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Instantaneous Local Circumstances of Solar and Lunar Eclipses

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ABSTRACT

The instantaneous local circumstances of an eclipse answer the question: Given a specific time and a location on or near the surface of the Earth, is a lunar or solar eclipse in progress, and if so, what are the eclipse conditions (i.e., magnitude or obscuration) at that time and location? The answer to this question is especially important for planning activities or operations in which illumination is critical. Eclipses can have a profound impact on the level of natural light, yet are often overlooked when predicting the illuminance. This technical note describes how to compute the instantaneous local circumstances of solar and lunar eclipses.

1. Introduction

The primary goal of eclipse computations is to determine the *circumstances* of individual eclipses; e.g., the type of eclipse, the date and time at which each phase of the eclipse begins and ends, and the region of the Earth from which the eclipse is visible. In the case of solar eclipses, circumstances can be *general* or *local*, the former providing information on global aspects of the eclipse, and the latter providing information for specific locations on the Earth. Local circumstances are especially important for central solar eclipses, in which the visibility of the annular or total phases is confined to narrow paths on or near the Earth's surface. In contrast, the circumstances of lunar eclipses are general in that they are the same for all locations on the Earth, provided that the Moon is visible from the location. General circumstances of eclipses are published in various sources such as canons (e.g., Espenak and Meeus 2006, 2009) and almanacs (e.g., *The Astronomical Almanac* 2011). Local circumstances are available in special publications (e.g., Espenak and Anderson 2008), online canons (e.g., *Eclipses Online* 2013), and software (e.g., *The Multiyear Interactive Computer Almanac* 2012). The methods for computing eclipse circumstances are well documented (see e.g., *Explanatory Supplement to the Ephemeris and the American Ephemeris and Nautical Almanac* 1961; Urban and Seidelmann 2012).

Typical local-circumstance tabulations answer the question: Given a specific eclipse and a location on the Earth, at what times do the various eclipse phases begin and end, and what are the eclipse conditions (e.g., obscuration, magnitude) at maximum eclipse? This technical note is concerned with another type of local circumstances, hereafter referred to as *instantaneous local circumstances*. They answer a different question: Given a specific time and a location on the Earth, is an eclipse in progress, and if so, what are the eclipse conditions at that time and location? Knowledge of the instantaneous local circumstances is especially important for planning activities or operations in which illumination is critical. For example, an activity may need to take advantage of the light of a Full Moon. Such an activity may be negatively affected, or even fail, if a total eclipse of the Moon is in progress at the time for which the activity is planned. Alternatively, an operation that must take place under the cover of darkness may be possible around the time of Full Moon if a total lunar eclipse is in progress. Knowing the instantaneous local circumstances of an eclipse is an important but often neglected part of predicting the illuminance.

The remainder of this note describes how to compute the instantaneous local circumstances of solar and lunar eclipses.

2. Solar Eclipses

The geometry of a solar eclipse is illustrated in, e.g., Urban and Seidelmann (2012) (section 11.2.4). A partial solar eclipse takes place when the Earth is within the outer part (penumbra) of the Moon's shadow, but not the inner part (umbra). A solar eclipse is central when the Earth is within the Moon's umbra or antumbra. If the eclipse is central at a particular time and location, and the height of the apex of the Moon's umbral shadow cone above the Earth's surface is less than or equal to the height of the location, the central eclipse is total. Alternatively, if the height of the apex of the Moon's umbral shadow cone above the Earth's surface is greater than the height of the location, the central eclipse is annular. The most dramatic change in surface illumination occurs during the period of total eclipse, when the sky becomes dark and the bright planets and stars become visible.

Computation of the instantaneous local circumstances of a solar eclipse requires *topocen-*

tric places of the Sun and Moon at the time and location of interest. A rigorous procedure for computing topocentric places of solar-system bodies can be found in chapter 7 of Urban and Seidelmann (2012). This procedure has been implemented in software (see Naval Observatory Vector Astrometry Software 2011). An approximate method for computing topocentric places is described in section D of *The Astronomical Almanac* (2011); this method can be used, for example, with the Sun and Moon ephemerides described by Bretagnon and Simon (1986) and Chapront-Touzé and Chapront (1991) respectively.

2.1. Phase of the Solar Eclipse

Figure 1 illustrates the positions of the Sun and Moon at the time of first (and last) contact. Thus, a solar eclipse—at least partial—is occurring if:

$$E < s_\mathrm{s} + s_\mathrm{m} \tag{1}$$

where E is the topocentric elongation of the Moon from the Sun (e.g., Urban and Seidelmann 2012, eq. 12.1), and s_s and s_m are the topocentric semidiameters of the Sun and Moon, respectively (e.g., Urban and Seidelmann 2012, eq. 10.34). Topocentric places of the Sun and Moon at the time and location of interest are used in computing these quantities.

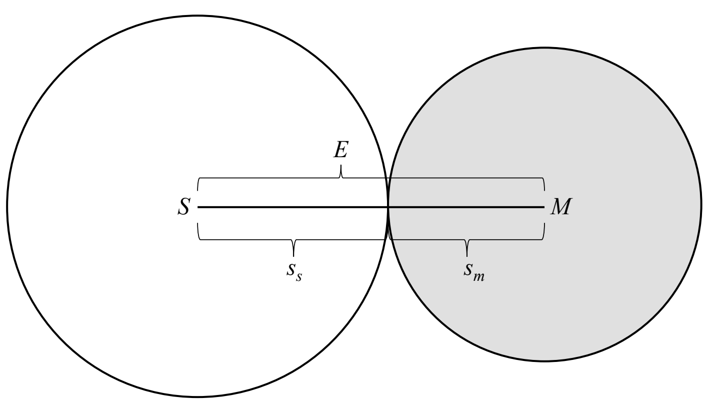

Fig. 1.—Solar-eclipse geometry at first and last contact (the orientation is arbitrary). S and M are the centers of the Sun and Moon, respectively. E is the elongation of the Moon from the Sun, and s_s and s_m are the semidiameters of the Sun and Moon, respectively. All quantities are topocentric.

Figure 2 illustrates the positions of the Sun and Moon at the time of second (and third) contact. The solar eclipse is central (annular or total) at the time and location of interest if:

$$E \leq |s_{\mathrm{m}} - s_{\mathrm{s}}|. \tag{2}$$

If this condition is satisfied, the central eclipse is total if:

$$s_{\mathrm{m}} \geq s_{\mathrm{s}}. \tag{3}$$

If the condition specified in equation 2 is satisfied, but the condition specified in equation 3 is not, the central solar eclipse is annular. The above conditions (equations 1–3) should be checked in sequence at each instant of time, stopping once a condition is not satisfied. The last satisfied condition indicates the phase of the eclipse.

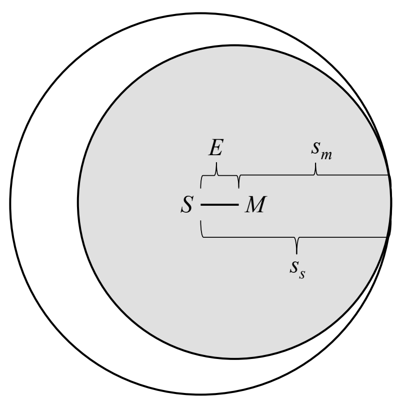

Fig. 2.—Solar-eclipse geometry at second and third contact. S and M are the centers of the Sun and Moon, respectively. E is the elongation of the Moon from the Sun, and s_{s} and s_{m} are the semidiameters of the Sun and Moon, respectively. All quantities are topocentric.

2.2. Obscuration of the Sun

If the first condition above (equation 1) indicates that a solar eclipse is happening at the time and location of interest, the obscuration of the Sun by the Moon, expressed as a fraction of the area of the Sun, can be computed by applying the geometry of circle-circle intersection (Weisstein 2005). First, compute the discriminant x:

$$x = (-E + s_{\mathrm{m}} + s_{\mathrm{s}})(E + s_{\mathrm{m}} - s_{\mathrm{s}})(E - s_{\mathrm{m}} + s_{\mathrm{s}})(E + s_{\mathrm{m}} + s_{\mathrm{s}}) \tag{4}$$

If x is negative, the Moon either entirely covers the Sun (total eclipse) at that instant or is totally within the disk of the Sun (annular eclipse) depending on the semidiameters of the bodies. Compute the obscuration (O_{s}) accordingly: If $s_{\mathrm{m}} \geq s_{\mathrm{s}}$, then $O_{\mathrm{s}} = 1.0$ (total eclipse), otherwise compute the ratio of the areas of the two bodies (annular eclipse):

$$O_{\mathrm{s}} = {s_{\mathrm{m}}}^2 / {s_{\mathrm{s}}}^2 \tag{5}$$

If x is positive (partial eclipse) or zero, compute the overlap area (A), which has the shape of a lens:

$$A = a + b - c \tag{6}$$

where:

$$\begin{aligned} a &= {s_{\mathrm{m}}}^2 \arccos\left(\left(E^2 + {s_{\mathrm{m}}}^2 - {s_{\mathrm{s}}}^2\right) / \left(2\,E\,s_{\mathrm{m}}\right)\right) \\ b &= {s_{\mathrm{s}}}^2 \arccos\left(\left(E^2 + {s_{\mathrm{s}}}^2 - {s_{\mathrm{m}}}^2\right) / \left(2\,E\,s_{\mathrm{s}}\right)\right) \\ c &= 0.5\sqrt{x} \end{aligned}$$

and then O_{s}:

$$O_{\mathrm{s}} = A / \pi {s_{\mathrm{s}}}^2. \tag{7}$$

3. Lunar Eclipses

The geometry of a lunar eclipse is illustrated in, e.g., Urban and Seidelmann (2012) (section 11.2.3). A penumbral lunar eclipse takes place when the Moon is within the outer part (penumbra) of the Earth's shadow, but not the inner part (umbra). A partial lunar eclipse occurs when a portion of the Moon is within the Earth's umbra. The eclipse is total when the Moon is completely within the umbra. The appearance of lunar eclipses is described in, e.g., Liu and Fiala (1992). Penumbral lunar eclipses are imperceptible to the human eye until at least 50-70% of the Moon's diameter is within the penumbra. Total lunar eclipses vary greatly in brightness depending on the Earth's atmospheric conditions—especially along the terminator—and the depth to which the Moon penetrates the umbra. The Danjon scale (see, e.g., Ottewell 1991; Espenak 2009) illustrates this broad range in brightness.

Computation of the instantaneous local circumstances of a lunar eclipse requires *apparent* (geocentric) *places* of the Sun and Moon at the time of interest. A rigorous procedure for computing apparent places of solar-system bodies can be found in chapter 7 of Urban and Seidelmann (2012). This procedure has been implemented in software (see Naval Observatory Vector Astrometry Software 2011). Ephemerides and methods for computing the apparent places of the Sun and Moon are also given by Bretagnon and Simon (1986) and Chapront-Touzé and Chapront (1991) respectively.

3.1. Phase of the Lunar Eclipse

Figure 3 illustrates the geometry at the various phases of a lunar eclipse. Let $\hat{\mathbf{r}}_{\rm as}$ be a unit vector pointing from the geocenter to the antisolar point (S'). This unit vector lies along the axis of the Earth's shadow cone, and is computed from:

$$\hat{\mathbf{r}}_{\rm as} = -(\mathbf{r}_{\rm s} \,/\, |\mathbf{r}_{\rm s}|) \tag{8}$$

where $\mathbf{r}_{\rm s}$ is the apparent equatorial position vector of the Sun. If necessary, $\mathbf{r}_{\rm s}$ can be computed from the apparent equatorial spherical coordinates (right ascension, declination, and distance) of the Sun (see, e.g., Urban and Seidelmann 2012, eq. 7.20). Let $\hat{\mathbf{r}}_{\rm m}$ be a unit vector pointing from the geocenter to the center of the Moon (M or M'). This unit vector is computed from:

$$\hat{\mathbf{r}}_{\rm m} = \mathbf{r}_{\rm m} \,/\, |\mathbf{r}_{\rm m}| \tag{9}$$

where $\mathbf{r}_{\rm m}$ is the apparent equatorial position vector of the Moon. If necessary, $\mathbf{r}_{\rm m}$ can be computed from the apparent equatorial spherical coordinates (right ascension, declination, and distance) of the Moon using the same method as used for obtaining $\mathbf{r}_{\rm s}$.

The angular distance of the center of the Moon from the antisolar point is:

$$\theta = \arccos\,(\hat{\mathbf{r}}_{\rm as} \cdot \hat{\mathbf{r}}_{\rm m}). \tag{10}$$

The angular radii of the penumbral and umbral shadows, respectively, at the distance of the Moon are:

$$f_{\rm pen} = 1.02\,(\pi'_{\rm m} + s_{\rm s} + \pi_{\rm s}) \tag{11}$$

$$f_{\rm umb} = 1.02\,(\pi'_{\rm m} - s_{\rm s} + \pi_{\rm s}) \tag{12}$$

where $\pi_{\rm s}$ is the equatorial horizontal parallax (see, e.g., Urban and Seidelmann 2012, section 1.3.6.1) of the Sun, and $\pi'_{\rm m}$ is the "reduced" equatorial horizontal parallax of the Moon (the lunar horizontal parallax at the mean radius of the Earth, assumed to be at latitude 45°). The factor of 1.02 accounts for the Earth's atmosphere (*Explanatory Supplement to the Ephemeris and the American Ephemeris and Nautical Almanac* 1961, section 9E). Here, and throughout section 3, the semidiameters $s_{\rm s}$ and $s_{\rm m}$ are computed using the apparent places; i.e., they are geocentric, not topocentric.

Then, a lunar eclipse is occurring at the time of interest—it is at least penumbral—if:

$$\theta < (f_{\rm pen} + s_{\rm m}) \equiv L_1. \tag{13}$$

The lunar eclipse is at least partial (the Moon is in the earth's umbral shadow) if:

$$\theta < (f_{\rm umb} + s_{\rm m}) \equiv L_2, \tag{14}$$

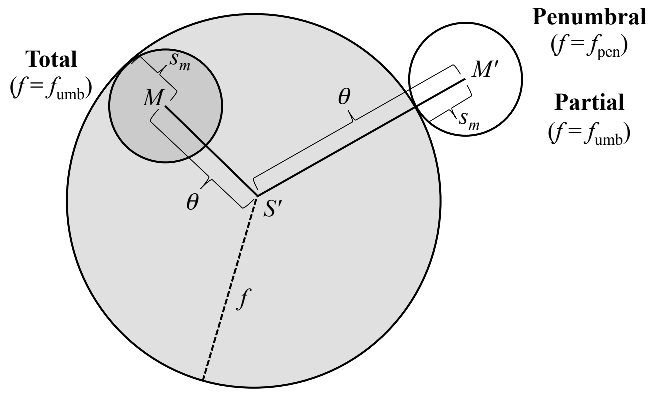

Fig. 3.—Lunar-eclipse geometry from a geocentric perspective. The axis of the Earth's shadow cone is orthogonal to the figure at S', the antisolar point. $\hat{\mathbf{r}}_{\mathrm{as}}$ (see section 3.1) lies along this axis and points from the geocenter to S'; $\hat{\mathbf{r}}_{\mathrm{m}}$ points from the geocenter to the center of the Moon (M or M'). θ is the angular distance between the antisolar point and the center of the Moon, and s_{m} is the semidiameter of the Moon. The Earth's shadow at the distance of the Moon has angular radius f: the radius of the penumbra is f_{pen} and the radius of the umbra is f_{umb}. The right side of the figure shows the Moon (M') at the beginning and end of penumbral eclipse (where $f = f_{\mathrm{pen}}$), and likewise for umbral eclipse (where $f = f_{\mathrm{umb}}$). The left side of the figure shows the Moon (M) at the beginning and end of total eclipse (where $f = f_{\mathrm{umb}}$). All quantities are geocentric. This figure adapted from figure 11.4 of Urban and Seidelmann (2012).

and the lunar eclipse is total if:

$$\theta \leq (f_{\mathrm{umb}} - s_{\mathrm{m}}) \equiv L_3 \tag{15}$$

(Urban and Seidelmann 2012, equation 11.133).

The conditions above (equations 13–15) should be checked in sequence at each instant of time, stopping once a condition is not satisfied; the last satisfied condition provides the phase of the eclipse. Finally, the lunar eclipse is visible from the location of interest if the Moon is above the horizon at the time of interest. The altitude or zenith distance of the Moon is obtained by transforming the topocentric coordinates of the Moon from the equatorial system to the horizon system (see, e.g., Urban and Seidelmann 2012, section 7.1.3.4).

3.2. Magnitude of the Lunar Eclipse

The magnitude of a lunar eclipse is the fraction of the Moon's diameter that is inside the Earth's penumbral or umbral shadow. In the former case, it is the penumbral magnitude; in the latter case it is the umbral magnitude. The penumbral magnitude is given by:

$$m_{\mathrm{pen}} = (f_{\mathrm{pen}} + s_{\mathrm{m}} - \theta) \,/\, 2\, s_{\mathrm{m}} \tag{16}$$

The umbral magnitude is set to zero when the lunar eclipse is in the penumbral phase. The umbral magnitude is:

$$m_{\mathrm{umb}} = (f_{\mathrm{umb}} + s_{\mathrm{m}} - \theta) \,/\, 2\, s_{\mathrm{m}} \tag{17}$$

The penumbral magnitude is set to zero when the lunar eclipse is in the umbral (partial or total) phase.

4. Conclusion

Procedures for computing the basic instantaneous local circumstances of solar and lunar eclipse have been given. It is only necessary to check the solar-eclipse conditions in section 2.1 around the time of New Moon; i.e., in equation (1), the limiting condition $E = s_{\mathrm{s}} + s_{\mathrm{m}}$ will not exceed approximately $0\overset{\circ}{.}6$. Similarly, it is only necessary to check the conditions for lunar eclipse (section 3.1) around time of Full Moon; in equation (13), the limiting condition $\theta = f_{\mathrm{pen}} + s_{\mathrm{m}}$ will not exceed approximately $1\overset{\circ}{.}7$. Checking the eclipse conditions only when necessary will save computing resources.

Knowing the instantaneous local circumstances is the first step in assessing the impact that an eclipse will have on the illuminance.

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