

# Software Rejuvenation For Secure Tracking Control Of Cyber-Physical Systems

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## Motivation

**Software rejuvenation (SR)** protects cyber-physical systems (CSPs) against **cyber attacks** on the run time code by **periodically refreshing** the system with an uncorrupted **software image**.

During SR, the system is in **open loop**, then this mechanism of protection may imply severe **issues** from the **control** perspective, such as **stability** and the inability to complete a mission (**tracking performances**).

## Goal

To propose a **secure tracking control scheme** based on **software rejuvenation** for nonlinear and linear systems and provide the general conditions that guarantee the property of **safety** and **liveness**.

## Introduction

### Software Engineering

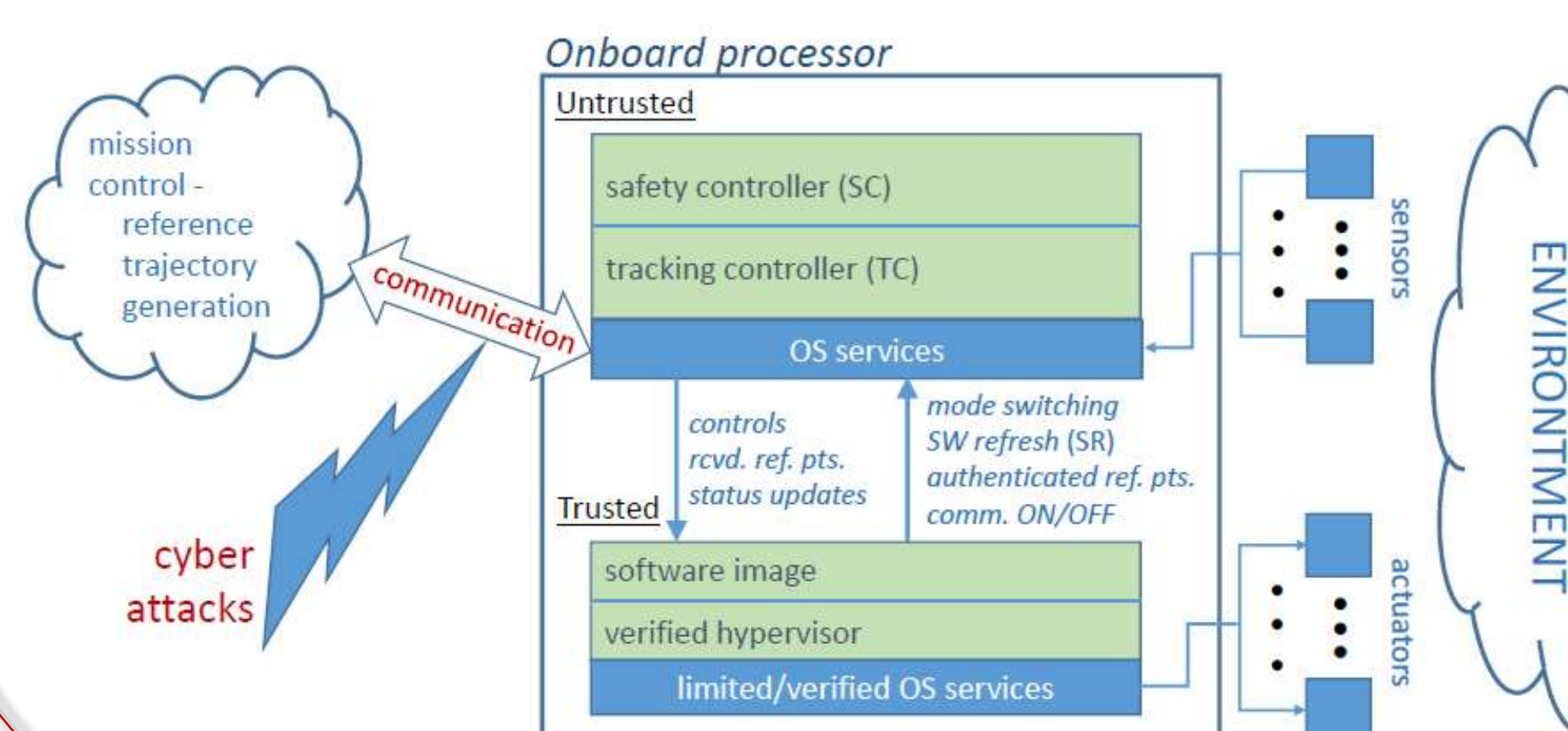
"...Software Rejuvenation is a periodic preemptive rollback of continuously running applications to prevent failures in the future." [Huang et al, 1995]

- I reboot;
- I restart the application from a clean internal state.

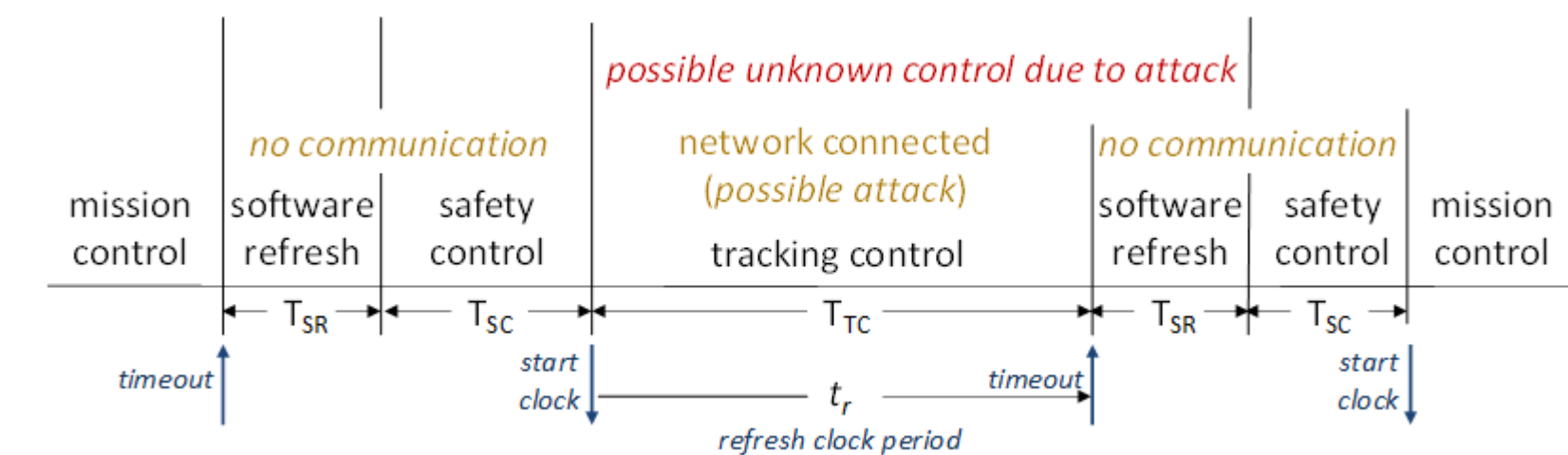
### Control System

- I Fault Tolerant Control;
- I Secure Control of CPS [Abdi et al 2018, Arroyo et al 2017]

## Attack Model and Architecture

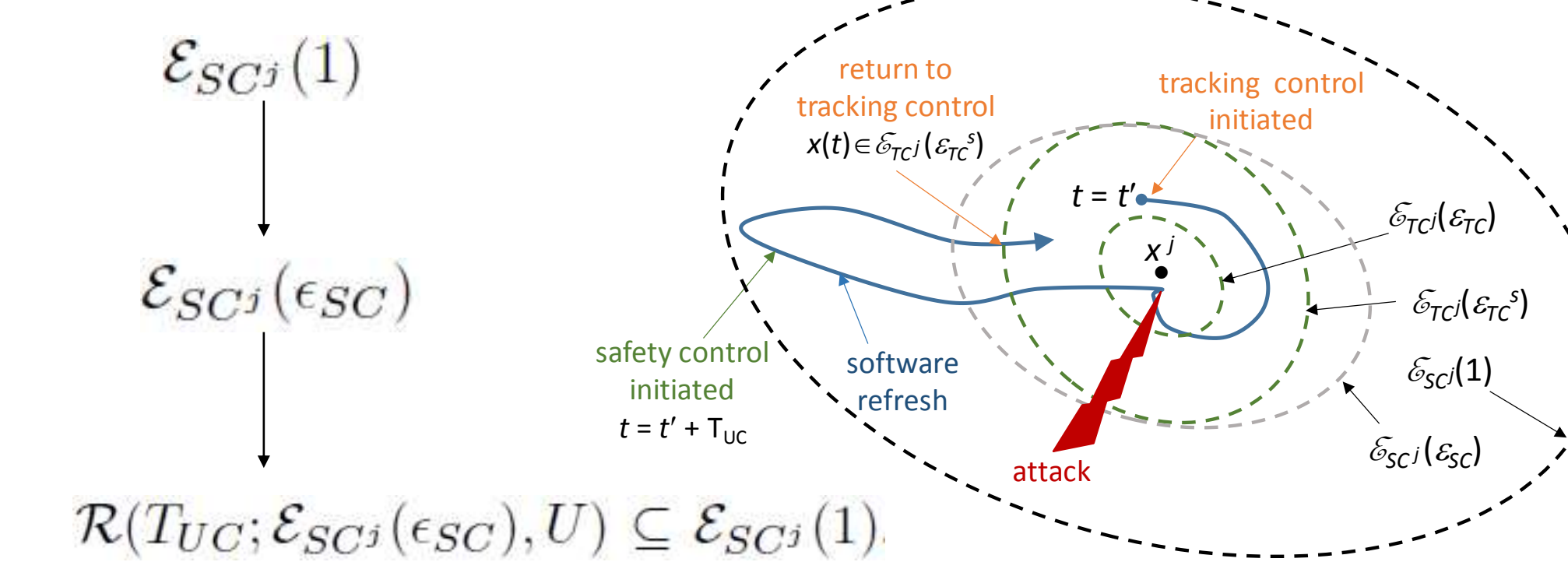


## How SR works



The refresh clock period has to guarantee that for any control input, the system cannot leave the safety set.

## Safety Control



## Lyapunov Functions and Invariant Sets

Controlled system:  $\dot{x} = f_\varphi(x) \triangleq f(x, \varphi(x))$ ,

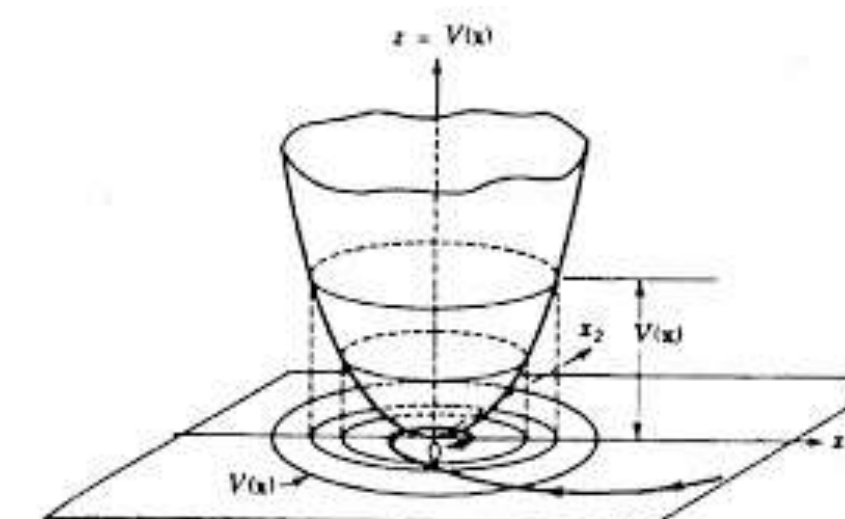
Lyapunov Function:  $V_\varphi: \mathbb{R}^n \rightarrow \mathbb{R}$   $\dot{V}_\varphi(x) = \frac{\partial V}{\partial x} \cdot f_\varphi(x) < 0$

Lyapunov level Set:  $\mathcal{E}_\varphi(\epsilon) = \{x \in \mathcal{N}_{V_\varphi}(x_{eq}) | V_\varphi(x) \leq \epsilon\}$

Positively Invariant Set:

$$\forall t > 0, \mathcal{R}(t; \mathcal{E}_\varphi(\epsilon), \varphi) \subseteq \mathcal{E}_\varphi(\epsilon).$$

$$0 < \epsilon' < \epsilon, \mathcal{E}_\varphi(\epsilon') \subseteq \mathcal{E}_\varphi(\epsilon)$$



## Two Fundamental Propositions

**Proposition 2.1:** Given system (1) with stabilizing controller  $\varphi$  for equilibrium state  $(x_{eq}, \varphi(x_{eq}))$  and Lyapunov function  $V_\varphi(x)$  as defined above, given  $\epsilon > 0$  for any  $\epsilon < \epsilon' \leq 1 \exists \gamma > 0 \exists \forall t \geq (\epsilon' - \epsilon)\gamma^{-1}$ ,

$$\mathcal{R}(t; \mathcal{E}_\varphi(\epsilon'), \varphi) \subseteq \mathcal{E}_\varphi(\epsilon). \quad (10)$$

**Proposition 2.2:** For any  $U \subseteq \mathcal{U}$  and any  $0 < \epsilon < \epsilon' \leq 1$ ,  $\exists T_U > 0 \exists \mathcal{R}(t; \mathcal{E}_\varphi(\epsilon), U) \subseteq \mathcal{E}_\varphi(\epsilon') \forall t < T_U$ .

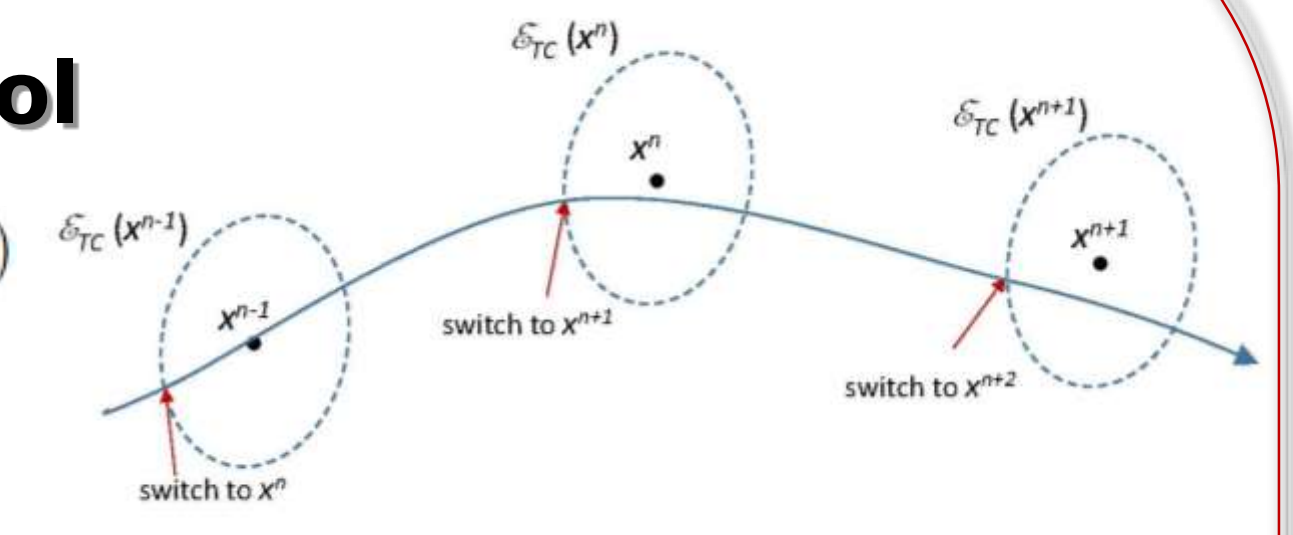
## Tracking Control

$$\mathcal{E}_{TC^j}(\epsilon_{TC}^s) \subseteq \mathcal{E}_{SC^j}(\epsilon_{SC})$$

sequence of reference points

$$x^1, \dots, x^J$$

$$\mathcal{E}_{TC^j}(\epsilon_{TC}) \subset \mathcal{E}_{TC^{j+1}}(\epsilon'), \exists t^{j+1} > 0 \exists \forall t \geq t_{j+1}, \mathcal{R}(t; \mathcal{E}_{TC^j}(\epsilon_{TC}), TC^{j+1}) \subset \mathcal{E}_{TC^{j+1}}(\epsilon_{TC}).$$



## Safety and Liveness

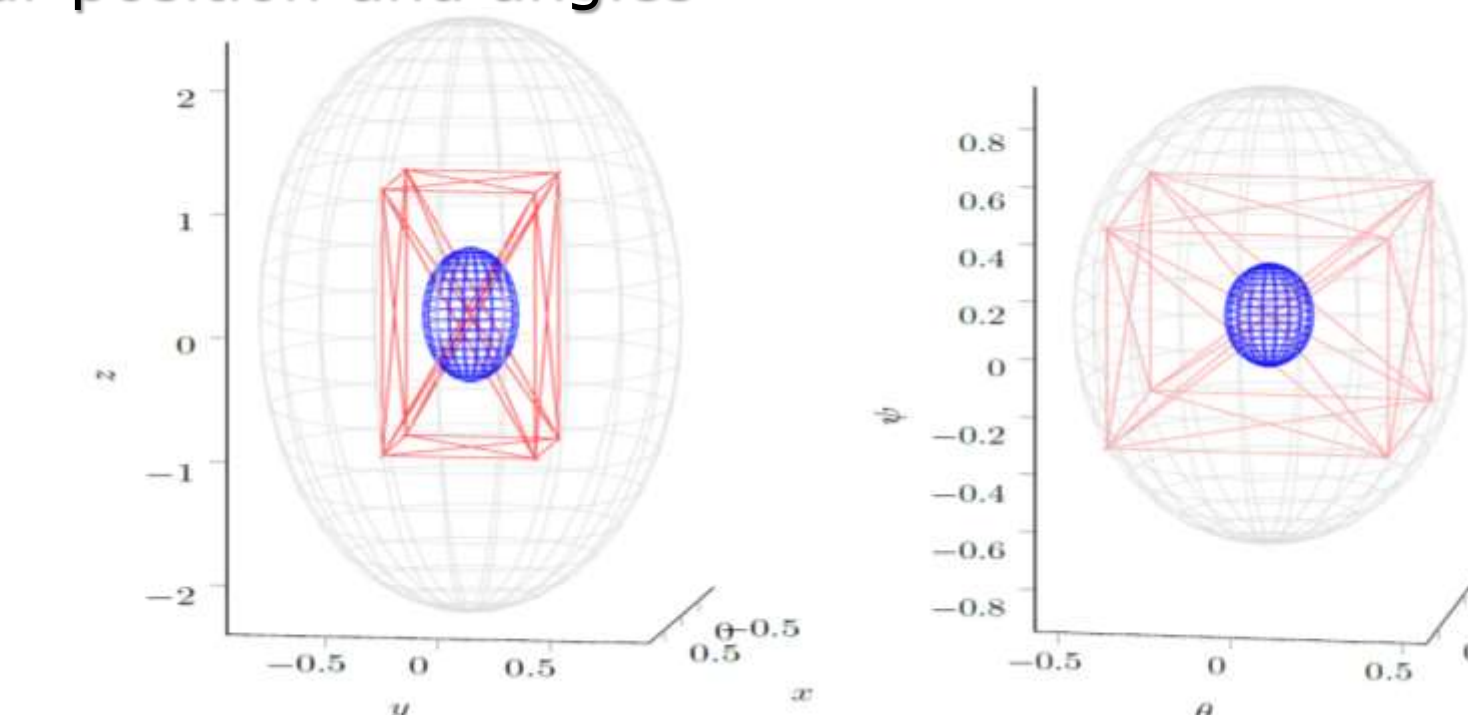
- **safety:** when  $x^j \rightarrow x^{j+1}$ , the system has to be in  $\mathcal{E}_{TC^{j+1}}(\epsilon_{TC}^s)$ ;
- **liveness:** in presence of software refresh, the tracking controller has to drive the system to  $\mathcal{E}_{TC^j}(\epsilon_{TC})$ .

## Example: quadrotor

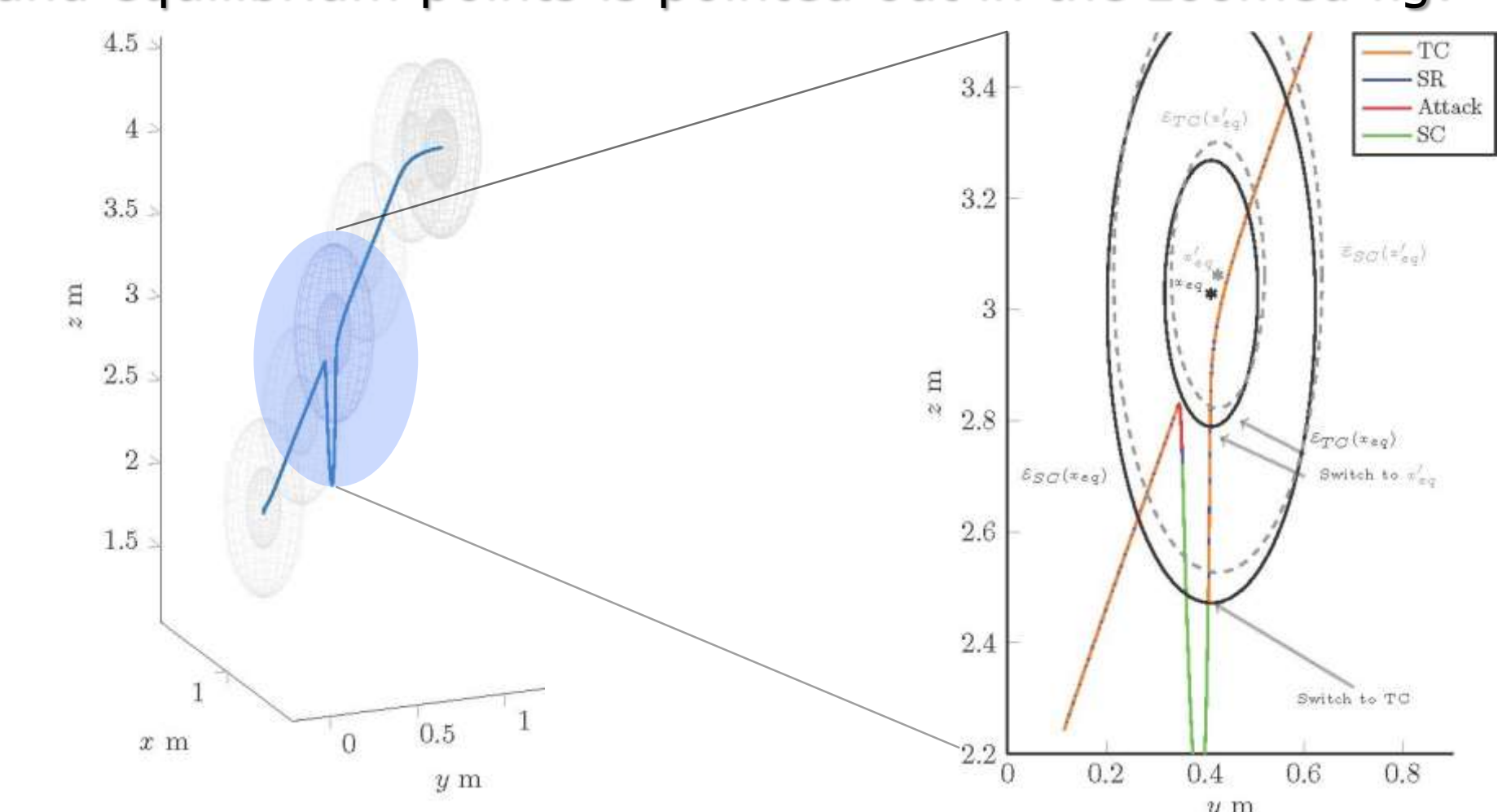
- 6 DOF quadrotor using the PX4 jMAVSIM quadrotor simulator
- Linearized model



Safety sets and Reachability Analysis for quadrotor's linear position and angles



Simulation results in presence of an attack. Details about the switching between the several controllers and equilibrium points is pointed out in the zoomed fig.



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