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A Difference in the Evaluation of Ephemerides Arising from Using the SPICE Format Specification Rather Than Export Format File Specification

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Abstract

The Jet Propulsion Laboratory (JPL) makes its planetary and lunar ephemerides available in two formats. The first is the export format devised by JPL's Solar System Dynamics group. The second is the SPICE Double Precision Array File format devised by JPL's Navigation and Ancillary Information Facility. JPL's requirement for the evaluation of these ephemerides is that they are precise to 0.5 mm (Newhall, 1989). A comparison of the export format and SPICE format versions of the same ephemeris found differences as large as 20 mm. The largest differences occur for Mercury, the planet with the smallest semi-major axis. It is shown that the differences arise not from a difference in the values of the coefficients in the files but because the export format specifies the use of an extended precision time argument while the time argument in the SPICE specification is double precision. Evaluating the SPICE format version of the ephemeris using an extended precision time argument reduces the differences in the evaluated positions to meet the 0.5 mm requirement.

1. The Problem

The Jet Propulsion Laboratory's (JPL) requirement for the evaluation of solar system ephemerides is that they are precise to 0.5 mm (Newhall, 1989). This requirement is approximately the precision of a 64 bit number on the scale of the solar system. It is several orders of magnitude more precise than the current accuracy of the ephemerides for all these bodies. For example, Folkner et al. (2014) provides, in Table 1, the best estimate accuracies in the DE430 ephemerides for the planets and the Moon. These are the solar system bodies whose positions are known with the most accuracy. Only the ephemeris of the Moon is accurate enough to be within three orders of magnitude of the precision requirement.

Table 1: Accuracy of the DE430 Solar System Ephemerides

Body	Accuracy
Mercury	20 m
Venus	15 m
Mars	5 m
Jupiter	400 m
Uranus	2500 km
Neptune	2000 km
Pluto	2500 km
Moon	4 cm

JPL makes its planetary ephemerides available in two formats: the export format (Standish & Newhall, 1988) and SPK kernels for use with the SPICE library (Acton, 1990). In light of the accuracy requirement it is expected that the values for the position and velocity of the JPL DE430 ephemerides agree to within 0.5 mm, regardless of the format.

The Astronomical Applications Department has relied on export format ephemeris files for use in producing its products. Two recent developments make the development of a ability to use SPK kernels desirable:

- The International Astronomical Union (IAU) has recently recommended the use of SPK kernels to distribute solar system ephemerides.
- The state-of-the-art ephemerides of satellites, other than the Moon, produced at JPL are available only as SPK kernels.

To meet these developments, a set of Fortran procedures were written to read SPK kernel ephemeris files. These procedures were validated by comparing the DE430 ephemerides evaluated using both their SPK kernel and export format file. The differences between the SPK and export format evaluations were determined for 100,000 positions and velocities each for the planets (except Earth), the Sun, the Earth-Moon barycenter (EMB), and Pluto with respect to the solar system barycenter. In addition, the differences were determined for 100,000

Table 2: The difference between the DE430 solar system ephemerides in export and SPK kernel formats.

Body	σ_x (mm)	Δx_{max} (mm)
Sun	0.0064	0.042
Mercury	23.	168.
Venus	17.	107.
EMB	15.	92.
Mars	12.	72.
Jupiter	6.5	39.
Saturn	4.8	31.
Uranus	3.4	20.
Neptune	2.8	17.
Pluto	2.2	11.
Earth	0.0075	0.043
Moon	0.50	3.1

positions and velocities each for the Earth and Moon with respect to the EMB. A constant interval of 4.01795 days beginning at the initial Julian Date for DE430 (228 7184.5) was used. The interval was chosen so that the entire time span of DE430 was sampled without repeating the same relative instant of the Chebyshev polynomials that make up the ephemerides.

A partial summary of the results are shown in Table 2. Here the standard deviation and maximum differences are given for the x-coordinate. The x-axis lies approximately in the plain of the ecliptic, which, in turn, is inclined approximately 3° to the solar system's invariable plane (Owen, 1990). Thus, the magnitude and error of the x-coordinate is the greatest for all of the ephemerides.

Only the difference in the positions of the Sun with respect to the solar system barycenter and the Earth with respect to the Earth-Moon barycenter meet the JPL requirement for precision in representation of the ephemerides.

In no case are the differences between the the evaluations of the ephemerides large enough to be significant compared to the accuracies of the ephemerides in Table 1. The difference between the export and SPK evaluations closest to being significant is the 1.1 orders of magnitude difference for the lunar ephemeris.

The naïve expectation is the magnitudes of the differences would be proportional to the magnitudes of the semi-major axes. Instead, except for the Sun with respect to the solar system barycenter and the Earth with respect to the Earth-Moon barycenter, the differences have an inverse relation to the semi-major axis. Comparing the magnitudes of the differences to the bodies' mean speeds with respect to the center of its coordinate system results in a high correlation of 0.96.

2. The Source of the Differences

Both the export format and the SPK kernel store the positions and velocities in the form of Chebyshev polynomial coefficients. The value of a parameter, y, for independent variable x evaluated using Chebyshev polynomials, up to order j is,

$$t = \frac{x - t_0}{2\, t_{span}} \tag{1}$$

$$y(x) = \sum_{i=0}^{j} a_i T_i(t) \tag{2}$$

where t_{span} is the length of the span of the independent variable covered by the Chebyshev polynomials, t_0 is value of the independent variable at the center of that span, and $T_i(t)$, is the value of the ith order Chebyshev polynomial evaluated at t. A property of parameters represented by Chebyshev polynomials (Rivlin, 1974) is

$$a_k \gtrapprox \sum_{i=k+1}^{\infty} a_i \tag{3}$$

That is, low order Chebyshev polynomials dominate the evaluation. The Chebyshev polynomials may be iteratively generated from

$$\begin{aligned} T_0(t) &= 1, \\ T_1(t) &= t \\ &= \frac{x - t_0}{2\, t_{span}}, \quad \text{and} \\ T_{i+1}(t) &= 2\, t\, T_i(t) - T_{i-1}(t) \\ &= 2\, \frac{x - t_0}{2\, t_{span}}\, T_i\left(\frac{x - t_0}{2\, t_{span}}\right) \\ &\quad -T_{i-1}\left(\frac{x - t_0}{2\, t_{span}}\right). \end{aligned} \tag{4}$$

Thus, the precision to which $T_i(t)$ can be determined is the same as the precision of x for $i > 0$.

For the export format x is a 128-bit floating-point number. But the SPICE system specifies a 64-bit floating-point number for x. So, the source of the differences could be the difference in the precision of the time argument used to evaluate the ephemerides. Two tests were performed to determine validate this hypothesis.

2.1. Comparison to the Mean Motion

If the source of the differences arises from the precision to which the independent variable is known, the mean value of the differences in the evaluated parameters is expected be proportional to the mean value of the first derivative of the evaluated parameter with respect to the independent variable.

Table 3: The ratio of the standard deviation in the differences between the export and SPK Kernel formats to the mean motion of the bodies.

Body	Mean Motion ($m\ s^{-1}$)	σ_r/n (s)
Sun	15	4.5×10^{-7}
Mercury	47843	3.8×10^{-7}
Venus	35005	5.8×10^{-7}
EMB	29785	5.9×10^{-7}
Mars	24133	5.2×10^{-7}
Jupiter	13067	6.0×10^{-7}
Saturn	9644	6.0×10^{-7}
Uranus	6800	5.4×10^{-7}
Neptune	5433	5.4×10^{-7}
Pluto	4743	3.5×10^{-7}
Earth	12	6.4×10^{-7}
Moon	1011	4.4×10^{-7}

The mean motions of the Earth and Moon are with respect to the Earth-Moon barycenter. All of the other mean motions are with respect to the solar system barycenter.

In this case, the mean differences in the positions should be proportional to the mean motion of the body. Table 3 shows that this is the case. This table shows the ratio of the standard deviation of the differences in the distances, σ_r to the mean motion, n for each of the solar system bodies. Although the values of n spans 3.6 orders of magnitude, the values of σ_r/n only spans 0.26 orders of magnitude. The mean value of $\langle \sigma_r/n \rangle = 5.2 \times 10^{-7}$s.

The greatest differences from the mean value, a factor of 1.5, occur for Mercury and Pluto. These two bodies have the largest eccentricities. The size of the difference in position evaluated from the export format and SPK kernel is expected to be proportional to the instantaneous speed. Thus, the differences between their mean and instantaneous speeds are the greatest. So, the value of n is the least representative for these two bodies. Also, more time is spent near apoapse than near periapse for a body on an eccentric orbit. So, the mean instantaneous speed will be lower than the mean speed for observations made at a uniform time interval. The result is the value of σ_r/n should be smaller for a body on a more eccentric orbit. This expectation is the case for Mercury and Pluto.

Assuming the precision of the time argument is the limiting factor in evaluating the SPK format ephemerides, then the slope of the envelope enclosing the values of the differences between the two formats, δx is

$$\delta x = n \times \Delta_{64} \qquad (5)$$

where Δ_{64} is the precision of a 64-bit floating point number. The ISO/IEC/IEEE (2011) standard provides a 64-bit number a 53-bit significand[1]. The machine epsilon, maximum rounding error is:

$$\begin{aligned} 2^{-53} &= \Delta_{64} \\ &\approx 1.1 \times 10^{-16}. \end{aligned}$$

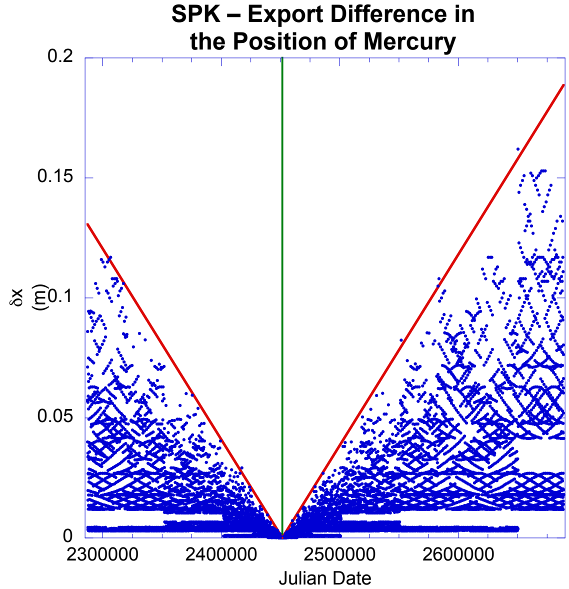

Figure 1: The difference in the position of Mercury evaluated from its export format and SPK format files for 100,000 barycentric positions.

Figure 1 shows the difference in the position of Mercury evaluated from its export format and SPK format files for 100,000 barycentric positions. the vertical green line segment marks the position of J2000.0 and the red line segments mark the position of the envelope estimated from Mercury's mean motion. A few of the observed differences do fall outside of the envelope. These outliers are to be expected because Mercury's instantaneous speed varies by ±18% from its mean speed.

3. Removing the Differences Between the Two Versions

The differences between the two versions is removed by modifying the SPK reading software to use a 128-bit time argument like the export format uses rather than use the SPICE specification of 64 bits. Encoding this change in the software and testing using the same set of test values used to generate Table 2 gives the differences between the export and SPK format files summarized in Table 4.

The size of σ_x and maximum Δx_{max} now increases with the magnitude of the semi-major axis of the object. For the planets and Pluto $\sigma_x/x \approx$

[1] The standard specifically calls for storing 52 bits with a 53rd unstored "implicit" bit. The level of the precision for a Julian date found by Kaplan et al. (2011) is consistent with a number that has a 53-bit significand once the result of rounding to the precision specified is included.

Table 4: The difference between the DE430 solar system ephemerides in export and SPK kernel formats where the interpreting software for both methods use extended precision time arguments.

Body	σ_x (mm)	Δx_{max} (mm)
Sun	0.0002	0.0014
Mercury	0.009	0.045
Venus	0.019	0.104
EMB	0.031	0.179
Mars	0.037	0.180
Jupiter	0.094	0.477
Saturn	0.272	1.19
Uranus	0.427	1.91
Neptune	0.523	2.86
Pluto	0.653	2.86
Earth	0.004	0.017
Moon	0.006	0.032

1.6×10^{-16} while $\Delta x_{max}/x \approx 8.5 \times 10^{-16}$. These values are consistent with differences arising from machine level truncation and rounding errors in the process of evaluating the positions.

The size of the standard deviation for Neptune and Pluto and the maximum differences for Saturn, Uranus, Neptune, and Pluto do exceed the 0.5 mm specification. This difference arises because the distances involved exceed the precision of a 64-bit floating point number[2].

3.1. The Choice of Epoch for $t = 0$

The choice of epoch for $t = 0$ substantially increases the precision of a 64-bit for the SPK kernel compared to the export format. For the SPK kernel $t = 0$ at J2000.0, JD 245 1545.0, while for the export format $t = 0$ at JD 0.0. The value of δt, the precision of t is approximately

$$\delta t \approx |t \times \Delta_{64}| \tag{6}$$

For the export format the independent variable is the Julian date. The range of Julian dates in DE430 is

$$\begin{aligned} \text{JD}\,228\,7184.5 &\leqslant t \leqslant \text{JD}\,268\,8976.5, \text{ or} \\ 197,612,740,800 \text{ s} &\leqslant t \leqslant 232,327,569,600 \text{ s}. \end{aligned}$$

For the SPK kernel the independent variable is seconds of time from J2000.0. For DE430 the range is

$$-14,200,747,200 \text{ s} \leqslant t \leqslant 20,514,081,600 \text{ s}.$$

The precision for a 64-bit time argument for the two formats at the extreme values for DE430 and

[2]Saturn's semi-major axis of 9.54 au is approximately 1.4×10^{15} mm.

Table 5: The precision for t for DE430 in export and SPK kernel formats.

Julian Date	export format δt (s)	SPK kernel δt (s)
228 7184.5	2.2×10^{-5}	1.6×10^{-6}
268 8976.5	2.6×10^{-5}	2.3×10^{-6}
245 1545.0	2.4×10^{-5}	0*
mean	2.4×10^{-5}	1.0×10^{-6}

*There is a small uncertainty at 0 because of the granularity of 64-bit numbers. This value is insignificant, $\sim 10^{-1038}$.

at the epoch J2000.0 are shown in Table 5. For the export format the precision is approximately constant because of the choice of a $t = 0$ epoch far from the time period of DE430. Thus, the mean precision of a 64-bit time argument for export format time is a factor of approximately 24 larger than for the SPK kernel.

The choice of epoch during the time period of the ephemeris also means that for a sub-period the use of a 64-bit time argument is adequate to evaluate the ephemerides and reach the required level of precision. This sub-period, t_s is approximately:

$$t_s \approx \frac{\sigma}{n\,\Delta_{64}} \tag{7}$$

where σ is the precision to which the ephemeris is to be evaluated. The period for $\sigma = 0.5$ mm is short compared to the length of DE430 ranging from ± 3 years for Mercury to ± 30 years for Pluto.

3.2. Units

The data in the export format file are stored in units of astronomical units and days, while the data in the SPK kernel are stored in units of kilometers and seconds. Some of the differences found might be attributable to round off and truncation errors arising from the conversion between units and the order in which the arithmetic operations are performed. None of these differences can be significant. So, no analysis was made to discover how much the differences in units contributes to the observed differences.

3.3. The Earth and Moon Ephemerides with Respect to the Earth-Moon Barycenter

The values of σ_x and maximum Δx_{max} for the Earth and Moon with respect to the Earth-Moon barycenter are *not* consistent with machine level truncation and rounding errors. These values, shown in Table 6 are approximately two orders of magnitude too large.

Table 6: The proportional differences for the Earth and Moon with respect to the Earth-Moon barycenter between the DE430 solar system ephemerides in export and SPK kernel formats where the interpreting software for both methods use extended precision time arguments.

Body	σ_x/x	$\Delta x_{max}/x$
Earth	9.2×10^{-13}	3.6×10^{-12}
Moon	1.6×10^{-14}	8.4×10^{-14}

One possible source of these differences is that the positions of the Earth and Moon are stored in different manners in the two formats. In the SPK kernel the positions of the Earth and Moon with respect to the Earth-Moon barycenter are stored. In the export format file, the *geocentric* position of the Moon is stored. the positions of the Earth and Moon with respect to the Earth-Moon barycenter has to be inferred from these data along with the Earth-Moon mass ratio.

Although the differences between the positions are too large to be consistent with machine error, they *are* approximately two orders of magntiude smaller than the 0.5 mm requirement for the ephemerides. So, no further analysis has been done to determine the source of the differences.

4. Results

The 0.5 mm precision requirement of the JPL planetary ephemerides is lost in evaluating the SPK kernel because SPICE specifies the use of a 64-bit floating point value for the independent variable. Replacing the 64-bit independent variable with a 128-bit one reduces the differences, in most cases, to the point where they are consistent with machine truncation and roundoff error in the evaluation.

The use of a 128-bit independent variable for the ephemeris of the Earth with respect to the EMB reduced the standard deviation in the differences by factor of approximately two and the standard deviation in the ephemeris of the Moon with respect to the EMB by a factor of about 100. The resulting standard deviations are too large by about two orders of magnitude to be consistent with errors arising from machine round off. They are approximately two orders of magnitude smaller than the JPL precision requirement and at least 3.8 orders of magnitude smaller than the accuracy of the ephemerides. So, no further analysis was made to discover the source of these differences.

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