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Uncertainty Associated with the Sediment Mobility Tool

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PURPOSE: The goal of this Coastal and Hydraulics Engineering Technical Note (CHETN) is to quantify the uncertainty associated with the forcing and sediment movement in the Sediment Mobility Tool (SMT).

INTRODUCTION: The beneficial use of dredged sediment through placement in the nearshore is a common practice, but answers to key questions about whether the sediment will move and where it is likely to go, remain challenging. The SMT was developed to assist coastal engineers and planners to site nearshore placement areas. SMT is a simple web application that can rapidly produce a preliminary assessment of how often sediment placed in the nearshore will be mobilized and the direction of sediment transport. Currently, the web application can be found on the Navigation Portal (*http://navigation.usace.army.mil*). The sediment mobility calculations used in the web application are detailed in *Evaluating Sediment Mobility for Siting Nearshore Berms* technical note by McFall et al. (2016), and the depth of closure calculations are detailed in *Calculating Depth of Closure Using WIS Hindcast Data* technical note by Brutsché et al. (2016). This technical note addresses the aleatory and epistemic uncertainties from the forcing and sediment movement in this model.

SMT PROCEDURE: To quantify the uncertainties associated with the SMT, the procedure of the model and potential sources of uncertainties with each step are identified.

1. **User Input**
   The user draws the shoreline and selects the proposed placement site. The web application uses shoreline angle to calculate the wave refraction during the wave transformation. The selected placement location allows the application to know which side of the shoreline is water and is used to determine the closest Wave Information Study (WIS) hindcast station.

   The user then inputs the median grain size (d_{50}) of the sediment to be placed, nearshore placement depth, longshore current 1 meter (m) above the bed, water temperature, and water salinity. The sediment mobility is calculated for a range of sediment grain sizes, but by allowing the user to define the median grain size, the mobility results are also calculated for that specific sediment size. The nearshore placement depth does not account for a mound or bar created by the nearshore placed sediment, nor does it account for tidal variations. Sediment mobility is calculated for a 10-year span; therefore, tidal depth variations are averaged out by the time span. The water temperature and salinity are used in the water density and viscosity calcuations.

   Sources of uncertainties in this step are related to user-provided information include drawing of the shoreline, estimating the longshore current, the nearshore placement depth, water temperature, and water salinity. These uncertainties can vary based on the user and can be potentially reduced by the user. Taken in the aggregate, the uncertainties pertaining to these user-provided inputs can be considered aleatory. The epistemic uncertainty in this step comes from the WIS hindcast of the offshore wave characteristics. It constitutes the main forcing parameter for the computation of the frequency of sediment mobility, and its uncertainty was considered herein.

2. **Wave Transformation**
The WIS wave hindcasts for 10 years (1 January 1990–1 January 2000) from the closest WIS station are transformed to the nearshore placement site. Only shoreward directed waves are transformed. The measured wave direction and height from the WIS station are transformed from offshore to the nearshore region using conservation of energy flux and Snell's Law. Shore parallel contours are assumed for the wave angle calculation. Although the shore parallel contours assumption is a commonly made assumption when bathymetric data are not available, this assumption constitutes a source of uncertainty.

3. **Sediment Mobility – Linear Wave Theory**
The sediment mobility is calculated two ways: linear wave theory and stream-function wave theory. The linear wave theory method uses the critical bed shear stress (τ_{cr}) to calculate the sediment mobility. The critical shear stress is estimated following a procedure given by Soulsby (1997) and Soulsby and Whitehouse (1997) as

$$D_* = d_{50} \left(\frac{g \left({}^{\rho_s}\!/_{\rho} - 1 \right)}{\nu^2} \right)^{1/3} \tag{1}$$

$$\theta_{cr} = \frac{0.30}{1 + 1.2\, D_*} + 0.55[1 - \exp(-0.020\, D_*)] \tag{2}$$

and

$$\tau_{cr} = \theta_{cr}\, g\, (\rho_s - \rho)\, d_{50} \tag{3}$$

where D_* is the dimensionless grain size, g is the gravitational acceleration, ρ_s is the sediment density, ρ is the water density, ν is the kinematic viscosity, θ_{cr} is the critical Shields parameter, and τ_{cr} is the critical shear stress. The critical shear stress is the threshold stress for which the sediment can be expected to be dislodged from the seabed for all greater shear stresses.

The maximum shear stress (τ_{max}) accounts for the wave- and current-induced shear stresses and is calculated as

$$\tau_m = \tau_c \left[1 + 1.2 \left(\frac{\tau_w}{\tau_c + \tau_w} \right)^{3.2} \right] \tag{4}$$

and

$$\tau_{max} = [(\tau_m + \tau_w \cos \phi)^2 + (\tau_w \sin \phi)^2]^{1/2} \tag{5}$$

where τ_m is the mean bed shear stress, τ_c is the current induced shear stress, τ_w is the wave induced shear stress, and ϕ is the angle between the wave and current directions. Additional equations used to calculate the noted variables are detailed in McFall et al. (2016).

Using linear wave theory, the magnitude of the wave orbital velocity is equal under the wave crest and trough. Waves become more asymmetric in the nearshore, but the aleatory uncertainty associated with the wave asymmetry is site specific and is not quantified in this report.

4. **Sediment Mobility – Stream-Function Wave Theory**

The second method used to calculate sediment mobility uses stream-function wave theory and the near-bottom velocity. The critical near-bottom velocity (u_{cr}) using nonlinear stream function wave theory is calculated with a procedure given by Ahrens and Hands (1998), which is based on research by Hallermeier (1980), as

$$u_{cr} = \sqrt{8\, g\, \gamma\, d_{50}} \tag{6}$$

for $d_{50} \leq 2.0\, mm$ and $\gamma = (\rho_s - \rho)/\rho$, where ρ_s is the sediment density, and ρ is the water density. Ahrens and Hands (1998) used Dean's (1974) stream function wave theory table (SFWT) to derive the following equations for the near-bottom wave-induced velocity from the wave crest ($u_{\max crest}$) and trough ($u_{\max trough}$) as

$$u_{\max crest} = \left(\frac{H}{T}\right)\left(\frac{h}{L_o}\right)^{-0.579} \exp\left[0.289 - 0.491\left(\frac{H}{h}\right) - 2.97\left(\frac{h}{L_o}\right)\right] \tag{7}$$

and

$$u_{\max trough} = -\left(\frac{H}{T}\right) \exp\left[1.966 - 6.70\left(\frac{h}{L_o}\right) - 1.73\left(\frac{H}{h}\right) + 5.58\left(\frac{H}{L_o}\right)\right] \tag{8}$$

where h is the water depth, H is the significant wave height, T is the peak wave period, and L_0 is the offshore wave length given by $L_0 = (g\, T^2)/2\, \pi$. The maximum near-bottom velocity was taken as $u_{max} = \max(|u_{\max crest}|, |u_{\max trough}|)$.

The epistemic uncertainty of the critical near-bottom velocity is discussed by Hallermeier (1980) and Ahrens and Hands (1998), and the uncertainty of the Equations 7 and 8 compared to the SFWT is discussed by Ahrens and Hands (1998). These predicted uncertainties are detailed in the subsequent section that quantifies the uncertainty associated with this method.

CASE STUDY SITE – VILANO BEACH, FLORIDA: To understand the cumulative uncertainty associated with the sediment mobility in the SMT, a previously modeled nearshore placement project at Vilano Beach, Florida, is used as a case study site. Two nearshore berms were constructed in an approximate depth of 3 m at Vilano Beach during the summer of 2015 (McFall et al. 2017), and the nearshore berms were modeled with the SMT and the Coastal Modeling System (CMS) (Brutsché et al. 2017). The previous modeling efforts at this site make it an ideal case study. The uncertainty of the frequency of sediment mobility is analyzed at this site because it is a key result of the SMT to assist coastal engineers and planners in understanding the temporal scale of how rapidly sediment placed in the nearshore will move.

UNCERTAINTY WITH MODEL INPUT: The uncertainty associated with user input parameters can be considered aleatory, and it is dependent on the user and the available data for a site.

Quantification of this uncertainty is outside the scope of the current study. The wave climate data extracted from the closest WIS station to a particular site constitutes the main forcing input that can impact the frequency of sediment mobility. The epistemic uncertainty of WIS was assessed by comparing observed measurements with the modeled hindcast values. Wave hindcasts for WIS stations in the Atlantic Ocean are computed with the WAVEWATCH III third-generation wave model (Tolman 2014). Statistical metrics that have been applied to WIS data to quantitatively evaluate model performance are discussed in Bryant et al. (2016). Bias, root mean-squared error (RMSE), and scatter index metrics were found useful to assess the uncertainty of the SMT. The RMSE for WIS was corrected for bias, which can be considered equivalent to the standard deviation of the difference between model predictions and measured observations or uncertainty. The scatter index (SI) was calculated by dividing the RMSE by the mean of the observed values. It represents the percentage of expected error of the model output. The study presented a test case for a location in the Great Lakes and reported values of 0.09 m, 0.28 m, and 42.73% for bias, RMSE, and SI, respectively. Since these metrics vary by location but were not available for the case study site used herein, they were considered instructive for the selection of uncertainty values for the current analysis. An uncertainty value of 0.25 m and error of 30% were adopted for the quantification of uncertainty associated with the SMT at the case study site.

UNCERTAINTY WITH WAVE TRANSFORMATION: The error induced by the wave transformation from the offshore to the nearshore site was estimated also using the case study site at Vilano Beach, Florida. Offshore wave hindcasts were transformed into the nearshore placement site using a bathymetric grid in CMS. The same wave hindcasts were transformed into the nearshore placement site using the shore parallel contours assumption of the SMT. The CMS-transformed waves were considered to be the true value for the estimation of the bias and uncertainty. The computed bias associated with the SMT for the significant wave height was 0.21 m while the uncertainty was 0.16 m.

UNCERTAINTY WITH SEDIMENT MOBILITY-LINEAR WAVE: The epistemic uncertainty of critical bed shear stress was quantified for quartz sand. The uncertainty associated with the bed shear stress is dependent on the wave-induced near-bottom velocity using linear wave theory. Aleatory uncertainty associated with true waves becoming asymmetric in the nearshore is site specific and is not included in this error analysis. However, it was possible to estimate the uncertainty associated to the computation of the critical shields parameter (Equation 2) by computing the error as the difference between the predicted and measured wave and current data presented in Soulsby and Whitehouse (1997) and calculating the standard deviation. The uncertainty ($\sigma_{\theta cr}$) of this critical threshold to induce sediment motion is calculated to be 0.017 m. This uncertainty was incorporated in the computation of the critical shear stress (Equation 3) by using a Monte Carlo approach where the θ_{cr} parameter was randomly sampled assuming a normal distribution with standard deviation $\sigma_{\theta cr}$. The uncertainty associated with θ_{cr} for combined waves and currents is presented, along with its mean, in Figure 1 over the range of $0.1 \leq D_* \leq 1000$ in the form of confidence intervals (CI) curves of 95%, 80%, and 68%.

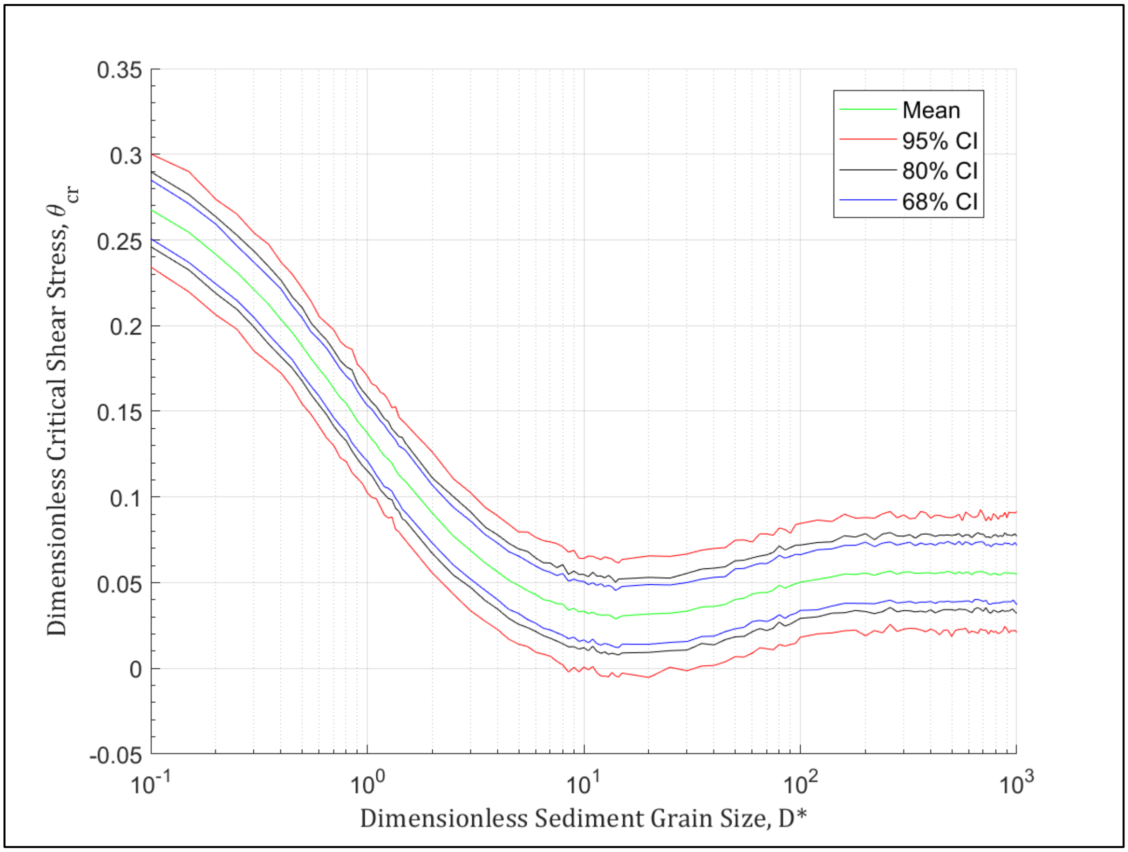

Figure 1. Confidence intervals for the dimensionless critical shear stress.

UNCERTAINTY WITH SEDIMENT MOBILITY-STREAM-FUNCTION WAVE: Hallermeier (1980) provides an error analysis for his entire method, not just the critical near-bottom velocity in Equation 6, but does note that the "Predicted critical velocities between 10 cm/s and 50 cm/s show ±10% agreement with a majority of test results for a level bed of quartz sand in oscillatory flow." Ahrens and Hands (1998) noted that with an assumed Gaussian distribution and literal interpretation of Hallermeier's statement suggest errors of approximately ±28% at the 95% confidence interval (CI). From this information, the uncertainty of the critical near-bottom velocity can be estimated as follows:

$$\sigma_{ucr} = u_{cr}\left(\frac{0.28}{1.96}\right) \tag{9}$$

where u_{cr} is the output of Equation 6 and 1.96 is the z-score corresponding to the upper limit of the 95% CI. The uncertainty was incorporated using a Monte Carlo approach where the output of the critical near-bottom velocity in Equation 6 was taken as the mean and modified by random sampling using the uncertainty, σ_{ucr} as follows:

$$U_{cr} = u_{cr} + \sigma_{ucr} Z_r \tag{10}$$

where $U_{cr}$is the sampled critical near bottom velocity for a particular iteration and Z_r is the randomly sampled z-score.

Ahrens and Hands (1998) conducted an error analysis of the predicted near-bottom velocity under the wave crest using Equation 7 to the SWFT and found a r^2 correlation coefficient of 0.99, N = 28, max error = 6.0%, and RMSE = 3.5%. For under the wave trough using Equation 8, he noted r^2 = 0.982, N = 28, max error = 11.1%, and RMSE = 5.7%. These fixed errors are within the range of the site-specific forcing uncertainty used in this study.

CUMULATIVE UNCERTAINTY: The sources of forcing and parameter uncertainties previously described were combined through a Monte Carlo procedure to assess the uncertainty of the SMT. The final product of the SMT is estimates of the frequency of sediment mobility (f_M) based on two methods: linear wave theory and stream-function wave theory. These estimates are provided for a number of predetermined sand grain size diameters of 0.1, 0.2, 0.3, 0.4, and 0.5 millimeters (mm), and the case study's median grain size of 0.33 mm, for which the critical shear stress and critical velocity are calculated. The f_M is calculated by estimating the fraction of WIS observations over a 10-year period that exceed the critical values.

Two approaches were considered for quantifying the wave forcing uncertainty in the model. One approach was to compute a total uncertainty by combining the uncertainty associated with WIS and the uncertainty associated with the wave transformation. The combined uncertainty can be computed as

$$\sigma_{Total} = \sqrt{\sigma_{WIS} + \sigma_{Transf}} \tag{11}$$

The total uncertainty assuming $\sigma_{WIS} = 0.25$ m and $\sigma_{Transf} = 0.16$ m was 0.30 m. This combined uncertainty can then be used as the shape parameter of the normal distribution in the random sampling scheme used to incorporate uncertainty in the forcing (significant wave height). The main drawback of this approach is that the uncertainty is assumed constant, which for low wave height values would result in proportionally higher uncertainties.

Since the SMT analysis does not focus on extremes and considers a continuous record length, most waves are relatively small. Wave heights in the range of magnitude of the uncertainty value would be frequently sampling negative values whose correction would change the shape of the distribution. This first method was deemed unsatisfactory; thus, a second approach was used which accounts for uncertainty proportional to the wave height. The second method was the favored approach to compute the uncertainty associated with the frequency of mobility. A proportional uncertainty of 30% was used in this study for the wave height.

Wave forcing parameters H and T are used in linear-wave theory for establishing the bottom wave orbital velocity and consequently computing the wave-related shear stress $\tau_w$in Equation 5. The application of the wave forcing parameters in stream function wave theory can be observed in Equations 7 and 8. The random sampling scheme for the input parameters H, T, and wave direction consisted of first sampling a value of significant wave height H from a normal distribution with $\mu = H$ and $\sigma = 0.30\mu$. Normal distribution fits were then applied to the subsamples of the parameters T and wave direction in WIS data set corresponding to the value of the sampled H. Wave period and direction values were sampled from these distributions so they correlate to the sampled wave height. If the exact sampled H was not found within the historical record, a range was established around that value based on one standard deviation to obtain historical H values from which to obtain the subsamples of wave periods and directions. If no historical values were found within this range, the random sampling was repeated for a second time; however, if still no data were found, then the initial historical H value was used. Sampled H values were also maintained within the historical range of the WIS data.

The analysis also accounted for the uncertainty contributed by the critical thresholds, τ_{cr} and u_{cr}, in the computation of the frequency of mobility. For instance, a value of θ_{cr} is sampled randomly from $N(\overline{\theta_{cr}}, \sigma_{\theta cr})$ and is used to compute τ_{cr} at each sampling iteration, where $\overline{\theta_{cr}}$ is obtained from Equation 2. Also, a value of u_{cr} is sampled from $N(\overline{u_{cr}}, \sigma_{ucr})$, where $\overline{u_{cr}}$ is computed using Equation 6.

At the case study site of Vilano Beach, Florida, a collection of 10,000 $u_{\max crest}$ and τ_{max} values were generated in the Monte Carlo sampling procedure for each of the 87,590 nearshore wave height values transformed from the WIS station, making it possible to develop histograms that account for the uncertainty contributed by the wave forcing and the equations. This process was repeated for the six different sand particle sizes, including the median grain size of 0.33 mm, considered in the SMT at the case-study site, yielding statistics for the frequency of mobility. Figures 2 and 3 illustrate the mean value and the uncertainty associated with the frequency of mobility for each sediment grain size using both linear wave theory (bed shear stress) and stream-function wave theory (near-bed velocity). The mean corresponds to the frequency of mobility shown in the SMT. These values are also shown in Table 1.

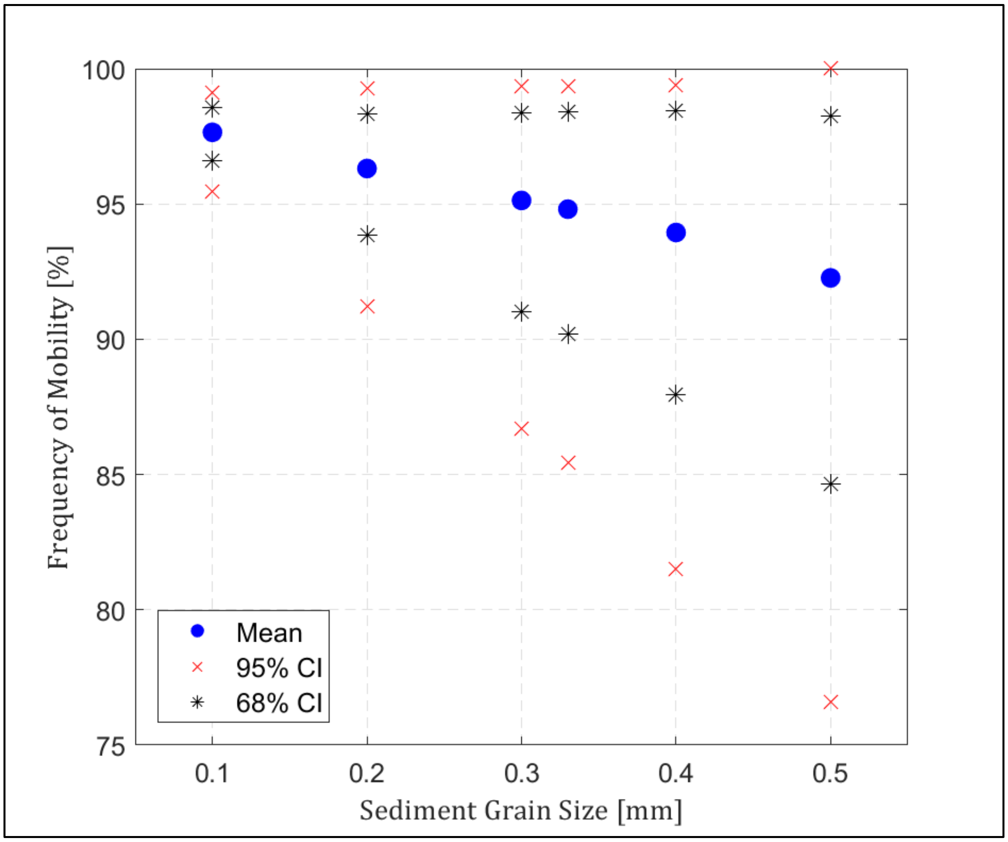

Figure 2. Confidence limits associated with the frequency of mobility using linear wave theory (bed shear stress) for several grain sizes. The mean corresponds to the frequency of mobility shown in the SMT.

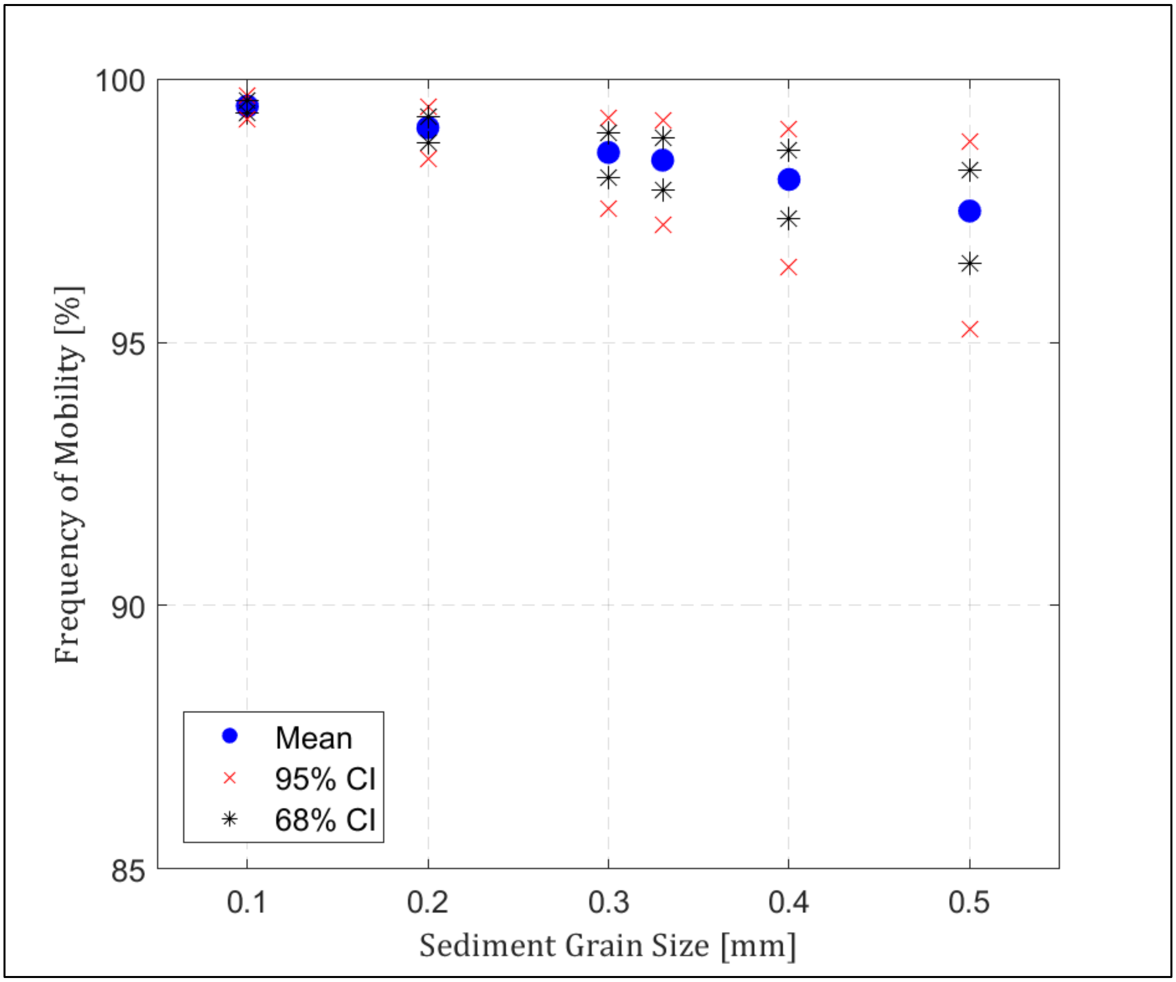

Figure 3. Confidence levels associated with the frequency of mobility using stream-function wave theory for several grain sizes.

Table 1. Confidence intervals for the frequency of mobility.

Grain size (mm)	Linear Wave Theory			Stream-Function Wave Theory		
	Mean	95% CI	68% CI	Mean	95% CI	68% CI
0.10	97.6%	95.5% - 99.1%	96.6% - 98.6%	99.5%	99.2% - 99.7%	99.4% - 99.6%
0.20	96.3%	91.2% - 99.3%	93.9% - 98.3%	99.1%	98.5% - 99.5%	98.8% - 99.3%
0.30	95.1%	86.7% - 99.3%	91.0% - 98.4%	98.6%	97.5% - 99.3%	98.1% - 99.0%
0.33	94.8%	85.4% - 99.4%	90.2% - 98.4%	98.5%	97.2% - 99.2%	97.9% - 98.9%
0.40	93.9%	81.5% - 99.4%	87.9% - 98.4%	98.1%	96.4% - 99.0%	97.3% - 98.6%
0.50	92.3%	76.6% - 100%	84.6% - 98.3%	97.5%	95.3% - 98.8%	96.5% - 98.3%

The mean frequency of mobility decreases with increased grain size diameter. This is expected as larger waves are required to mobilize larger sediments, which come less frequently. The uncertainty analysis results show that the uncertainty also increases with increase in grain size. The frequency of mobility calculated using linear wave theory (maximum bed shear stress) show less separation in the upper confidence limits compared to the lower confidence limits. This is likely due to the high mean frequency of mobility, resulting in upper confidence limits that approach the physical limit for frequency of mobility of 100%. The uncertainty associated with the frequency of mobility calculated using stream-function wave theory (near-bed velocity) is more balanced around the mean, and tends to have a narrower confidence interval (Figures 2 and 3).

SUMMARY: The epistemic uncertainty associated with the forcing and sediment movement in the SMT has been quantified. The uncertainty was analyzed using a case study site at Vilano Beach, Florida. The cumulative uncertainty from each step of the SMT has been accounted for in the confidence interval including the offshore wave conditions, wave transformation to the nearshore, critical thresholds for sediment motion, maximum bed shear stress, and maximum near-bottom velocity. A collection of 10,000 bed shear stress and near-bed velocities were generated using a Monte Carlo sampling procedure for each of the 87,590 nearshore wave height values.

Certain trends were visible in the resulting confidence intervals for the frequency of mobility. The range of the confidence interval increased with increased sediment grain size. This was consistent for both the 95% and 68% confidence intervals for both wave theories. The frequency of sediment mobility for the linear wave theory method (bed shear stress) tended to have an increased confidence interval spread compared to the stream-function method (near-bed velocity). Additionally, the upper confidence limit using the linear wave theory method showed less separation from the mean than the lower confidence limit. This is likely due to high frequencies of mobility and the physical restriction that the frequency of mobility cannot exceed 100%.

For the case study site's median grain size (d_{50}) of 0.33 mm using linear wave theory, the frequency of sediment mobility was 94.8% with 95% confidence limits of 85.4% and 99.4%. Using stream-function wave theory, the median grain size is estimated to be mobilized by 98.5% of the waves with 95% confidence limits of 97.2% and 99.2%. The maximum difference between the mean and a 95% confidence limit using linear wave theory was 15.7% and using stream-function wave theory was 2.2%. The limited epistemic uncertainty is shown with the narrow confidence intervals of the frequency of sediment mobility, which indicates the robustness of the SMT for preliminary engineering studies to site nearshore placement areas for dredged sediment.

ADDITIONAL INFORMATION: This CHETN was prepared as part of the US Army Corps of Engineers (USACE) Coastal Inlets Research Program (CIRP) and Regional Sediment Management (RSM) Program by Dr. Brian C. McFall, Mr. Victor M. Gonzalez, Mr. Efrain Ramos-Santiago, Dr. Norberto C. Nadal-Caraballo, and Dr. Katherine E. Brutsché, US Army Engineer Research and Development Center, Coastal and Hydraulics Laboratory, Vicksburg, MS. Questions pertaining to this CHETN may be directed to Dr. Brian McFall (*Brian.C.McFall@usace.army.mil*), the USACE CIRP Program Manager, Dr. Tanya M. Beck (*Tanya.M.Beck@usace.army.mil*), or the USACE RSM Program Manager, Dr. Katherine E. Brutsché (*Katherine.E.Brutsche@usace.army.mil*). Additional information regarding CIRP may be obtained from the CIRP website *http://cirp.usace.army.mil/,* and additional information about RSM can be obtained from the RSM website *http://rsm.usace.army.mil/*.

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