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Functional mapping approach to incoporate epistemic uncertainty in system reliability assessment

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Abstract

Reliability analysis of aerospace systems requires the consideration of multiple failure modes, multiple inter-disciplinary interactions, and various sources of epistemic uncertainty arising from data and modeling inadequacies. Effective reliability analysis requires accurate modeling of the uncertainty sources and accurate reliability computation, while maintaining computational affordability. In this project, we successfully developed an effective functional mapping approach to include epistemic uncertainty sources in reliability analysis. In this approach, auxiliary variables are introduced to bring both aleatory and epistemic uncertainty sources to the same level of the stochastic analysis. This approach was extended to sensitivity analysis of multidisciplinary models and time-dependent simulations, in order to quantify the contributions of aleatory and epistemic sources to the uncertainty in the model output. Leveraging the functional mapping approach, adaptive surrogate modeling techniques were developed for problems with spatial and temporal variability, coupled multi-disciplinary models, and reliability analysis with multiple limit states. Since multi-disciplinary simulations of realistic systems are expensive, the adaptive surrogate modeling approach was further extended to identify the input setting, the individual disciplinary analysis, the particular time instant, and the particular surrogate model to improve, at each step of the surrogate training process, thus minimizing the effort in multiphysics simulations. Several types of surrogate modeling approaches were developed: adaptive Kriging, support vector machines, neural networks, and probability-space models such as copula and mixture distribution models, to adapt to different aspects of the problem. The estimation of model error using calibration tests and extrapolation to prediction for untested configurations was significantly enhanced by focusing on model form error in the governing equation instead of the discrepancy in the model output. While the discrepancy term cannot be used in the prediction for untested configuration, the model form error can be used as long as the governing physics is the same. This idea was successfully developed through Bayesian state estimation to estimate the model form errors, and a surrogate model was developed to relate the system states to the model form errors; this surrogate model was then used to predict the model errors in the untested configuration for dynamic, coupled multi-disciplinary systems. The proposed methods were successfully illustrated for the reliability analysis of a hypersonic vehicle panel, using a coupled model of four disciplinary analyses, namely aerodynamics, aerodynamic heating, heat transfer, and structural deformation.

Keywords: Multi-disciplinary analysis, reliability, uncertainty, sensitivity, model error, surrogate modeling, hypersonic vehicle panel.

I. Project Overview and Objectives

The overall goal of this project was to investigate a systematic, rigorous and affordable computational approach to *estimate the reliability of structural systems* subjected to combined and extreme environments, in the presence of multiple sources of *epistemic uncertainty*, namely, data and model uncertainties, in addition to aleatory uncertainty (natural variability). The following objectives were pursued in order to achieve this goal:

- 1. Investigate a functional mapping approach to effectively include data and model uncertainties in reliability estimation with respect to individual damage mechanisms.
- 2. Investigate the combination of functional mapping and Bayesian networks to estimate system-level reliability, considering multiple damage mechanisms.
- 3. Expand the functional mapping approach to include epistemic uncertainty in the description of variability over space and time.
- 4. Expand the functional mapping approach to include heterogeneous information through a Bayesian network-based integration methodology, and to quantify the relative contributions of aleatory and epistemic uncertainty sources to the reliability assessment.

The methods developed and investigated through the four objectives were assessed using several illustrative problems of gradually increasing complexity. In Year 1, we investigated the reliability analysis of a curved beam under various epistemic uncertainty sources, and under spatial and temporal variations of loads and properties. In subsequent years, we investigated the reliability analysis of a hypersonic vehicle panel, briefly described below.

Hypersonic vehicle panel: A rigid, curved panel representing a deformed or post-buckled hypersonic aircraft panel is shown in Figure 1. This is a quasi-static, partial version of a 4-

discipline coupled aerothermoelastic problem (aerodynamics, aero-heating, heat transfer, and structural deformation), with the structural analysis removed, to avoid consideration of a fully coupled problem. The output quantities of interest are (1) temperature distribution in the panel and (2) instability of the panel.

The reliability analysis objective is to compute the probability of the output temperature T_{str} exceeding a threshold value at single or multiple locations and the probability that the time to instability is less than a required time interval. Epistemic uncertainty in the random field modeling of spatial variability in the input pressure and temperature needs to be



Figure 1. Hypersonic vehicle panel

considered. The problem can be solved at different levels of complexity, starting from deterministic, uniform pressure and temperature, to different variations of random field representation. The output temperature distribution is computed through a finite difference solution of a differential equation. This example leveraged ongoing in-house research at AFRL, where the focus was on developing Bayesian calibration and validation techniques for uncertainty quantification of a four-discipline coupled analysis of a hypersonic vehicle panel.

II. Research Accomplishments

A few of the technical accomplishments are highlighted in the subsections below.

- Including epistemic uncertainty in reliability analysis
- Efficient surrogate modeling for reliability analysis with temporal variability
- Sensitivity analysis of epistemic uncertainty
- Reducing epistemic uncertainty in reliability analysis with multiple limit state functions
- Adaptive surrogate modeling in multi-disciplinary reliability analysis
- Reliability analysis of hypersonic vehicle panel under epistemic uncertainty
- Model form error estimation and extrapolation to untested configuration

A. Including Epistemic Uncertainty in Reliability Analysis

In this accomplishment, the representation of various epistemic uncertainty using functional mapping and likelihood-based approaches was studied.

(1) Functional Mapping Approach

The traditional method for handling epistemic uncertainty is to implement a double-loop procedure, where realizations of aleatory uncertainty depend on the realizations of epistemic uncertainty. The double loop procedure can be denoted as "stochastic mapping" (as shown in Fig. 2a), i.e., for a specific value of epistemic uncertainty, we get a distribution of the random variable. In other words, a single value of the epistemic uncertainty leads to a random variable or uncertain quantity follows certain distribution, but not a single value. This is what leads to expensive nesting in uncertainty quantification computation, since two loops of sampling are required, an outer loop for the distribution parameters and an inner loop for the random variable.



Figure 2a Stochastic Mapping



Functional mapping can overcome this challenge by creating a one-to-one relationship between specific realizations of epistemic parameters and corresponding specific realizations of random variables. Note that for a given value of epistemic uncertainty, a unique value of random variable is obtained corresponding to a CDF value. This is the basic sampling approach in the Monte Carlo method, known as the "inverse CDF" approach (as indicated in Fig. 2b). We can write this relationship for a normal random variable X as $X = F_X^{-1}(u \mid \mu_X, \sigma_X)$, where u is the CDF value. Note that u is a realization of the uniform random variable U, ranging from 0 to 1. Thus we can write the functional mapping between X and (μ_X , σ_X in the form $X = h(U, \mu_X, \sigma_X)$. More generally, we can write X = h(U, p) which defines a one-to-one functional mapping between the distribution parameters p and the random variable X. This means that, with the help of an auxiliary uniform random variable U, a sample realization of a random variable X can be related to the corresponding sample realizations of distribution parameters p by a single-level representation, instead of a nested two-level representation. This simple idea is proving to be very powerful in integrating multiple sources of uncertainty in reliability analysis in an efficient and effective manner.

(2) Data uncertainty

In practical applications, it is common to have only sparse point data and/or interval data on an input variable X, thus causing uncertainty in its PDF. Both parametric and non-parametric approaches have been developed to address the issue of data uncertainty caused due to the presence of limited data, which are discussed below

(a) Parametric approach

In a parametric approach, an input variable is represented using a distribution type and distribution parameters. The presence of limited data causes uncertainty regarding distribution type and parameters. In the Bayesian approach, this uncertainty is expressed using probability distributions for distribution type and parameters.

<u>Distribution parameter uncertainty</u>: Let a dataset **D** for a variable X consist of n point data $p_i(i=1 \text{ to } n)$ and m interval data $[a_j,b_j]$ (j=1 to m). The likelihood function for the distribution parameters Θ can be constructed as

$$L(\mathbf{\theta}) = \prod_{i=1}^{n} f_X(X = p_i | \mathbf{\Theta} = \mathbf{\theta}) \prod_{j=1}^{m} [F_X(X = b_j | \mathbf{\Theta} = \mathbf{\theta}) - F_X(X = a_j | \mathbf{\Theta} = \mathbf{\theta})]$$
(1)

where $f_X(x)$ and $F_X(x)$ represent the PDF and CDF of a variable X respectively. From the likelihood function, the PDFs of the distribution parameters are obtained using Bayes' theorem.

<u>Distribution type uncertainty</u>: Two approaches are available to handle distribution type uncertainty - (1) Composite distribution of possible distribution types using Bayesian Model Averaging (BMA), or (2) Single distribution type that best describes the data using Bayesian Hypothesis Testing (BHT). The weights for averaging or selection can be computed by comparing the likelihoods of distribution types (in the presence of uniform prior probabilities). (b) Non-parametric approach

As opposed to the parametric approach, the non-parametric approach does not assume any particular distribution type or distribution parameters but the PDF is constructed using interpolation techniques. This approach uses a single PDF to represent the combination of both aleatory and epistemic uncertainty.

Let a dataset **D** for a variable *X* consist of *n* point data $p_i(i=1 \text{ to } n)$ and *m* interval data $[a_j,b_j]$ (j=1 to m); the domain of *X* is discretized into *Q* points to model the non-parametric distribution. Let the PDF values at these discretized points be equal to q_i ($i=1, 2, \dots, Q$). Since $\mathbf{q} = q_i$ ($i=1, 2, \dots, Q$) is unknown, they can be estimated by solving the following optimization problem:

$$\max_{\mathbf{q}} L(\mathbf{q}) = \prod_{i=1}^{n} f_{X}(X = p_{i} | \mathbf{q}) \prod_{j=1}^{m} [F_{X}(X = b_{j} | \mathbf{q}) - F_{X}(X = a_{j} | \mathbf{q})]$$

$$s.t. \mathbf{q} \ge 0; f_{X}(x) \ge 0; \int f_{X}(x) \, \mathrm{d}x = 1$$
(2)

After obtaining the PDF values at these discretized points, interpolation techniques are used to estimate the PDF values at any other input values.

(3) Model uncertainty

The representation of different types of model uncertainty such as model parameter uncertainty and model discrepancy were investigated.

(a) Model parameter uncertainty

Model parameter uncertainty represents the uncertainty in the model parameters due to either natural variability or limited data or both. The three possible scenarios of model parameter uncertainty are -(1) model parameter is deterministic but unknown (epistemic uncertainty), (2) model parameter is stochastic with known distribution parameters (aleatory uncertainty), and (3) model parameter is stochastic with unknown distribution parameters (aleatory and epistemic uncertainty).

If a model parameter is deterministic but unknown, it can be estimated using available data using model calibration procedure. The uncertainty (epistemic) arises in the estimation due to limited available data. As the amount of data increases, the uncertainty in the estimation of model parameters decreases. The likelihood-based approach used for estimating the distribution parameters in the data uncertainty section can be used to estimate model parameters.

Model parameters that are associated with aleatory uncertainty (probability distributions) and with fixed distribution parameters, can typically be considered as input variables for reliability analysis and the techniques used for quantification of uncertainty in inputs (parametric and non-parametric approaches) can also be used for model parameters. If the distribution parameters are unknown, then model calibration procedure can be carried out to estimate the distribution parameters using available data.

(b) Model discrepancy

Model discrepancy represents the combined error introduced due to the assumptions and simplifications made in building a model (model form error) as well as the errors that arise in the methodology adopted in solving the model equations (numerical solution errors). Different types of numerical solution errors exist such as discretization error, round-off error, and truncation error. Suppose $g_{obs}(\mathbf{X})$, $g_{model}(\mathbf{X})$, and $\delta(\mathbf{X})$ represent the observation, model prediction and model discrepancy respectively. For a given $\mathbf{X} = \mathbf{x}$, the three quantities are related as $g_{obs}(\mathbf{x}) = g_{model}(\mathbf{x}) + \delta(\mathbf{x})$. The quantification of the model discrepancy is achieved by comparing the predictions from the simulation model with experimental observations at specific values of input variables \mathbf{X} . In the Kennedy O' Hagan (KOH) approach, a Kriging model is used to represent model discrepancy and its parameters are calibrated along with system model parameters.

(c) Reliability analysis errors

Different types of errors that arise in carrying out reliability analysis such as surrogate uncertainty and uncertainty quantification (UQ) error are also included in the investigation, as below.

<u>Surrogate uncertainty</u>: The uncertainty associated with the prediction of a surrogate is called surrogate uncertainty. For example, the prediction of a GP model is a Gaussian distribution with parameters dependent on the input. When a GP surrogate is used, the prediction at any input is a Gaussian distribution with parameters dependent on the input. In most cases, only the mean predictions are used to estimate the failure probability. When the accuracy of the surrogate is high (i.e. the uncertainty of prediction is low), the treatment of using the mean predictions works well, and results in a single value of the reliability estimate. If the accuracy of the model prediction is low, it becomes necessary to also include the prediction uncertainty for reliability estimation. In order to quantify the effects of surrogate uncertainty on reliability analysis, an uncertainty quantification problem can be formulated as shown in Fig. 3.



Figure 3 Effects of surrogate uncertainty on reliability analysis

<u>Monte Carlo Simulation (MCS) error</u>: MCS error represents the error due to the use of limited number of Monte Carlo samples for uncertainty propagation. The MCS error, also referred to as Uncertainty Quantification (UQ) error, is quantified as the difference between the empirical CDF (constructed using Monte Carlo samples after uncertainty propagation) and the true CDF of the output quantity of interest.

More detailed discussions about the representation of various sources of epistemic uncertainty and their inclusion in reliability analysis can be found in **[J1, J2]** listed in Section III.

B. Efficient Surrogate Modeling Approach for Reliability Analysis with Temporal Variability [J3]

Based on the spatial and temporal variability modeling in accomplishment A, we developed a single-loop Kriging (SILK) surrogate modeling method for time-dependent reliability analysis in [J4]. Current surrogate modeling methods for time-dependent reliability analysis implement a double-loop procedure, with the computation of extreme value response in the outer loop and optimization in the inner loop. The computational effort of the double-loop procedure is quite high even though improvements have been made to improve the efficiency of the inner loop. In the proposed method, the optimization loop used in current methods is completely removed. A single surrogate model is built for the purpose of time-dependent reliability assessment. Training points of random variables and over time are generated at the same level instead of at two separate levels. The surrogate model is refined adaptively based on a learning function modified from time-independent reliability analysis and a newly developed convergence criterion. This will reduce the epistemic uncertainty in reliability analysis in the most effective way. Strategies for building the surrogate model are investigated for problems with and without stochastic processes. The efficiency of the developed SILK method is verified using numerical examples. As shown in Table 1, the new proposed SILK method is much more efficient than current available reliability analysis methods (i.e. Rice, Independent EGO, Mixed EGO) while the accuracy is also better than current methods.

Method	Number of function evaluations	$p_f(t_0, t_s) ~(\times 10^{-4})$	Error (%)
SILK	18.35	1.08	0.92
Rice	1017	0	100
Independent EGO	212	1.31	20.18
Mixed-EGO	69	1.09	0
MCS	5×10 ⁸	1.09	N/A

 Table 1. Results of a numerical example

C. Sensitivity Analysis of Epistemic Uncertainty in Reliability Analysis (C2)

Since there are multiple sources of epistemic uncertainty that may affect the reliability analysis result, in **[C2]**, global sensitivity analysis methods are developed using the functional mapping approach to quantify contributions of various sources of epistemic uncertainty on the uncertainty of reliability analysis results.

Global sensitivity analysis requires a one-to-one relationship between the inputs and the outputs. Here, inputs refer to the epistemic uncertainty sources and output refers to the failure probability. For a given realization of uncertain distribution type, uncertain distribution parameters and uncertain model parameters, the failure probability is represented by an unconditional PDF due to surrogate uncertainty and MCS error. As GSA requires one-to-one relationship, the uncertainty in the failure probability estimate due to surrogate uncertainty and MCS error can be represented using an auxiliary variable, which then results in a one-to-one relationship between epistemic inputs and failure probability. Note that the auxiliary variable in the following equation represents the combined contributions of both surrogate uncertainty and MCS error.

$$U_{SU,MCS} = F_{p_f}^U(p_f \mid \boldsymbol{\Theta} = \boldsymbol{\theta}, \mathbf{D}_{\mathbf{T}} = \mathbf{d}_{\mathbf{T}}, \boldsymbol{\Psi} = \boldsymbol{\varphi}) = \int_{-\infty}^{p_f} f_{p_f}^U(w \mid \boldsymbol{\Theta} = \boldsymbol{\theta}, \mathbf{D}_{\mathbf{T}} = \mathbf{d}_{\mathbf{T}}, \boldsymbol{\Psi} = \boldsymbol{\varphi})dw$$
(3)

In Eq. (3), $U_{SU,MCS}$ represents the auxiliary variable for the combined contribution of surrogate uncertainty and MCS error. $F_{p_f}^U$ and $f_{p_f}^U$ represents the CDF and PDF of the unconditional distribution of the failure probability. Θ , D_T and Ψ refers to the vector of uncertain model parameters, distribution type uncertainty variables and uncertain model parameters. θ , d_{T} and ϕ represent their realizations respectively. To further separate the uncertainty contributions due to surrogate uncertainty and MCS error, we can introduce two auxiliary variables. Thus, for a given realization of uncertain distribution type, uncertain distribution parameter, uncertain model parameter, auxiliary variable for surrogate uncertainty and the auxiliary variable for MCS error, failure probability results in a point estimate; resulting in a one-to-one mapping between the epistemic inputs and failure probability estimate. As distribution type uncertainty is a discrete variable, it cannot directly be used in GSA. To overcome this issue, we use the CDF of distribution type uncertainty, which is a continuous distribution. For illustration, consider a random variable with unknown distribution type but with two possible candidate distribution types are available such as Normal and Type 1 Extreme Value Distribution (EVD) with corresponding probability mass functions of 0.2 and 0.8 respectively. We can generate samples from this discrete distribution is by using its CDF, which follows a uniform distribution between 0 and 1. Several samples can be drawn from this distribution; for all samples below 0.2, a normal distribution is assumed else a Type 1 EVD is assumed. Thus, the discrete variable is converted to a continuous variable and included in sensitivity analysis framework. The overall procedure for generating deterministic failure probability estimates from the epistemic inputs is given below.

• Generate samples of distribution type uncertainty, uncertain distribution parameters and uncertain model parameters. Using these realizations of epistemic inputs, a failure probability estimate is first estimated by including only surrogate uncertainty; this results in a PDF of failure probability estimate.

• Generate a sample from this failure probability PDF after considering surrogate uncertainty by generating a sample from its corresponding auxiliary variable.

• When MCS error is considered, the failure probability sample obtained in the previous step (after considering surrogate uncertainty) becomes a PDF. From this PDF, a sample of failure probability estimate is generated using the auxiliary variable associated with MCS

error. Using a generated sample of the auxiliary variable associating with MCS error, a sample of the failure probability estimate can be obtained.

Thus, a one-to-one relationship is obtained between the epistemic inputs and failure probability estimate. Then, sensitivity analysis can be carried out using any of the existing techniques. Table 2 gives the global sensitivity analysis results of a cantilever beam problem.

Variable	First-order sensitivity index			
$\mu_{\scriptscriptstyle P}$	0.48			
$\sigma_{\scriptscriptstyle P}$	0.086			
d_P	0.277			
u _{su}	0.049			
<i>u_{MCS}</i>	0.105			

Table 2. First order sensitivity indices of epistemic variables towards failure probability

D. Reducing Epistemic Uncertainty in Reliability Analysis with Multiple Limit State Functions (J3, J8)

In engineering application problems, multiple surrogate models are usually built to perform system reliability analysis. How to efficiently reduce the epistemic uncertainty due to surrogate modeling is a challenging problem. Current limit state surrogate modeling methods for system reliability analysis usually build surrogate models for failure modes individually or build composite limit states. In practical engineering applications, multiple system responses may be obtained from a single setting of inputs as shown in Fig. 4. In such cases, building surrogate models individually will ignore the correlation between different system responses and building composite limit states may be computationally expensive since the nonlinearity of composite limit state is usually higher than individual limit states. In [J3], we propose a new efficient Kriging surrogate modeling approach (EKSA) for system reliability analysis by constructing composite Kriging surrogates through the selection of Kriging surrogates constructed individually and Kriging surrogates built based on singular value decomposition (SVD). The resulting composite surrogate model will combine the advantages of both types of Kriging surrogate models and thus reduce the number of required training points. A new stopping criterion and a new surrogate model refinement strategy are proposed to reduce the epistemic uncertainty effectively and thus further improve the efficiency of this approach. The surrogate models are refined adaptively with high accuracy near the active failure boundary until the proposed new stopping criterion is satisfied. Fig. 5 shows the comparison of the learned limit states from the proposed EKSA method and that from current available methods. Following that, Table 3 gives the number of function evaluations required by different methods. The results show that the proposed method can reduce the epistemic uncertainty more effectively than current methods.



(c) Composite surrogate model

Table 3. Results Comparison of a Series System Example						
	ILS-CL		CLS		EVSA	MCS
	AK-MCS	EGRA	AK-MCS	EGRA	EKSA	MC5
$\hat{p}^s_{\scriptscriptstyle f}$	0.0812	0.0801	0.0420	0.0328	0.0828	0.0835
NOF	8+44.78	8+49.67	8+189	8+180	8+24.03	1×10 ⁶
$\mathcal{E}(\%)$	2.75	4.07	49.7	60.72	0.84	N/A

Fig. 5. Comparison of learned composite limit state from EKSA and true composite limit state

Table 3. Results Comparison of a Series System Example

Note: "NOF" is "Number of function evaluations", which is given as "Number of initial training points"+ "Number of added training points".

E. Adaptive surrogate modeling for multi-disciplinary reliability analysis [J11, J12]

An adaptive surrogate modeling framework is developed for the reliability analysis of coupled multi-disciplinary systems (e.g., a hypersonic vehicle panel) with spatio-temporal variability. The Kriging surrogate modeling method in conjunction with singular value decomposition is first employed to replace the original computer simulation models in the multidisciplinary analysis. Due to the limited computational resources, the initial surrogate models may not accurately represent the original physics simulation models, which results in errors in the reliability analysis of the hypersonic vehicle panel. A methodology is developed to analyze the effects of surrogate model uncertainty on the results of reliability analysis by considering the variability of the system response over space and time, and to adaptively allocate the computational resources to improve the accuracy of reliability analysis. A four-step resource allocation procedure is developed to determine at what input setting, which discipline, when, and which surrogate model improvement to allocate the computational resources to. The result comparisons of the proposed method and Monte Carlo simulation demonstrate that the proposed adaptive surrogate modeling method is able to efficiently and accurately assess the reliability of the panel subjected to failure modes of deformation and over-heating.

Fig. 6 gives the overall flowchart of the proposed MDRA framework for the panel. There are mainly three steps, namely *initial surrogate modeling*, *uncertainty quantification* (UQ) and error *analysis* of the failure probability estimate, and *uncertainty reduction* of the failure probability estimate.



Fig. 6. Overview of the proposed framework

Step 1: *Initial surrogate modeling* – Initial surrogate models are built for the coupling and response variables by following the method presented in Ref. **[J12]**.

Step 2: *UQ and error analysis of failure probability estimate* – Since the initial surrogate models may not accurately represent the original computer simulation models, we first quantify the uncertainty in the failure probability estimate due to surrogate model uncertainty. Based on this, we develop an approach to estimate the error of reliability analysis with the consideration of the variability over space and time.

Step 3: Uncertainty reduction of failure probability estimate – If the error of reliability analysis can satisfy the accuracy requirement, we report the failure probability estimate. Otherwise, the surrogate models need to be refined. In this step, a new uncertainty reduction approach is proposed to effectively refine the surrogate model by adaptively allocating the computational resources to surrogate models of coupling and response variables.

- 1. At which realization of input random variables and stochastic processes to refine the surrogate models?
- 2. Even for a fixed realization of random variables and stochastic processes, the uncertainty of the system safety state is the combined effect of multiple disciplinary models. The question that needs to be answered is which disciplinary surrogates to refine?
- 3. Since the surrogate model uncertainty will propagate and accumulate over time, we need to determine the best time instant at which to refine the disciplinary surrogates identified from the last step.
- 4. For the identified discipline at a specific time instant, there are multiple response surrogate model and coupling surrogate models. In this step, we need to identify which surrogate model to refine.



Fig.7 Flowchart of uncertainty reduction

The methodology and results are presented in detail in Refs. [J11, J12].

F. Reliability Analysis of Hypersonic Vehicle Panel under Epistemic Uncertainty (J12)

The above developed reliability analysis and surrogate modeling methods are applied to the reliability analysis of a panel structure on a hypersonic aircraft vehicle as depicted in Fig. 1. As the vehicle is subjected to hypersonic flow, an attached oblique shock is created at the forebody leading edge. This resulted aerodynamic pressure causing elastic deformation of the panel, which feeds back to alter the aerodynamic pressure on the panel. The panel is also subjected to aerothermal effects from aerodynamic heating. The aerothermal component is coupled with the

aeroelastic component. The stability of the panel structure in the hypersonic flow is studied. Fig. 8 shows the coupled aerothermoelastic response of this panel, which is characterized by the four interacting disciplines including aerodynamic pressure, aerodynamic heating, heat transfer, and structural deformation.



Fig. 8. Multidisciplinary analysis of a hypersonic aircraft panel

Two failure modes of the hypersonic aircraft panel are considered, namely deformation (instability) and heating. In the failure mode of deformation, the panel is defined as failure if the deformation over space is negative and passes a certain threshold. In the failure mode of heating, failure of the panel occurs when the maximum temperature on the panel is larger than a threshold. The panel is failed is any failure mode happens. Based on these definitions, we define the reliability of the panel as follows

$$R(t) = \Pr\{v(\mathbf{d}, \tau) > \varepsilon_{v} \cap T(\mathbf{d}, \tau) < \varepsilon_{\tau}, \forall \tau \in [0, t], \mathbf{d} \in \Omega_{\mathbf{d}}\}$$
(4)

where [0, t] = [0, 0.5] seconds is the time interval of interest, $\varepsilon_v = -0.075$ is the threshold of deformation which is a negative value, and $\varepsilon_T = 470 K$ is threshold of temperature.

Table 4 gives the random variables and stochastic processes of the hypersonic aircraft panel.

Table 4. Random variables and stochastic processes of the hypersonic anerart paner						
Variable	Distribution	Mean	Standard Deviation	Correlation		
M	Gaussian	10.2	0.01	N/A		
$T_{0}\left(\mathrm{K} ight)$	Gaussian	313	1	N/A		
h	Lognormal	0.0031	2×10 ⁻⁵	N/A		
а	Lognormal	0.046	2×10 ⁻³	N/A		
AoA (degree)	Gaussian	5.125	0.05	Eq. (5)		
$A_t(Km)$	Gaussian	31.5	0.1	Eq. (6)		

Table 4. Random variables and stochastic processes of the hypersonic aircraft panel

$$\rho_{AoA}(t_1, t_2) = \exp(-((t_2 - t_1) / \zeta_{AoA})^2)$$
(5)

where $\zeta_{AoA} = 0.05 \ s$ is the correlation length of angle of attack.

$$\rho_A(t_1, t_2) = \exp(-((t_2 - t_1) / \zeta_A)^2)$$
(6)

where $\zeta_{A} = 0.05 s$ is the correlation length of altitude.

Fig. 9 gives the time-dependent reliability analysis results of the hypersonic panel without considering epistemic uncertainty sources. Fig. 10 gives the reliability analysis results after incorporating the epistemic uncertainty sources. The considered uncertainty sources include model uncertainty and data uncertainty. The results show that the epistemic uncertainty sources affect the reliability analysis results significantly.



Fig. 9. Time-dependent failure probability of the multidisciplinary system



Fig. 10. Time-dependent failure probability of the panel after considering epistemic uncertainty

G. Model error extrapolation to untested configuration [C10]

The above methods assume that the model error has been quantified before inclusion in the reliability analysis. This would be true if tests were conducted on the system of interest. However, the prediction system of interest is often different from the tested configuration. Therefore, a methodology is developed to estimate the discrepancy in the model output of untested coupled multi-physics systems, based on tests of related systems. Model predictions often exhibit discrepancy with respect to experimental observations, due to assumptions and approximations in the model. Bayesian approaches for estimating discrepancy in single models have been studied in the past. In this paper, we approach the problem of discrepancy prediction in coupled multiple models (especially multi-disciplinary models) using Bayesian state estimation methods. The proposed state estimation-based approach is found to have significant advantages over the previously studied Kennedy-O'Hagan method, in the estimation of discrepancies of hidden states, and in the identification of the sources of these discrepancies, namely model form errors. We adopt a substructuring-based approach to take advantage of the weak coupling where appropriate. The proposed approach is illustrated for a four-discipline problem related to aero-thermo-elastic response prediction of a hypersonic aircraft panel.

The proposed methodology involves Bayesian estimation of parameters and model form errors (MFEs) in multi-disciplinary models of dynamic systems, and transferring these estimates from the tested structure to an untested one. First, a combined state and parameter estimation approach is developed, using state estimation to determine the MFEs, and MCMC-based parameter estimation to determine the model parameters. Next, artificial neural network (ANN) surrogate modeling is used to model the relationship between MFEs and system states. Finally, the model error in the untested system is predicted using the estimated model parameters from the first step and the ANN models for MFE-s from the second step.

Step 1. Simultaneous estimation of model parameters and MFEs

We assume that the analyst is unaware of the type of MFE-s and the specific equations affected by them. The underlying assumption is that all the disciplinary models are corrupted by modeling errors. The MFE-s are accounted for by introducing additive terms to the system model (Eq. 1) in the following manner:

$$dX_{i}(t) = a_{i} \{X_{1:n}(t), \theta, t\} dt + \mathbf{b}(t) dW_{i}(t) dt, X_{i}(0) = X_{i,0}; i = 1, 2, ..., n; 0 \le t \le T$$
(7)

Here, $dW_i(t)$ is a $n_{X,i} \times 1$ vector of derivatives of Brownian motion, and $\mathbf{b}(t)$ is a diagonal matrix of size $N_{X,i} \times N_{X,i}$, with the *r*-th diagonal term denoting the intensity of the *r*-th white-noise random process $dW_i^{(r)}(t)$. The coupled systems of stochastic differential equations represented by Eq. 7 is discretized in time using Ito-Taylor schemes (Kloeden and Platen, 1992) leading to the following discrete-time systems of equations:

$$\boldsymbol{X}_{i,k} = \boldsymbol{g}_i \left(\boldsymbol{X}_{1:n,k-1}, \boldsymbol{\theta}, t_k, \hat{\boldsymbol{\xi}}_{i,k} \right); i = 1, 2, ..., n; k = 1, 2, ..., N_T$$
(8)

Here, $\hat{\xi}_{i,k}$ is a vector of zero-mean Gaussian random variables with $\mathbb{E}\left[\hat{\xi}_{i_1,k_1}\hat{\xi}_{i_1,k_1}^T\right] = \Sigma_{\hat{\xi}}$, and $\mathbb{E}\left[\hat{\xi}_{i_1,k_1}\hat{\xi}_{i_2,k_2}^T\right] = 0$, if $i_1 \neq i_2$ and $k_1 \neq k_2$, $i_1, i_2 = 1, 2, ..., n$, and $k_1, k_2 = 1, 2, ..., N_T$. We estimate the system states *X* and the model parameters θ simultaneously, using combined state and parameter estimation, with $p(\theta)$ as the prior pdf of the model parameters. The proposed procedure is based on the Metropolis-Hastings MCMC algorithm. In the absence of MFE-s and the associated process noise terms dW_i , the likelihood can be evaluated directly as $p(\mathbf{y}_{1:N_T} | \theta) = \prod_{k=1}^{N_T} p(\mathbf{y}_k | \theta)$. In this study, to account for the presence of the process noise terms, we evaluate the likelihood using Bayesian state estimation as

$$p(\mathbf{y}_{1:N_T} \mid \boldsymbol{\theta}) = \prod_{k=1}^{N_T} \iint p(\mathbf{y}_k \mid \mathbf{x}_k, \boldsymbol{\theta}) p(\mathbf{x}_k \mid \mathbf{x}_{k-1}, \boldsymbol{\theta}) p(\mathbf{x}_{k-1} \mid \mathbf{y}_{1:k-1}, \boldsymbol{\theta}) d\mathbf{x}_k d\mathbf{x}_{k-1}$$
(9)

Of the three terms inside the integral in the RHS of Eq. 7, the first term $p(\mathbf{y}_k | \mathbf{x}_k, \theta)$ can be evaluated using the measurement model [Eq. 2], the second term $p(\mathbf{x}_k | \mathbf{x}_{k-1}, \theta)$ can be evaluated using the process model, and the final term $p(\mathbf{x}_{k-1} | \mathbf{y}_{1:k-1}, \theta)$ can be determined using Bayesian state estimation with the process and measurement models. In this study, we adopt the ensemble Kalman filter (EnKF) for state estimation. By substituting the estimated values of system states $X_i; i = 1, 2, ..., n$, in the system model, we evaluate the term $\varepsilon_i(t); i = 1, 2, ..., n$, which represents the MFE-s in the *n* disciplinary equations. See **[C10]** for details.

Step 2. Formulating an ANN-based relationship between MFEs and system states

In Step 1, we presented a numerical approach to evaluate the MFEs in the tested configuration. The MFEs were estimated as random processes, and each MFE represents a scalar time-history of an external input associated with a single scalar governing equation. We now present an ANN-based approach to associate individual MFEs with the system states influencing it. This permits the transfer of the MFEs estimated from one configuration to another, as long as both configurations share the same governing equations. This approach can be adopted for partial differential equations discretized using finite difference as well as finite element schemes.

Consider the governing system of equations associated with one discipline of the multitdiscipline system of equations. The disciplinary equation is in the form of a partial differential equation, which can be discretized using the finite element framework into a system of ordinary differential equations. If the finite element mesh consists of elements of the same type, dimensions, and parameters, the system of equations governing the nodal degrees-of-freedom do not vary from one node to another. We assume that the form of the MFEs also do not vary across nodes.

Step 3. Response prediction for an untested system

We assume that the untested system, whose response is to be predicted, and the tested system share a common governing system of equations, but the two systems may differ in geometry, external inputs, and boundary conditions. This implies that the model parameters $\theta \sim p(\theta | y_{1:N_T})$ estimated in Step 1, and the ANN models representing the MFE-s from Step 2, can be used to update the prediction system as well. We note that if the test and prediction systems differ in boundary conditions, the MFE-s associated with the elements at the boundary of the prediction system are not updated.

Figure 11 gives an illustration of model discrepancy prediction for the untested system, considering two output quantities, temperature and deformation; these are outputs of different disciplinary models. The plots show that the predicted discrepancy agrees well with the actual discrepancy in this problem. The details of the methodology and the numerical results are presented in **[C10]**.





III. Work Products

The work products include 13 archival journal publications and 10 conference publications. Detailed information of these publications are given as below.

Archival Journal Publications

[J1] Nannapaneni, S., and Mahadevan, S., "Reliability Analysis under Epistemic Uncertainty," *Reliability Engineering & System Safety*, Vol. 155, No. 11, pp. 9–20, 2016.

[J2] Nannapaneni, S., Hu, Z., and Mahadevan, S., "Uncertainty Quantification in Reliability Estimation with Limit State Surrogates," *Structural and Multidisciplinary Optimization*, Vol. 54, No. 6, pp. 1509-1526, 2016.

[J3] Hu, Z., and Mahadevan, S., "Uncertainty Quantification of Time-Dependent Reliability Analysis in the Presence of Parametric Uncertainty," ASCE-ASME *Journal of Risk and Uncertainty in Engineering Systems*, Vol. 2, No. 3, page 031005 (11 pages), 2016.

[J4] Hu, Z., and Mahadevan, S., "A Single-Loop Kriging Surrogate Modeling for Time-Dependent Reliability Analysis," *Journal of Mechanical Design*, ASME, Vol. 138, No. 6, pp. 061406 (10 pages), 2016.

[J5] Devathi, H., Hu, Z., and Mahadevan, S., "Snap-Through Buckling Reliability Analysis under Spatiotemporal Variability and Epistemic Uncertainty", *AIAA Journal*, 54 (12), pp. 3981-3993, 2016.

[J6] Hu, Z., and Mahadevan, S., "Resilience Assessment Based on Time-Dependent System Reliability Analysis", *Journal of Mechanical Design*, ASME, 138(11), page 111404 (12 pages), 2016.

[J7] Hu, Z., and Mahadevan, S., "Global sensitivity analysis-enhanced surrogate (GSAS) modeling for reliability analysis", *Structural and Multidisciplinary Optimization*, 53 (3), 501-521, 2016.

[J8] Hu, Z., Nannapaneni, S., and Mahadevan, S., "Efficient Kriging Surrogate Modeling Approach for System Reliability Analysis", *Artificial Intelligence for Engineering Design*, *Analysis and Manufacturing*, Vol. 31, No. 2, pp. 143-160, 2017.

[**J9**] Hu, Z., Ao, D., and Mahadevan, S., "Calibration experimental design considering field response and model uncertainty," Computer Methods in Applied Mechanics and Engineering, Vol. 38, No. 5, pp. 92-119, 2017.

[J10] Hu, Z., and Mahadevan, S., "Time-Dependent Reliability Analysis Using a Vine-ARMA Load Model," ASCE-ASME *Journal of Risk and Uncertainty in Engineering Systems*, Vol. 3, No. 1, pp. 011007 (12 pages), 2017.

[J11] Hu, Z., and Mahadevan, S., "Adaptive Surrogate Modeling for Time-Dependent Multidisciplinary Reliability Analysis," *Journal of Mechanical Design*, ASME, 2018, 140(2), 021401.

[J12] Hu, Z., and Mahadevan, S., "A surrogate modeling approach for reliability analysis of a multidisciplinary system with spatio-temporal output," *Structural and Multidisciplinary Optimization*, Vol. 56, No. 3, pp. 553-569, 2017.

[**J13**] Hu, Z., Mahadevan, S., and Ao, D., "Uncertainty aggregation and reduction in structure– material performance prediction," *Computational Mechanics*, Vol. 61, Nos. 1-2, pp. 237-257, 2018.

Conference Proceedings Publications

[C1] Zhen Hu, Sankaran Mahadevan, and Xiaoping Du, 2015, "Uncertainty Quantification in Time-Dependent Reliability Analysis method " The ASME 2015 International Design Engineering Technical Conferences (IDETC) and Computers and Information in Engineering Conference (CIE), August 2-7, 2015 in Boston, MA.

[C2] Hu, Z., and Mahadevan, S., "Sensitivity Analysis-Based Surrogate Modeling of Limit States," Proceedings, AIAA 2016 SciTech Conference, San Diego, CA, 2016.

[C3] Devathi, H., Hu, Z., and Mahadevan, S, 2016, "Modeling Epistemic Uncertainty in the Representation of Spatial and Temporal Variability in Reliability Analysis," Proceedings, AIAA 2016 SciTech Conference, San Diego, CA, 2016.

[C4] Hu, Z., and Mahadevan, S., "Resilience Assessment Based on Time-Dependent System Reliability Analysis," Proceedings, ASME 2016 International Design Engineering Technical Conferences (IDETC) and Computers and Information in Engineering Conference (CIE), Charlotte, NC, 2016.

[C5] Hu, Z., and Mahadevan, S, "Time-Dependent Reliability Analysis Using a New Multivariate Stochastic Load Model," Proceedings, ASME 2016 International Design Engineering Technical Conferences (IDETC) and Computers and Information in Engineering Conference (CIE), Charlotte, NC, 2016.

[C6] Nannapaneni, S., Hu, Z., and Mahadevan, S, "Uncertainty Quantification in Metamodel-Based Reliability Prediction," Proceedings, ASME 2016 International Design Engineering Technical Conferences (IDETC) and Computers and Information in Engineering Conference (CIE), Charlotte, NC, 2016.

[C7] Hu, Z., and Mahadevan, S., "Reliability Analysis Considering Tail Dependence over Space and Time," Proceedings, AIAA 2017 SciTech Conference, Grapevine, Texas, 2017.

[C8] Nannapaneni, S., Hu, Z., and Mahadevan, S, 2017, "Uncertainty Aggregation in Reliability Analysis," Proceedings, AIAA 2017 SciTech Conference, Grapevine, Texas, 2017.

[C9] Hu, Z., and Mahadevan, S., "Global Sensitivity Analysis using Efficient Distribution Surrogates," Proceedings, AIAA 2019 SciTech Conference, San Diego, California, 2019.
[C10] Subramanian, A., and Mahadevan, S., "Model Error Propagation in Coupled Multi-Physics Systems," Proceedings, AIAA 2019 SciTech Conference, San Diego, California, 2019.