begin{aligned}
y_{ij}^{\mathrm{II}} &\sim \mathsf{Mott}(f_i^{\mathrm{II}}) \quad \forall i \in \{1,\ldots,13\} \text{ and } j \in \{1,2,\ldots,n_i^{\mathrm{II}}\} \\
\log\left(f^{\mathrm{II}}\right) &= L^{\mathrm{II}} \eta^{\mathrm{II}} \\
L^{\mathrm{II}} &= \mathsf{cholesky\ decompose}\left(K(x \mid \alpha^{\mathrm{II}}, \rho^{\mathrm{II}})\right) \\
\eta_i^{\mathrm{II}} &\sim \mathsf{normal}(0,1) \quad \forall i \in \{1,\ldots,13\} \\
\alpha^{\mathrm{II}} &\sim \mathsf{half\ normal}(0,1) \\
\rho^{\mathrm{II}} &\sim \mathsf{inv\ gamma}(.1,.1)
\end{aligned}
$$

Mott pdf:

$$p(a \mid b) = \frac{1}{2b}\left(\frac{a}{b}\right)^{-1/2} e^{-(a/b)^{1/2}}$$

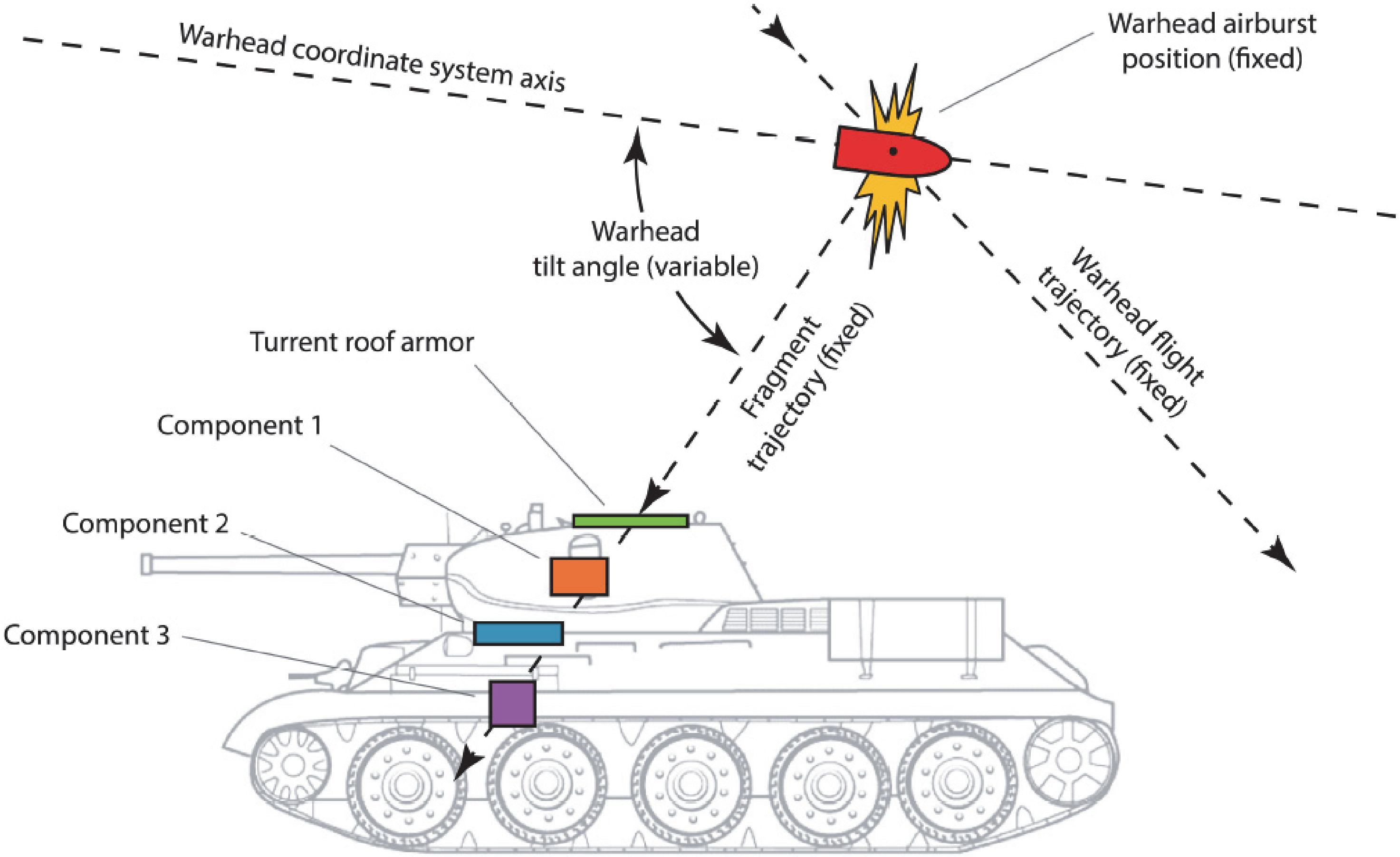
N. Mott, E. Linfoot, A Theory of Fragmentation, in: Fragmentation of Rings and Shells: The Legacy of N.F. Mott, Shock Wave and High Pressure Phenomena, Springer Berlin Heidelberg, Berlin, Heidelberg, 2006, p. 11 (2006).

Experiment 1: Warhead Airburst



Notional data

Experiment 2: Armor Coupon Experiment



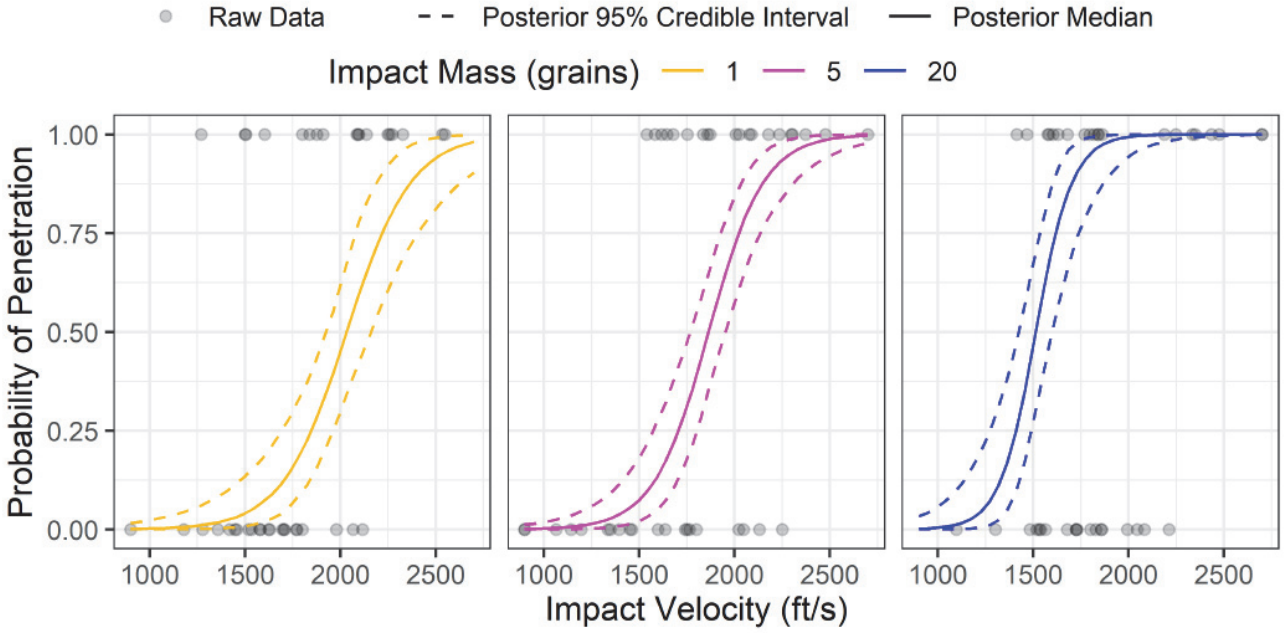
Experiment 2: Armor Coupon Experiment

Model 3: Probability of penetration model

$$
\begin{aligned}
y_k^{\text{III}} &\sim \mathsf{Bernoulli}(\mu_k^{\text{III}}) \quad \forall k \in \{1, 2, \ldots, n^{\text{III}}\} \\
\mathsf{logit}(\mu_k^{\text{III}}) &= \beta_A^{\text{III}} + \beta_B^{\text{III}} m_k + \beta_C^{\text{III}} v_k + \beta_D^{\text{III}} m_k v_k \\
\beta_A^{\text{III}}, \beta_B^{\text{III}}, \beta_C^{\text{III}}, \beta_D^{\text{III}} &\sim \mathsf{normal}(0, 10)
\end{aligned}
$$

Experiment 2: Armor Coupon Experiment

Model 3: Probability of penetration model



Notional data

Experiment 2: Armor Coupon Experiment

Model 4: Residual mass model

$$u_j \sim \text{exponential}(\tau_j)$$

$$\log(\tau_j) = \beta_E + \beta_F m_j + \beta_G v_j + \beta_H m_j v_j$$

$$\beta_E, \beta_F, \beta_G, \beta_H \sim \text{normal}(0, 1)$$

Experiment 2: Armor Coupon Experiment

Model 5: Residual velocity model

$$w_j \sim \text{normal}(\gamma_j, \sigma)$$

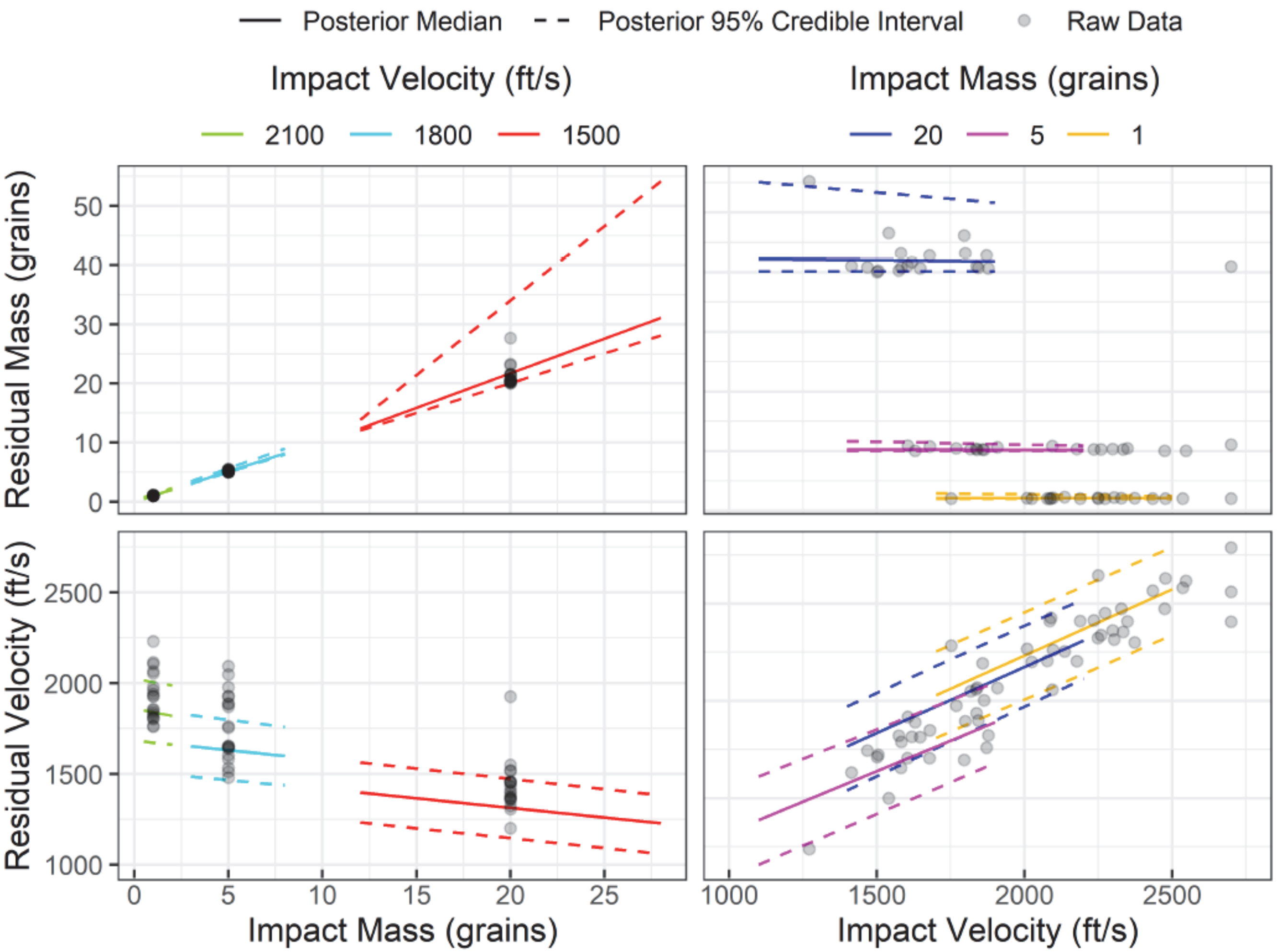
$$\gamma_j = \beta_P + \beta_Q m_j + \beta_R v_j + \beta_S m_j v_j$$

$$\beta_P, \beta_Q, \beta_R, \beta_S \sim \text{normal}(0, 10)$$

$$\sigma \sim \text{normal}(0, 10)$$

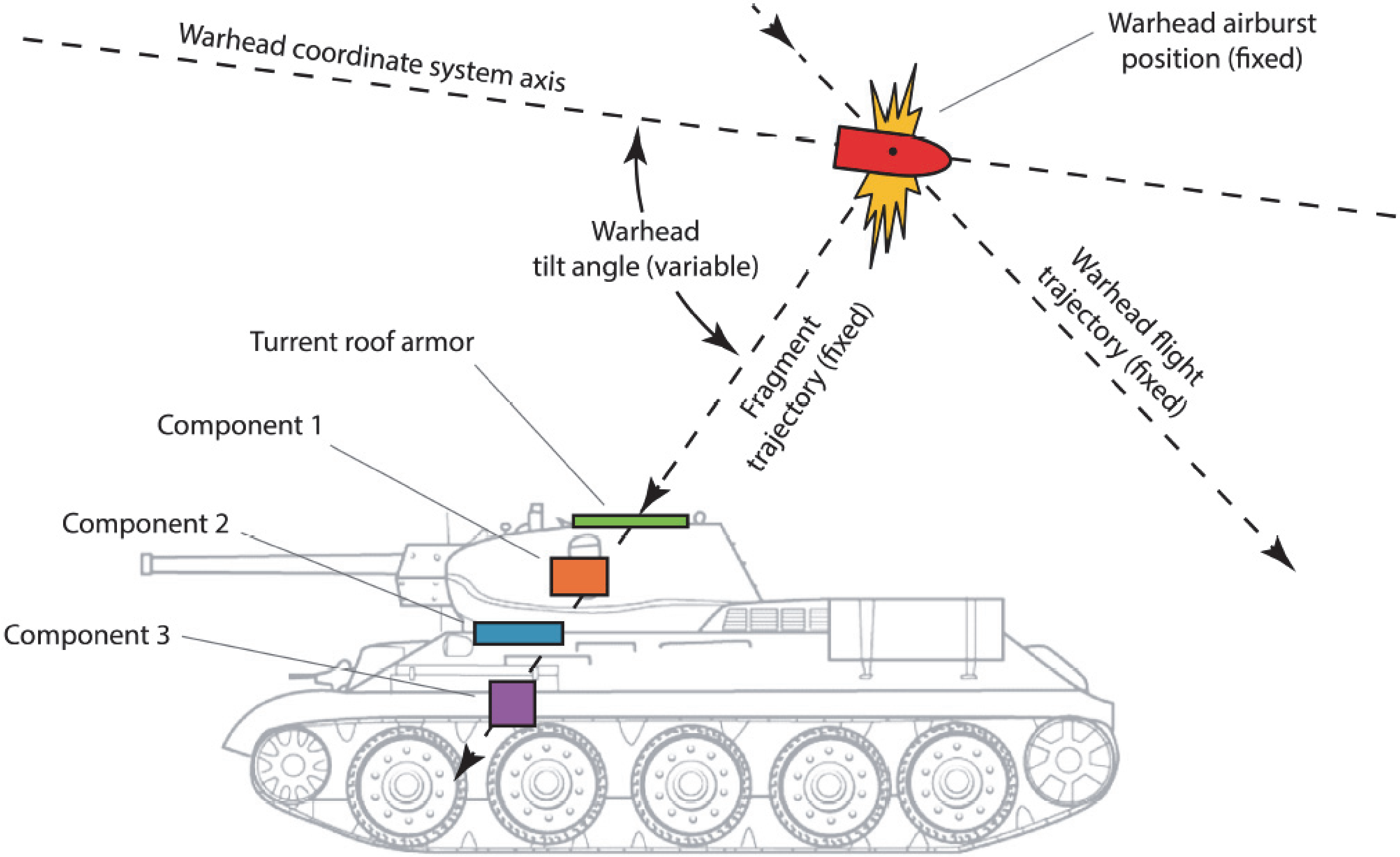
Experiment 2: Armor Coupon Experiment

Models 4 and 5: Residual mass and velocity models



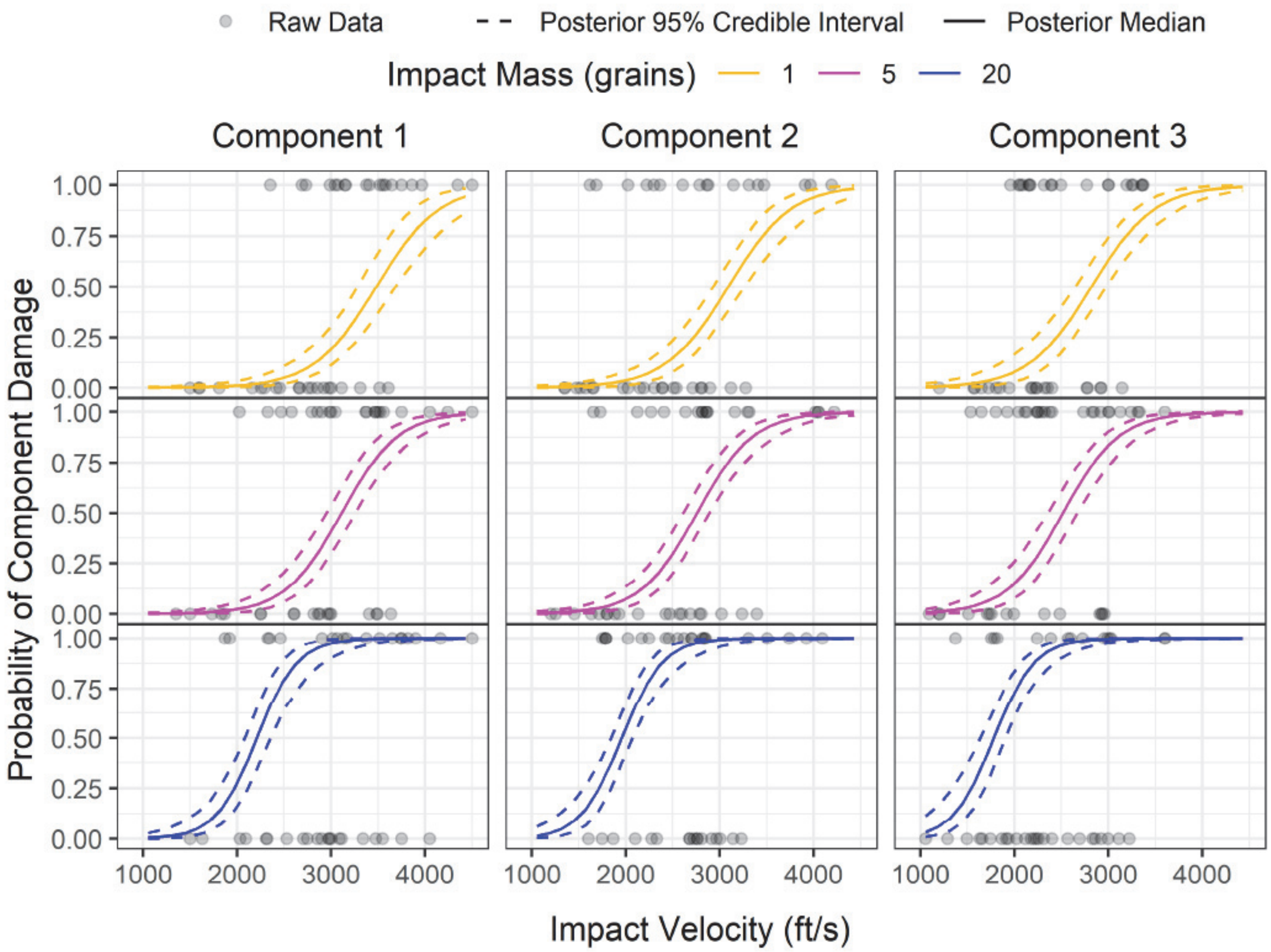
Notional data

Experiment 3: Component Damage Experiment



Experiment 3: Component Damage Experiment

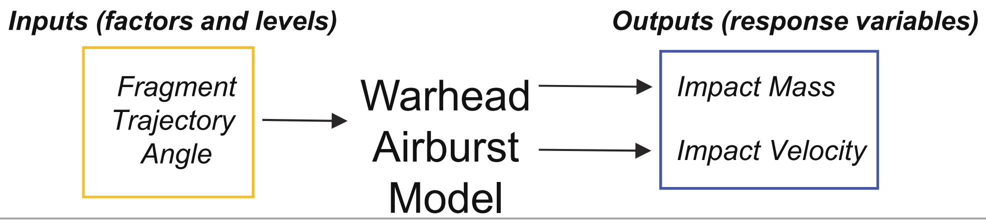
Model 6: Probability of component damage model



Notional data

Propagation of Uncertainty

First Model



Inputs (factors and levels)

Fragment Trajectory Angle

Warhead Airburst Model

Outputs (response variables)

Impact Mass

Impact Velocity

Second Model

Inputs (factors and levels)

Impact Mass

Impact Velocity

Armor Coupon Model

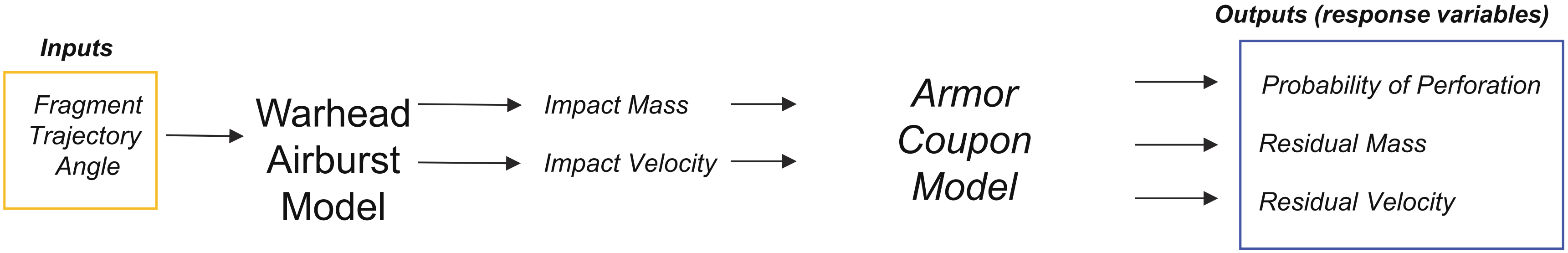
Outputs (response variables)

Probability of Perforation

Residual Mass

Residual Velocity

Propagation of Uncertainty from First to Second Model



Inputs

Fragment Trajectory Angle

Warhead Airburst Model

Impact Mass

Impact Velocity

Armor Coupon Model

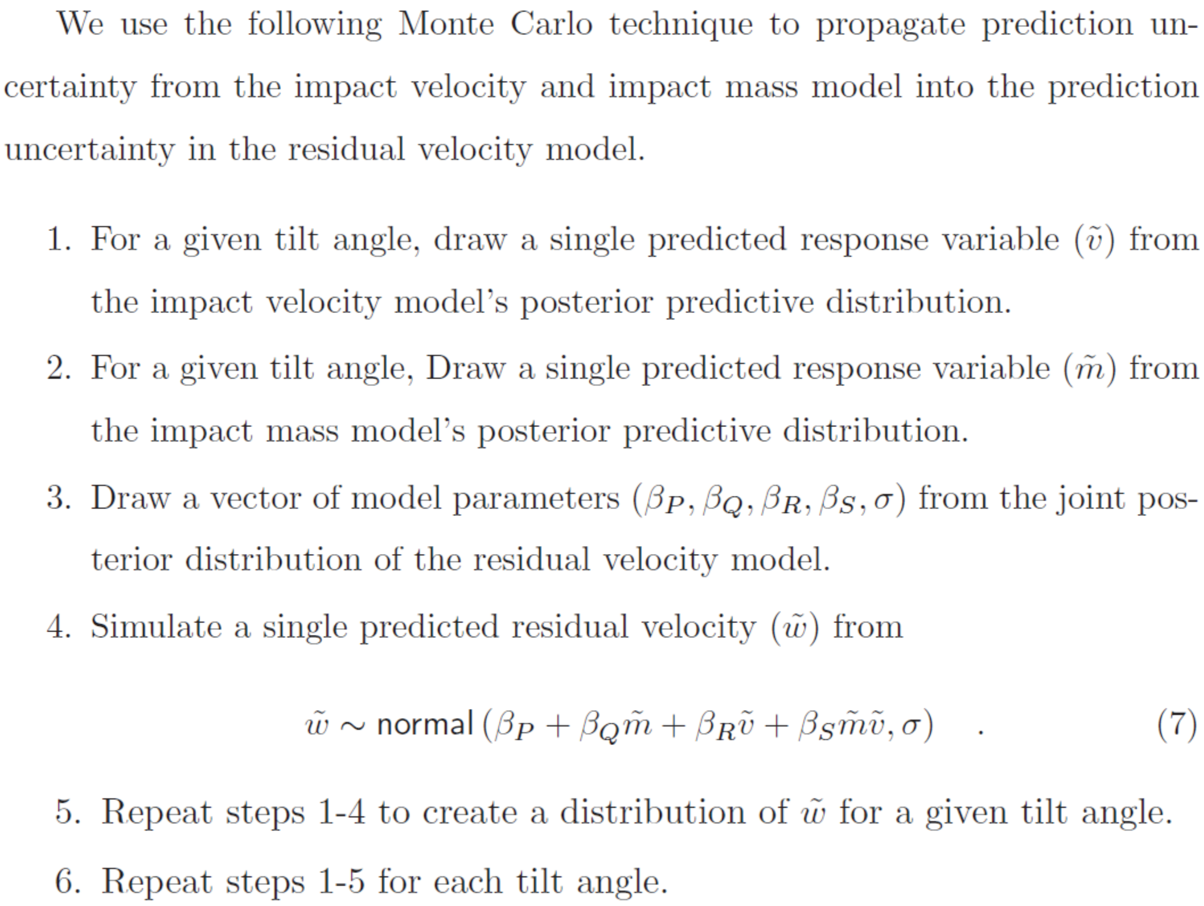
Outputs (response variables)

Probability of Perforation

Residual Mass

Residual Velocity

Propagation of Uncertainty



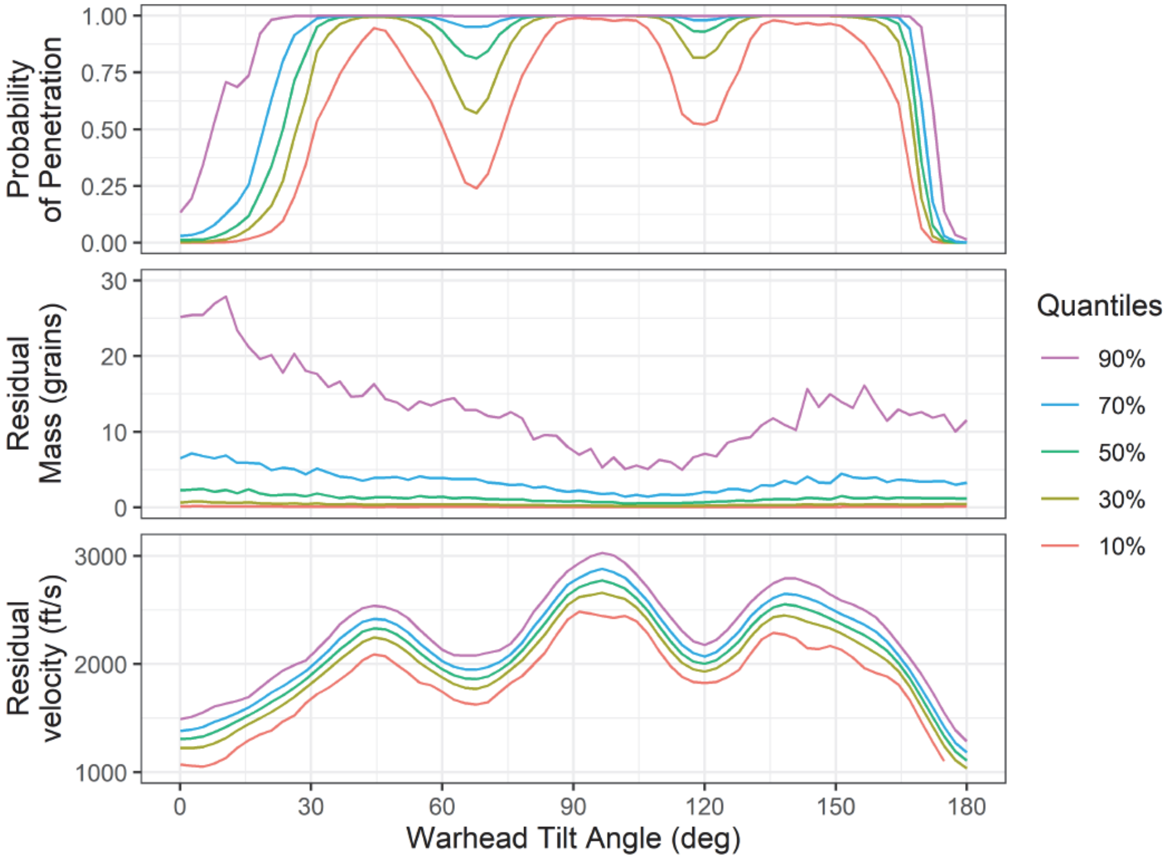
We use the following Monte Carlo technique to propagate prediction uncertainty from the impact velocity and impact mass model into the prediction uncertainty in the residual velocity model.

1. For a given tilt angle, draw a single predicted response variable ($\tilde{v}$) from the impact velocity model's posterior predictive distribution.
2. For a given tilt angle, Draw a single predicted response variable ($\tilde{m}$) from the impact mass model's posterior predictive distribution.
3. Draw a vector of model parameters ($\beta_P, \beta_Q, \beta_R, \beta_S, \sigma$) from the joint posterior distribution of the residual velocity model.
4. Simulate a single predicted residual velocity ($\tilde{w}$) from

$$\tilde{w} \sim \mathsf{normal}\left(\beta_P + \beta_Q \tilde{m} + \beta_R \tilde{v} + \beta_S \tilde{m}\tilde{v}, \sigma\right) \quad . \tag{7}$$

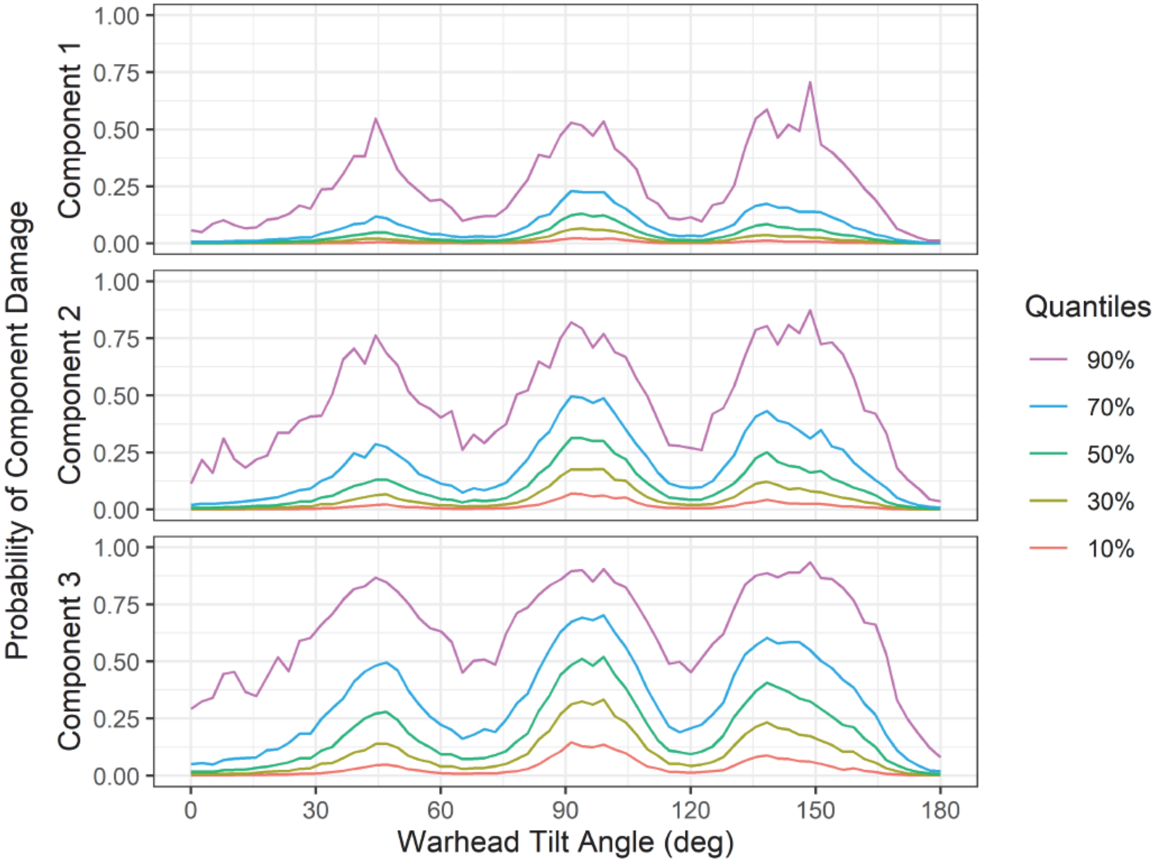
5. Repeat steps 1-4 to create a distribution of $\tilde{w}$ for a given tilt angle.
6. Repeat steps 1-5 for each tilt angle.

Propagation of Uncertainty – Stage 1



Notional data

Propagation of Uncertainty – Stage 2



Notional data

Propagation of Uncertainty – Stage 3

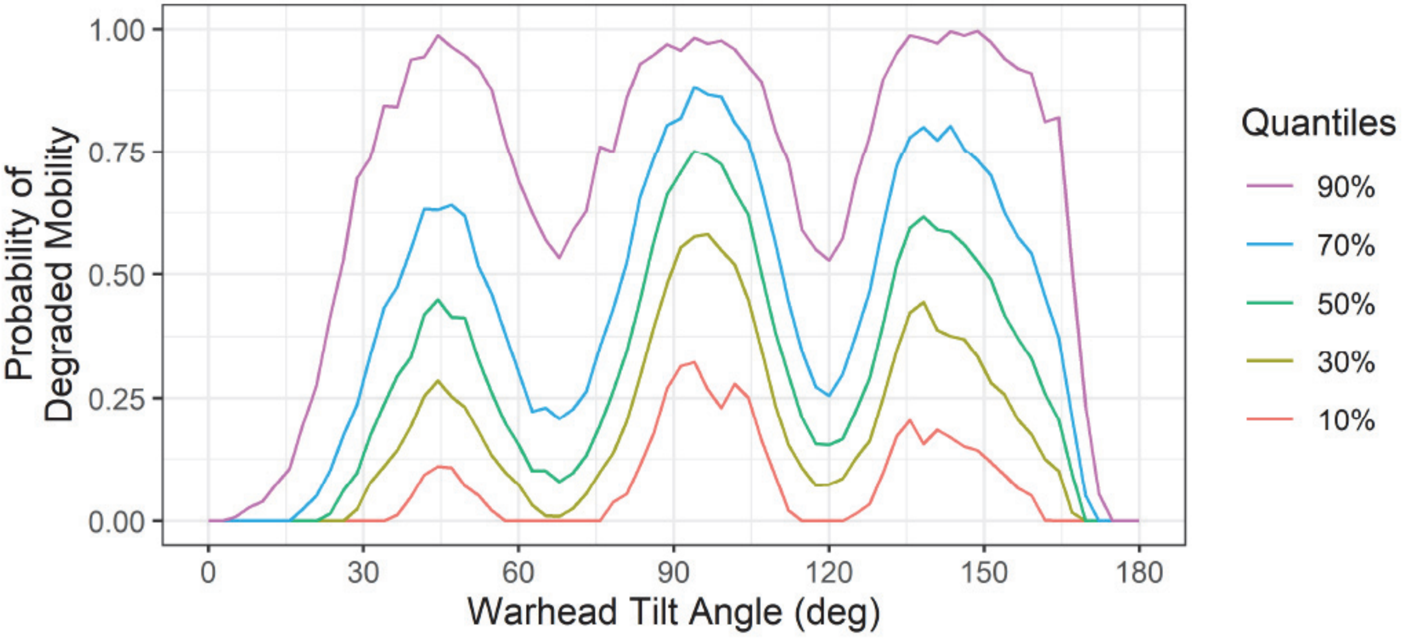
$$p_{mds/cd} = 1 - \left(1 - p_{c1d/h}\right)\left(1 - p_{c2d/h}\right)\left(1 - p_{c3d/h}\right)$$

Probability of
mobility degraded
state
given prob of
component damages

Probability of
component 1
non-dysfunction

Probability of
component 2
non-dysfunction

Probability of
component 3
non-dysfunction



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14. ABSTRACT
A vulnerability assessment, which evaluates the ability of an armored combat vehicle and its crew to withstand the damaging effects of an anti-armor weapon, presents a unique challenge because vehicles are expensive and testing is destructive. This limits the number of full-up-system-level tests to quantities that generally do not support meaningful statistical inference. The prevailing solution to this problem is to obtain test data that is more affordable from sources which include component- and subsystem-level testing. This creates a new challenge that forms the premise of this paper: how can lower-level data sources be connected to provide a credible system-level prediction of vehicle vulnerability? This paper presents a case study that demonstrates an approach to this problem that emphasizes the use of fundamental statistical techniques—design of experiments, statistical modeling, and propagation of uncertainty—in the context of a combat scenario that depicts a ground vehicle being engaged by indirect artillery.

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