Final Technical Report

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I. Project Overview and Objectives

The overall goal of this project was to investigate a systematic, rigorous and affordable computational approach to ***estimate the reliability of structural systems*** subjected to combined and extreme environments, in the presence of multiple sources of ***epistemic uncertainty***, namely, data and model uncertainties, in addition to aleatory uncertainty (natural variability). The following objectives were pursued in order to achieve this goal:

1. Investigate a functional mapping approach to effectively include data and model uncertainties in reliability estimation with respect to individual damage mechanisms.
2. Investigate the combination of functional mapping and Bayesian networks to estimate system-level reliability, considering multiple damage mechanisms.
3. Expand the functional mapping approach to include epistemic uncertainty in the description of variability over space and time.
4. Expand the functional mapping approach to include heterogeneous information through a Bayesian network-based integration methodology, and to quantify the relative contributions of aleatory and epistemic uncertainty sources to the reliability assessment.

The methods developed and investigated through the four objectives were assessed using several illustrative problems of gradually increasing complexity. In Year 1, we investigated the reliability analysis of a curved beam under various epistemic uncertainty sources, and under spatial and temporal variations of loads and properties. In subsequent years, we investigated the reliability analysis of a hypersonic vehicle panel, briefly described below.

Hypersonic vehicle panel: A rigid, curved panel representing a deformed or post-buckled hypersonic aircraft panel is shown in Figure 1. This is a quasi-static, partial version of a 4-discipline coupled aerothermoelastic problem (aerodynamics, aero-heating, heat transfer, and structural deformation), with the structural analysis removed, to avoid consideration of a fully coupled problem. The output quantities of interest are (1) temperature distribution in the panel and (2) instability of the panel.

The reliability analysis objective is to compute the probability of the output temperature T_{str} exceeding a threshold value at single or multiple locations and the probability that the time to instability is less than a required time interval. Epistemic uncertainty in the random field modeling of spatial variability in the input pressure and temperature needs to be

Figure 1. Hypersonic vehicle panel

considered. The problem can be solved at different levels of complexity, starting from deterministic, uniform pressure and temperature, to different variations of random field representation. The output temperature distribution is computed through a finite difference solution of a differential equation. This example leveraged ongoing in-house research at AFRL, where the focus was on developing Bayesian calibration and validation techniques for uncertainty quantification of a four-discipline coupled analysis of a hypersonic vehicle panel.
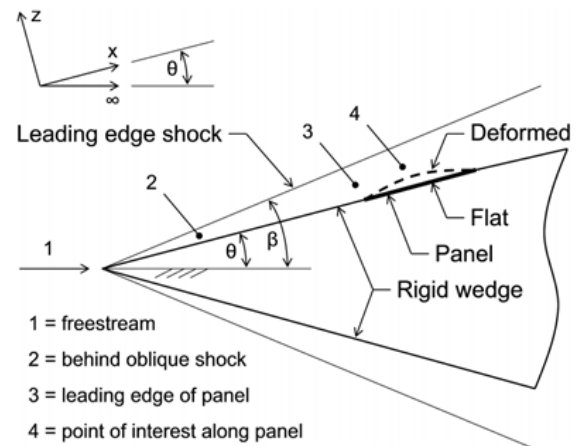
3

II. Research Accomplishments

A few of the technical accomplishments are highlighted in the subsections below.

- Including epistemic uncertainty in reliability analysis
- Efficient surrogate modeling for reliability analysis with temporal variability
- Sensitivity analysis of epistemic uncertainty
- Reducing epistemic uncertainty in reliability analysis with multiple limit state functions
- Adaptive surrogate modeling in multi-disciplinary reliability analysis
- Reliability analysis of hypersonic vehicle panel under epistemic uncertainty
- Model form error estimation and extrapolation to untested configuration

A. Including Epistemic Uncertainty in Reliability Analysis

In this accomplishment, the representation of various epistemic uncertainty using functional mapping and likelihood-based approaches was studied.

(1) Functional Mapping Approach

The traditional method for handling epistemic uncertainty is to implement a double-loop procedure, where realizations of aleatory uncertainty depend on the realizations of epistemic uncertainty. The double loop procedure can be denoted as "stochastic mapping" (as shown in Fig. 2a), i.e., for a specific value of epistemic uncertainty, we get a distribution of the random variable. In other words, a single value of the epistemic uncertainty leads to a random variable or uncertain quantity follows certain distribution, but not a single value. This is what leads to expensive nesting in uncertainty quantification computation, since two loops of sampling are required, an outer loop for the distribution parameters and an inner loop for the random variable.
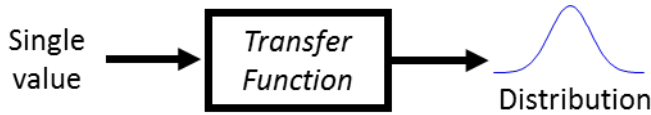

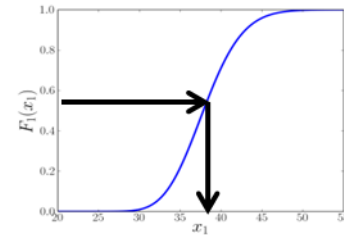**Figure 2a Stochastic Mapping**   **Figure 2b Inverse CDF**

Functional mapping can overcome this challenge by creating a one-to-one relationship between specific realizations of epistemic parameters and corresponding specific realizations of random variables. Note that for a given value of epistemic uncertainty, a unique value of random variable is obtained corresponding to a CDF value. This is the basic sampling approach in the Monte Carlo method, known as the "inverse CDF" approach (as indicated in Fig. 2b). We can write this relationship for a normal random variable X as $X = F_X^{-1}(u \mid \mu_X, \sigma_X)$, where u is the CDF value. Note that u is a realization of the uniform random variable U, ranging from 0 to 1. Thus we can write the functional mapping between X and (μ_X , σ_X in the form $X = h(U, \mu_X, \sigma_X)$. More generally, we can write $X = h(U, \boldsymbol{p})$ which defines a one-to-one functional mapping between the distribution parameters $\boldsymbol{p}$ and the random variable X. This means that, with the help of an auxiliary uniform random variable U, a sample realization of a random variable X can be related to the corresponding sample realizations of distribution parameters $\boldsymbol{p}$ by

4