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Structural Approach to Distributed Optimization

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14. ABSTRACT
A central component of distributed optimization algorithm design is the case-by-case design of algorithms that solve distributed optimization problems by crafting algorithms that satisfy certain conditions. This research aimed to address this shortcoming. In this research several milestones have been achieved:
(a) We showed that distributed optimization algorithms can all be written as a mixture of average tracking dynamics and gradient feedback,
(b) We showed that we can relax the fundamental assumption of convexity in several of these works,
(c) As for the average tracking for the distributed optimizaiton, we developed tools and techniques to study the averaging dynamics, these tools include infinite flow property, P* chains, and balanced networks,
(d) We study a very specific application of distributed optimization and optimization problems to power networks, and we show that relaxation of those problems lead to convex problems with guaranteed performance.

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Accomplishments During Entire Proposal: (i) A central component of distributed optimization algorithm design is the case-by-case design of algorithms that solve distributed optimization problems by crafting algorithms that satisfy certain conditions. The following papers outline the general set-up where a general design protocol can be utilized to provide a general structural approach to solve such problems:

- Touri, Behrouz, and Bahman Gharesifard. "Continuous-time distributed convex optimization on time-varying directed networks." *Decision and Control (CDC), 2015 IEEE 54th Annual Conference on. IEEE, 2015.*

The details of the general approach is on the final phases of preparation and will be submitted for publication in SIAM Journal of Control and Optimization.

(ii) New general algorithms that solve distributed optimization problems. Using the methodological thinking of the proposed general approach, we have proposed new algorithms that solve distributed optimization problems. The following papers outline the proposed algorithm:

- Touri, Behrouz, and Bahman Gharesifard. "Saddle-point dynamics for distributed convex optimization on general directed graphs." *Decision and Control (CDC), 2016 IEEE 55th Conference on. IEEE, 2016.*
- Touri, Behrouz, and Bahman Gharesifard. "A Modified Saddle-point Dynamics for Distributed Convex Optimization on General Directed Graphs." To appear in *IEEE Transactions on Automatic Control.*

(iii) The common assumptions in many of the current distributed optimization algorithms is that the objective functions of the agents are all convex. This assumption is relaxed in the following publications:

- Tatarenko, Tatiana, and Behrouz Touri. "On local analysis of distributed optimization." *American Control Conference (ACC), 2016. IEEE, 2016.*

- Tatarenko, Tatiana, and Behrouz Touri, "Non-Convex Distributed Optimization." *IEEE Transactions on Automatic Control, vol. 62, no. 8, pp. 3744-3757, Aug. 2017.*

(iv) The subcontractor at the University of California San Diego (Behrouz Touri) have worked on extending the results on distributed optimization to design of distributed optimization solvers over random networks. We speculated that a variation of the push-sum based distributed optimization problem can tackle this problem. However, to show this, we needed to understand the mechanism on how the push-sum algorithm works over random networks (without distributed optimization component to it). One of the major accomplishments over the review period was to show that the push-sum algorithm achieves average consensus over random networks (under general conditions on the stochastic variation of the network). We used several results in products of random stochastic matrices, martingale theory, and concentration inequalities to show this. We are now investigating the use of this result on formulating a robust distributed optimization solver that can be used over random networks and has provable performance guarantees. This work is reported in the following publication:

- Rezaeinia P, Gharesifard B, Linder T, Touri B. "Push-sum on random graphs: almost sure convergence and convergence rate." IEEE Transactions on Automatic Control. 2019 Jul 17.
- Rezaienia P, Gharesifard B, Linder T, Touri B. Convergence Rate of Push-Sum Algorithms on Random Graphs. In 2018 IEEE Conference on Decision and Control (CDC) 2018 Dec 17 (pp. 4218-4223). IEEE.

Recently, we were be able to use these results to show that the push-sum based distributed optimization algorithm works over random networks and it is reported in the conference publication:

- Rezaeinia, Pouya, Gharesifard, Bahman, and Touri, Behrouz, "Distributed Optimization over Random Graphs." 7th IFAC Workshop on Distributed Estimation and Control in Networked Systems, August 27-28, 2018, Groningen, Netherlands.

(v) We studied the application of optimization and distributed optimization for home energy management systems as well as power flow optimization over networks. In particular, we addressed the fundamental flaw in many of the recent studies where almost all the current algorithms to optimize the scheduling of energy resources incorporating storage units are either computationally intractable or they result in simultaneous charging and discharging of storage (battery) units which is physically impossible with the current battery technology. As a first step to find distributed solvers for such a problem, we addressed this fundamental issue and formulated a computationally tractable optimization problem that ensures non-simultaneous charging and discharging. This work is reported in:

- Garifi, K., Baker, K. and Touri, B., “Convex Relaxation of Grid-Connected Energy Storage System Models with Complementarity Constraints in DC OPF.” Submitted for publication at IEEE Transactions on Smart Grid.
- Garifi, K., Baker, K., Christensen, D. and Touri, B., "Stochastic Model Predictive Control for Demand Response in a Home Energy Management System," IEEE Power and Energy Society General Meeting, Portland, OR, 2018.