Cosmological Induction

[Omitted for blind review][†‡]
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Although Hume's problem of induction is well known, it has not been implemented explicitly in cosmology. This paper fills that lacuna in the literature and then applies the result to achieve two further goals: first, to explain why Hume's problem is particularly acute for multiverse cosmological models, and second, to demonstrate that John Norton's material theory of induction contains ambiguities that require careful elucidation, lest the material theory be susceptible to Hume's problem, despite Norton's assertions to the contrary.

1. Introduction. Hume's problem of induction has important philosophical consequences for many scientific endeavors. Here I implement it in cosmology, a task that, to my knowledge, has not been undertaken in the literature. I argue that the problem is particularly acute in the multiverse setting, and that the problem may still lurk beneath the surface for John Norton's material theory of induction, despite Norton's claims to the contrary.

In Section 2, I motivate the discussion by introducing the Cosmological Principle, which cosmologists use routinely for induction. Section 3 contains a careful reconstruction of Hume's problem. This reconstruction enables us to transform the temporal version of the problem (as Hume originally presented it) into the spatial version of the problem (as it appears in cosmology). The conversion occurs in Section 4. In Section 5, I argue that the spatial version of the problem reveals that multiverse cosmological models are vulnerable to the problem in a novel way. In Section 6, I implement the disentangled spatial and temporal flavors of induction to demonstrate that the material theory of induction faces a dilemma: either it is rendered inert, or it is susceptible to Hume's problem. Finally, in Section 6, I address the critic who maintains that Hume's problem is "old news" in philosophy of science and that all sciences must, of necessity, take on some risk of induction.

† To contact the author, please write to: [Omitted for blind review].

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2. The Cosmological Principle. Cosmological induction occurs whenever a cosmologist proposes a relation—either of similarity or of dissimilarity—between the observed piece of the cosmos and the unobserved pieces. Cosmological induction is commonly justified by the Cosmological Principle.[1]

The Cosmological Principle is the assertion that "the universe is spatially homogeneous and isotropic at large scales."[2] It is the inductive principle of choice for cosmologists, as is made clear by the following series of passages. The passages come, respectively, from Hermann Bondi, Steven Weinberg, and Frank Wilczek:

> [W]e may consider the simplicity postulate as a basis for the cosmological principle. This is the point of view adopted by the proponents of general relativity and is effectively that in our present ignorance of the universe *progress may most easily be made by assuming, purely as a working hypothesis*, that the large-scale structure of the universe is as simple as possible and hence it is uniform.[3]

> There remains the possibility that the universe is not homogeneous and isotropic after all. It might be homogeneous but not isotropic, as in the model of K. Gödel. However, the cosmic microwave radiation…appears to be highly isotropic…The real reason, though, for our adherence to the Cosmological Principle *is not that it is surely correct, but rather, that it allows us to make use of the extremely limited data* provided to cosmology by observational astronomy.[4]

> The most profound result of observational cosmology has been to establish the Cosmological Principle: that the same laws apply to all parts of the observed Universe, and moreover matter is—on average—uniformly distributed throughout. *It seems only reasonable*, then, to think that the observed laws are indeed universal, *allowing no meaningful alternative*, and to seek a unique explanation for each and every aspect of them.[5]

These passages span almost five decades, and their authors are of the highest prominence. Nevertheless, as indicated by the italicized portions of the passages, they each lack a strong foundational justification for the Cosmological Principle.[6]

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[1] In Section 5, focus will shift from the Cosmological Principle (which cosmologists often employ in single-universe models) to a generalized version of cosmological induction employed in multiverse models.
[2] Beisbart (2009), p. 176.
[3] Bondi (1960), p. 13. Italics added for emphasis.
[4] Weinberg (1972), p. 408. Italics added for emphasis.
[5] Wilczek (2007), p. 46. Italics added for emphasis.
[6] For additional discussion concerning the difficulty involved with justifying the Cosmological Principle, see, for example, Beisbart (2009) and Beisbart & Jung (2006), and the references contained within those articles.

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The justification sought would permit the extension of knowledge from a small part of the universe to the whole, or, at a minimum, to nearby-but-unobserved portions. Cosmologists and philosophers of science have produced literature aiming to justify cosmological induction, the standard strategy for which has been enumerative induction. A sample of such literature is as follows:

> The Copernican principle is basically a philosophical assumption, but this does not make it unscientific…The Copernican principle must be tested, of course: we cannot accept any guiding principle if it predicts things that conflict with observations. Everywhere we look in the Universe, *we observe that its appearance in any one region is very similar to that in another with the same cosmological age*…In each case, the properties are the same everywhere they can be measured. This fact has a name: the Universe is homogeneous.[7]

> [E]ven if we are never able to make observations concerning events outside the presently observable region, *our knowledge of the presently observable region may permit us to make justifiable inferences* concerning events in other parts of the universe.[8]

> [C]onsider matter or radiation that left our past light cone at about the time of decoupling—as it might be, a cosmic background photon. That photon is estimated to now be about 40 billion light-years from us (the distance being measured along the cosmic time-slice t = now). Vast indeed. This vastness emphasizes how risky is the induction from CP's holding in the observable universe…to its holding throughout any such region…To put the point very simply, *in terms of enumerative induction over spacetime regions*: the observable universe is such a small fraction of such regions, that it is risky to claim it is a fair sample.[9]

These authors collectively communicate the difficulty surrounding the justification of cosmological induction. Nonetheless, the suggestion (gathered collectively from the emphasized portions of the provided passages) is that enumerative induction (in the form of the Cosmological Principle) completes the needed inductive task because, as might be said colloquially, the Cosmological Principle has "worked so far."

The Cosmological Principle has held for already-gathered observations. It has never failed. So we should expect its continued success. Cosmologists and philosophers of science offer

[7] Schutz (2003), pp. 347-348. Italics removed from the word "homogeneous" to avoid confusion regarding the remaining italicized portion, which has been added for emphasis. Do not let the shift to the Copernican Principle (from the Cosmological Principle) detract from the relevant point: in either case, the reasoning offered by cosmologists for establishing homogeneity involves the same process of enumerative induction.
[8] Knobe, Olum, and Vilenkin (2006), p. 55. Italics added for emphasis.
[9] Butterfield (2013), p. 17. Italics added for emphasis.

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precisely this rationale, as exemplified in the second set of passages. The fact that Schutz (in the quoted passage) deals with the Copernican Principle—a relative of the Cosmological Principle—via enumerative induction provides additional evidence that cosmologists resort to that very tactic when pressed to provide justification for their cosmological induction. It is the standard justificatory strategy for induction in many alternative cosmological frameworks, including multiverse cosmology. Olum and Vilenkin are multiverse cosmologists, and they explicitly employ enumerative induction in the quoted passage. The two sets of passages show, in both temporal and theoretical contexts, that enumerative induction plays a meaningful role in cosmology.

Relying on enumerative induction as the justification of further inductive inferences leads straight toward a familiar philosophical foe. In brief, attempts to justify cosmological induction through enumerative induction summon a variant of Hume's notorious inductive problem. In its traditional form, Hume's problem provides a clear depiction of the difficulty associated with justifying enumerative induction. But that traditional form plays out over temporal induction. That same difficulty, but played out over spatial induction, arises when we try to justify the Cosmological Principle. To see this, and further to display the severity of the problem for multiverse cosmologists, we turn to Hume's argumentative template.

3. Hume's Problem of Induction. Although well known, the following passage from Hume is worth presenting explicitly because it reveals that Hume conceptualizes the problem as inherently temporal in scope:

> [A]ll our experimental conclusions proceed upon the supposition, that the future will be conformable to the past. To endeavor, therefore, the proof of this last supposition by probable arguments, or arguments regarding existence, must be evidently going in a circle, and taking that for granted, which is the very point in question…To say it is experimental, is begging the question. For all inferences from experience suppose, as their foundation, that the future will resemble the past, and that similar powers will be conjoined with similar sensible qualities…It is impossible, therefore, that any arguments from experience can prove this resemblance of the past to the future; since all these arguments are founded on the supposition of that resemblance. Let the course of things be allowed hitherto ever so regular; that alone, without some new argument or inference, proves not, that, for the future, it will continue so. In vain do you pretend to have learned the nature of bodies from your past experience.[10]

[10] Hume (1999), pp. 115-117.

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Hume repeatedly and emphatically refers directly to the temporal nature of the problem. The transition of Hume's argumentative thrust to the cosmological (spatial) setting begins with identifying the original argumentative structure.

Any enumerative induction, in reaching a conclusion about some future experience, follows a standard template. That template can be represented as follows:

(1) Every past experience *A* has been *B*.
—————(T)
(C) The next experience *A* will be *B*.

Note that the premise (1) is a compressed, exhaustive conjunction of (presumably) many individual experiences, all of which exhibit experience *A* being *B*, without any *A* that are not *B*. Listing those experiences as separate premises would more clearly mark the induction to the next experience as *enumerative*.

As Hume's passage indicates in several places, the key feature of the argument is that any justification for enumerative induction from experience includes a rule of inference such as "the future will be conformable to the past." Hume then argues that this rule cannot be justified except circularly. The lettered line of separation (T) in the template represents that rule of inference; it is a universal schema that connects the past experiences to the inferred future experience. It is labeled T to suggest that the inference rule operates across *times*:

T: What has been so will continue to be so.

Justification for the inference thus depends upon the justification for rule T. Such justification, notes Hume, only can be of the following sort:

(1) Rule T succeeds for the past.
—————(T)
(C) Rule T succeeds for the future.

Hume thus argues that all justification for T rests upon a circularity. The result is a failure of all attempts to justify the inference overall. Breaking the circularity requires access to the future, which is necessarily an inaccessible time.

4. The Spatial Problem of Induction. We now demonstrate that cosmologists face the same justificatory impasse, save for one small adjustment: the cosmological inferences to be justified

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through enumerative induction span *place* rather than *time*.[11] This simple difference is not trivial. Hitherto, the literature has not recognized the difference explicitly. The lacuna leaves incompletely grounded any claim that Hume's problem is ubiquitous in scientific endeavors, since some scientific endeavors (such as cosmology) involve induction in the spatial dimension rather than in the temporal dimension. In this section, the lacuna is filled, thereby restoring the argumentative force of Hume's conclusion in at least one branch of science (cosmology). Perhaps the argumentation presented here will apply to other sciences, as well, but that investigation lies beyond the present focus.

Cosmologists seek justification for the inference to a claim such as "Unobserved place *A* is *B*." Varying Hume's original template summons the spatial correlate:

(1) Every observed place *A* is *B*.
——————(P)
(C) Unobserved place *A* is *B*.

As before, we make the rule of inference explicit. The rule needs to link observed places to unobserved places. The result is rule P:

P: What is here so is there so.

P (so labeled to suggest that the inference rule operates across *places*) is the correlate of rule T in Hume's original argument. Importantly, just as the class "has been so" (from rule T) continually grows as time proceeds, so too the class "is here so" (from Rule P) continually grows as observations are gathered.

Justification for the inference now depends upon the justification for rule P. But, just as in Hume's original version, such justification only can be circular:

(1) Rule P succeeds for here.
——————(P)
(C) Rule P succeeds for there.

The only means of justifying rule P is via rule P itself. As in Hume's original formulation with respect to the future, breaking the circularity depends upon the ability to access the other places ("there"). But such places are inaccessible from "here," just as the future is inaccessible from the present.

[11] I use "place" rather than some more specific term (such as "spacetime region," "miniverse," etc.) to preserve the fact that each cosmological theory—and especially all of the variants involved in the multiverse tradition—describes its pieces differently.

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To see Hume's problem reborn cosmologically, the relationship can be displayed clearly as follows, with correlations distinguished:

	Hume (across times)	Cosmology (across places)
Inference	Every **past experience** *A* **has been** *B*. ——————(**T**) The **next experience** *A* **will be** *B*.	Every **observed place** *A* **is** *B*. ——————(**P**) **Unobserved place** *A* **is** *B*.
Rule	**T**: What **has been** so **will continue to be** so.	**P**: What **is here** so **is there** so.
Justification	Rule **T** succeeds for **the past**. ——————(**T**) Rule **T** succeeds for **the future**.	Rule **P** succeeds for **here**. ——————(**P**) Rule **P** succeeds for **there**.

Table 1. The relationship between Hume's classic problem of induction and the cosmological correlate.

We see here a general problem for cosmological inference via enumerative induction. It is the difficulty underlying the lack of justification for the Cosmological Principle.[12] The cosmologists quoted in Section 2 recognized, but struggled to articulate, the difficulty with justifying the Cosmological Principle. The preceding analysis pinpoints it.

5. Cosmological Induction in the Multiverse. I turn now to a specialized cosmological setting in which the justification for spatial induction is even more difficult to obtain than in the general setting presented in Section 4. Multiverse models, which are rising to prominence in contemporary cosmology, will be seen to be foundationally speculative when the results from Section 4 are applied to them.

In most sciences, the uniformity of experience staves off concern for the temporal version of Hume's problem. As each new *A* proves to be *B*, the task of addressing Hume's problem gets pressed incrementally further away. However, in the case of multiverse cosmology, there can be no uniformity of experience. Cosmological induction in the multiverse setting amounts to making an inductive inference to the population from a sample of size one. This means that one cannot obtain even premise (1)—"Rule P succeeds for here"—in the multiverse context.

A multiverse is nothing more than a collection of actually-existing parts, here called 'miniverses.'[13] Our universe is one of those miniverses. The multiverse hypothesis imposes three conditions on the miniverses:

[12] To see this, substitute "homogeneous and isotropic on large scales" for *B*.

[13] The term 'miniverse' is borrowed from John Earman. His 2009 contribution to the "Philosophy of Cosmology: Characterising Science and Beyond" conference at Oxford is the first (and only) place I have encountered it. (Manuscript cited in references.) Its usefulness (in my opinion) makes it only a matter of time before it (or a substitute term for the same concept) will become standard jargon in the discipline.

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(i) The miniverses are topologically connected to each other.
(ii) The miniverses are causally disconnected from each other.[14]
(iii) The miniverses may exhibit different combinations of laws, parameters, and constants.

These conditions are in place to separate "scientific" multiverse hypotheses from "philosophical" multiverse hypotheses, such as the modal realism hypothesis of David Lewis.[15] The present focus is on the "scientific" group advocated by cosmologists in the contemporary scientific literature. Note that condition (iii) is what separates the multiverse hypothesis from many single-universe (henceforth, *standard*) cosmological accounts. Standard cosmological models may posit topologically connected, causally disconnected regions within just one universe, but the regions in question are taken to be unified under a single set of laws, parameters, and constants.

Multiverse cosmological theories imply the actual existence of miniverses that are completely inaccessible from our universe. Hence, these theories face a steep – perhaps cliff-like – justificatory slope of the sort that other scientific theories do not face. Nonetheless, the literature pays little attention to making precise the challenge associated with justifying the multiverse hypothesis.[16] The results of Section 4 can be built upon to demonstrate precisely the difficulty involved.

In standard cosmological accounts, inference takes place (with the help of the Cosmological Principle) from the ever-increasing observed regions to the unobserved regions of the universe. The number (or volume) of observed spacetime regions increases as observations are collected, but even still, as Butterfield notes in the passage quoted earlier, there remains considerable risk when inferring from that small sample to the rest of the universe.

In the multiverse case, the induction takes place from our universe to other miniverses. Yet the gathering of observations within our universe, while still perhaps offering fodder for inductive inference *within* it via the Cosmological Principle, does nothing with respect to offering confirmation for the properties *outside* of it. Thus, our increasing success within our universe provides no help in justifying cosmological induction in the multiverse setting. Any gathered information applies merely to our universe. The shift to the multiverse scenario *reduces* the size of the sample to the smallest possible size (one), while at best leaving the

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[14] Note that causal connection (and hence 'causal disconnection') is defined by the theorist in question. Ancients, for example, would differ markedly from contemporary theorists with respect to what counts as causal connectedness in their multiverse models. For further details, see [omitted for blind review].
[15] For example, Lewis's modal realism hypothesis does not satisfy clause (i).
[16] There are many attacks against multiverse cosmological accounts in the literature. However, most involve discussions about what should (or should not) be considered "scientific." For example, see Kragh (2012), pp. 38-42, for an introduction to the interaction between Popper's falsifiability demarcation criterion and the multiverse hypothesis. For one account of a multiverse theorist claiming to meet the challenge set by the falsifiability criterion, see Smolin (2007). The present concern is that the literature lacks an account that spells out *precisely* why multiverse theories are so difficult to justify.

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population size unchanged (infinite).[17] The difficulties inherent in justifying the Cosmological Principle for the single-universe theorist are thereby compounded for the multiverse theorist trying to justify cosmological induction.

The situation is worse still. Further exacerbating the difficulty is the stipulation that we *necessarily* have access to exactly one miniverse. Multiverse cosmologists themselves lay down this tenet, and it is captured by condition (ii). This is the crucial difference that guarantees amplification of the already-difficult justificatory problem. Whereas single-universe advocates inherit no restrictions on the possibility of acquiring further evidence with which to strengthen their inductive inference – even if that strengthening is meager – multiverse theorists are perpetually trapped in a situation wherein no acquired evidence can strengthen their inductive inference at all.

Thus, the result of Section 4 is uniquely acute in the multiverse setting. In most of the mature sciences, including traditional cosmology, iteration after iteration of experience is implemented to strengthen rule T (or rule P for traditional cosmologists). Each new experience *A* that proves to be *B* contributes to belief in rule T. Each new place *A* proving to be *B* contributes to belief in rule P. Despite this, Hume tells us that we commit a fallacy when reasoning this way. Let us label this fallacy the "Weight of History" fallacy.[18] We cannot accumulate enough confirmation to avoid Hume's problem. Hence, assuming that our past experience constitutes "enough experience" to justify our use of rules T or P is committing the Weight of History fallacy. Nonetheless, scientists have discounted the Weight of History fallacy for centuries now, because the sciences in question continue to produce results. For example, in traditional cosmology, rule P is adopted without foundational justification as observations stream in from the cosmos. This much was clear from the series of passages provided in Section 2.

However, in the multiverse setting, cosmologists arrive at Hume's problem *first*—immediately—rather than *eventually*. In attempts to justify the use of enumerative induction between our universe and the other miniverses, the very first step invokes Hume's problem. One cannot accrue some evidence about the similarity (or dissimilarity) between our universe and the other miniverses. *A fortiori*, that process cannot be repeated to gather more evidence. No evidence gathered in our universe provides *any* confirmation about the properties of other miniverses. In the multiverse cosmological case, one never gets a second bit of experience with which to strengthen rule P. Rule P operates as an inference from a sample of size one, which immediately triggers the need to solve the spatial version of Hume's problem. The resulting situation is so dire that multiverse theorists cannot even put themselves in position to commit the Weight of History fallacy. That is what separates the multiverse case from the standard

[17] Certainly there are model-by-model considerations that need to be differentiated from each other. But the overwhelming majority of multiverse models involve an infinite number of miniverses. See [omitted for blind review].
[18] This label was coined by [omitted for blind review] in personal conversation.

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cosmological case. In the multiverse case, the *premise* in the attempted justification cannot be obtained, let alone the conclusion.

6. The Material Theory of Induction. John Norton maintains that Hume's problem can be escaped by adopting a material theory of induction whereby "inductive inferences are warranted by facts."[19] By employing the preceding analysis of cosmological (spatial) induction, we can see that Norton's material theory of induction requires further exposition, lest it not only fail to escape Hume's problem, but tacitly operate via principles that it condemns, specifically in terms of the implementation of universal schemas. Previous objections to the material theory tend to focus on identifying a Hume-like regress within the material theory.[20] Of note is that the extant objections leverage the temporal and logical flavors of inductive inference.[21] Of further note is that, in addition to addressing the temporal and logical versions of a Hume-like regress within the material theory, Norton partially deflects the involvement of Hume's problem at all concerning the material theory: "If failing to solve Hume's problem is sufficient to damn the material theory, then we must also damn all other accounts."[22] The preceding discussion of spatial inference can be leveraged to identify ambiguities lurking in Norton's presentation heretofore of the material theory of induction. These ambiguities stand in the way of the material theory's ability to escape of Hume's problem, or to deflect it, in the manner Norton hopes. Specifically, the material theory presents ambiguities in terms of the *domain* from which, and *range* into which, inductive inference proceeds. Additional ambiguity emerges with respect to the material theory's ampliative component. In what follows, I use the preceding distinction between spatial and temporal induction to sketch an initial attempt to disambiguate these components within the material theory. The result appears to leave the material theory in a dilemma: it either takes on a much weaker form, or it falls prey to Hume's problem by adopting the type of universal schema that it claims to eschew.

In its simplest characterization, the material theory's reliance upon facts appears to open pathways that are closed to formal theories (which, like enumerative induction, rely on universal schema to power inferences): "Since the facts are contingent, there are no universal warrants. No system of inductive logic holds universally; each holds only in the limited domain in which the warranting facts are true."[23] This is the underpinning for Norton's foundation that apparently separates the material theory from formal theories and, hence, from Hume's problem. Norton's approach suggests that, because the argument for enumerative induction (for example) proceeds

[19] Norton (2014), p. 672.
[20] See Norton (2014) and the Epilog (pp. 1-4) of Norton (2018 ms.) for full details concerning the "regress" objections.
[21] See Norton (2014). On p. 697, Norton identifies the first type of objection as one that "plays out in time." Then, on p. 681, he identifies the second type of objection as one that "plays out in the logical space of our present justifications."
[22] Norton (2018 ms. - Epilog), p. 2.
[23] Norton (2014), p. 674.

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necessarily upon universal rules (presented as rule T and rule P in previous sections), attempts to justify it are bound to fall prey to Hume's problem. Likewise, because the material theory instead avoids universal rules (by opting for reliance upon contingent facts), Norton believes that it slips out of the grips of Hume's problem.

Although the material theory's novelty appears to be its grounding in facts, the true delineator between the material theory and formal theories of induction is *not* its reliance upon facts. Inductive inference of any type relies upon facts. Formal theories quite clearly rely upon facts: the temporal version of enumerative induction presented earlier, for example, takes facts from the past as its input and yields an inference about a future potential fact. And the spatial version of enumerative induction, with cosmology as its exemplar instantiation, takes facts from the present location and yields an inference about a potential fact from an unexplored location. The material theory's uniqueness manifests not in its reliance upon facts, but upon its eschewing of all universal schemas.

But eschewing universal schemas comes at a cost. The material theory lacks explicit denotation of the "limited domain in which the warranting facts are true," which obfuscates the material theory's reach. More specifically, the material theory fails to include details that identify the *range* into which the material theory licenses inferences from its *domain* of facts. This role in formal theories is played by the universal schema. Rule T, for example, explicitly specifies that the range for the inductive inference extends into the set of future potential facts. The very purpose for including a universal inference rule is to connect the domain of facts to the range of inferences licensed by those facts. The material theory's avoidance of universal schemas leaves unspecified the range into which inferences are licensed from a given domain of facts.

Norton offers clues about the intended range of inferences authorized by the material theory. He notes that the material theory is ampliative: "From the premise that all past winters have been snowy in the same location, we infer inductively that the next winter will be snowy there."[24] This indicates that the range of inference lies beyond the domain of facts itself. Additionally, however, he notes that

> "all induction is local. It is contextual. The chemical facts that authorize these inductive inference [sic] are truths of a particular domain of chemistry. They warrant a local mini-logic, peculiar to the context, in which evidence of chemical similarity and of a few samples warrants the generalizations indicated. This local mini-logic resembles the universal schema of enumerative induction. But the resemblance is superficial. There will, no doubt, be other domains in which other facts will warrant inferences that also resemble enumerative induction. The inferences of each domain will be distinct, carrying

[24] Norton (2018 ms. – Chapter 2), p. 8.

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> their own unique restrictions that do not derive from universal schema, and bearing their own unique form of inductive risk."[25]

This latter passage suggests that the range of inferences reflects back onto the domain itself, in a local self-contained "mini-logic." On one hand, Norton appears to indicate that the range of inductive inference for the material theory extends beyond the domain of material facts, and on the other hand, he appears to suggest that the range of inductive inference is contained within the local contexts of the domain of material facts themselves.[26]

Investigating spatial inference from the perspective of the material theory makes tangible the "range" concern. Let us try to instantiate the content of the quotes from the previous paragraph into a consistent depiction of the material theory. First, let us assume some local place p_0. Further, for the sake of charitable argumentation, let p_0 exhibit some extent, to the effect that it captures Norton's intent when he describes the material theory operating "locally," "contextually," and as a "domain" with "unique restrictions." In order to accommodate the material theory being ampliative over p_0, "ampliative" must be interpreted to mean either

(a) Inductive inferences licensed by the material theory extend from a domain of facts within p_0 to a range of other potential facts which would extend p_0 to p_{0+}, where p_{0+} occupies the very same local place as p_0, or
(b) Inductive inferences licensed by the material theory extend from a domain of facts within p_0 to a range of other potential facts which would extend p_0 to p_1, where p_1 is a local place that wholly contains p_0, but is not coextensive with p_0.

Interpretation (a) essentially limits inductive inference to a predefined spatial region, within which the material theory permits inferences based upon the factual content of that spatial region. Interpretation (b) permits inferences to extend beyond the predefined spatial region, with such inferences being licensed by the factual content of the initial predefined spatial region. Simple examples of inferences of each of these options would be (a) inferring that some uninspected sample of salt A within p_0 has crystallographic form B, because inspected samples of salt A within p_0 have crystallographic form B, or (b) inferring that the large-scale matter distribution in p_1 is homogeneous, because the large-scale matter distribution in p_0 is homogeneous.[27] Interpretation (a) and interpretation (b) each present problematic scenarios for the material theory.

[25] Norton (2018 ms. – Chapter 1), p. 23.
[26] Of note is that Norton even uses the word "domain" itself several times in the longer quoted passage. The point of contention is the ambiguity of this usage, since explicit discussion of the domain and range of inductive inference is not present in Norton's published accounts of the material theory.
[27] These examples arise directly within Norton's discussion of the material theory: for (a), see Norton (2018 ms. – Chapter 1), and for (b) see Norton (2018 ms. – Chapter 2).

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Interpretation (a)

Interpretation (a), conceptually, matches a multiverse scenario wherein p_0 and p_{0+} represent the spatial extent of a single miniverse. Within these restrictions, the material theory only licenses inferences from the "here" domain into a range that also remains "here"; no inferences into the "there" range are licensed, because we have no warranting facts about "there." In and of itself this appears to be innocuous, and perhaps even Norton's intended meaning for "ampliative" in terms of matching his description of a "local mini-logic." Indeed, Norton notes that the warranting facts that license inferences "hold true only in limited domains, so that there are many of them and the inductive logic each warrants has local applicability only."[28]

However, pursuing this interpretation of "ampliative" quickly results in the realization that the material theory loses all of the reach that is typically associated with inference. To see why, simply reconfigure interpretation (a) for the temporal variety, wherein the amplification of inferences temporally involves licensing inferences from t_0 to t_{0+}, where t_0 and t_{0+} are co-temporal and represent time up to t_0. When looking at the temporal aspect of this interpretation of "ampliative," the material theory now only licenses inferences that remain within the "has been" category; no inferences about "will continue to be" are licensed, because we have no warranting facts to connect "has been" to "will continue to be."

For example, from the domain of the present year of 2019, the material theory would permit the inference that the winter of 1986 was snowy, if we had no record of it but we did have record of 1983, 1984, 1985, 1987, 1988, and 1989 being snowy winters. This is so because under interpretation (a) of "ampliative," the range for the inductive inference extends to 2019. But we would not be licensed to infer that the winter of 2020 will be snowy, because that reaches beyond the temporal bound of 2019, thereby extending beyond the range of licensed inferences.

This result renders the material theory rather inert; it takes away all predictive power in both the temporal and spatial dimensions. The isolation of temporal and spatial induction from each other removes of the confounds that come with conflating the two, and placing the material theory under inspection of this sort makes clear that the material theory loses a substantial portion of its effectiveness under interpretation (a) of "ampliative."

Interpretation (b)

Interpretation (b) of "ampliative" involves much more freedom, since the inferences licensed by the material theory would be unrestricted by a predefined and static spatial plot. This freedom helps escape the inertness that resulted from interpretation (a). However, it is unclear how the ampliative nature of the material theory in this setting is any different from the ampliative nature of the formal theories that Norton tries to create distance from. Specifically,

[28] Norton (2018 ms. – Chapter 2), p. 2.

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interpretation (b) appears to lead to the material theory's "warranting facts" requiring the assistance of a universal schema in order for induction to proceed.

To see why, we first recall that the universal schemas in formal accounts of induction serve to connect gathered facts to some inferred potential fact. Rule P, for example, connects facts about observed places *A* being *B* to the potential fact that an unobserved place *A* is *B*. Rule P is what licenses the inference. In the material theory, warranting facts replace the universal schema: "In a material theory, the warranting fact is, by supposition, distinct from the conclusion of the inductive inference."[29] It takes the place of rule T or rule P in standard enumerative induction; that is the only role left after the premises (gathered facts) and the conclusion (the inference) are stricken out as candidates. The warranting facts are the connection between the background facts and the inference.

Next, we note that the warranting facts arise strictly within the domain of an inference – that is, strictly within the set of facts that are non-inferences themselves. Norton describes this as follows: "[inductive inferences] are only good in so far as the inferences are carried out in domains that are factually hospitable to the inferences. The facts that make the domain hospitable are the facts that warrant the inference." With respect to the *range* of inductive inferences, the preceding establishes that in the material theory, the warranting facts cannot subsist strictly within the range. They must be part of the domain of facts *from which* inductive inferences arise. In the present inspection of interpretation (b), this means that the warranting facts must be contained within p_0. When we combine this with a portion of the material theory discussed in the inspection of interpretation (a) – specifically, that the inductive logic arising from warranting facts has "local applicability only" – we see that the warranting facts both arise in, and apply to, only the domain of inference (p_0 in the present setting).

But now, under interpretation (b) of "ampliative," the supposition is that the warranting facts of p_0 license inferences about potential facts in p_1. Norton explains that "In a formal theory of induction, that assessment of the risk becomes an assessment of the reliability of the inference schema used…In a material theory of induction, things are quite different. The warrant for an induction is a fact and we assess and then control the inductive risk by exploring and developing the fact."[30] Since we have established that the warranting facts apply only to p_0, some further connector is needed in order to reach into a range that includes p_1. The existing facts – the warranting facts – from p_0 cannot license inferences into a range that involves p_1. Something beyond those warranting facts from p_0 must connect the domain (p_0) with the range (p_1) for the inference to proceed. This can only be some sort of inductive rule overlaid upon the facts of p_0, something to the effect of Rule M:

M: The material facts within p_0 warrant inference to potential facts within p_1.

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[29] Norton (2014), p. 676.
[30] Norton (2018 ms. – Chapter 1), p. 24.

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Rule M looks suspiciously like rules T and P. Specifically, rule M itself is not a material fact, which means that we cannot generate justification for it within the material theory itself via the "scaffolding and archway" processes that Norton describes as the means by which the material theory gives rise to the totality of our scientific propositions.[31] Rule M is a universal schema. If the material theory is to be "ampliative" in the sense of interpretation (b), it requires something like rule M; it requires a universal schema. Alas, including such a universal schema would make it susceptible to Hume's problem.[32]

The challenge that remains for Norton's development of the material theory is to articulate precisely the domain and the range involved in inferences licensed by the material theory. Synthesizing an account that provides clear definitions for the domain and range of inference, while simultaneously permitting the material theory to be ampliative yet unsusceptible to Hume's problem, would set the material theory of induction far apart from its (potentially) lesser competitors.

7. The Legacy of the Problem. Beyond Norton's material theory of induction, there are several other common dismissals of Hume's result.[33] The attitude uniting most of those dismissals is perhaps best captured with the "No Miracles" response: Hume's problem actually is not a problem. The fact that science works so remarkably well is justification enough for the continued use of enumerative induction (or other forms of inductive inference). Correspondingly, the objection to the present work could be that cosmologists do not fare any worse than other scientists in their use of induction. Put another way, the same old philosophical problem should not warrant further discussion.

The response, as revealed by the foregoing sections, is twofold. First, in some subfields, science is fast approaching a state in which Hume's lesson cannot be ignored. Multiverse theorists are grappling with cosmological induction in a novel setting where Hume's problem emerges far sooner. It is not unreasonable to expect other scientific disciplines to encounter novel manifestations of Hume's problem as well.

Second, the recalcitrance of Hume's problem is such that it demands fresh attention. Decades ago, Salmon made the same request:

> I, too, have faith that the scientific method is especially well suited for establishing knowledge of the unobserved, but I believe this faith should be justified. It seems to me

[31] See Norton (2014) pp.685-688 and Norton (2018 ms. – Epilog) for details of the "archway" example.
[32] Perhaps even more consequential would be discerning whether the material theory is in fact material, rather than formal, in the assessment of interpretation (b) holds. But that discussion is postponed to a later time, since the sketch presented here offers other possible escapes for the material theory beyond interpretation (b).
[33] See Howson (2003), pp. 15-19 for a very efficient list of the most common. Howson's structure follows the template laid forth on pp. 12-54 of Salmon (1966).

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> extremely important that some people should earnestly seek a solution to this problem concerning the foundations of scientific inference.[34]

The explicit extension of the problem from the temporal dimension to the spatial dimension is meant as a contribution to Salmon's cause. Perhaps it will elicit new perspectives on the problem both from scientists and from philosophers of science. The emergence of the contemporary multiverse hypothesis signals, perhaps for the first time, the need for scientists to address the problem directly. The history of science tells us of scientists—and even Hume himself—neglecting the problem in favor of pursuing further confirmation for their inductive inferences. Multiverse cosmologists cannot do likewise. This makes Salmon's words even more relevant now than they were when he published them almost a half century ago.

[34] Salmon (1966), p. 56.

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