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6. AUTHORS

7. PERFORMING ORGANIZATION NAMES AND ADDRESSES
University of Washington
Office of Sponsored Programs
4333 Brooklyn Ave NE Box 359472
Seattle, WA 98195 -9472

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					609-921-6415

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INVESTIGATOR(S):

Name: Ph.D Steven Brunton
Email: sbrunton@uw.edu
Phone Number: 6099216415
Principal: Y

Organization: **University of Washington**
Address: Office of Sponsored Programs, Seattle, WA 981959472
Country: USA

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Submitted By: Ph.D Steven Brunton Email: sbrunton@uw.edu
Phone: (609) 921-6415

STEM Degrees: 2 **STEM Participants:** 6

Major Goals: The overall objective of this work is to use machine learning control (MLC) to explore new flow regimes and behaviors and then use model identification techniques, to identify parsimonious and interpretable models that characterize the underlying flow physics. Machine learning constitutes a growing set of data-driven optimization techniques that are ideal for the modeling and control of high-dimensional, nonlinear, and multi-scale systems, such as are found in fluid dynamics. Further, sparse regression techniques have the potential to identify models that are both physically interpretable and generalize beyond the training data. This work will provide new computational methods to analyze data from fluid simulations and experiments, and will also result in a better understanding of the fundamental structure and interaction physics of unsteady fluid flows.

The modeling and control of fluid flows remains a grand challenge problem of the modern era, with potentially transformative scientific, technological, and industrial impact. Indeed, better understanding of complex flow physics may enable drag reduction, lift increase, mixing enhancement, and noise reduction in domains as diverse as transportation, energy, security and medicine. Fluid dynamics is a canonically difficult problem because of strong nonlinearity, high-dimensionality, and multi-scale physics; both modeling and control may be thought of as extremely challenging optimization problems. Recent advances in machine learning and sparse optimization are revolutionizing how we approach these traditionally intractable problems. We envision that these methods will enable the discovery of novel flow physics as well as practical new control strategies to achieve improved performance in engineering flows. At the end of this work, we will have learned a tremendous deal about important canonical flows. But moreover, we will have developed a framework to control and characterize fluids that improves with increasing data, positioning it to capitalize on the big data revolution. Improved data-driven modeling and control of fluid flows has the potential to significantly advance numerous scientific, engineering, and industrial efforts, resulting in drag reduction, lift increase, mixing enhancement, and noise reduction.

Accomplishments: During the first year, my lab has focused on developing powerful extensions to the sparse identification of nonlinear dynamics (SINDy) algorithm to incorporate the effect of actuation and control and to identify models in the low-data limit and in response to abrupt changes to the dynamics, which are expected during the application of active control. Initial results are extremely promising, indicating that SINDy models may be identified with extremely limited data, depending on measurement quality, and the resulting models are lean enough to be used for effective model predictive control (MPC), even in nonlinear systems. We have also applied the SINDy modeling framework to more complex fluid flows with broadband frequency content, namely the fluidic pinball, which consists of three independently rotating cylinders in a triangular configuration. In this flow, we have identified extremely simple and interpretable models that involve two coupled nonlinear spring-mass-damper oscillators with nonlinear damping. This is an encouraging result, since this is a natural generalization of the

models obtained for the single cylinder flow, indicating that more complex, and even turbulent, flows may be characterized by a few dominant nonlinear oscillators, in the right coordinates. Finally, we have also investigated optimal sensor placement for flow estimation and other machine learning control algorithms, including the use of deep neural networks for model predictive control to maintain high performance control in a mode-locked laser.
During the second year, my lab has focused on developing optimization techniques to extend the sparse identification of nonlinear dynamics (SINDy) algorithm to more complex, high-dimensional systems with a diversity of observed behaviors, which are common features of unsteady fluid dynamics. First, we have developed a theoretical foundation for the sparse optimization approach used in the original SINDy paper, testing its limitations and extending it to other sparse optimization problems, such as compressed sensing, regularization, robust filtering, etc., which arise in a variety of signal processing applications. Next, we developed a randomized linear algebra software package to efficiently extract modal decompositions in a scalable framework. On the science side, we have extended SINDy to work on hybrid dynamical systems, where the dynamics switch between multiple distinct dynamical regimes, which will be useful for multiphase flows. We have also shown improved learning of dynamics via neural networks by constraining the network to enforce Runge-Kutta time stepping constraints, enabling significant improvements to de-noising. Finally, we have used SINDy to learn discrepancies between a controlled experiment and an idealized Hamiltonian, demonstrating how to include partial knowledge of the physics to improve the learning process, resulting in improved control.
Approach

- Apply SINDy to the complex fluidic pinball flow, consisting of three independent cylinders;
- Extend SINDy to incorporate actuation and control and identify models in the low-data limit;
- Develop effective model predictive control based on SINDy models;
- Explore sensor placement for maximally extracting flow information for models and control;
- Develop deep MPC algorithms for nonlinear control of a mode-locked laser.
- Develop new sparse optimization techniques for more robust performance
- Extend SINDy approach to model hybrid dynamical systems
- Explore randomized linear algebra for modal decompositions at scale
- Use known constraints to improve simultaneous de-noising and discovery of dynamics
- Learn discrepancy between models and experimental data for control

Accomplishments for Reporting Period

- SINDy for model predictive control in the low-data limit
- SINDy for detecting abrupt system changes
- SINDy applied to the fluidic pinball
- Deep model predictive control for self-tuning fiber lasers
- Sparse sensor placement optimization for flow reconstruction
- Improved sparse optimization framework
- Sparse identification of hybrid dynamical systems
- Randomized algorithms for modal extraction at scale
- Neural network de-noising and discovery with time-stepper dynamics
- Learning model discrepancies from data for control

Training Opportunities: Graduate Students Involved During Reporting Period

- Krithika Manohar (Ph.D., defended June 2018)
- Markus Quade (visiting Ph.D. student from U Potsdam, visiting under DAAD fellowship)
- Kardindan Kaheman (Ph.D.student)
- Jared Callaham (Ph.D.student)
- Thomas Mohren (Ph.D. student)
- Isabel Scherl (Ph.D.student)
- Benjamin Strom (Ph.D.student, defended March 2019)
- Thomas Baumeister (visiting Masters student from TU Munich)

Postdoctoral Researchers Involved During Reporting Period

- Aditya Nair

Acting Assistant Professor Involved During Reporting Period

- Kazuki Maeda

Results Dissemination: • Loiseau, Deng, Pastur, Morzinski, Noack, Brunton, “Sparse reduced-order modeling of the fluidic pinball,” GDR Controle des ecollements, 2017.
• Kaiser, Kutz, Brunton, “Sparse identification of nonlinear dynamics for model predictive control in the low-data limit,” Proceedings of the Royal Society A, 474(2219), 2018.
• Baumeister, Brunton, Kutz, “Deep learning and model predictive control for self-tuning model-locked lasers,” Journal of the Optical Society of America, 35(3):617—626, 2018.
• Quade, Abel, Kutz, Brunton, “Sparse identification of nonlinear dynamics for rapid model recovery,” Chaos, 28 (063116), 2018.
• Manohar, Brunton, Kutz, Brunton, “Data-driven sparse sensor placement for reconstruction,” IEEE Control Systems Magazine, 38(3):63—86, 2018.
• Erichson, Brunton, Kutz, “Randomized matrix decompositions using R,” Journal of Statistical Software, 89(11): 1—48, 2019.
• Mangan, Askham, Brunton, Kutz, Proctor, “Model selection for hybrid dynamical systems via sparse regression,” Proceedings of the Royal Society A, 475(20180534), 2019.
• Zheng, Askham, Brunton, Kutz, Aravkin, “A unified framework for sparse relaxed regularized regression: SR3,” IEEE Access, 7(1):1404—1423, 2019.
• Rudy, Kutz, Brunton, “Deep learning of dynamics and signal-noise decomposition with time-stepping constraints,” Journal of Computational Physics, 396:483—506, 2019
• Rudy, Brunton, Kutz, “Smoothing and parameter estimation by soft-adherence to governing equations,” Journal of Computational Physics, 398:108860, 2019
• Kaheman, Kaiser, Strom, Kutz, Brunton, “Learning discrepancy models from experimental data,” CDC, 2019.
• Brunton, Noack, Koumoutsakos, “Machine Learning for Fluid Mechanics,” To appear in Annual Review of Fluid Mechanics, 2019.

Honors and Awards: • Callaham: DOD NDSEG Graduate Fellowship, 2019
• Brunton: Presidential Early Career Award in Science and Engineering (PECASE), 2019
• Brunton: SIAM CSE Early Career Prize, 2019
• Brunton: College of Engineering Junior Faculty Award, 2018
• Brunton: Promotion to Associate Professor, 2018
• Manohar: Accepted NSF Postdoctoral Fellowship to work at Caltech, 2018

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Technology Transfer: • Related patent: Jose Nathan Kutz, Steven Brunton, Xing Fu, “Tuning Multi-Input Complex Dynamic Systems Using Sparse Representations of Performance and Extremum-Seeking Control,” US Patent Number 9,972,962, May 2018

PARTICIPANTS:

Participant Type: PD/PI
Participant: Steve Brunton
Person Months Worked: 1.00 **Funding Support:**
Project Contribution:
International Collaboration:
International Travel:
National Academy Member: N
Other Collaborators:

Participant Type: Graduate Student (research assistant)
Participant: Krithika Manohar
Person Months Worked: 4.00 **Funding Support:**
Project Contribution:
International Collaboration:
International Travel:
National Academy Member: N
Other Collaborators:

Participant Type: Graduate Student (research assistant)
Participant: Kardindan Kaheman
Person Months Worked: 15.00 **Funding Support:**
Project Contribution:
International Collaboration:
International Travel:
National Academy Member: N
Other Collaborators:

Participant Type: Graduate Student (research assistant)
Participant: Ben Strom
Person Months Worked: 3.00 **Funding Support:**
Project Contribution:
International Collaboration:
International Travel:
National Academy Member: N
Other Collaborators:

Participant Type: Graduate Student (research assistant)
Participant: Isabel Scherl
Person Months Worked: 3.00 **Funding Support:**
Project Contribution:
International Collaboration:
International Travel:
National Academy Member: N
Other Collaborators:

Participant Type: Graduate Student (research assistant)
Participant: Thomas Mohren
Person Months Worked: 2.00 **Funding Support:**
Project Contribution:
International Collaboration:
International Travel:
National Academy Member: N
Other Collaborators:

Participant Type: Postdoctoral (scholar, fellow or other postdoctoral position)
Participant: Aditya Nair
Person Months Worked: 4.00 **Funding Support:**
Project Contribution:
International Collaboration:
International Travel:
National Academy Member: N
Other Collaborators:

Participant Type: Other (specify)
Participant: Kazuki Maeda
Person Months Worked: 2.00 **Funding Support:**
Project Contribution:
International Collaboration:
International Travel:
National Academy Member: N
Other Collaborators:

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Project Summary - W911NF-17-1-0422
(Reporting Period: August 2017 – July 2019)

Uncovering Nonlinear Flow Physics with Machine Learning Control and Sparse Modeling

Steven L. Brunton
Department of Mechanical Engineering
University of Washington, Seattle, Washington, 98195

Objective

The overall objective of this work is to use machine learning control (MLC) to explore new flow regimes and behaviors and then use model identification techniques, to identify parsimonious and interpretable models that characterize the underlying flow physics. Machine learning constitutes a growing set of data-driven optimization techniques that are ideal for the modeling and control of high-dimensional, nonlinear, and multi-scale systems, such as are found in fluid dynamics. Further, sparse regression techniques have the potential to identify models that are both physically interpretable and generalize beyond the training data. This work will provide new computational methods to analyze data from fluid simulations and experiments, and will also result in a better understanding of the fundamental structure and interaction physics of unsteady fluid flows.

During the first year, my lab has focused on developing powerful extensions to the sparse identification of nonlinear dynamics (SINDy) algorithm to incorporate the effect of actuation and control and to identify models in the low-data limit and in response to abrupt changes to the dynamics, which are expected during the application of active control. Initial results are extremely promising, indicating that SINDy models may be identified with extremely limited data, depending on measurement quality, and the resulting models are lean enough to be used for effective model predictive control (MPC), even in nonlinear systems. We have also applied the SINDy modeling framework to more complex fluid flows with broadband frequency content, namely the *fluidic pinball*, which consists of three independently rotating cylinders in a triangular configuration. In this flow, we have identified extremely simple and interpretable models that involve two coupled nonlinear spring-mass-damper oscillators with nonlinear damping. This is an encouraging result, since this is a natural generalization of the models obtained for the single cylinder flow, indicating that more complex, and even turbulent, flows may be characterized by a few dominant nonlinear oscillators, in the right coordinates. Finally, we have also investigated optimal sensor placement for flow estimation and other machine learning control algorithms, including the use of deep neural networks for model predictive control to maintain high performance control in a mode-locked laser.

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Approach

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- Develop new sparse optimization techniques for more robust performance
- Extend SINDy approach to model hybrid dynamical systems
- Explore randomized linear algebra for modal decompositions at scale
- Use known constraints to improve simultaneous de-noising and discovery of dynamics
- Learn discrepancy between models and experimental data for control

Relevance to Army

The modeling and control of fluid flows remains a grand challenge problem of the modern era, with potentially transformative scientific, technological, and industrial impact. Indeed, better understanding of complex flow physics may enable drag reduction, lift increase, mixing enhancement, and noise reduction in domains as diverse as transportation, energy, security and medicine. Fluid dynamics is a canonically difficult problem because of strong nonlinearity, high-dimensionality, and multi-scale physics; both modeling and control may be thought of as extremely challenging optimization problems. Recent advances in machine learning and sparse optimization are revolutionizing how we approach these traditionally intractable problems. We envision that these methods will enable the discovery of novel flow physics as well as practical new control strategies to achieve improved performance in engineering flows. At the end of this work, we will have learned a tremendous deal about important canonical flows. But moreover, we will have developed a framework to control and characterize fluids that improves with increasing data, positioning it to capitalize on the big data revolution. Improved data-driven modeling and control of fluid flows has the potential to significantly advance numerous scientific, engineering, and industrial efforts, resulting in drag reduction, lift increase, mixing enhancement, and noise reduction.

Accomplishments for Reporting Period

- **SINDy for model predictive control in the low-data limit**

The data-driven discovery of dynamics via machine learning is currently pushing the frontiers of modeling and control efforts, and it provides a tremendous opportunity to extend the reach of model predictive control. However, many leading methods in machine learning, such as neural networks, require large volumes of training data, may not be interpretable, do not easily include known constraints and symmetries, and often do not generalize beyond the attractor where models are trained. These factors limit the use of these techniques for the online identification of a model in the low-data limit, for example following an abrupt change to the system dynamics. In this work, we extend the recent sparse identification of nonlinear dynamics (SINDY) modeling procedure to include the effects of actuation and demonstrate the ability of these models to enhance the performance of model predictive control (MPC), based on limited, noisy data. SINDY models are parsimonious, identifying the fewest terms in the model needed to explain the data, making them interpretable, generalizable, and reducing the burden of training data. We show that the resulting SINDY-MPC framework has higher performance, requires significantly less data, and is more computationally efficient and robust to noise than neural network models, making it viable for online training and execution in response to rapid changes to the system. SINDY-MPC also shows improved performance over linear data-driven models, although linear models may provide a stopgap until enough data is available for SINDY.

Relevant figures: Figure 1, Figure 1, Figure 1

- **SINDy for detecting abrupt system changes**

Big data have become a critically enabling component of emerging mathematical methods aimed at the automated discovery of dynamical systems, where first principles modeling may be intractable. However, in many engineering systems, abrupt changes must be rapidly characterized based on limited, incomplete, and noisy data. Many leading automated learning techniques rely on unrealistically large data sets, and it is unclear how to leverage prior knowledge effectively to re- identify a model after an abrupt change. In this work, we propose a conceptual framework to recover parsimonious models of a system in response to abrupt changes in the low-data limit. First, the abrupt change is detected by comparing the estimated Lyapunov time of the data with the model prediction. Next, we apply the sparse identification of nonlinear dynamics (SINDy) regression to update a previously identified model with the fewest changes, either by addition, deletion, or modification of existing model terms. We demonstrate this sparse model recovery on several examples for abrupt system change detection in periodic and chaotic dynamical systems. Our examples show that sparse updates to a previously identified model perform better with less data, have lower runtime complexity, and are less sensitive to noise than identifying an entirely new model. The proposed abrupt-SINDy architecture provides a new paradigm for the rapid and efficient recovery of a system model after abrupt changes.

Relevant figures: Figure 1

- **SINDy applied to the fluidic pinball**

This work applies a sparse gray-box modeling procedure recently proposed by the same authors to the fluidic pinball, a new benchmark for nonlinear flow control. This procedure relies on experimentally available quantities, such as time-resolved sensor measurements and optional non-time-resolved PIV snapshots. Its application to the fluidic pinball illustrates the versatility of the present approach and its ability to identify human-interpretable nonlinear low-order models. These low-order models may then be used for nonlinear model-based control.

Relevant figures: Figure 1

- **Deep model predictive control for self-tuning fiber lasers**

Self-tuning optical systems are of growing importance in technological applications such as mode-locked fiber lasers. Such self-tuning paradigms require intelligent algorithms capable of inferring approximate models of the underlying physics and discovering appropriate control laws in order to maintain robust performance for a given objective. In this work, we demonstrate the first integration of a deep-learning (DL) architecture with model predictive control (MPC) in order to self-tune a mode-locked fiber laser. Not only can our DL-MPC algorithmic architecture approximate the unknown fiber birefringence, it also builds a dynamical model of the laser and appropriate control law for maintaining robust, high-energy pulses despite a stochastically drifting birefringence. We demonstrate the effectiveness of this method on a fiber laser that is mode-locked by nonlinear polarization rotation. The method advocated can be broadly applied to a variety of optical systems that require robust controllers.

Relevant figures: Figure 1

- **Sparse sensor placement optimization for flow reconstruction**

Optimal sensor placement is a central challenge in the design, prediction, estimation, and control of high-dimensional systems. High-dimensional states can often leverage a latent low-dimensional representation, and this inherent compressibility enables sparse sensing. This article explores optimized sensor placement for signal reconstruction based on a tailored library of features extracted from training data. Sparse point sensors are discovered using the singular value decomposition and QR pivoting, which are two ubiquitous matrix computations that underpin modern linear dimensionality reduction. Sparse sensing on a tailored basis is contrasted with compressed sensing, a universal signal recovery method in which an unknown signal is reconstructed via a sparse representation on a universal basis. Although compressed sensing can recover a wider class of signals, we demonstrate the benefits of exploiting known patterns in data with optimized sensing. In particular, drastic reductions in the required number of sensors and improved reconstruction are observed in examples ranging from facial images to fluid vorticity fields. Principled sensor placement may be critically enabling when sensors are costly and provides faster state estimation for low-latency, high-bandwidth control.

Relevant figures: Figure 1, Figure 1, Figure 1

- **Improved sparse optimization framework**

Regularized regression problems are ubiquitous in statistical modeling, signal processing, and machine learning. Sparse regression in particular has been instrumental in scientific model discovery, including compressed sensing applications, variable selection, and high-dimensional analysis. We propose a broad framework for sparse relaxed regularized regression, called Sparse Regularized Relaxed Regression (SR3). The key idea is to solve a relaxation of the regularized problem, which has three advantages over the state-of-the-art: (1) solutions of the relaxed problem are superior with respect to errors, false positives, and conditioning, (2) relaxation allows extremely fast algorithms for both convex and nonconvex formulations, and (3) the methods apply to composite regularizers such as total variation (TV) and its nonconvex variants. We demonstrate the advantages of \SR3 (computational efficiency, higher accuracy, faster convergence rates, greater flexibility) across a range of regularized regression problems with synthetic and real data, including applications in compressed sensing, LASSO, matrix completion, TV regularization, and group sparsity. To promote reproducible research, we also provide a companion Matlab package that implements these examples.

Relevant figures: Figure 10, Figure 11

- **Sparse identification of hybrid dynamical systems**

Hybrid systems are traditionally difficult to identify and analyze using classical dynamical systems theory. Moreover, recently developed model identification methodologies largely focus on identifying a single set of governing equations solely from measurement data. We have developed a new methodology, Hybrid-Sparse Identification of Nonlinear Dynamics (Hybrid-SINDy), which identifies separate nonlinear dynamical regimes, employs information theory to manage uncertainty, and characterizes switching behavior. Specifically, we utilize the nonlinear geometry of data collected from a complex system to construct a set of coordinates based on measurement data and augmented variables. Clustering the data in these measurement-based coordinates enables the identification of nonlinear hybrid systems. This methodology broadly empowers nonlinear system identification without constraining the data locally in time and has direct connections to hybrid systems theory. We demonstrate the success of this method on numerical examples including a mass-spring hopping model and an infectious disease model. Characterizing complex systems that switch between dynamic behaviors is integral to overcoming modern challenges such as eradication of infectious diseases, the design of efficient legged robots, and the protection of cyber infrastructures.

Relevant figures: Figure 12, Figure 13

- **Randomized algorithms for modal extraction at scale**

Matrix decompositions are fundamental tools in the area of applied mathematics, statistical computing, and machine learning. In particular, low-rank matrix decompositions are vital, and widely used for data analysis, dimensionality reduction, and data compression. Massive datasets, however, pose a computational challenge for traditional algorithms, placing significant constraints on both memory and processing power. Recently, the powerful concept of randomness has been introduced as a strategy to ease the computational load. The essential idea of probabilistic algorithms is to employ some amount of randomness in order to derive a smaller matrix from a high-dimensional data matrix. The smaller matrix is then used to compute the desired low-rank approximation. Such algorithms are shown to be computationally efficient for approximating matrices with low-rank structure. We have developed the R package rsvd, along with a tutorial introduction to randomized matrix decompositions. Specifically, randomized routines for the singular value decomposition, robust principal component analysis, interpolative decomposition, and CUR decomposition are discussed. Several examples demonstrate the routines, and show the computational advantage over other methods implemented in R.

Relevant figures: Figure 14, Figure 15, Figure 16

• **Neural network de-noising and discovery with time-stepper dynamics**

A critical challenge in the data-driven modeling of dynamical systems is producing methods robust to measurement error, particularly when data is limited. Many leading methods either rely on denoising prior to learning or on access to large volumes of data to average over the effect of noise. We propose a novel paradigm for data-driven modeling that simultaneously learns the dynamics and estimates the measurement noise at each observation. By constraining our learning algorithm, our method explicitly accounts for measurement error in the map between observations, treating both the measurement error and the dynamics as unknowns to be identified, rather than assuming idealized noiseless trajectories. We model the unknown vector field using a deep neural network, imposing a Runge-Kutta integrator structure to isolate this vector field, even when the data has a non-uniform timestep, thus constraining and focusing the modeling effort. We demonstrate the ability of this framework to form predictive models on a variety of canonical test problems of increasing complexity and show that it is robust to substantial amounts of measurement error. We also discuss issues with the generalizability of neural network models for dynamical systems and provide open-source code for all examples.

Relevant figures: Figure 17, Figure 18, Figure 19, Figure 20, Figure 21, Figure 22

• **Learning model discrepancies from data for control**

First principles modeling of physical systems has led to significant technological advances across all branches of science. For nonlinear systems, however, small modeling errors can lead to significant deviations from the true, measured behavior. Even in mechanical systems, where the equations are assumed to be well-known, there are often model discrepancies corresponding to nonlinear friction, wind resistance, etc. Discovering models for these discrepancies remains an open challenge for many complex systems. In this work, we use the sparse identification of nonlinear dynamics (SINDy) algorithm to discover a model for the discrepancy between a simplified model and measurement data. In particular, we assume that the model mismatch can be sparsely represented in a library of candidate model terms. We demonstrate the efficacy of our approach on several examples including experimental data from a double pendulum on a cart. We further design and implement a feed-forward controller in simulations, showing improvement with a discrepancy model.

Relevant figures: Figure 23, Figure 24

Collaborations and Technology Transfer

- European collaboration with Jean Christophe Loiseau and Bernd R. Noack (ENSMA and LIMSI): Machine learning control and sparse model identification of unsteady flow systems of increasing complexity.
- European collaboration with Markus Quade and Markus Abel (U Potsdam, Germany): Sparse model identification after abrupt system changes.
- European collaboration with Thomas Baumeister (TU Munich, Germany): Deep model predictive control of a laser system (visiting from TU Munich).
- European collaboration with Sebastian Peitz and Michael Dellnitz (Paderborn, Germany): Deep model predictive control of a fluid flow past a circular cylinder.
- European collaboration with Christian Walter (Clausthal, Germany): Dynamic mode decomposition for turbomachinery.
- European collaboration with Urban Fasel (ETH,Zurich): Unsteady aeroelastic modeling for aerial vehicles.
- European collaboration with Petros Koumoutsakos (ETH, Zurich) and Bernd Noack (LIMSI, France) to explore uses and limitations of machine learning for fluid mechanics.
- Related patent: Jose Nathan Kutz, Steven Brunton, Xing Fu, "Tuning Multi-Input Complex Dynamic Systems Using Sparse Representations of Performance and Extremum-Seeking Control," US Patent Number 9,972,962, May 2018

Resulting Journal Publications During Reporting Period

- Loiseau, Deng, Pastur, Morzinski, Noack, Brunton, "Sparse reduced-order modeling of the fluidic pinball," *GDR Controle des ecollements*, 2017.
- Kaiser, Kutz, Brunton, "Sparse identification of nonlinear dynamics for model predictive control in the low-data limit," *Proceedings of the Royal Society A*, **474**(2219), 2018.
- Baumeister, Brunton, Kutz, "Deep learning and model predictive control for self-tuning model-locked lasers," *Journal of the Optical Society of America*, **35**(3):617—626, 2018.
- Quade, Abel, Kutz, Brunton, "Sparse identification of nonlinear dynamics for rapid model recovery," *Chaos*, **28**(063116), 2018.
- Manohar, Brunton, Kutz, Brunton, "Data-driven sparse sensor placement for reconstruction," *IEEE Control Systems Magazine*, **38**(3):63—86, 2018.
- Erichson, Brunton, Kutz, "Randomized matrix decompositions using R," *Journal of Statistical Software*, **89**(11):1—48, 2019.
- Mangan, Askham, Brunton, Kutz, Proctor, "Model selection for hybrid dynamical systems via sparse regression," *Proceedings of the Royal Society A*, **475**(20180534), 2019.
- Zheng, Askham, Brunton, Kutz, Aravkin, "A unified framework for sparse relaxed regularized regression: SR3," *IEEE Access,* **7**(1):1404—1423, 2019.
- Rudy, Kutz, Brunton, "Deep learning of dynamics and signal-noise decomposition with time-stepping constraints," *Journal of Computational Physics*, **396**:483—506, 2019
- Rudy, Brunton, Kutz, "Smoothing and parameter estimation by soft-adherence to governing equations," *Journal of Computational Physics*, **398**:108860, 2019

- Kaheman, Kaiser, Strom, Kutz, Brunton, "Learning discrepancy models from experimental data," *CDC*, 2019.
- Brunton, Noack, Koumoutsakos, "Machine Learning for Fluid Mechanics," *To appear in Annual Review of Fluid Mechanics*, 2019.
- Callaham, Maeda, Brunton, "Robust flow reconstruction from limited measurements via sparse representation", *arXiv:1810.06723,* 2019.
- Scherl, Strom, Shang, Williams, Polagye, Brunton, "Robust principal component analysis for particle image velocimetry," *arXiv:1905.07062,* 2019.

Graduate Students Involved During Reporting Period

- Krithika Manohar (Ph.D., defended June 2018)
- Markus Quade (visiting Ph.D. student from U Potsdam, visiting under DAAD fellowship)
- Kardindan Kaheman (Ph.D.student)
- Jared Callaham (Ph.D.student)
- Thomas Mohren (Ph.D. student)
- Isabel Scherl (Ph.D.student)
- Benjamin Strom (Ph.D.student, defended March 2019)
- Thomas Baumeister (visiting Masters student from TU Munich)

Postdoctoral Researchers Involved During Reporting Period

- Aditya Nair

Acting Assistant Professor Involved During Reporting Period

- Kazuki Maeda

Awards, Honors and Appointments

- Callaham: DOD NDSEG Graduate Fellowship, 2019
- Brunton: Presidential Early Career Award in Science and Engineering (PECASE), 2019
- Brunton: SIAM CSE Early Career Prize, 2019
- Brunton: College of Engineering Junior Faculty Award, 2018
- Brunton: Promotion to Associate Professor, 2018
- Manohar: Accepted NSF Postdoctoral Fellowship to work at Caltech, 2018

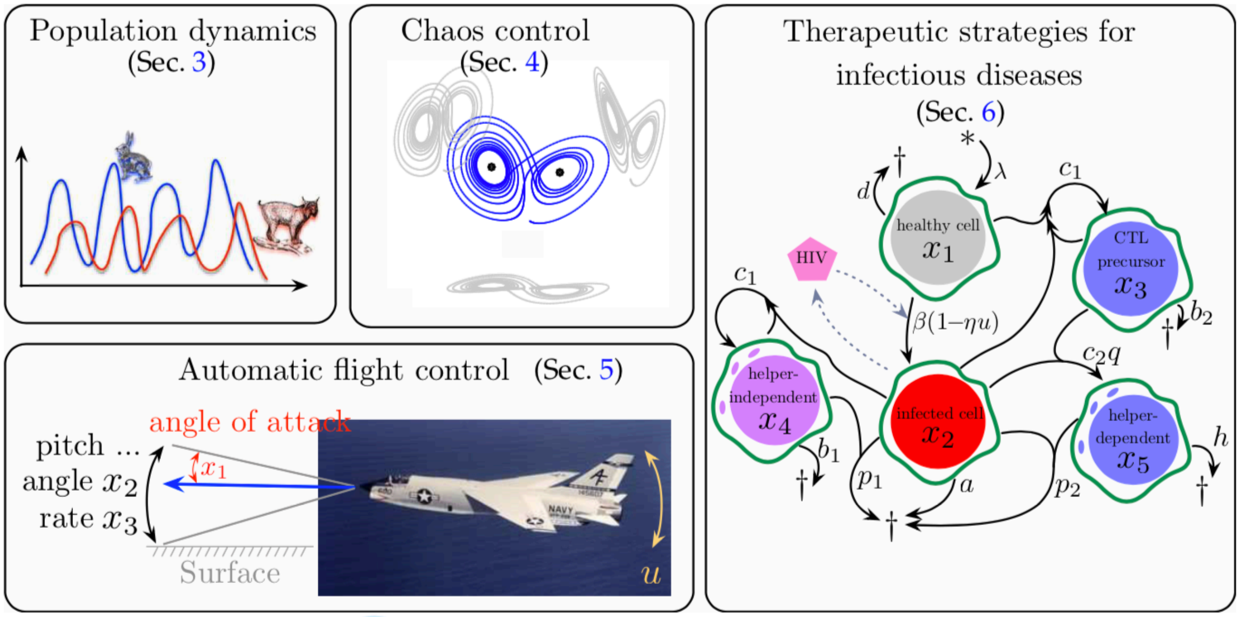


Figure 1. Overview of examples of SINDy with model predictive control.

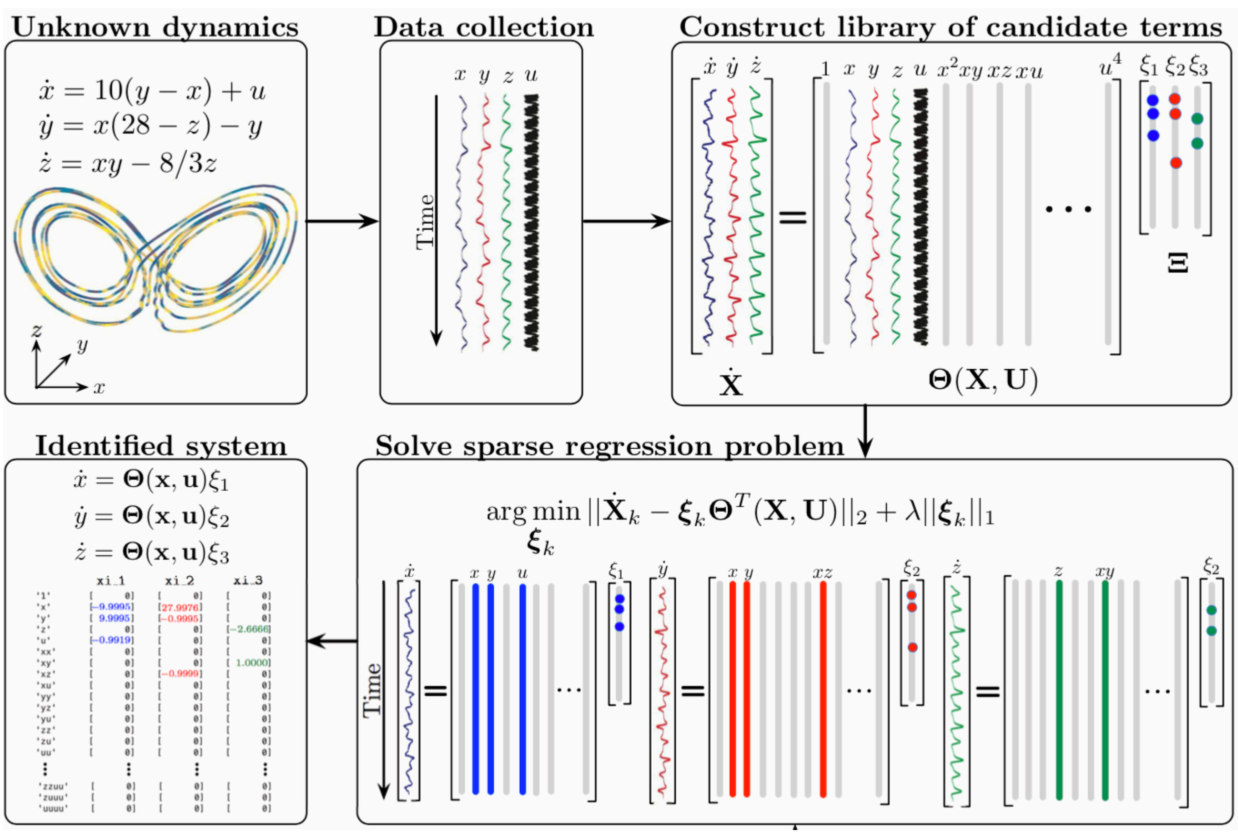


Figure 2. Schematic overview of SINDy with actuation and control.

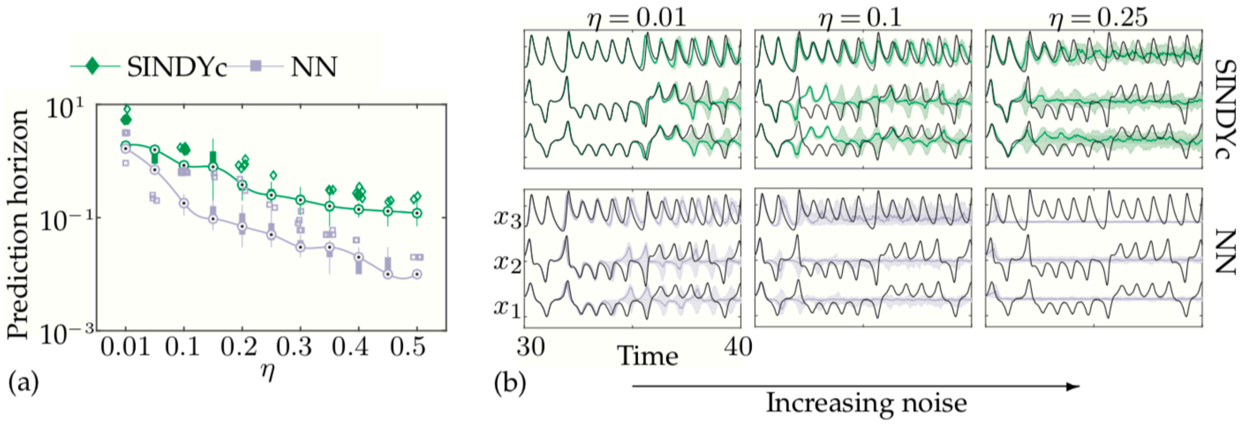


Figure 3. Prediction horizon of SINDy models is more robust to sensor noise than neural network models.

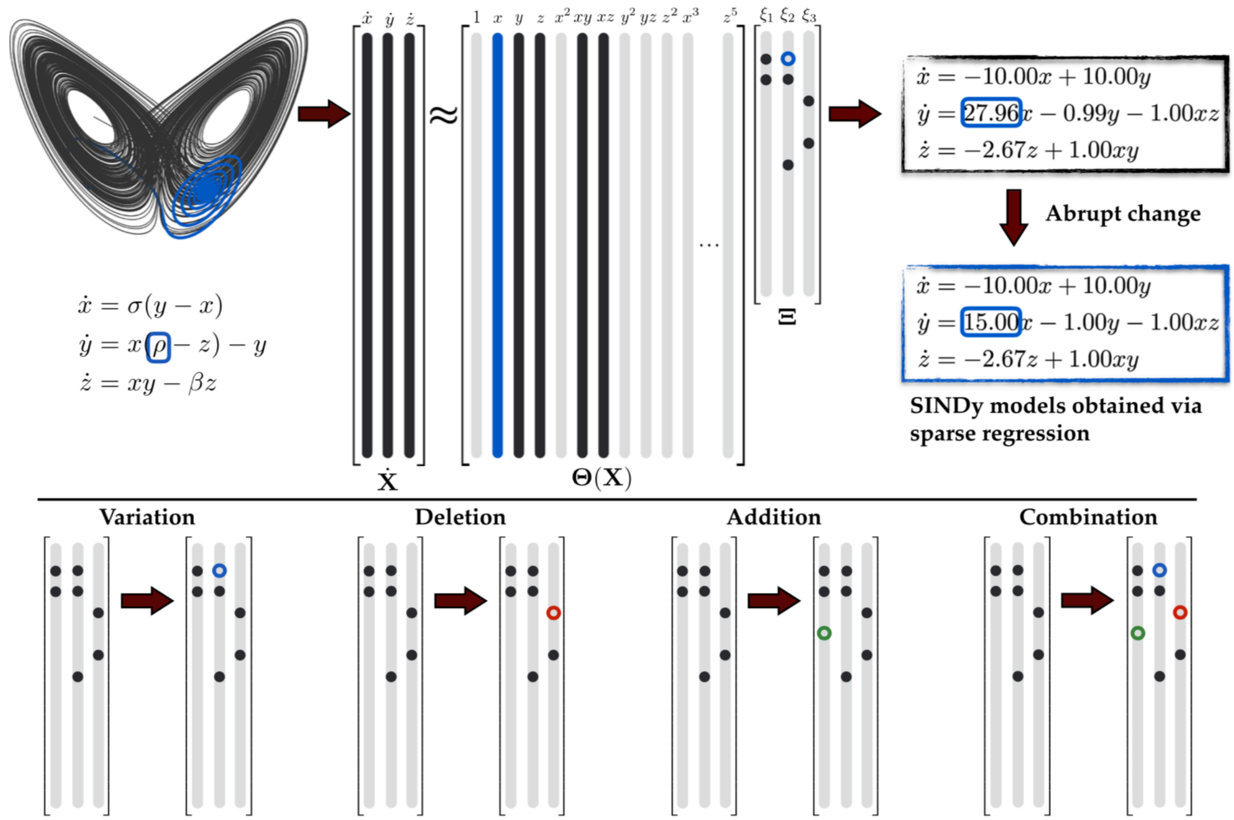
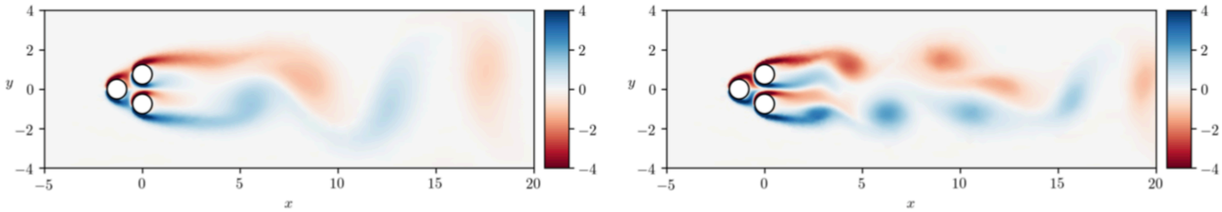


Figure 4. Overview of how to use SINDy to detect new models after abrupt system changes.



(a) $Re = 60$ (b) $Re = 120$

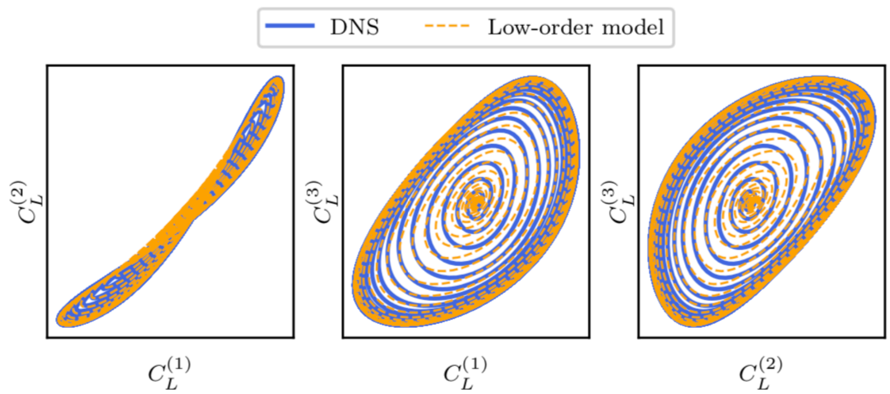


Figure 5. SINDy model for fluidic pinball model. Lift and drag on each cylinder are accurately predicted with a simple coupled nonlinear oscillator model.

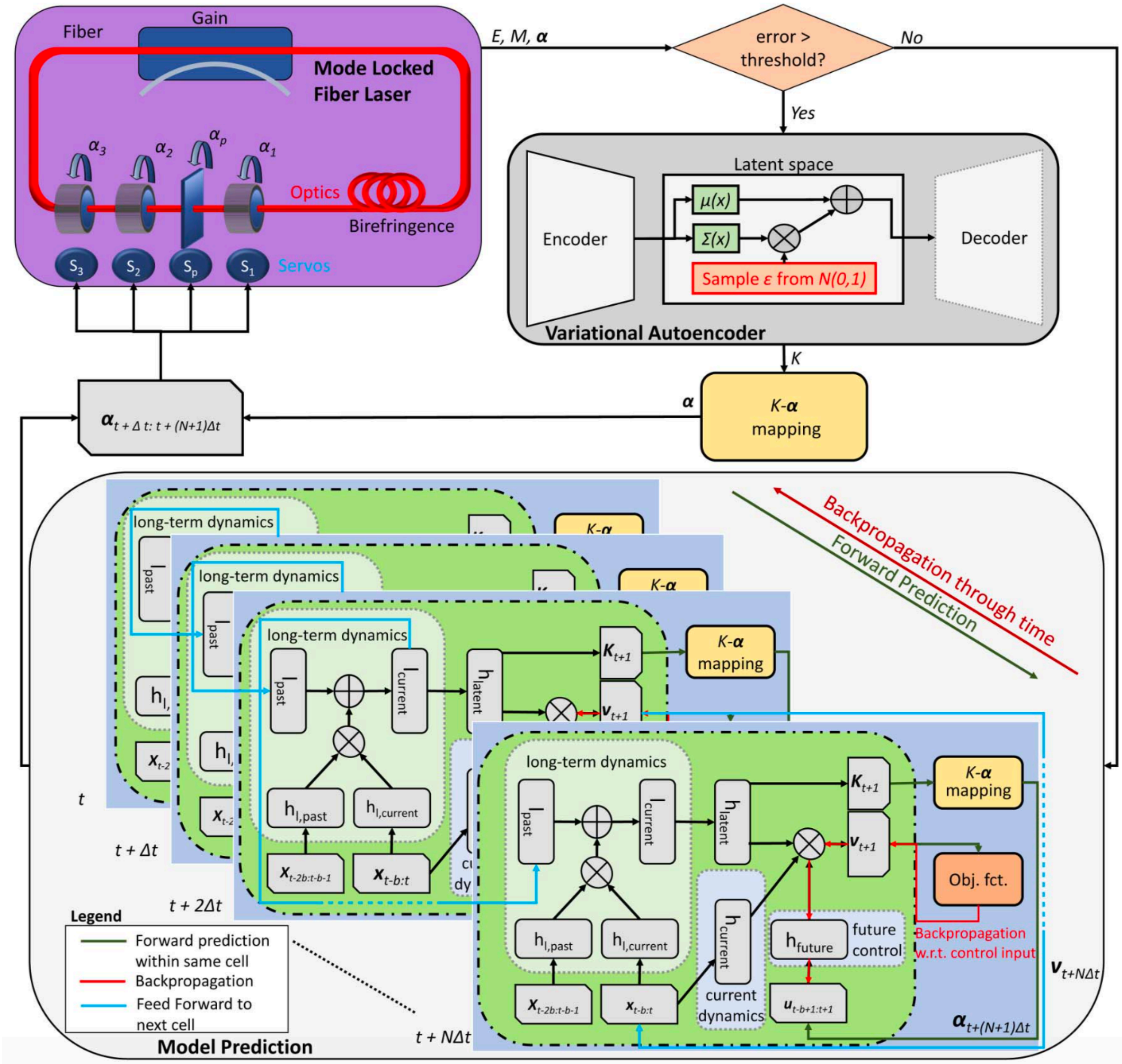


Figure 6. Deep model predictive control architecture for the mode-locked laser.

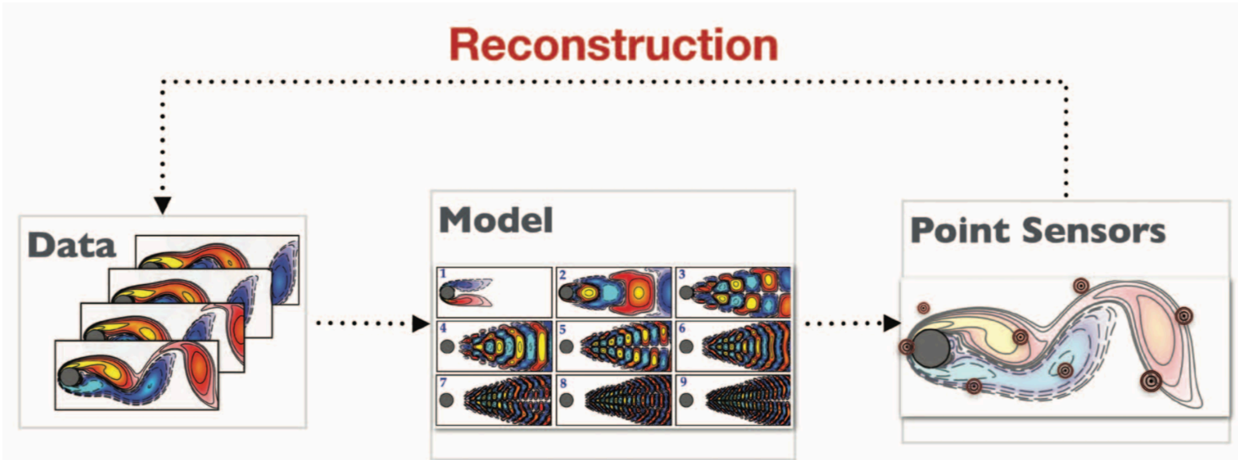


Figure 7. Schematic overview of sensor placement for reconstruction.

Mathematical Formulation of Sensor Selection

Many physical systems are described by a high-dimensional state $\mathbf{x} \in \mathbb{R}^n$, yet the dynamics evolve on a low-dimensional attractor that can be leveraged for prediction and control. Thus, a state $\mathbf{x}$ that evolves according to nonlinear dynamics $\dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}(t))$ will often have a compact representation in a transform basis Ψ. On a universal basis $\Psi \in \mathbb{R}^{n \times n}$, such as Fourier or wavelet bases, $\mathbf{x}$ may have a *sparse* representation

$$\mathbf{x} = \Psi \mathbf{s} \qquad \mathbf{s} \in \mathbb{R}^r, \tag{S1}$$

where $\mathbf{s}$ is a sparse vector indicating which few modes of Ψ are active. On a tailored basis $\Psi_r \in \mathbb{R}^{n \times r}$, such as a proper orthogonal decomposition basis, $\mathbf{x}$ may have a *low-rank* representation

$$\mathbf{x} = \Psi_r \mathbf{a} \qquad \mathbf{a} \in \mathbb{R}^r. \tag{S2}$$

The central challenge in this work is to design a measurement matrix $\mathbf{C} \in \mathbb{R}^{p \times n}$ consisting of a small number ($p \ll n$) of optimized measurements

$$\mathbf{y} = \mathbf{C}\mathbf{x} \qquad \mathbf{y} \in \mathbb{R}^p, \tag{S3}$$

that facilitate accurate reconstruction of either $\mathbf{s}$ or $\mathbf{a}$, and hence $\mathbf{x}$. Combining (S1) and (S3) yields

$$\mathbf{y} = (\mathbf{C}\Psi)\mathbf{s} = \Theta \mathbf{s}, \tag{S4}$$

which is referred to as the *compressed sensing* problem, while combining (S2) and (S3) yields

$$\mathbf{y} = (\mathbf{C}\Psi_r)\mathbf{a} = \Theta \mathbf{a}. \tag{S5}$$

In both cases, effective measurements $\mathbf{C}$ given a basis Ψ or Ψ_r are chosen so that the operator Θ is well conditioned for signal reconstruction. Thus, it is possible to solve for the sparse coefficients $\mathbf{s}$ or the low-rank coefficients $\mathbf{a}$ given the measurements $\mathbf{y}$, either by ℓ_1 minimization in (S4) or pseudo-inverse of Θ in (S5), respectively. The goal of this work is to optimize the measurements in $\mathbf{C}$. Moreover, in many physical applications, it is desired that $\mathbf{C}$ consists of rows of the identity matrix corresponding to individual point sensors of individual components of $\mathbf{x}$.

Figure 8. Mathematical framing of sensor placement problem.

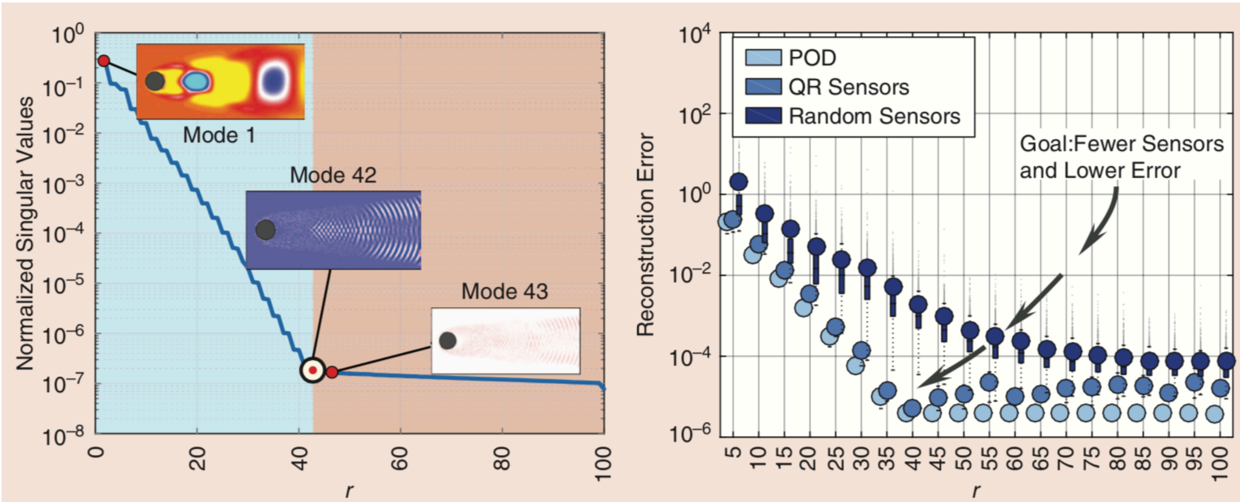


Figure 9. Optimal sparse sensor placement for the flow past a cylinder results in significantly better performance than random sensors (right).

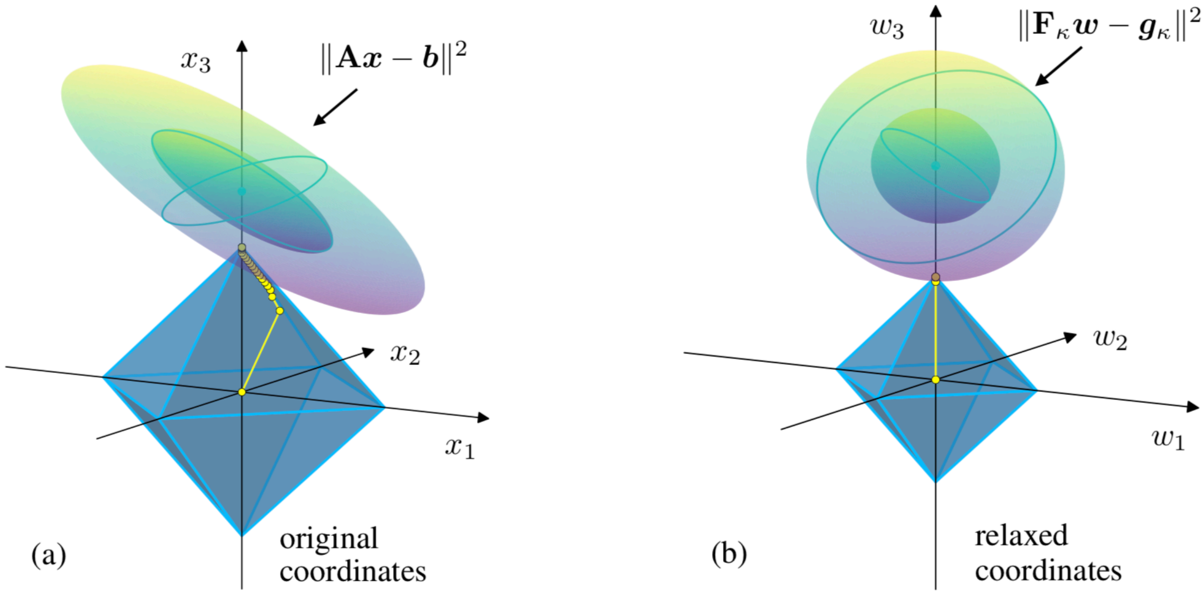


Figure 10. (a) Traditional sparse optimization path, and (b) proposed new relaxed optimization.

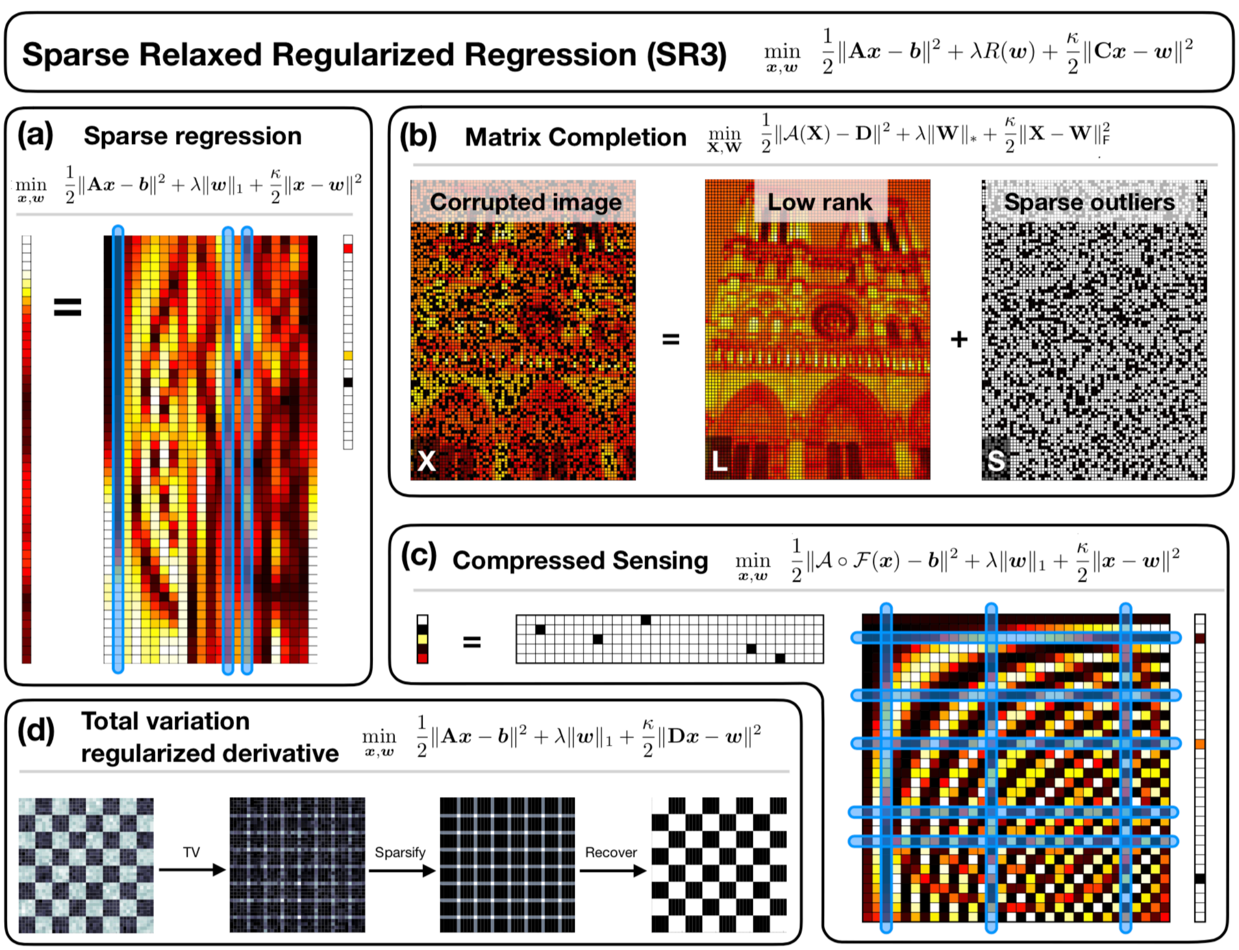


Figure 11. Common optimization applications where SR3 method improves performance.

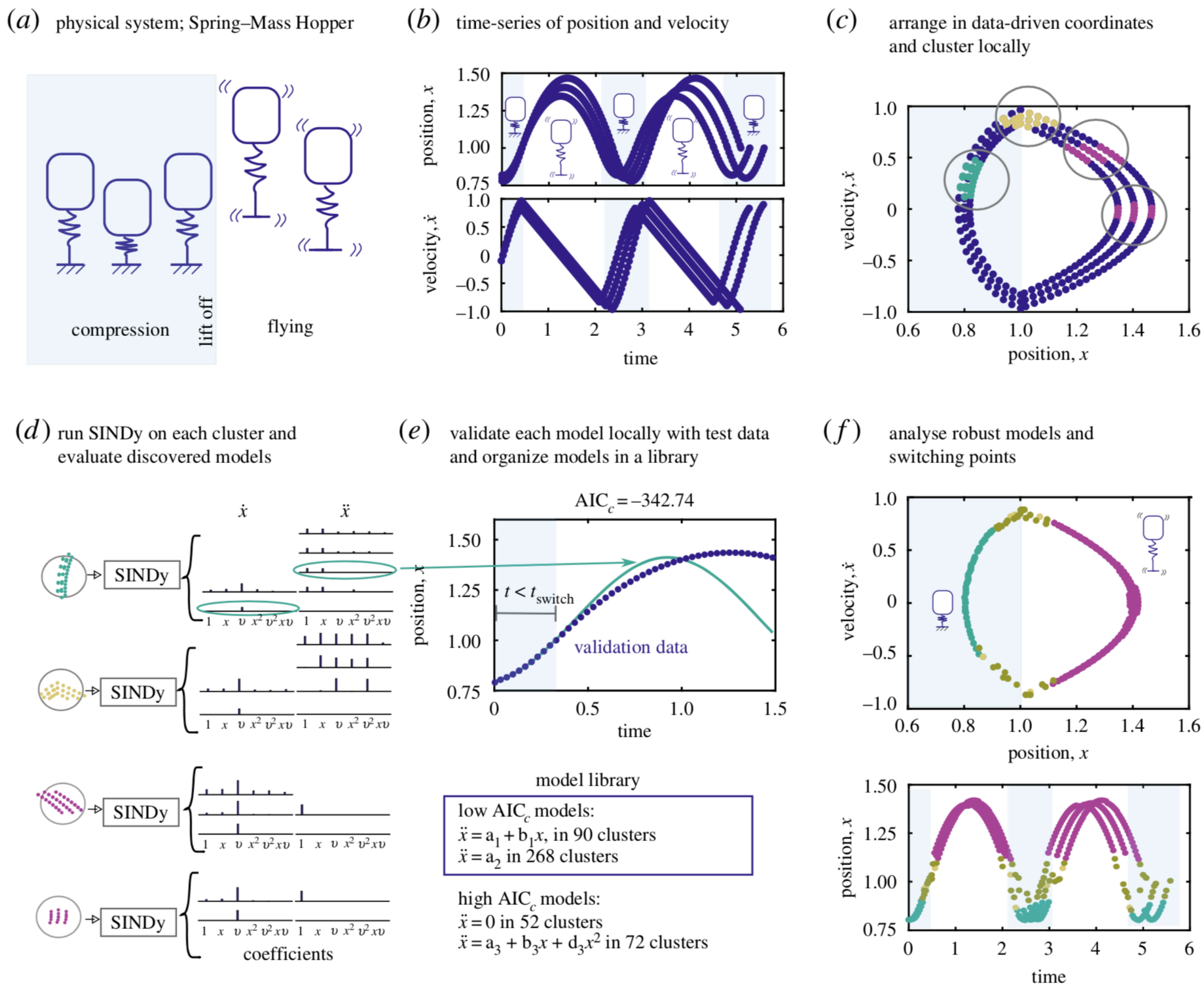


Figure 12. Schematic of SINDy approach extended to hybrid systems, demonstrated using the spring-mass-hopper system.

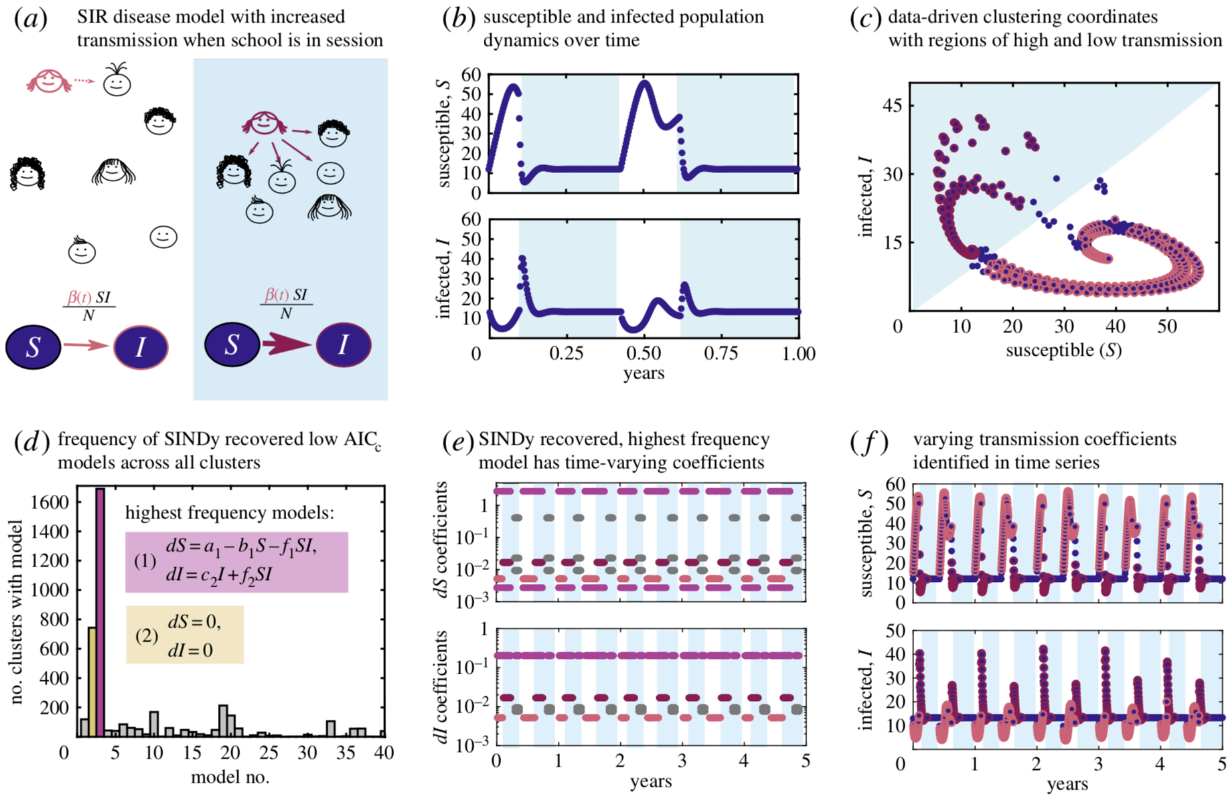


Figure 13. Hybrid SINDy approach applied to disease dynamics.

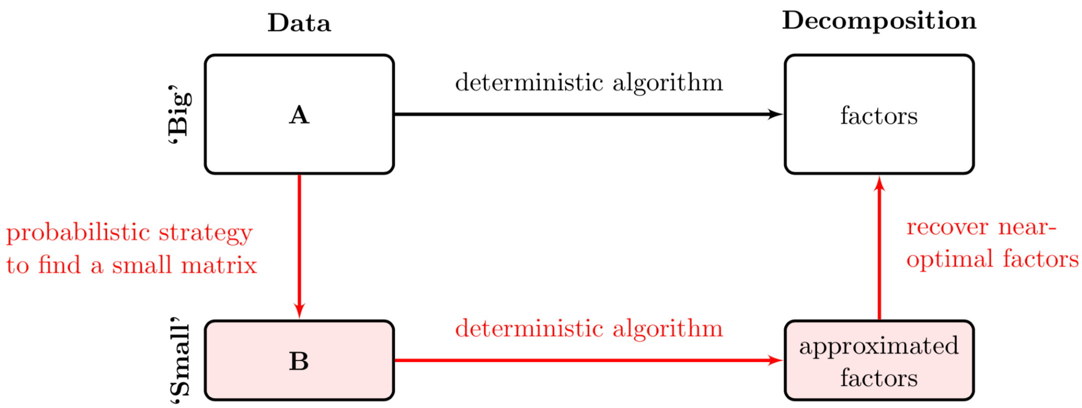


Figure 14. Randomized algorithms provide a probabilistic strategy for modal decomposition.

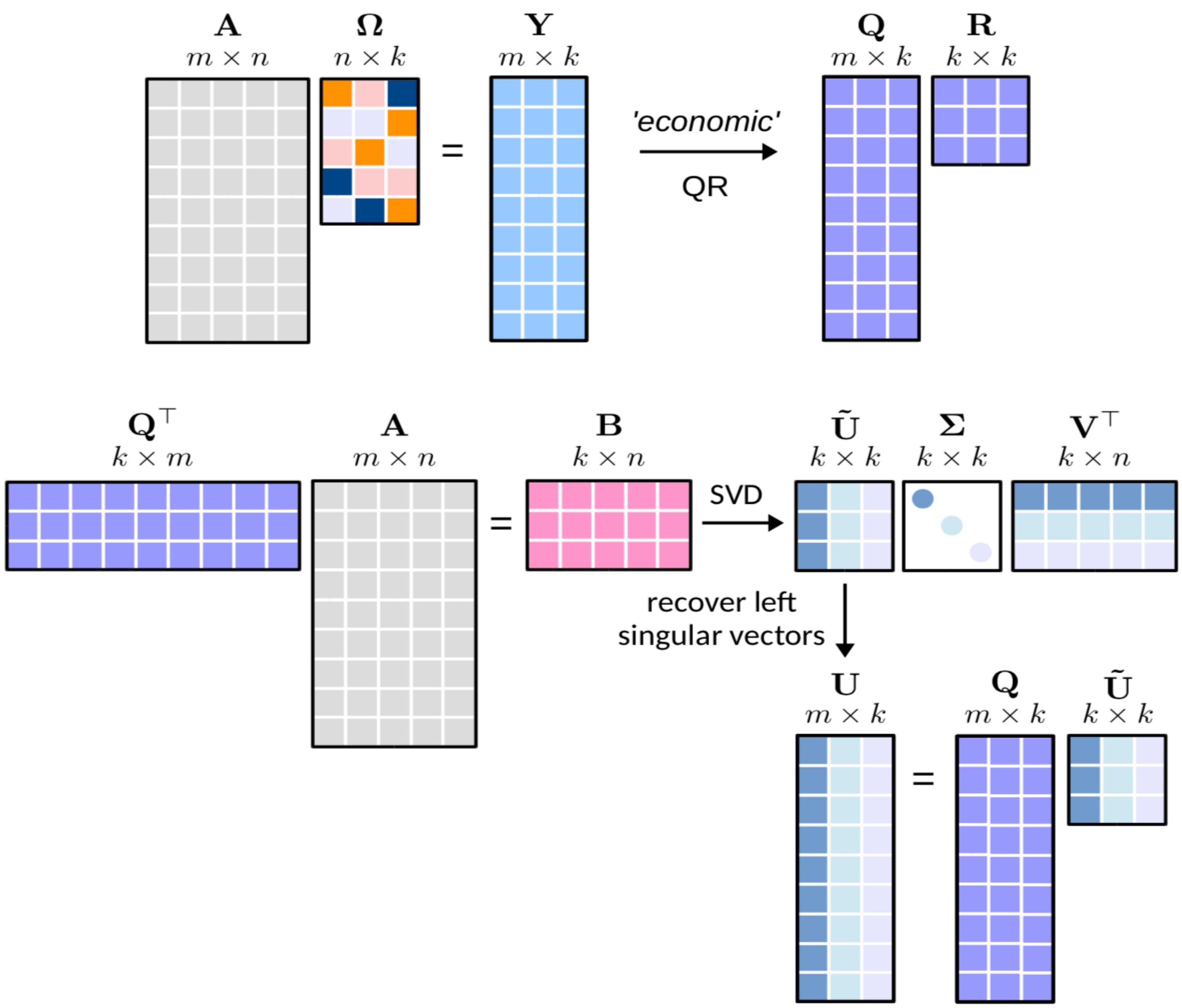


Figure 15. Example of randomized singular value decomposition algorithm.

(a) Original image.

(b) SVD. (nrmse=0.121)

(c) rSVD using $q = 0$. (nrmse=0.165)

(d) rSVD using $q = 2$. (nrmse=0.122)

Figure 16. Performance of randomized SVD on image compression.

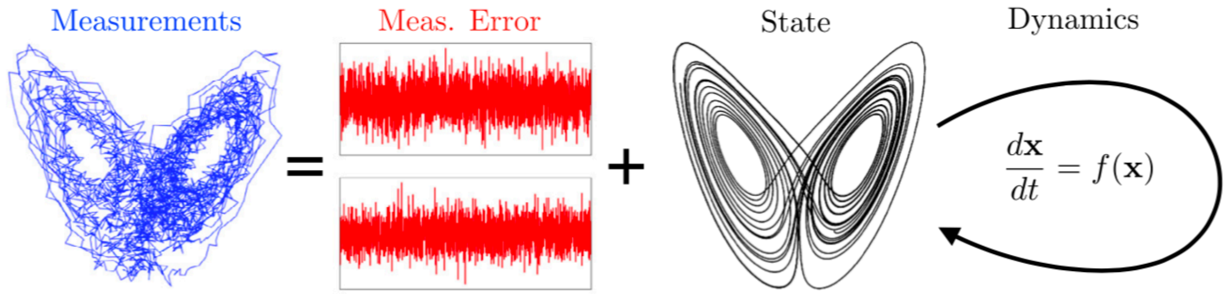


Figure 17. Illustration of simultaneous de-noising and discovery of dynamics.

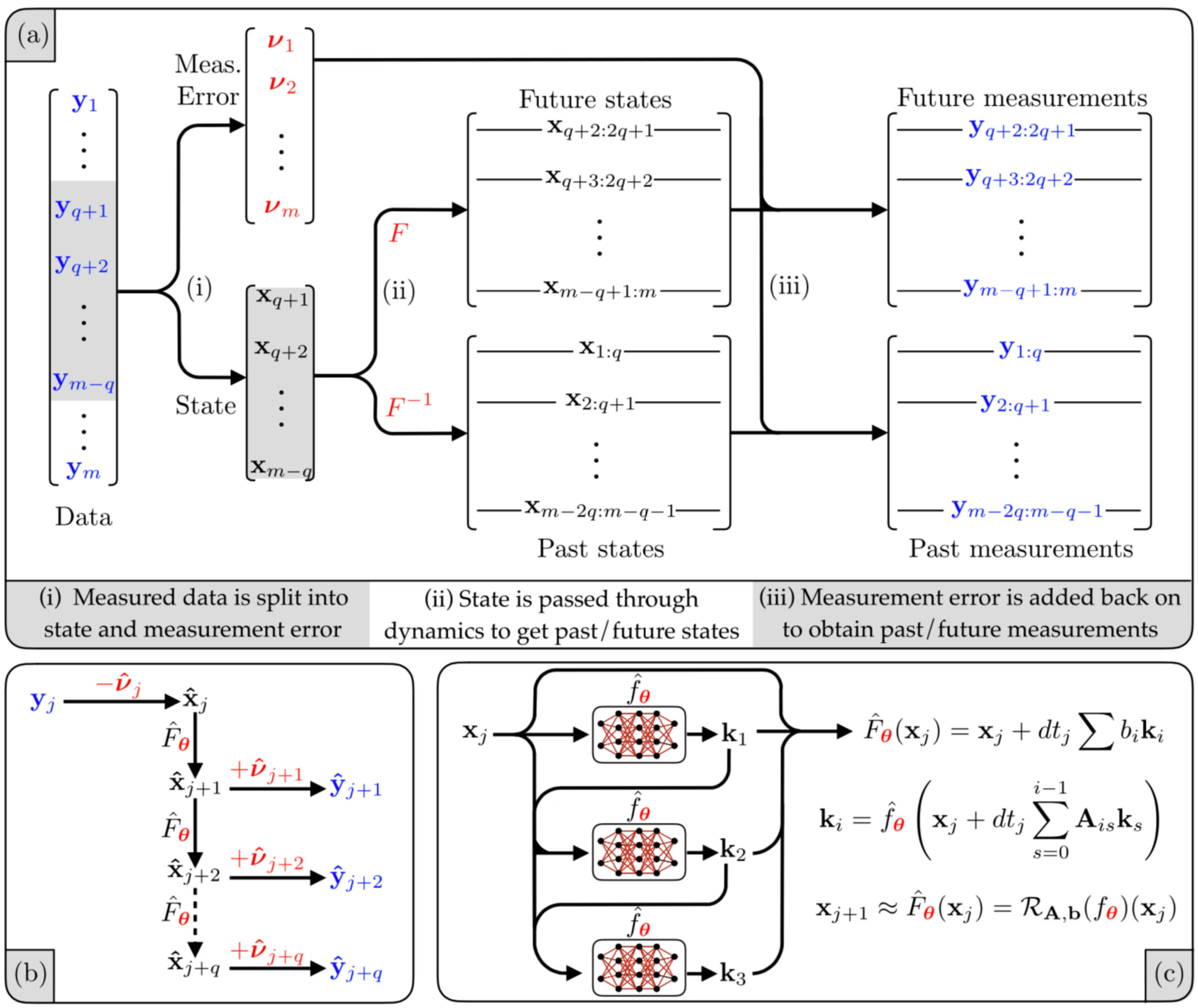


Figure 18. Schematic of how to enforce Runge Kutta integrator constraints to denoise and discover dynamics.

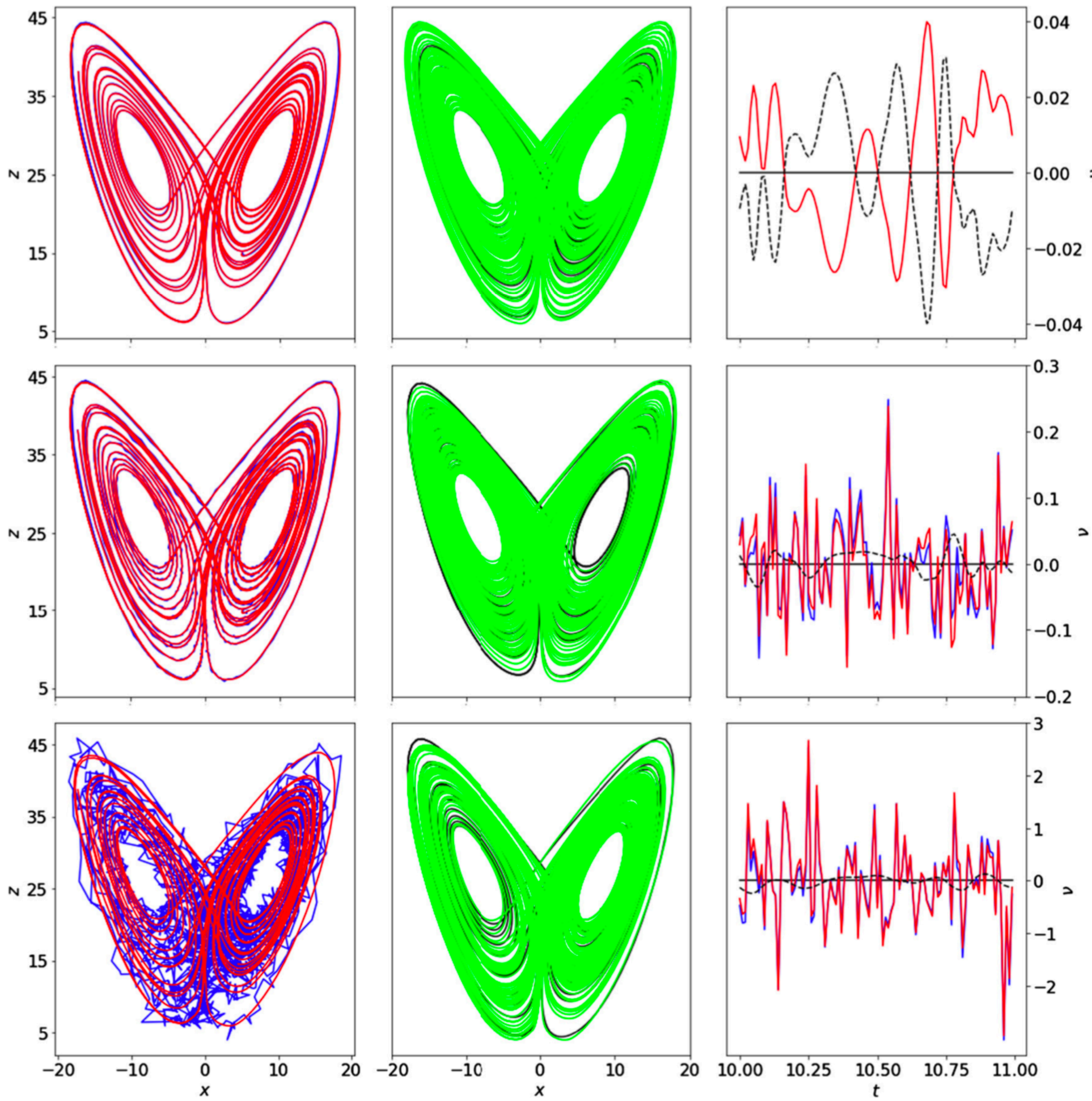


Figure 19. Denoising performance on Lorenz system.

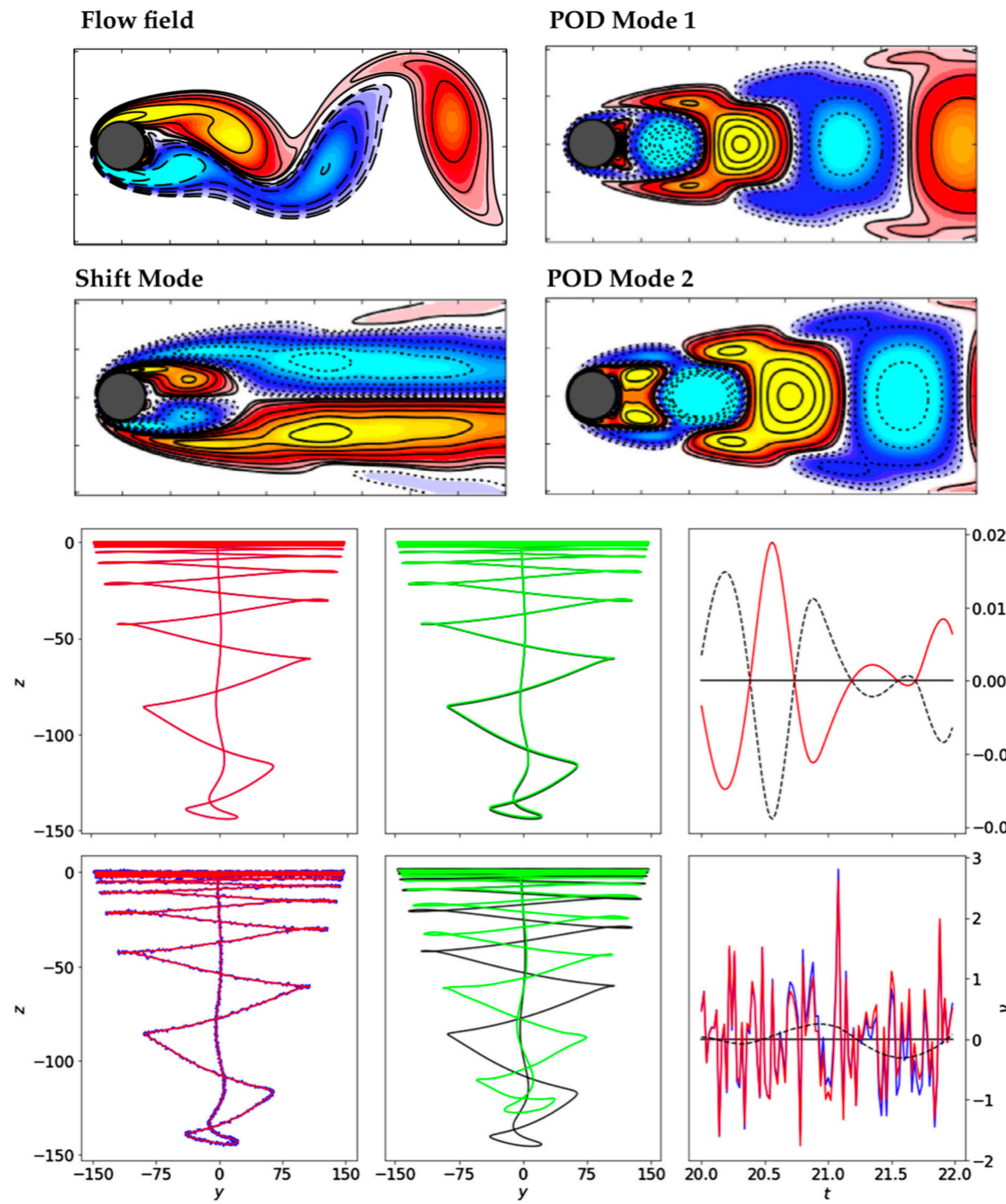


Figure 20. Denoising performance on fluid flow example.

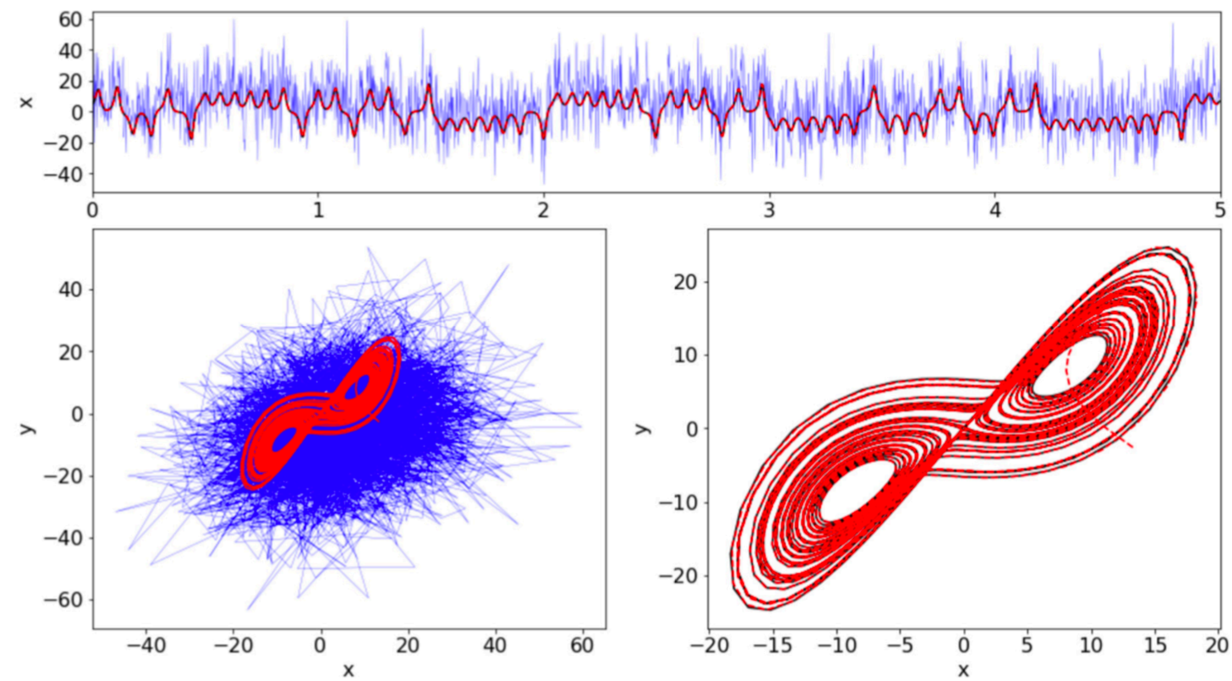


Figure 21. Denoising performance on Lorenz system with biased noise.

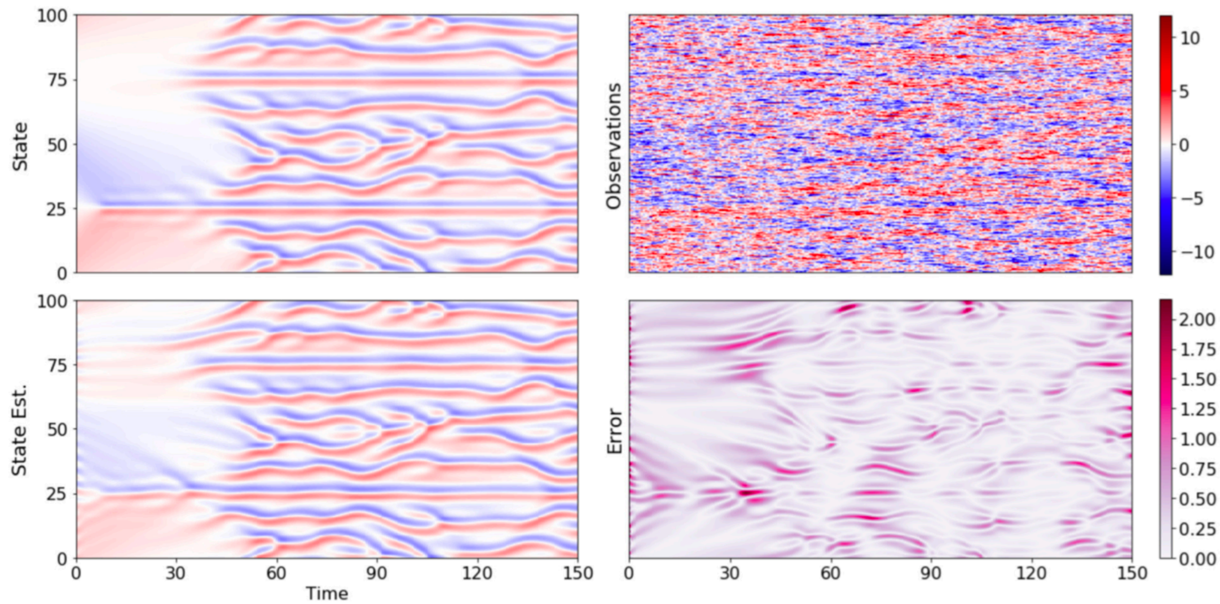


Figure 22. Denoising performance on Kuramoto Sivashinsky example.

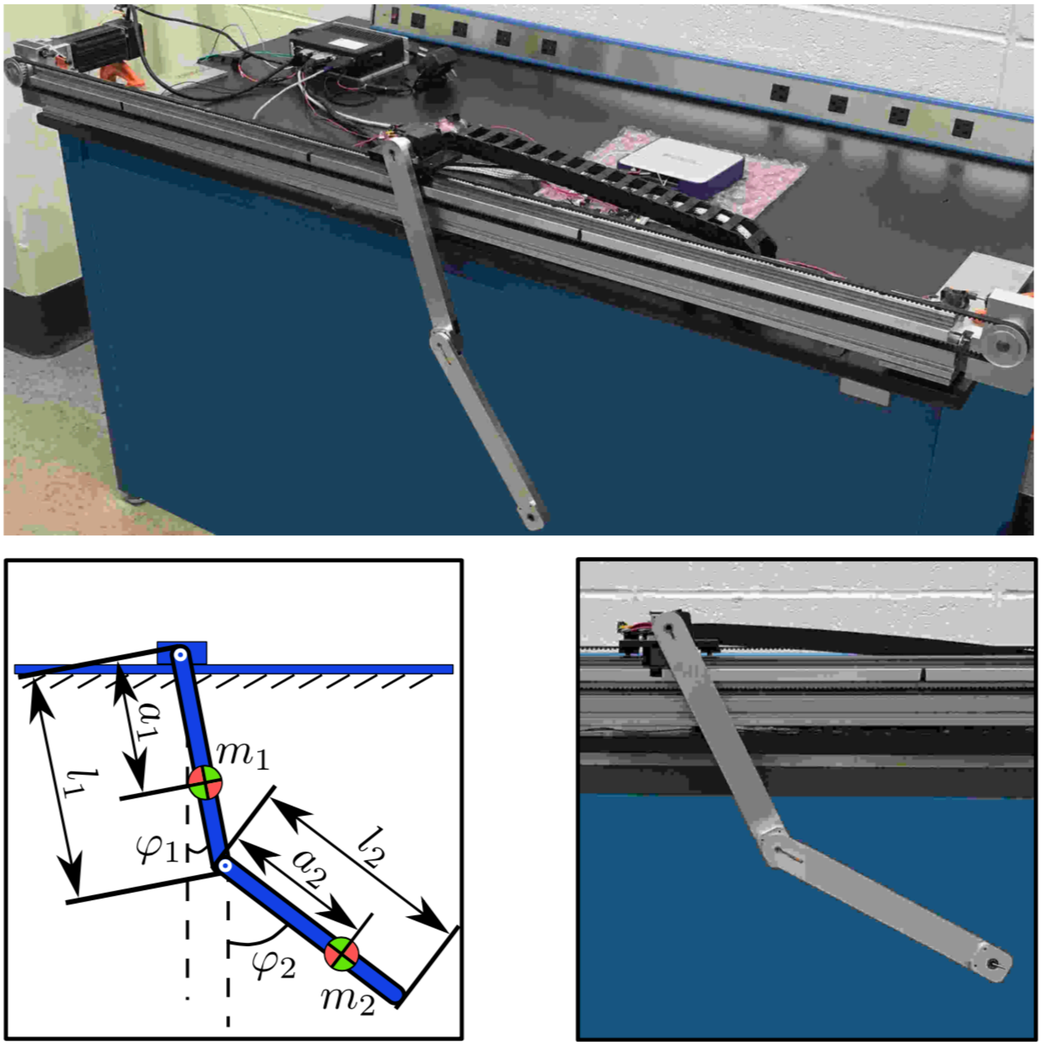


Figure 23. Double pendulum on cart experiment to test discrepancy modeling.

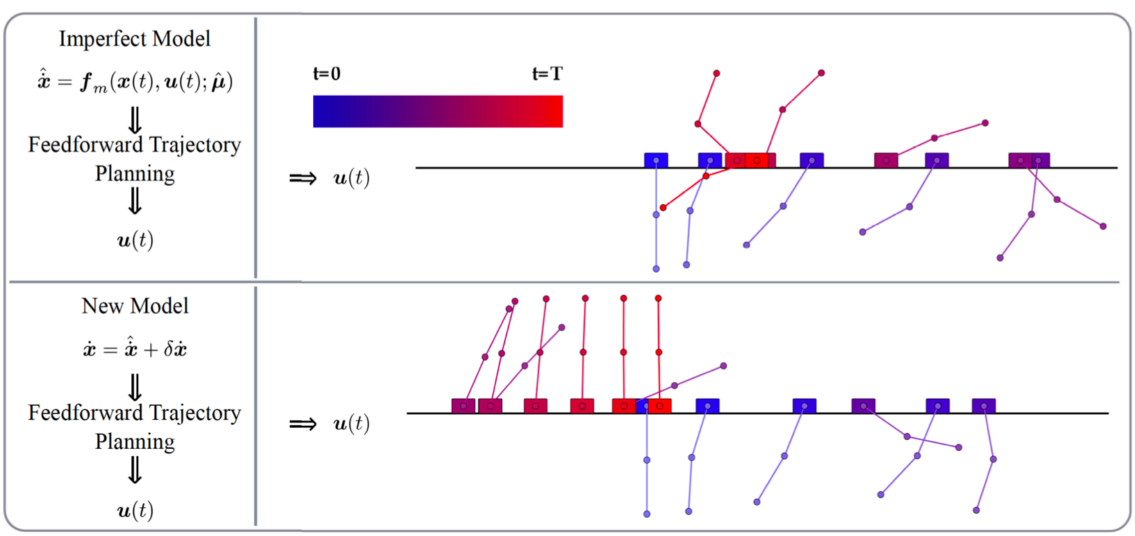


Figure 24. Simulated control performance with and without discrepancy model.