

USNO/AA Technical Note 2014-05  
Preliminary Analysis of the Mount Wilson Sunset Observations

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## Abstract

A total of 250 observations of the times of sunset were made from Mount Wilson, California from 1987 Mar. 16 through 1991 Nov. 10. Along with the time of observation, the current atmospheric temperature, dew point, and pressure at the observer were recorded. An initial reduction to a subset of that data was performed by J. Bangert in 2007. The weather data are used to determine the horizontal refraction, computed using the Hohenkerk & Sinclair (1985) algorithm and compared to the timing data. The observed sunsets all occurred over the Pacific Ocean giving a uniform, level, depressed horizon. This note examines the resulting (O – C) values for the time of sunset. They show a significant  $41 \pm 58$  s delay between the observed and computed time of sunset. These values also show a significant positive skew to the sunset times and evidence of extreme refraction events. The systematic differences found here are attributed not to a problem with the Hohenkerk & Sinclair algorithm, but to possible boundary layer lapse rates differing from the standard rate used in the reduction. The boundary layer lapse rate difference is thought to arise from the contact of the relatively cold water of the Pacific Ocean with the overlying atmosphere. Boundary layer lapse rates differing from that of the rest of the troposphere are not included in the Hohenkerk & Sinclair algorithm. Finally, a set of suggested future actions to improve the refraction algorithm is made.

## 1. Introduction

The effect of the Earth’s atmosphere on electromagnetic radiation is a concern in most forms of navigation. Aside from inertial navigation, position is determined from the observation of some framework of distant reference sources. These observations are made using some form of radiation. For observations on or near the Earth’s surface this radiation necessarily has to pass through the Earth’s atmosphere, which affects the signal received by the observer. The effect of the atmosphere is usually divided into two parts:

1. time-delay, an increase in the length of time it takes for the radiation to travel from the reference object to the observer, and
2. refraction, a displacement between the geometric and apparent position of the reference object.

These two aspects of the effect of the atmosphere on radiation are complementary. But, they are usually discussed separately, because most methods of navigation only require knowledge of one or the other of these phenomena. The aspect of refraction is discussed in this technical note, but it should be understood that its physics applies as well to time-delay.

The refraction angle, the amount of displacement between the apparent and geometric places caused by refraction, is a function of the bulk physical properties of the medium along the path the photon traverses, the angle of incidence, and the wavelength of the photon. The atmosphere’s properties are a function of its pressure, temperature, and composition<sup>1</sup>.

The larger the angle of incidence, the larger the refraction angle. At the zenith the angle of incidence is zero, an absolute minimum. Thus, the refraction angle is also at a minimum. Snell’s law (e.g. Jenkins & White, 1976, pg. 12) requires that this minimum refraction angle also be zero. This property makes the zenith distance, the angular distance between a point on the sky and the zenith, a natural parameter for determining the effect of refraction.

Near the horizon, both the zenith distance and the rate of change with zenith distance of atmospheric properties along a photon’s path are large. So, the determination of the refraction angle becomes problematic. At the same time, observations at large zenith distances can be important. For example, these observations may be used to establish a horizon as a reference, they are required to establish the position of landmarks when sailing in sight of land, and the direct light from bright objects such as the Sun and the Moon is still visible after they have set geometrically, which has a profound

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<sup>1</sup>The amounts of water vapor and carbon dioxide in addition to nitrogen and oxygen.

effect on illuminance levels. Better determination of the refraction angle at large zenith angles also leads to a reduced uncertainty in the refraction angle at smaller zenith angles, allowing a more accurate determination of the position of all reference sources used for navigation.

The purpose of this technical note is to document a preliminary analysis of sunsets observed from the Mount Wilson Observatory, California with the purpose of extracting refraction data from the timing of this phenomenon. The immediate goal is to determine the quality of the data. A secondary goal is to make an assessment of what else may be needed in terms of analysis, future observations, and software to reduce the uncertainty arising from atmospheric refraction in Astronomical Applications Department products.

## 2. Description of the Data

The Mount Wilson sunset observations consist of 250 observations made by L. Rarogiewicz from 1987 Mar. 16 through 1991 Nov. 10 (Rarogiewicz, 1994). They were made from two nearby spots on Mount Wilson. These locations are tabulated in Table 1. The estimated uncertainty in the position is  $\pm 1''$  each in latitude and longitude and  $\pm 1$  m in height. The notes accompanying the observations do not indicate whether the latitude is geodetic or geocentric. At the observed latitude, the geocentric latitude is only  $1''$  smaller than the geodetic, so the difference is insignificant. The observer states that the only corrections he made were to convert watch time to WWV, so a conservative estimate for the uncertainty in the time of observation is 2 s.

All 250 observations include the time of sunset<sup>2</sup>. The time of the contact of the heliocenter with the horizon is included for 217 observations, and 109 observations include the time of the contact of the sun’s lower limb with the horizon. A total of 246 observations were made using binoculars, and 4 were made with the unaided eye. The observer included the local temperature ( $^{\circ}\text{F}$ ) and pressure (in Hg) for all observations, and dew point ( $^{\circ}\text{F}$ ) for 249 observations.

The weather data were taken within 5 min. of the time of observation from the National Weather Service (NWS) station at Mount Wilson. The first observing site is contiguous with the weather station and the second site is approximately 160 m away from the first site. Thus, the weather data should be accurate within the accuracy of the NWS instruments. This information is important since the rate of change of the refraction angle with the the temperature at a zenith distance of  $90^{\circ}$  is  $-14'' \text{ K}^{-1}$ . However temperature can vary significantly over

<sup>2</sup>Apparent contact of the Sun’s upper limb with the horizon.

relatively short distances, and absolute calibration of temperature is difficult<sup>3</sup>.

### 2.1. Categories

The observer categorized the observations into five quality categories: *A* – Best, *B* – Good, *C* – Fair, *Q* – Questionable, and *U* – Unknown quality. These categories are based on the observer’s subjective estimate of the conditions under which the observations were made and how well the observations were timed. Rarogiewicz (1994) states that the *Q* observations “may be (but not necessarily) questionable” and the *U* observations “might be super-good or lousy, but have no way to tell which!”

The observer also categorized the observations as to whether or not the *Novaya Zemlya* effect was observed. The *Novaya Zemlya* effect is a mirage caused by a strong thermal inversion. The photons from the source are trapped in a light duct between two “surfaces” of total internal reflection. This effect allows light from the source to be seen at a much greater zenith angle than normal. The *Novaya Zemlya* effect is marked by a “square shape” to the Sun and considerably reduced brightness from atmospheric extinction. The observer’s categories are: *0* – No effect observed or noted, *P* – Possibly present, and *N* – *Novaya Zemlya* effect explicitly noted down in observation notes. The difference between the *Novaya Zemlya* effect and a large refraction angle is discussed in Appendix A.

Table 2 summarizes the number of observation in each category.

### 2.2. The Horizon

The observer used the Pacific Ocean for the horizon. Figure 1 shows the California coast in the vicinity of Mount Wilson. As seen at Mount Wilson the geometry of the California coast is such that the sun’s azimuth needs to be less than approximately  $276^{\circ}$  to set over the ocean. Thus, observations can only be made only during the fall and winter seasons, from approximately 7 Sept. through 5 Apr. Actual observations cover the period from 19 Sept. through 24 Mar.

At the observer’s height, a mean sea level horizon is  $\sim 150$  km distant, not including refraction. The horizon is shown in Fig. 1 as a blue arc Any significant deviation from a flat, mean sea level horizon is unlikely other than the Channel Islands. Their highest peak may stick up as much as  $0^{\circ}3'$  above the horizon as seen from Mount Wilson. They are located at azimuths between  $258^{\circ}$  and  $264^{\circ}$ . They are also situated primarily beyond the horizon with only their peaks visible to the observer. Thus, they

<sup>3</sup>Most thermometers are calibrated to an absolute accuracy of  $\sim 1\text{--}2$  K.

Table 1: Locations of Mount Wilson sunset observations.

No. Obs.	Latitude	Longitude	Height (m)
189	N 34° 13' 34"6	W 118° 03' 56"2	1739
61	N 34° 13' 37"0	W 118° 04' 07"1	1725

Table 2: Distribution of observations by quality and existence of the *Novaya Zemlya* effect for Mount Wilson sunset observations.

<i>Novaya</i> <sup>a</sup> <i>Zemlya</i>	Observation Quality <sup>a</sup>					Total
	<i>A</i>	<i>B</i>	<i>C</i>	<i>Q</i>	<i>U</i>	
<i>0</i>	40	33	15	66	24	178
<i>P</i>	17	15	7	12	4	55
<i>N</i>	9	3	3	2	0	17
Total	66	51	25	80	28	250

<sup>a</sup> See § 2.1 for explanation.

will rarely pose any significant problem in determining the time of sunset. The Sun sets in this range of azimuths from approximately 6 to 21 October and approximately 21 February to 6 March. Ten observations were made during one of these periods. All ten observations were made on dates where only the north-western edge of Santa Cruz Island might have affected the observed time of sunset.

Up to about two kilometers above the Earth's surface, the atmospheric temperature and lapse rate<sup>4</sup> may be affected by the character of that surface. For the observer at Mount Wilson, the distance the light travels within 2 km of the surface is approximately 310 km. The path of a photon from the setting Sun is primarily over the Pacific Ocean. The final section of the light path travels overland for approximately 127 km near the equinox to approximately 49 km near the solstice. Except near the solstice, the overland distance a photon travels to Mount Wilson is 75 km or more. Thus, two, possibly quite different, temperature environments may need to be considered to determine the accumulated refraction along the light path.

### 3. Preliminary Investigation

J. Bangert (2010, private communication) made a reduction of the Mount Wilson observations. He determined the expected time of sunset using an early prototype of the SLAC 2.0 code. At that time the algorithm used was the one used in Hohenkerk & Sinclair (1985)<sup>5</sup>. In this reduction,

<sup>4</sup>The change in temperature with height above the Earth's surface.

<sup>5</sup>Since that time, the SLAC refraction model has been updated to use a revised version (Hohenkerk & Sinclair, 2008) along with revised values for the numerical constants, and an Earth model that takes into account its flattening.

Bangert assumed the lapse rate was the NOAA et al. (1976) *U.S. Standard Atmosphere, 1976*, henceforth USSA76, value of  $6.5^\circ\text{C km}^{-1}$ . The solar azimuths at sunset were not determined, so this analysis cannot determine if the Channel Islands might affect the time of sunset beyond the crude estimate of the dates when the sunset should occur near this feature. The data are in the form of a file of  $(O - C)$  values in minutes. The code made no attempt to use either the contact of the lower limb or heliocenter with the horizon. The values in this file are analyzed here. Table 3 gives the statistics for the  $(O - C)$ s<sup>6</sup> broken down into each of the possible subcategories. Each of the individual categories have rather small numbers of observations and rather broad standard deviations, so combining them would be advantageous.

One item of interest is whether the observer's perceived quality of the observation has any significant effect on the timing of sunset. Only the  $0$ -type observations are numerous enough that they can be divided up by quality type. Figure 2a. shows the distribution of each quality type of the  $0$ -type data. The data are divided into bins 20 s wide. There is significant variation from bin-to-bin, which may indicate that the bins are too narrow to smoothly represent the data given the total number of observations. Increasing the width of the bins, however, would reduce the detail to the point that few inferences can be drawn. Instead, the uncertainty will be reduced by combining similar quality categories.

The most numerous types,  $A$  and  $Q$  observations, show a significant peak at somewhat greater than 0 s. The estimated most likely values are 10 s and 20 s, respectively. They also both have significantly fatter tails in the positive direction. Thus, these two groups will be combined. The average value of

<sup>6</sup>Bangert's values for the  $(O - C)$ s in minutes have been converted to seconds.



Figure 1: The geometry of the California coast near Mount Wilson. Taken from Google Maps.

the combined group is 40 s with a standard deviation of 61 s and median value of 31 s.

The less numerous  $B$ ,  $C$ , and  $U$  observations do not have a conspicuous peak, but, they all have long tails to positive values in the  $(O - C)$ s. The combination of these three groups has an average value of 42 s with a standard deviation of 55 s and median value of 43 s.

The two combined groups have similar values for both their mean value and standard deviation. Only the median is significantly different. The distributions of these two groups is shown in Fig. 2b. The two combined groups are surprisingly similar. There still appears to be significant bin-to-bin variation, which may be a sign that the statistical variation is too large to let the distribution be smoothly modeled by the existing number of observations. It may also explain the difference in the medians of the two combined groups. So, the combined groups will be treated as a single one here (see Fig. 3), but a larger set of similar observations may necessitate their being examined separately in the future.

Both of the two *Novaya Zemlya* categories have too few observations to make meaningful comparisons between the different quality categories. However, except for the four  $P$  observations that are in the  $U$  quality category, the mean observed sunset times of each quality category are all closer to the mean of their *Novaya Zemlya* category than they

are to the other categories and the mean values are all well separated as shown in Fig. 3.

The sunset time of the mean  $(O - C)$  for each of the *Novaya Zemlya* categories is approximately 1 to 1.5  $\sigma$  later than the previous one. Table 4 gives the statistical properties, including the median and skewness, for each of the *Novaya Zemlya* types and the full set of observations. For a normal distribution, the skewness is 0. A positive skewness indicates more observations at values greater than the mean value compared to a normal distribution. All three categories have a significant positive skewness, but the  $P$  and  $N$  categories are more skewed than the  $\theta$  observations. The kurtosis for the  $\theta$  category indicates that it is significantly more peaked than a normal distribution while both the  $P$  and  $N$  categories have broader peaks than a normal distribution.

The median values are all significantly smaller than the mean values of the distribution as expected from the positive skewness of the distribution of the observations. The mode of the  $\theta$ -type observations occurs at a yet smaller value of about 20 s. There are too few  $P$  and  $N$ -type observations to estimate their modes reliably, but inspection by eye indicates that they should be lower than their respective median values.

Table 3: Preliminary statistics for  $(O-C)$ s of sunset times by category for Mount Wilson sunset observations.

Category	No.	Mean (s)	$\sigma$ (s)	Maximum (s)	Minimum (s)
All	250	68	80	382	-140
<i>A0</i>	40	50	56	217	-17
<i>B0</i>	33	43	50	133	-50
<i>C0</i>	15	65	61	188	-24
<i>Q0</i>	66	35	62	180	-140
<i>U0</i>	24	28	56	135	-125
<i>AP</i>	17	100	64	233	10
<i>BP</i>	15	151	83	308	31
<i>CP</i>	7	126	79	253	37
<i>QP</i>	12	104	66	208	24
<i>UP</i>	4	42	94	112	-94
<i>AN</i>	9	227	96	382	101
<i>BN</i>	3	161	47	209	114
<i>CN</i>	3	197	29	230	172
<i>QN</i>	2	172	55	211	133
All <i>A</i>	66	87	88	382	-17
All <i>B</i>	51	82	80	308	-50
All <i>C</i>	25	98	77	253	-24
All <i>Q</i>	80	49	70	211	-140
All <i>U</i>	28	29	61	135	-125
All <i>0</i>	178	41	58	217	-140
All <i>P</i>	55	114	77	308	-94
All <i>N</i>	17	203	77	382	101

## 4. A Possible Explanation

The existence of a significant positively skewed tail in the time of sunset for all three *Novaya Zemlya* categories is an indication that the lapse rate within the surface boundary layer is usually lower than the average USSA76 rate used to predict the time of sunset. There may even be a thermal inversion, a layer in which the lapse rate is positive. The *Novaya Zemlya* effect is a form of mirage (see Appendix A) that occurs when a thermal inversion is particularly strong. It is of interest here because the atmospheric conditions that create this type of mirage also lead to significantly later times for sunset. A lower than average lapse rate is all that is required for a late sunset, but the *Novaya Zemlya* effect requires a strong thermal inversion. Are the observations in the *N* category truly examples of this effect? What evidence is there to determine whether or not these observations are actual occurrences of it?

There is evidence for a temperature inversion. Using the USSA76 value for the lapse rate, the difference between the estimated air temperature at

the ocean surface,  $T_{sa}$ , and the air temperature at the observer,  $T_0$ , should be about

$$\begin{aligned} T_{sa} - T_0 &= -(-1732 \text{ m}) \times (0.0065 \text{ K m}^{-1}) \\ &= 11.2 \text{ K} \end{aligned} \quad (1)$$

where 1732 m is the mean height of the two observation sites. Table 5 gives the mean surface water temperatures,  $T_{sw}$ , near Los Angeles during the fall and winter reported by Osborn (2013)<sup>7</sup>. Since the annual variation is small, it is assumed that the day-to-day variation from the mean monthly temperature is also small.

The estimated difference,

$$\Delta T = T_{sa} - T_{sw}, \quad (2)$$

between the value for the air temperature at the sea surface and the estimated ocean surface water temperature as functions of the phenomenon category and month are given in Table 6. Values without an estimated uncertainty arise because there was only a single event of that type during the month.

<sup>7</sup>These values are compiled from data from the National Oceanographic Data Center.

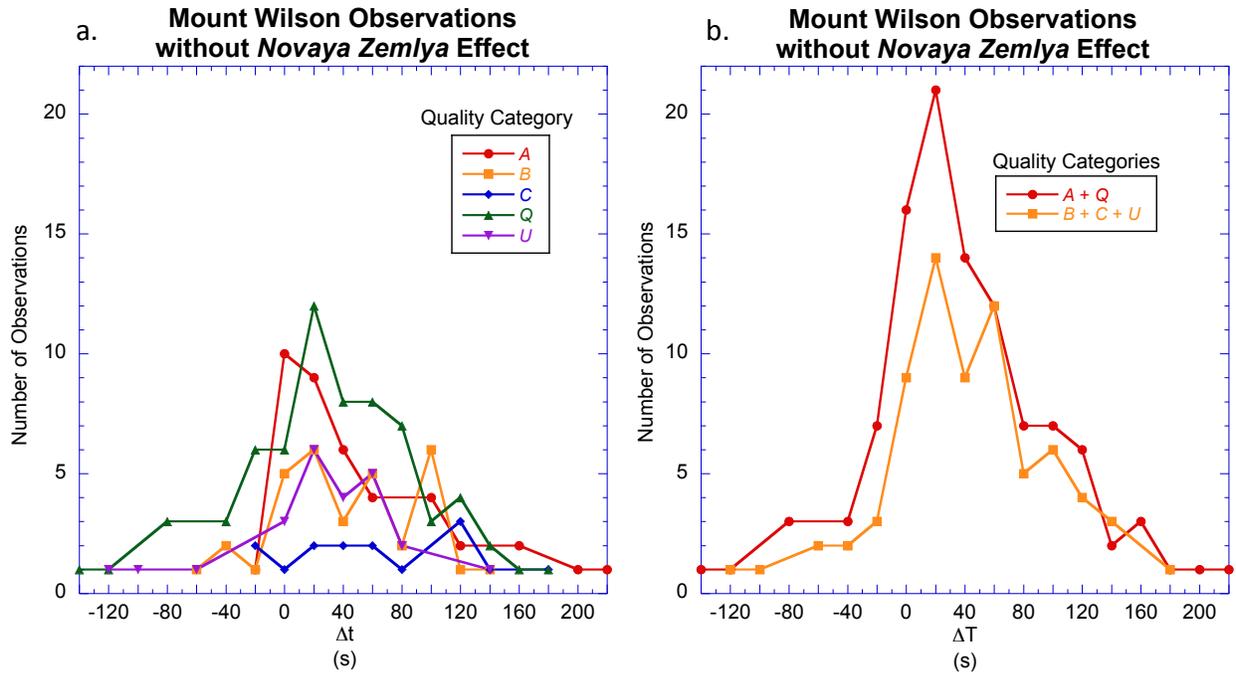


Figure 2: The distribution of  $(O - C)$ s of sunset observations without the *Novaya Zemlya* effect by quality estimate.

Table 4: The statistical properties of the  $(O - C)$ s of sunset times for the Mount Wilson sunset observations.

Group	Mean (s)	Median (s)	$\sigma$ (s)	Skewness
$\theta$	41	35	58	0.032
$P$	114	101	77	0.283
$N$	204	190	77	0.845
All	68	53	80	0.738

There were no  $N$ -type events observed during the month of February. There are four immediate observations that can be made:

1. The ocean surface water temperature is always significantly lower than the value of the estimated surface air temperature.
2. The ocean surface water temperatures are lower than the mean air temperature observed at Mount Wilson for the months of September and October.
3.  $\Delta T$  for the  $P$ -type phenomena is usually greater than for the  $\theta$ -type phenomena and, in turn,  $\Delta T$  for the  $N$ -type phenomena is usually greater than for the  $P$ -type phenomena<sup>8</sup>.
4. There is a positive correlation between the value of  $\Delta T$  and the mean seawater temperature.

Thus, the evidence supports the possibility that temperature inversions existed when the  $P$  and  $N$ -type observations were made. Furthermore, the

<sup>8</sup>Two of the three cases where this generalization does not hold are based on a single observation. The standard deviations are large enough that the chance of overlap is high. So the observed exceptions may be the result of small number statistics.

evidence supports the hypothesis that a stronger inversion existed when  $N$ -type observations were made.

The different phenomena are not distributed uniformly over time. The observation season was broken into four approximately equal time periods, and the number of each type of phenomenon in each period was determined. These values are given as percentages of the number of observations during each period to normalize for the unequal number of observations made. The results are tabulated in Table 7. The  $P$  and  $N$ -type observations are particularly concentrated in the first quarter when the ocean surface water temperature and  $\Delta T$  are greatest. The enhancement of the  $P$ -type observations in the first quarter compared to the last quarter is a factor of 3.9, and the enhancement of the  $N$ -type for the same periods is a nearly identical 4.0.

The working hypothesis to explain these correlations is: The true lapse rate is smaller than the USSA76 value of  $6.5^\circ\text{C km}^{-1}$ . A smaller lapse rate would result in a lower air temperature at the surface of the ocean. Hence, the atmosphere along the ray path would be denser. The results are a larger

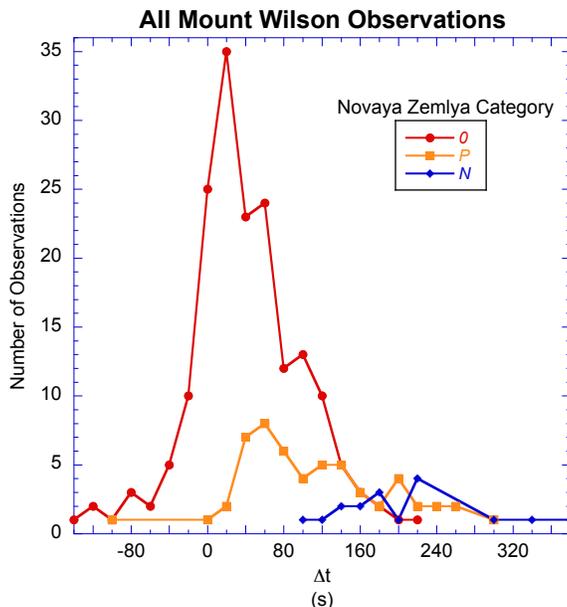


Figure 3: The distribution of  $(O - C)$ s of all sunset observations made at Mount Wilson.

Table 5: The mean water temperature at the ocean’s surface near Los Angeles compiled from data from the National Oceanographic Data Center.

Time Period	$T$ °C
Sept.	19
Oct.	19
Nov.	18
Dec.	16
Jan.	14
Feb.	14
Mar.	16
Annual	17

angle of refraction and later observed time of sunset. But the mean air temperature at the ocean surface would still be significantly higher than the ocean water surface temperature in Sept. and Oct. with a lapse rate of  $0 \text{ K m}^{-1}$ . A portion of the observations were made when there was a temperature inversion (negative lapse rate) in the boundary layer<sup>9</sup>. Such an inversion would also result in an increase in the refraction angle and later observed time of sunset. The systematic shift in the mean  $(O - C)$  time of sunset from 0 may be explained by an average lapse rate smaller than the USSA76 rate<sup>10</sup>. Temperature inversions may account for the long tail in the distribution.

<sup>9</sup>Possibly created by the air at the ocean surface being significantly cooled by its contact with the ocean.

<sup>10</sup>An alternative explanation, that has not been addressed, is: Was the correct value used for the estimate of the geometric dip as viewed from Mount Wilson? This possibility is one of a number of details that need to be addressed in a more thorough investigation.

Table 6: The mean difference between the estimated air temperature and mean sea surface temperature.

Month	Phenomenon Category		
	$\theta$ $\Delta T$ (°C)	$P$ $\Delta T$ (°C)	$N$ $\Delta T$ (°C)
Sept.	$13 \pm 4$	$15 \pm 3$	$17 \pm 2$
Oct.	$13 \pm 7$	$17 \pm 5$	19
Nov.	$5 \pm 4$	$9 \pm 5$	$10 \pm 3$
Dec.	$2 \pm 7$	$7 \pm 5$	$9 \pm 1$
Jan.	$6 \pm 4$	$5 \pm 4$	$9 \pm 2$
Feb.	$5 \pm 5$	4	—
Mar.	$7 \pm 6$	$12 \pm 2$	11

Are the  $N$ -type observations truly the result of the *Novaya Zemlya* effect? This effect is an extreme example of a temperature inversion requiring the formation of an optical duct in the atmosphere. Observations suggest that this effect is not uncommon at far northern latitudes (Lehn, 1979). In fact, Sampson (1993) estimates the occurrence of the *Novaya Zemlya* effect at about 10% during the winter in Edmonton, Canada, nearly the same proportion of the  $N$ -type observations made during the first and second quarters at Mount Wilson. But Mount Wilson is located in the low mid-latitudes and the mean temperature is much higher. The sunsets with the four largest  $(O - C)$ s were more than 300 s later than expected, which required a refraction angle more than  $1^\circ 25'$  greater than the expected value. The largest  $(O - C)$  was 382 s requiring an increase in the refraction angle of  $1^\circ 59'$ . The light path through the Earth’s atmosphere was an additional 177 km. This additional distance

Table 7: The distribution of observations by phenomenon category over different time periods.

Time Period	Number of Observations	% $\theta$	% P	% N
Sept. 19 – Nov. 4	62	51.6	38.7	9.7
Nov. 5 – Dec. 20	83	69.9	20.5	9.6
Dec. 1 – Feb. 5	64	81.3	15.6	3.1
Feb. 6 – Mar. 24	41	87.8	9.8	2.4
All	250	71.2	22.0	6.8

is greater than the distance from the observation point on Mount Wilson to the nominal point where the light ray would be tangent to the Earth’s surface refraction. These large refraction angles suggest that at least some of the  $N$ -type observations were actual examples of the *Novaya Zemlya* effect.

## 5. Results

The observer of sunset time at Mount Wilson made 250 observations over 1700 days. These observations were only made during the months of Sept. through Mar. to assure the Sun set over the distant, level horizon of the Pacific Ocean. The observer sorted these observations into five quality categories and whether or not he thought the *Novaya Zemlya* effect was present.

A preliminary analysis of these observations finds:

- Aside from stronger peaks for the  $A$  and  $Q$  quality category observations, there does not appear to be any significant difference in the quality categories. So, the assignment of these categories adds little, if any, useful information to these data.
- The categories  $\theta$ ,  $P$ , and  $N$  do identify the occurrence of some phenomenon causing the time of sunset to be significantly delayed. Thus, these categories are useful. The  $N$  may even indicate a true *Novaya Zemlya* effect, while the  $P$  possibly indicates a significant thermal inversion, but not one strong enough to provide a light duct.
- The observations are consistent with a lower than average lapse rate or, in extreme cases, a thermal inversion. Such a situation may be caused by a warmer mass of air over the cool surface of the ocean. The mean ocean surface temperature during the months of observations is consistent with this hypothesis. The data suggests that such a state is not a rare occurrence. Section 6 lists a number of additional analyses that may be able to improve our knowledge of astronomical refraction and the role it plays in navigation.
- The most important conclusion that can be arrived at from this analysis is that non-standard

conditions that significantly affect the refraction angle occur frequently. It is likely that such non-standard conditions are linked with the weather conditions, which change with locality and season. Table 3 of Hohenkerk & Sinclair (2008) shows that the refraction angle is particularly sensitive to the lapse rate. At a zenith distance of  $90^\circ$ , a change in the lapse rate of  $0.5 \text{ K km}^{-1}$  for the entire troposphere results in a change of  $-17''$  in the refraction angle. The boundary layer in which a non-standard lapse rate is most likely to exist is only about 0.02–0.18 the depth of the troposphere, but it contains about 0.03–0.28 of its mass, and the lapse rate in the boundary layer can vary from the dry adiabatic lapse rate of  $9.8 \text{ K km}^{-1}$  to large negative values in a thermal inversion, a range more than 20 times greater than that explored by Hohenkerk & Sinclair. Furthermore, analysis of mirages by Fraser & Mach (1976) shows that the lapse rate can vary widely within the boundary layer. Thus, the main source of uncertainty here is the boundary layer lapse rate structure.

## 6. Future Work

There are numerous tests that can be done to confirm, and extend these initial conclusions. Listed here are those tests, in approximate order of execution that are currently envisioned:

1. The data were analyzed here using an early prototype of the refraction functions found in SLAC 2.0. The analysis should be repeated using the current version of those functions with checks to determine if the Channel Islands might have affected the observed time of sunset.
2. The preliminary Mount Wilson analysis only used the time of sunset observations, contact of the upper limb of the Sun with the horizon. This data set also includes 109 observations of contact of the lower limb of the Sun with the horizon and 217 observations of contact of the center of the Sun with the horizon. These data should be analyzed as well. Com-

parison of the time of lower limb contact with the time of upper limb contact could be useful in determining changes in refraction over short time scales, approximately 120 s. Analysis of the time of center contact may also contain such information, provided the center line was readily identifiable as claimed by the observer (Rarogiewicz, 1994).

3. In addition to the Mount Wilson sunset observations, there are 254 observations of sunrise and 135 observations of sunset made at Edmonton, Alberta, Canada available for analysis (Sampson 2010, private communication). These observations should be analyzed using the same basic methods.
4. At least some records of relevant parameters such as the ocean surface water temperature and air temperature near the ocean's surface should be available. Obtaining these data would allow a more accurate estimate of the refraction angle to be made, increasing the accuracy of the subsequent analysis.

## A. Mirages

Mirages are generally classified by how they affect sighting of foreground objects (Fraser & Mach, 1976). In general, four properties are used in this classification:

1. Image position (above, 'superior', or below, 'inferior', the primary image)
2. Upright or inverted image
3. Number of images
4. Image shape (vertically compressed, 'stooping', or elongated 'towering' or 'looming').

Each of these phenomena is a result of the lapse rate structure near the Earth's surface. This classification scheme is not generally useful for astronomical refraction. However,

- Understanding the differences between the different types of mirages is critical in establishing the position of the horizon and, hence, the dip.
- The amount of stoop or towering of a mirage is a direct indicator of both the refraction angle and its rate of change with zenith distance near the horizon.
- When these phenomena occur, extracting data on the lapse rate is possible, at least in principal. Thus, analysis of the shape of the Sun or Moon near the horizon might be used to determine the lapse rate.

In particular, whether the mirage is stooping or towering is a direct indication of the astronomical refraction near the horizon.

If the lapse rate near the surface is greater than normal, then the density of the atmosphere near the surface is less than it would be under the nor-

mal lapse rate. The result is less than normal refraction. Surface objects in the distance stoop below their geometric positions, astronomical objects that would normally be below the horizon are not visible, and the sunset occurs earlier than expected. An example of this condition is over a hot surface such as a road on a clear, hot day.

If the lapse rate is less than normal or there is a thermal inversion, then the density of the atmosphere near the surface is greater than it would be under the normal lapse rate. The result is greater than normal refraction. Surface objects in the distance will tower above their geometric positions, astronomical objects that would normally be below the horizon will be visible, and the sunset occurs later than expected. An example of this condition is over a cool surface such as the ocean.

If the temperature inversion lapse rate is particularly steep, then total internal reflection can occur forming a light duct in the atmosphere (Lehn, 1979). Under these conditions, the object may be observed several degrees below the geometric horizon. This phenomenon is called the *Novaya Zemlya* effect. This name is taken from the first known record of it made by a 16th century ship's captain wintering at the island of *Novaya Zemlya*. The ship's log records a sunrise at a time when the Sun was geometrically nearly  $5^\circ$  below the horizon.

Sawatzky & Lehn (1975) speculate that the *Novaya Zemlya* effect may have led the Vikings to discover Iceland and Greenland. The east coast of Iceland is about 410 km,  $3^\circ 7'$  along the Earth's surface, from the west coast of the Faroe Islands, and the east coast of Greenland is about 290 km,  $2^\circ 6'$  from the west coast of the Iceland. Thus, the coasts of these two islands should be visible from the previous point of land under *Novaya Zemlya* conditions. Hence, the vikings that settled these areas may have seen their destinations before ever setting sail.

The *Novaya Zemlya* effect is likely a continuous extension of the enhanced refraction caused by a strong temperature inversion. It is not clear whether its onset is accompanied by a sudden increase in the refraction angle. It is clear that its physics are distinct from normal refraction. Two distinguishing characteristics of the *Novaya Zemlya* effect are: the Sun appears to be 'square' from extreme compression of its image in the light duct, and its brilliance is significantly reduced from atmospheric absorption. These two characteristics are rarely noted, however. Analysis of numerical experiments is required to ascertain other methods to distinguish the *Novaya Zemlya* effect from refraction associated with a normal looming mirage. Sampson (1993) indicates that this effect is not rare.

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