Astronomical Applications Department Technical Note 2015-03 Desirability of Upgrading the Ephemerides of the Uranian Satellites Used in the Publication of *The Astronomical Almanac*

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Abstract The ephemerides of the Uranian satellites currently used in the production of *The Astronomical Almanac* are the GUST 86 ephemerides (Laskar & Jacobson, 1987, *Astron. Astrophys.*, **188**, 212). These ephemerides are nearly thirty years old. During the intervening years other ephemerides of the Uraninan satellites have been published. Three of these ephemerides were examined as possible replacements to the GUST 86 ephemerides. There were slight differences between the values produced over an approximately ten year period, but those differences were still significantly smaller than the RMS uncertainty in the observations of the satellites. So, there is no compelling reason to replace the current ephemerides. It may still be reasonable to replace the GUST 86 ephmerides with the Jacobson (2014, *Astron. J.*, **148**, 76) ephemerides for the Uranian satellites to ensure a self-consistent system.

1 The Current Status

The Astronomical Almanac provides data on the five largest Uranian satellites: (U I) Ariel, (U II) Umbriel, (U III) Titania, (U IV) Oberon, and (U V) Miranda. These data consist of the apparent orbits at the time of opposition, the mean sidereal periods, and the times of greatest northern elongation throughout the year.

The source of these data are the GUST 86 (Laskar & Jacobson, 1987) ephemerides of the Uranian satellites. The GUST 86 ephemerides are in the form of a satellite theory and were fit to 4122 ground-based optical observations made between 1911 and 1986 and 311 Voyager 2 optical navigation observations made between November 1985 and February 1986. The mean errors and RMS differences between the observed and ephemeris positions of the ground-based observations are shown in Table 1. The second row in the table gives the mean error and RMS differences for "modern" observations, that is those observations made after 1970.0. The errors for the Voyager 2 data are given in pixels and lines, so they are difficult to interpret.

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Table 1 The ground-based observations used in the Laskar & Jacobson (1987) ephemerides of the satellites of Uranus.

Mean Era	Number of Obs.	Mean X (arcsec)	Mean Y (arcsec)	σ X (arcsec)	σ Y (arcsec)
1966.2	4122	0.006	0.006	0.142	0.131
1980.7	2496	−0.006	−0.002	0.077	0.078

2 Modern Ephemerides

In the 28 year interval since the GUST 86 ephemerides were published a number of new ephemerides of the Uranian satellites have been published. In particular there are the ephemerides of Lainey (2008), Emelyanov & Nikonchuk (2013), and Jacobson (2014). All three sets of ephemerides are numerical integrations of the orbits rather than a satellite theory. Thus,

- they are potentially more accurate,
- they have a limited time span, and
- to get the position at an arbitrary instant, the position must be interpolated.

The following subsections summarizes each of these three ephemerides.

2.1 Lainey (2008)

The Lainey (2008) LA 07 ephemerides were fit using the DE406 planetary ephemerides. In addition to the Voyager 2 observations, the ephemerides were fit to ground-based observations of the satellites from 1948 through 2003, summarized in Table 2.

The mean errors and RMS differences for the ground-based observations are summarized in Table 3. Modern observations make up more than 99.9% of the observations for

Table 2 The ground-based observations used in the Lainey (2008) ephemerides of the satellites of Uranus.

Body		Number of Observations	Era Initial	Final	Mean
Ariel	(U I)	2123	1948	1999	1986.6
Umbriel	(U II)	2126	1948	2001	1986.4
Titania	(U III)	2621	1948	2005	1988.6
Oberon	(U IV)	2656	1948	2005	1988.8
Miranda	(U V)	1820	1948	1998	1987.0

Table 3 The mean errors in the Lainey (2008) ephemerides of the satellites of Uranus.

Body		Right Ascension (arcsec)	Declination (arcsec)
Ariel	(U I)	0.0018±0.0662	0.0079±0.1069
Umbriel	(U II)	0.0006±0.0680	0.0097±0.1020
Titania	(U III)	−0.0019±0.1096	0.0068±0.1214
Oberon	(U IV)	−0.0022±0.1154	−0.0004±0.1245
Miranda	(U V)	−0.0057±0.2083	−0.0394±0.3761

all five satellites. So, no separate comparison for modern observations is needed. Lainey is the only author to breakdown the uncertainty by satellite. The RMS differences in right ascension of LA 07 compared to the modern observations of GUST 86 are approximately 85% of GUST 86 for Ariel and Umbriel approximately 143% of GUST 86 for Titania and Oberon and approximately 270% of GUST 86 for Miranda. The RMS differences in declination of LA 07 compared to the modern observations of GUST 86 are approximately 133% of GUST 86 for Ariel and Umbriel approximately 160% of GUST 86 for Titania and Oberon and approximately 480% of GUST 86 for Miranda. The large RMS differences for Miranda are not unexpected because its orbit is near Uranus and it is small, hence, dim and difficult to observe.

Lainey does not provide the period over which these ephemerides were integrated. But, the output for the ephemerides of the Uranian satellites from the *Institut de mécanique célest et de calcul des éphémérides* (IMCCE) web site[1] indicate that the available time span of these ephemerides is 1 Jan. 1995 through 15.7 Apr. 2015 UTC. Thus, these ephemerides do not provide the required coverage for future Astronomical Applications products and will not be considered further.

2.2 Emelyanov & Nikonchuk (2013)

The Emelyanov & Nikonchuk (2013) ephemerides were fit using the INPOP10 planetary ephemerides. They were fit using ground based observations, summarized in Table 4, and Voyager 2 observations. The weighted mean era of observation is about May 1968 and the RMS difference between the observed and ephemeris position for all observations is

[1] http://www.imcce.fr/hosted_sites/saimirror/nssreq7he.htm

Table 4 The ground-based observations used in the Emelyanov & Nikonchuk (2013) ephemerides of the satellites of Uranus.

Body		Number of Observations	Era Initial	Final
Ariel	(U I)	5469	15 Sept. 1847	4 Jan. 2008
Umbriel	(U II)	5510	2 Oct. 1847	3 Jan. 2008
Titania	(U III)	10,077	16 Feb. 1787	4 Jan. 2008
Oberon	(U IV)	15,325	16 Feb. 1787	4 Jan. 2008
Miranda	(U V)	3997	15 Feb. 1948	4 Jan. 2008

$0\overset{\prime\prime}{.}43$. Of these observations 73.2% were made over the period of 1975 to 2008. The mean era for these observations is approximately March 1991 and the RMS difference in their positions is $0\overset{\prime\prime}{.}24$, about three times the GUST 86 RMS difference.

Approximately 39% of the modern observations used for these ephemerides came from Veiga et al. (2003). These observations have a particularly large RMS difference of $0\overset{\prime\prime}{.}29$. Removing them from the RMS difference of the modern observations does reduce the their RMS difference to $0\overset{\prime\prime}{.}19$, but their is still more than twice the RMS difference of GUST 86.

These ephemerides were integrated over the period from 1787 to 2031.0. Thus, adopting them for use *The Astronomical Almanac* also means they will need to be replaced in no more than 15 years.

2.3 Jacobson (2014)

Jacobson (2014) made a general solution of the ephemerides and orientation of the Uranian system. Not only did he solve for the ephemerides and masses of the five major satellites, he determined values for the mass, J_2 and J_4 zonal harmonics of Uranus' gravitational field, the ephemeris of U XV, Puck, the orientation of Uranus' pole of rotation, and the orientation of the Uranian rings. The satellite ephemerides were integrated from 1900 through 2100 using the DE430 planetary ephemerides.

These ephemerides were fit to 14,755 ground-based observations of the major satellites, U I through U V, made between 1911 and 2013 and 349 observations of U XV made between 1994 and 2004. In addition, 457 Voyager spacecraft observations of the major satellites and 65 observations of Puck were used as well. Table 5 summarizes the root mean square uncertainty of the ground-based observations of the major satellites.

The overall RMS uncertainties of the ground-based observations for Jacobson (2014) are approximately the same as those for GUST 86. The RMS uncertainty of the modern observations, on or after 1975.0, of Jacobson are a factor of about 1.4 greater. Also, the RMS uncertainty in the Voyager observations are a factor of about 1.5 greater. The increase in these uncertainties should *not* be taken as an indication

Table 5 The ground-based observations used in the Jacobson (2014) ephemerides of the satellites of Uranus.

Observation Type	Mean Epoch	Number X	Number Y	RMS X (″)	RMS Y (″)
Filar micrometer	1921.2	363	353	0.309	0.279
Photographic	1977.4	5575	5590	0.173	0.168
CCD	1997.2	10,278	10,279	0.086	0.096
Transit	2001.0	227	227	0.153	0.183
Mutual events	2007.5	37	38	0.011	0.016
Stellar occultation	2002.0	58		0.150	
TOTAL	1989.2	16,538	16,487	0.131	0.132
TOTAL modern	1993.2	14,755	14,702	0.106	0.105

that the Jacobson ephemerides are inferior to the GUST 86 ephemerides for two reasons:

1. The extent of the modern observations from the Voyager era is much longer for the Jacobson ephemerides. The GUST 86 modern observations only cover the ten years prior to the Voyager encounter. while the Jacobson observations cover the period from ten years prior to 28 years after the Voyager encounter.
2. The Voyager observations are still the most accurate set of observations of the Uranian satellites, but the much larger number of modern era ground-based observations for Jacobson (14,728 compared to 2496) reduces the impact of the Voyager observations.

Thus, the Jacobson ephemerides are not inferior to those of GUST 86.

3 Comparison of Positions

A direct comparison of the positions was made. In this comparison the geocentric X- and Y-offsets with respect to the center of Uranus was computed for each of the main Uranian satellites using each of the three ephemeris services. The interval used started on 1 Jan. 2010 and continued through 27 Sept. 2019. The step between position determinations was seven days, so the total number of positions for each satellite was 509.

Table 6 shows the mean differences, RMS uncertainties, maximum differences, and minimum differences of Jacobson (2014) and Emelyanov & Nikonchuk (2013) with respect to GUST 86. Table 7 shows the same statistics for GUST 86 and Emelyanov & Nikonchuk with respect to Jacobson. The dimensions for all quantities are in arcseconds.

Except for the mean difference in the Y-offset for Umbriel for Emelyanov & Nikonchuk, all of the mean differences are a few mas, and a factor of ten to twenty smaller than the RMS uncertainty in the differences. The Emelyanov & Nikonchuk mean Y-offset is $\sim 10^{-2}$ arcsec and at least a factor of 2.4 smaller than its RMS uncertainty. Thus, there are no significant differences in the mean positions of these three ephemerides.

Furthermore, except for the mutual event observations used in Jacobson (2014), the ground observation RMS uncertainties are a factor of two or more greater than the RMS uncertainties between the ephemerides. The mutual event observations RMS uncertainties are on the order of one fifth the RMS uncertainties between the ephemerides. These observations account for less than 0.3% of the observations in the solution. Thus, there are no significant differences in the positions observed from the ground and any of these three ephemerides.

The comparison of these three ephemerides to observations and each other gives no compelling reason why one should be adopted over the other.

4 Recommendation

Direct comparison of the GUST 86, Emelyanov & Nikonchuk (2013), and Jacobson (2014) with each other *and* observations provides no compelling reason to prefer one over another. The Emelyanov & Nikonchuk ephemerides also suffer from the fact that they extend only until 2031.0, so they would have to be replaced in no more than 15 years.

It may still be worthwhile to replace the GUST 86 ephemerides with the Jacobson (2014) ephemerides for the Uranian satellites because the Jacobson model also updates both the mass of Uranus and the orientation of Uranus' pole. The updated mass will most likely be incorporated into the DE planetary ephemerides. And the updated orientation may be adopted by the IAU working group on cartographic coordinates and rotational elements. Thus, using the Jacobson ephemerides would be more consistent with other sources of data used in *The Astronomical Almanac* than continuing with GUST 86.

Satellite ephemerides developed at the Jet Propulsion Laboratory (JPL), which include all those developed by Jacobson are available from the JPL's Navigation and Ancillary Information Facility (NAIF) web site in the form of SPK kernels. SPK kernel data can be evaluated by the SPICE Toolkit[2], the CALCEPH library[3], or by SPICE_mod[4].

Files for the Emelyanov & Nikonchuk ephemerides are *not* available. Ephemerides that can be evaluated at an arbitrary instant can be constructed for the Emelyanov & Nikonchuk ephemerides in a manner similar to that now used for the ephemerides of dwarf planets and asteroids using data from the IMCCE website. The software used to make this conversion is in need of better documentation (Hiton , 1999)

[2] Developed and maintained by NAIF.

[3] Developed and maintained by the Insitut de mécanique céleste et de calcul des éphémérides (IMCCE).

[4] A Fortran 95 module under development in the Astronomical Applications Department N3AA.

Table 6 Statistical comparison with respect to GUST 86. Geocentric offset with respect to the center of Uranus.

Body		Compared Ephemeris	Mean Difference (arcsec)		RMS Uncertainty (arcsec)		Maximum Difference (arcsec)		Minimum Difference (arcsec)	
			X	Y	X	Y	X	Y	X	Y
Ariel	(U I)	Jacobson	0.002	−0.004	0.042	0.043	0.505	0.045	−0.231	−0.616
		Emelyanov & Nikonchuk	0.004	−0.003	0.014	0.005	0.029	0.007	−0.022	−0.015
Umbriel	(U II)	Jacobson	0.001	−0.003	0.049	0.051	0.499	0.064	−0.242	−0.587
		Emelyanov & Nikonchuk	0.003	−0.015	0.035	0.036	0.064	0.043	−0.049	−0.085
Titania	(U III)	Jacobson	0.002	−0.004	0.050	0.053	0.457	0.077	−0.234	−0.613
		Emelyanov & Nikonchuk	0.000	−0.009	0.035	0.045	0.067	0.060	−0.070	−0.092
Oberon	(U IV)	Jacobson	0.002	−0.005	0.053	0.061	0.440	0.109	−0.220	−0.632
		Emelyanov & Nikonchuk	0.001	−0.007	0.017	0.034	0.038	0.051	−0.039	−0.070
Miranda	(U V)	Jacobson	0.002	−0.004	0.042	0.040	0.512	0.034	−0.246	−0.592
		Emelyanov & Nikonchuk	0.002	−0.004	0.027	0.011	0.049	0.022	−0.054	−0.025

Table 7 Statistical comparison with respect to Jacobson (2014). Geocentric offset with respect to the center of Uranus.

Body		Compared Ephemeris	Mean Difference (arcsec)		RMS Uncertainty (arcsec)		Maximum Difference (arcsec)		Minimum Difference (arcsec)	
			X	Y	X	Y	X	Y	X	Y
Ariel	(U I)	GUST 86	−0.002	0.004	0.042	0.043	0.231	0.616	−0.505	−0.045
		Emelyanov & Nikonchuk	0.003	0.001	0.042	0.043	0.227	0.607	−0.514	−0.048
Umbriel	(U II)	GUST 86	−0.001	0.003	0.049	0.051	0.242	0.587	−0.499	−0.064
		Emelyanov & Nikonchuk	0.002	−0.011	0.040	0.041	0.226	0.593	−0.486	−0.048
Titania	(U III)	GUST 86	−0.002	0.004	0.050	0.053	0.234	0.613	−0.457	−0.077
		Emelyanov & Nikonchuk	−0.001	−0.005	0.039	0.042	0.231	0.596	−0.493	−0.055
Oberon	(U IV)	GUST 86	−0.002	0.005	0.053	0.061	0.220	0.632	−0.440	−0.109
		Emelyanov & Nikonchuk	−0.001	−0.005	0.039	0.042	0.231	0.596	−0.493	−0.055
Miranda	(U V)	GUST 86	−0.002	0.004	0.042	0.040	0.246	0.592	−0.512	−0.034
		Emelyanov & Nikonchuk	0.000	0.000	0.050	0.041	0.272	0.598	−0.524	−0.039

and the software should be reviewed to assure it is correct and robust.

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