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A high order truly multi-dimensional semi-Lagrangian approach for Vlasov simulations

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14. ABSTRACT
Accomplishments/New Findings. The PI, together with her group members,have been involved in the following research activities in the development, analysis and applications of efficient, robust and high order numerical approaches for kinetic and fluid simulations. In particular, the following topics are being pursued.
1. Development of highly accurate and e cient numerical methods for the Vlasov equation, see our publications [6, 7, 9, 10, 12, 13, 14, 17, 18].
2. Error analysis of integral deferred correction method for solving stiff problems,such as singular perturbation problems. Specific schemes we consider are implicit Runge-Kutta (RK) methods for stiff problems and implicit-explicit RK methods for temporal multi-scale problems, see our publications [1, 16].
3. Development and analysis of high order asymptotic preserving schemes for kinetic equations in the hyperbolic limit, see our publications [11].
4. Development and analysis of high order maximum principle preserving and positivity preserving methods for convectiondiffusion equations and for the Euler system, see our publications [2, 3].
5. Development of high order DG scheme for hyperbolic net and network problems,see our publications [4, 8].

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Final Report:
A High Order Truly Multi-dimensional Semi-Lagrangian Approach for Vlasov Simulations

FA9550-16-1-0179

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Status/Progress of the Project FA9550-16-1-0179 (07/15/2016-07/14/2018)

Objectives. This is a two-year project from Dr. Jingmei Qiu. We propose to develop a high order truly multi-dimensional semi-Lagrangian approach for Vlasov simulations. The project was planned as a three-year project. However, the project will be terminated in July 14th, 2018 (one year earlier than planned), because of the PI (Qiu)'s move from University of Houston to University of Delaware.

Status of effort. In year one of the project, the PI has made her effort in developing robust, efficient and high order *truly multi-dimensional* semi-Lagrangian discontinuous Galerkin methods for the Vlasov equation with her group members. In particular, we have strong and promising numerical evidence showing that the proposed semi-Lagrangian discontinuous Galerkin method are highly accurate and super efficient. In particular, with comparable performance, the proposed scheme only takes about 20% CPU time as the classical Runge-Kutta discontinuous Galerkin method under the same setting. In year two of the project, the PI and her group members further developed an adaptive version of the semi-Lagrangian discontinuous Galerkin method in time stepping sizes and apply it to the nonlinear Vlasov system. Further efficiency gain is computationally observed. The algorithm with automatic mesh adaptivity is currently under development. Successful algorithm design will have great potential to plasma simulations that are crucial to many application problems that are of air force interest.

Accomplishments/New Findings. The PI, together with her group members, have been involved in the following research activities in the development, analysis and applications of efficient, robust and high order numerical approaches for kinetic and fluid simulations. In particular, the following topics are being pursued.

1. Development of highly accurate and efficient numerical methods for the Vlasov equation, see our publications [6, 7, 9, 10, 12, 13, 14, 17, 18].

2. Error analysis of integral deferred correction method for solving stiff problems, such as singular perturbation problems. Specific schemes we consider are im-

plicit Runge-Kutta (RK) methods for stiff problems and implicit-explicit RK methods for temporal multi-scale problems, see our publications [1, 16].

3. Development and analysis of high order asymptotic preserving schemes for kinetic equations in the hyperbolic limit, see our publications [11].
4. Development and analysis of high order maximum principle preserving and positivity preserving methods for convection-diffusion equations and for the Euler system, see our publications [2, 3].
5. Development of high order DG scheme for hyperbolic net and network problems, see our publications [4, 8].

Personnel Supported. The PI is supported by this grant via one month summer salary per year. Besides, one postdoc associate (Dr. Xiaofeng Cai) is supported.

Publications. There are seventeen papers published/accepted in top journals in the field, since the project has been funded in 2016. In additional, one more paper is being submitted.

1. Error Estimate of Integral Deferred Correction Implicit Runge-Kutta method for Stiff Problems, S. Boscarino, J.-M. Qiu, Mathematical Modelling and Numerical Analysis, v50 (2016), Pages 1137-1166.
2. Parametrized Positivity Preserving Flux Limiters for the High Order Finite Difference WENO Scheme Solving Compressible Euler Equations, T. Xiong, J.-M. Qiu, Z. Xu, Journal of Scientific Computing, v67 (2016), Pages 1066-1088.
3. High Order Maximum Principle Preserving Finite Volume Method for Convection Dominated Problems, P. Yang, T. Xiong, J.-M. Qiu and Z. Xu, Journal of Scientific Computing, v67 (2016), Pages 795-820.
4. Notes on RKDG methods for shallow-water equations in canal networks, M. Briani, B. Piccoli, J.-M. Qiu, Journal of Scientific Computing, v68 (2016), Pages 1101-1123.
5. An Adaptive WENO Collocation Method for Differential Equations with Random Coefficients, W. Guo, G. Lin, A. Christlieb and J.-M. Qiu, MDPI, Special Issue "New Trends in Applications of Orthogonal Polynomials and Special Functions", v4 (2016), Pages 29.
6. A conservative semi-Lagrangian HWENO method for the Vlasov equation, X. Cai, J. Qiu and J.-M. Qiu, Journal of Computational Physics, v323 (2016), Pages 95-114.
7. High Order Mass Conservative Semi-Lagrangian Methods for Transport Problems, J.-M. Qiu, Handbook of Numerical Methods for Hyperbolic Problems: Part A, Chapter 16.

8. Numerical methods for hyperbolic nets and networks, S. Canic, M.L. Delle Monache, B. Piccoli, J.-M. Qiu and J. Tambaca, Handbook of Numerical Methods for Hyperbolic Problems.

9. An h-adaptive RKDG method for the Vlasov-Poisson system, H. Zhu, J. Qiu and J.-M. Qiu, Journal of Scientific Computing, v69 (2016), Pages 1346-1365.

10. A High Order Multi-Dimensional Characteristic Tracing Strategy for the Vlasov-Poisson System, J.-M. Qiu and G. Russo, Journal of Scientific Computing, v71 (2017), Pages 414-434.

11. A Hierarchical Uniformly High Order DG-IMEX Scheme for the 1D BGK Equation, T. Xiong and J.-M. Qiu, Journal of Computational Physics, v336 (2017), Pages 164-191.

12. An h-adaptive RKDG method for the two-dimensional incompressible Euler equations and the guiding center Vlasov model, H. Zhu, J. Qiu and J.-M. Qiu, Journal of Scientific Computing, v73 (2017), Pages 1316-1337.

13. A high order conservative semi-Lagrangian discontinuous Galerkin method for two-dimensional transport simulations, X. Cai, W. Guo and J.-M. Qiu, Journal of Scientific Computing, v73 (2017), Pages 514-542.

14. A High Order Semi-Lagrangian Discontinuous Galerkin Method for Vlasov-Poisson Simulations Without Operator Splitting, X. Cai, W. Guo and J.-M. Qiu, Journal of Computational Physics, v354 (2018), Pages 529-551.

15. Finite volume HWENO schemes for nonconvex conservation laws, X. Cai, J. Qiu and J.-M. Qiu, Journal of Scientific Computing, v75 (2018), Pages 65-82.

16. Implicit-Explicit Integral Deferred Correction Methods for Stiff Problems and Applications to Partial Differential Equations, B. Sebastiano, J.-M. Qiu and G. Russo, SIAM Journal of Scientific Computing, v40 (2018), Pages A787–A816.

17. High Order Multi-dimensional Characteristics Tracing for the Incompressible Euler Equation and the Guiding-center Vlasov Equation, T. Xiong, G. Russo and J.-M. Qiu, Journal of Scientific Computing, https://doi.org/10.1007/s10915-018-0705-y.

18. A high order semi-Lagrangian discontinuous Galerkin method for the two-dimensional incompressible Euler equations and the guiding center Vlasov model without operator splitting, X. Cai, W. Guo and J.-M. Qiu, Journal of Computational Physics, submitted.

Participation/presentations at meetings, conferences, seminars, etc. The PI has actively participated in national and international conferences and meetings, especially those related to the topic of the proposed project on "kinetic and plasma simulations". These include the following.

1. Seminar, School of Mechatronical Engineering, Beijing Institute of Technology, Beijing, China, July 2nd, 2018.
2. Seminar, School of Mathematical Sciences, Xiamen University, Xiamen, China, June 26th, 2018.
3. Seminar, School of Mathematical Sciences, Minnan Normal University, Zhangzhou, China, June 15th, 2018.
4. Seminar, Department of Mechanics and Aerospace Engineering, Southern University of Science and Technology, Shenzhen, China, June 13th, 2018.
5. Seminar, School of Mathematical Sciences, University of Science and Technology of China, Hefei, China, June 4th, 2018.
6. The Fourth International Workshop on the Development and Application of High-Order Numerical Methods, Nanjing, China, June 2nd, 2018.
7. DelMar Numerics Day 2018, University of Delaware, Newark, DE, May 5th, 2018.
8. Colloquium, Department of Mathematical Sciences, Rensselaer Polytechnic Institute (RPI), NY, April 30th, 2018.
9. Numerical Aspects of Hyperbolic Balance Laws and Related Problems, University of Ferrara, Italy, April 17th, 2018.
10. Scientific Computing seminar, Temple University, Philadelphia, Jan. 31st, 2018.
11. AWM and SIAM New Faculty Speaker Series, University of Delaware, Dec. 11th, 2017.
12. The workshop on Kinetic Theory and Fluid Mechanics: theoretical and computational aspects, University of Toulouse, Nov. 8th, 2017.
13. AMS Sectional meeting, Denton, TX, Sep. 9th, 2017.
14. Seminar, Department of Physics and Astronomy, University of Delaware, Newark, DE, August 25th, 2017.
15. Seminar, Department of Physics and Astronomy, University of Delaware, Newark, DE, August 11th, 2017.
16. Program review, Air Force Office of Scientific Research Computational Mathematics, Arlington, VA, August 16th, 2017.
17. Colloquium, University of Delaware, Newark, DE, May 18th, 2017.
18. Colloquium, Rutgers University-Camden, NJ, April 24th, 2017.

19. Scientific Computing seminar, University of Delaware, Newark, DE, January 6th, 2017.
20. Poster, Frontiers in Applied and Computational Mathematics, Brown University, January 4th, 2017
21. Program review, Air Force Office of Scientific Research Computational Mathematics, Arlington, VA, August 8th, 2016.