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14. ABSTRACT
The Secretary Problem (SP) is an abstraction of an optimal stopping problem often framed as a decision maker (DM) choosing from a queue of applicants. The "applicants" may represent interviewees, prospective spouses, sensor readings, food sources, or military targets. In its simplest form, a decision maker observes applicants one at a time and must choose a point at which to select a single applicant based only on the relative ranks of the applicants observed up to that point; payoff may be based on maximizing the probability of selecting the best applicant or minimizing the expected rank of the selected applicant. This paper develops methods for a cheating applicant to find advantageous positions in the queue that either improve the applicant's probability of being selected or aid in evading selection. A variant of the SP is explored that allows for non-selection of a poor applicant. It is demonstrated that a statistical test of hypothesis is ineffective for detecting this means of deception, so the advantage is on the side of the cheating applicant.

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Improving a Cheating Applicant's Chances In the Secretary Problem

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Abstract

The Secretary Problem (SP) is an abstraction of an optimal stopping problem often framed as a decision maker choosing from a queue of applicants. In its classical and simplest form, a decision maker (DM) observes applicants one at a time and assesses the relative ranks of the applicants based only on knowledge of the applicants observed up to that point with no information about applicants yet to be presented; the DM must choose a point at which to select a single applicant or be forced to select the last applicant. SP solutions are generally based on a payoff function that involves either maximizing the probability of selecting the lowest-rank applicant or minimizing the expected relative rank of the selected applicant.

This paper extends work by D.B. Glass to determine how a cheating applicant can find advantageous positions in the queue that improve the applicant's probability of being selected. This paper analyzes the properties of the classical two-phase solution for the SP in the context of cheating. Exact and approximate methods are developed for finding advantageous positions for the classical two-phase solutions for the SP.

This paper explores how a cheating applicant can use this information to improve its chances of either being selected or evading selection. Strategies are explored for the case where a cheating applicant has only approximate knowledge of its own rank.

Cheating strategies are developed for an extremely efficient solution by Chow et al. as well as for a new variant of the SP that allows for non-selection of any applicant rather than be forced to select a poor applicant.

It is demonstrated that a statistical test of hypothesis is ineffective for detecting cheating, which means the advantage is on the side of the cheating applicant. However, a decision maker could potentially use knowledge of the advantageous positions to identify and counter cheating activity when a cheating applicant attempts to maneuver into positions in the queue that either increase or decrease its probability of selection.

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Due to the large number of figures and tables, they are provided at the ends of sections for readability of the text.

I. Introduction

The so-called **Secretary Problem (SP)** is a well-known example of an optimal stopping problem. Variants of the SP have been studied extensively since the 1960s and closely related problems were addressed even earlier. Ferguson [Ferguson 1989] has provided a comprehensive history of the problem. The SP and its variants and generalizations have been referred to by many names, including the Best Choice, Marriage, Hiring, Dowry, and Suitor Problems. The SP itself is a variant of the "Game of Googol" described by Martin Gardner in 1960 in *Scientific American*.

The version of the SP most often referred to as the "**Classical Secretary Problem (CSP)**" is framed as follows:

- A decision maker (DM) is to select a *single* applicant from a pool of *N* applicants with the goal of optimizing a given *payoff function*.

- The applicants:
    - Have ranks ranging from **1 (the best)** to ***N* (the worst)** that are *relative*, meaning that any two applicants may be compared to determine which is better but their true values are unknown to the DM. Rank ties are not allowed.
    - Are presented one-by-one to the DM and observed as a queue in random order, where the order is random in the sense that any ordering of the applicants is equiprobable.

- The DM:
    - Knows *N*.
    - Learns only the *apparent rank* of an applicant, relative to previously observed applicants, at the time of presentation.
    - Must select or not select a single applicant at the time of presentation.
    - Cannot recall a previously observed applicant.
    - Must select the last applicant if no selection is made before that point.

- The DM **stops** when the selection decision is made.

Keeping with common conventions in the discussion of this area, this paper uses the roles of "DM" and "applicant." The neutral pronoun "it" is used because the DM and/or applicants may not be carbon-based entities.

Rank and Payoff

The term **apparent rank** used above was introduced by D.V. Lindley [Lindley 1961] to distinguish the DM's knowledge of an applicant's rank *at the time that applicant is observed* from its **true rank** with respect to the entire queue. Note that "true rank" and "apparent rank" are relative ranks of the applicants in their queue. Both the true rank and apparent rank are relative ranks; these ranks are distinct from **true values** (also called actual or absolute values) for the desirability of the applicant to the DM.[1] Of course, the DM may use a true value-based scoring system to arrive at the relative ranks.

The DM bases its selection strategy on a predetermined **payoff function**. By far, the most studied version of the CSP uses the "**Nothing But The Best (NBTB)**" payoff function [Lindley 1961], [Ferguson 1989], [Seale 1997], [Bearden 2004]. The DM receives a payoff of 1 if the best applicant is selected and 0 otherwise. The goal of the DM is to maximize the expected[2] payoff, that is, maximize the probability of selecting *only the top applicant*: the applicant with true rank 1.

Another payoff function which has received less attention is the relative rank itself of the selected applicant [Chow 1964]. In this case, the goal of the DM is to achieve **Minimum Expected Relative Rank (MinERR)** of the selected applicant. Rather than insisting on obtaining only the top applicant, MinERR payoff is based on achieving the lowest rank of the selected applicant *on average*.

Versions of the Secretary Problem

In the CSP, the DM cannot look ahead and cannot go back. At any position in the randomly ordered queue, the decision to select or not select a single applicant is based solely on the relative ordering of the applicants observed up to that point and the DM cannot select a previously observed candidate. However, the CSP is only a narrow part of the overall area of SP research. A few of the variations and generalizations of the SP found in the literature are as follows:

- *N* may be unknown to the DM.
- Applicants may be considered in groups.
- The quality of applicants may be provided as true values rather than as a relative rank ordering.
- The distribution of the quality of applicants may be known.
- The DM may be competing against a second player that manipulates the order of the applicant queue.
- The DM may be granted full or limited ability to recall previously observed applicants.

[1] The term "value" depends on the specific situation. "Value" could be an actual monetary amount, in which case the DM would consider a higher positive amount to be better, or "value" could be a cost in which case lower is better. Alternatively, the "value" could be a score on an achievement test (where higher is better) or a time in a running race (where lower is better).

[2] The terms **expected**, **average**, and **mean** are used interchangeably in this paper and refer to the *arithmetic mean* of a set of values.

- The payoff function may involve selecting the second, third, middle, worst, or other applicant instead of the very best.
- The payoff function may require selecting all of the *k* best applicants instead of only a single applicant.

The SP and its variants and generalizations are simply abstractions framed in familiar terms. These optimal stopping problems, as noted by Broder et al. [Broder 2007], are "only tangentially" related to hiring, dating, marriage, etc. The SP and related problems have been used or investigated in connection with many applications such as load balancing in multicore processors [Moh 2014], online data sampling by means of a robot that scores visual samples to decide where to drop sensors as it moves through its world [Girdhar 2009], sensing a cyber-physical system to decide when to time an attack on sensor signals [Krotofil 2014], a two-player game of deception involving missiles and decoys [Castañón 2014], and animals foraging or searching for mates [McNamara 2012].

The reasonable range of values of N depends on the underlying problem abstracted as a CSP. Job interviews frequently involve 5, 10, 20, or more applicants. A brief Internet search of postings by real estate agents and buyers indicated that prospective buyers might look at 10, 25, or even 70 properties, and, in a strong seller's market, the decision must often be made practically on the spot so there is no opportunity to "recall a previous applicant." Foraging and hunting (to include military sorties and other missions) might reasonably involve tens or hundreds of "applicants." If monitoring a physical process, then the number of sampled values that represent the "applicants" could range from being very small to millions or billions.

Appendix A gives an example of applying various CSP solutions to a notional physical system where the DM is an operator, the "applicants" are sensor readings, and the DM's goal is to shut the system down at the safest point based on the readings. The applicant queue has a length of $N = 100$ and immediate action must be taken at the best possible point in time so there is no opportunity to "recall a previous applicant."

Classical Two-Phase Solution for the Classical Secretary Problem

The CSP and its closest variations have received so much attention perhaps because of their remarkably elegant solutions and results. For NBTB payoff, the optimal algorithm is a simple two-phase solution described as follows. The DM chooses a *start position* in the queue. The start position (denoted by s) is the boundary between the two phases:

- *Observation-only phase*: The DM simply observes the relative ranks of the first $s-1$ applicants presented to the DM *without selecting any of them*.

- *Observation-and-selection phase*: The DM continues to observe the applicants as they are presented but, starting with the applicant in position s, the DM *selects the first applicant observed to have lower rank* than any previously observed candidate in the queue. If no candidate has been selected before position N is reached, then, per the rules of the CSP, the DM *must select the last candidate* regardless of its rank.

It is assumed that $s \geq 2$ because $s = 1$ results in the very first applicant in the queue being selected regardless of rank.

This solution for the CSP is so ubiquitous in the literature that it is referred to in this paper as the **Classical Two-Phase Solution (C2PS)**.[3]

Payoff Functions

For NBTB payoff, the value of s that maximizes the probability of selecting the best candidate is closely approximated by N/e, where e = 2.718281828… (the base of the natural logarithm); thus, the optimal start position is approximately 0.368N. (See, for example, [Ferguson 1989]). Remarkably, the constant e appears again in the values of two payoff functions. First, the NBTB payoff itself (the *probability of selecting the best candidate*) approaches $1/e \approx 0.368$. Second, the *expected relative rank* achieved with that start position is approximately $N/(2e)$ [Bajnok 2015]. The fact that the solution itself and the values of two payoff functions involve the fundamental constant e may account for the literature's overwhelming focus on this version of the CSP.

While the NBTB solution maximizes the probability of selecting the best candidate, it does not achieve the *minimum expected relative rank* of the selected candidate. The payoff function based on MinERR (that is, achieving the lowest average relative rank-selected applicant) has received less attention. Chow, Moriguti, Robbins, and Samuels [Chow 1964] presented an algorithm that achieves an expected relative rank of at most 3.8695 *regardless of the number of applicants*. This result is revisited at the end of this section and discussed in more detail in Section IX.

For the C2PS, the start position s that minimizes the expected relative rank and the resulting expected relative rank itself *both* are closely approximated by $\sqrt{N}$.[4] Note that this

[3] As noted in the definition of the version of the CSP studied in this paper, it is assumed that the DM can assess only the relative ranks of the applicants as they are observed, but not the true values for the quality of the applicants and the distribution of those values. Intuitively, it would seem that knowing the true values and their distributions would improve the DM's payoff. However, Allaart and Islas [Allaart 2016] found that, if the distributions of the applicants are not identically distributed, then the sharp lower bound on the probability of selecting the best candidate is the same as that when only relative ranks are known. They stated that "… in the worst case and for large N, a gambler who has full information can do no better than a gambler who observes only relative ranks."

[4] Unlike the triply remarkable NBTB payoff, the MinERR payoff appears to be only doubly remarkable because the probability of selecting the applicant with the best relative rank does not involve the square root of N. Fitting a trendline to the results for N = 10 to 1,000 (sampling ten values per decade) yields a "power law" fit for the probability of selecting the best applicant that is $0.8757/N^{0.31}$ (R^2 = 0.9914). The probabilities were calculated using the expressions for exact probabilities that are presented in Section IV.

start position and resulting expected relative rank are optimal only in the context of the C2PS. Other solutions, such as Chow's solution mentioned above, are discussed later that improve on this expected relative rank of the selected applicant.

The basis for the two "square root" phenomena associated with the C2PS for MinERR payoff is usually attributed to Bearden [Bearden 2005]; see for example, [Bajnok 2015]. However, there are two discrepancies between Bearden's model and the CSP model used in this paper. First, Bearden's model is "true value-based" where applicant qualities are drawn i.i.d. from uniformly distributed values in the interval [0, 1] rather than being known to the DM only as their relative ranks 1, 2, … N. Second, as is common with true value-based models, the value function is reversed from the relative rank; that is, a higher true value in Bearden's model corresponds to a lower rank in other CSP models.

Bearden's results regarding these two "square root" phenomena are summarized as follows. He derived an expression for the expected true value as a function of N and the start position (his expression would be $V_N(s) = (2sN - s^2 + s - N)/(2sN)$ in the notation of this paper) and showed that the start position that maximizes the expected true value $V_N(s)$ is closely approximated by $s = \sqrt{N}$. Bearden denoted the maximum expected true value $V_N(\sqrt{N})$ by V^*; the transformation $(1-V^*)N+1$ maps Bearden's true values in the interval [0, 1] where "higher is better" onto the interval [1, N] where "lower is better." The expression $(1-V^*)N+1$ simplifies to an expected relative rank of $\sqrt{N}$ + ½. Thus, both the optimal start position and the expected relative rank in Bearden's model were shown to be closely approximated by $\sqrt{N}$.

In spite of the differences between Bearden's model and the "lower is better" relative rank-based CSP addressed in this paper, the two "square root" phenomena can be demonstrated to hold for the latter model as shown in Figure 1. The actual optimal start position s and the resulting expected relative rank are plotted as fractions of the square root of the number of applicants; these values were calculated using the expressions for exact probabilities presented in Section IV. Both of these ratios are shown to closely approach 1 as N increases. Furthermore, the trendline for the optimal start position s was found to be $0.9945N^{0.4988}$ (R^2 = 0.9871) and the trendline for the expected relative rank using the optimal start position was $1.0197N^{0.4897}$ (R^2 = 0.9963) which also showed that both values are closely approximated by $\sqrt{N}$.

Examples of the Classical Secretary Problem and its Classical Two-Phase Solution

Two examples are presented to demonstrate the two-phase solution for the CSP and its results for NBTB and MinERR payoffs. Each example is based on a queue of applicants identified with their true ranks relative to the entire queue. The examples also list parenthetically the apparent ranks in the order observed by the DM up to that point in the queue. Apparent ranks and their properties are discussed in more detail in Section IX.

Example 1:

Suppose N = 10 applicants are presented to the DM in this order: 8, 1, 5, 3, 2, 4, 6, 10, 9, 7. These ranks are the true ranks, which are not known to the DM unless the DM reaches the last applicant. (The apparent ranks of the applicants, as they are encountered in the order observed by the DM, would be 1, 1, 2, 2, 2, 4, 6, 8, 8, 7.)

For NBTB payoff, the DM sets the start position at $N/e = 10/e \approx 3.68$ rounded up to s = 4. In the observation-only phase, the DM observes the first $s-1$ applicants and learns that applicant #2 (the second applicant in the queue) has the lowest relative rank of those three applicants; the DM does not know, unfortunately, that in fact applicant #2 has the lowest rank in the entire queue. The DM then continues into the observation-and-selection phase beginning with the applicant in position 4, and would select the first applicant with relative rank lower than that of applicant #2; however, no such applicant exists and the DM is forced to select the last applicant. The DM receives a NBTB payoff of 0 for selecting the applicant with rank 7.

For MinERR payoff, the DM sets the start position at $\sqrt{N} = \sqrt{10} \approx 3.16$ rounded down to s = 3. In the observation-only phase, the DM observes the first two applicants and learns that applicant #2 has the lowest relative rank of those two applicants. In the observation-and-selection phase, the DM is forced to select the last applicant and thus receives a payoff of 7 for selecting the applicant with rank 7.

In both cases, the DM reached the last applicant before making a selection so the DM knows the true ranks of the selected applicant and thus knows the payoffs for those selections.

(End of example)

Example 2:

Suppose $N = 10$ applicants are presented to the DM with their true ranks in this order: 8, 10, 5, 6, 9, 2, 1, 4, 7, 3. (The apparent ranks of the applicants, as they are encountered in the order observed by the DM, would be 1, 2, 1, 2, 4, 1, 1, 3, 6, 3.)

For NBTB payoff, the DM sets the start position at $s = 4$ as in Example 1. In the first phase, the DM observes the first three applicants and notes that applicant #3 has the lowest rank. In the second phase, the DM observes and discards applicants #4 and #5. The next applicant, #6 with rank 2, has lower rank than any previously observed candidate and thus is selected by the DM. The DM does not know the rank of the selected applicant relative to the entire queue but only that applicant #6 has the lowest rank relative to those observed up to that point in the queue and thus has apparent rank 1. Even though the second-best applicant in the entire queue was selected, the DM still receives a NBTB payoff of 0.

For MinERR payoff, the start position is again set at $s = 3$. In the first phase, the DM observes the first two applicants and learns that applicant #1 has the lowest relative rank of those two applicants. In the second phase, the DM selects the first applicant that has better rank than applicant #1 and so applicant #3 is selected. Although applicant #2 has apparent rank 1 it has true rank 5 and so the DM receives a payoff of 5.

In both cases, the DM did not reach the last applicant so the DM does not know the true ranks of the selected applicants and thus does not know the payoffs for those selections.

(End of example)

These examples are given to demonstrate the application of the C2PS with the two payoff functions used in this paper. The two queues used in these examples happened to result in worse-than-expected performance. On average, the optimal NBTB start position for the C2PS would select the applicant with rank 1 approximately $1/e \approx$ 36.8% of the time; for $N = 10$ the actual probability of selecting the best applicant is 39.87%. On average, the optimal MinERR start position for the C2PS would select applicants with expected relative rank approximately $\sqrt{N}$; for $N = 10$ the predicted expectation is $\sqrt{10} \approx 3.16$ but the actual expectation for the rank is 2.933. These actual values were calculated using the expressions for exact probabilities presented in Section IV.

As noted in the examples, unless the DM reaches and selects the last applicant the true rank of the selected applicant and thus the payoff is unknown to the DM. Section XI discusses the limits of the DM's knowledge based on the apparent rank of the selected applicant.

Chow's Solution for the Classical Secretary Problem

It has been noted already that the C2PS has an optimal start position that minimizes the expected relative rank achievable with that algorithm, but the C2PS does not achieve the actual minimum expected relative rank. The approach described below, however, does reach the actual minimum.

Chow et al. [Chow 1964] presented an algorithm achieves the stunning result that an expected relative rank of at most 3.8695 can be achieved regardless of the number of applicants. To emphasize this point, this result can be stated even more plainly: Chow's solution for MinERR payoff can select an applicant with an average rank less than 4 whether the number of applicants is 10, 100, 1,000, or 1,000,000. The administration of Chow's algorithm is more complicated than the C2PS in that it requires a set of values to be calculated for each position for a given value of N. However, these values are constants and are easily stored or calculated; Appendix D provides such a list.

While cited frequently in the literature, there seems to have been little detailed analysis or application of Chow's solution. Chow's solution is discussed in detail in Section IX and is used in several other sections and appendices.[5]

Regardless of the algorithm or payoff function, almost all published investigations of the SP and its variants seem to have been from the viewpoint of *optimizing the DM's selection process*. There appears to be a single publication, by D.B. Glass, that looks at the SP from the viewpoint of *maximizing an applicant's chances of being selected* [Glass 2012]. For brevity, this work by D.B. Glass is referred to in this paper simply as *Glass*.

Outline of Paper

The remainder of this paper proceeds as follows.

Section II provides a detailed glossary of the notation and symbols used in this paper.

Sections III-VII investigate and extend the results pioneered in *Glass*, which are based on the assumption that the DM uses the C2PS and always chooses the start position that optimizes NBTB payoff. Specifically, the contribution of this paper in this area is to outline the limits of a single **cheating applicant (CA)** to choose an advantageous position in the applicant queue presented to the DM under the full range of possible start positions that optimize the desired NBTB or MinERR payoff. The discussion is framed in terms of payoff functions and tradeoffs in search times. These results expand on the work in *Glass* to apply to both smaller and larger start positions than those chosen to maximize NBTB payoff. This paper outlines the options available to a CA when the DM uses the C2PS and introduces the concepts of advantageous positions from the point of view of the applicant. The critical position that delimits the advantageous positions can be determined directly

[5] Perhaps Chow's solution has not captured the attention of the research community because it does not involve any named fundamental constants.

through simulation or by the use of the exact expressions for probability of selection but it can also be closely estimated through simpler means.

It is generally assumed in the SP literature that an applicant is competing *for* selection but these principles are also immediately applicable to the complementary problem. A CA would not seek but rather *evade* selection if there were negative consequences to selection. This would be the case if the applicants were, for example, food sources or military targets. The applicant with rank 1 would, of course, benefit the most from an effective evasion strategy. This concept is introduced in Section V and is addressed in subsequent sections.

Section VIII exercises the approaches used in this paper for the C2PS by introducing another variant of the SP that allows for *non-selection of the last applicant* if its rank is unacceptably high. Although this variant is introduced primarily to demonstrate the approaches used in this paper, this discussion has value in its own right as the option of non-selection is commonly used but its consequences have apparently received little formal attention previously.

Due to the optimal performance of Chow's solution for MinERR payoff, it is the basis for several sections in this paper instead of the C2PS. Section IX discusses Chow's solution in detail and presents methods for an applicant to cheat Chow's solution to either facilitate selection or evade selection.

A CA must know its own rank in the queue in order to select advantageous positions. Section X addresses the situation where the CA does not know its exact rank but can *estimate* it within lower and upper bounds. This would apply, for example, in the case where the CA knows the distribution of the population of applicants.

Section XI addresses the properties of the *apparent rank* of selected applicants, as the DM generally does not know that applicant's true rank. Section XII explores the use of the apparent rank in a statistical test of hypothesis to attempt to distinguish between honest-only queues and queues with CAs.

Appendix A demonstrates the various CSP solutions discussed in this paper, without consideration of cheating, in the context of a *model of a sampled physical system*. The physical system discussed in Appendix A is used to explore the robustness of these solutions to a violation of the assumptions of the CSP where drifting characteristics of the system are introduced that translate into changes in the distribution of the ranks of the "applicants."

Appendix B presents a two-phase solution for the CSP for MinERR payoff by Bajnok and Semov [Bajnok 2015] that achieves a smaller expected rank (approximately $\frac{e}{2}\ln N$) than does the C2PS for MinERR payoff but is simpler to administer than Chow's solution. Thus, it is presented in some detail in this paper. Cheating is not discussed in the context of this solution.

Due to the spectacular performance of Chow's solution even under the constraint of being prohibited from recalling previously observed applicants, Appendix C compares its effectiveness to that of SP variants that allow *complete or partial recall* of previously observed applicants. Cheating is not addressed in this Appendix.

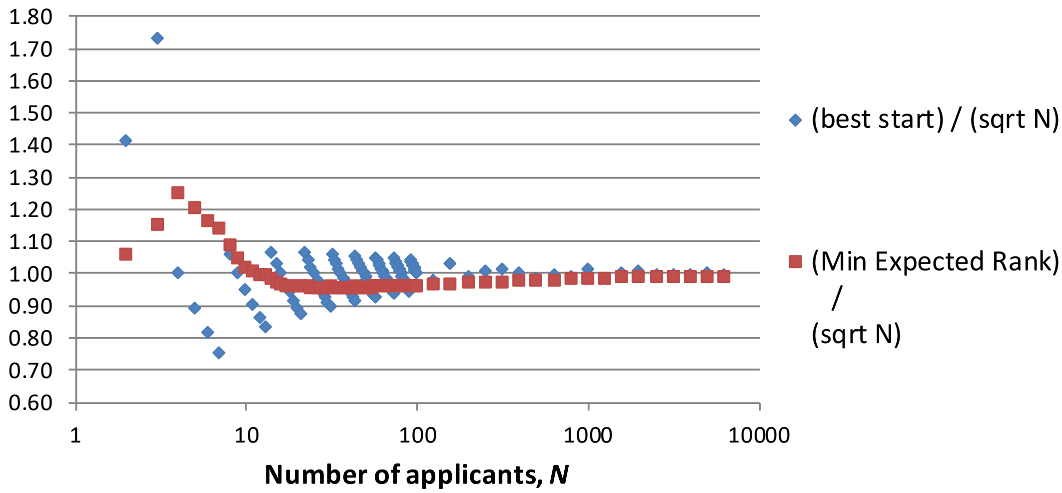

Figure 1. Best start position and resulting expected rank for C2PS for MinERR, as ratios of $\sqrt{N}$.

II. Notation and Dramatis Personae

The following symbols, acronyms, and abbreviations are used in this paper. Terms are listed roughly in the order of first use but that order is modified in some cases to group like terms.

SP	Secretary Problem.
CSP	Classical Secretary Problem (as defined in Section I).
C2PS	Classical Two-Phase Solution for the CSP.
SPANS	Secretary Problem Allowing Non-Selection (as defined in Section VIII).
Glass	The referenced work by D.B. Glass [Glass 2012].
DM	Decision Maker, tasked to observe a queue of applicants and select a single applicant with the goal of optimizing a given payoff function.
CA	Cheating Applicant, the only applicant with the ability to choose or influence its position in the queue. (Plural: CAs)
CA r	Cheating Applicant with rank r. For example, CA 3 is a cheating applicant with relative rank 3.
N	Number of applicants in the queue. The value of N is known to the DM and CA.
p	Position in the queue.
Applicant #p	Applicant in position p. For example, applicant #4 is the applicant in position 4.
True value	The actual (absolute) score or assessment of an applicant by the DM, or the actual payoff to the DM for selecting a given applicant.
r	The true rank (true relative rank) of an applicant in a queue of N applicants, where 1 is the best and N is the worst. The term "rank" alone refers to the true rank. The DM may use true values to arrive at the true ranks.
y_p	Apparent rank of applicants observed up to position p. The apparent rank of the applicant at position p is the relative rank of that applicant compared to only the applicants observed by the DM to that point.

s — Start position in the C2PS. It is the boundary between the observation-only and observation-and-selection phases and begins the second phase. The value of s is assumed to be known to the CA.

Pr{A} — Probability that event A occurs.

Pr{A ∩ B} — Joint probability that event A *and* event B occur.

Pr{A | B} — Conditional probability of event A *given* event B occurs.

Pr{select rank r}
Probability that DM selects the applicant with rank r. N and s are implicit.

Pr{select position p}
Probability that DM selects the applicant in position p of the queue. N and s are implicit.

Pr{select rank r ∩ position p}
Probability that DM selects the applicant with rank r *and* the applicant is in position p. N and s are implicit.

Pr{select rank r | position p}
Conditional probability that DM selects the applicant with rank r *given* the applicant is in position p. N and s are implicit.

Pr{select rank r | position p ∩ selection occurs}
Conditional probability that DM selects the applicant with rank r *given* the joint event that the applicant is in position p *and* the DM makes a selection of any applicant (as discussed in Section VIII). N and s are implicit.

NBTB — "Nothing But The Best" payoff function: the DM receives a payoff of 1 for selecting the applicant with rank 1, and 0 otherwise. The DM's goal is to maximize the expected payoff, which is Pr{select rank 1}.

MinERR — "Minimum Expected Relative Rank" payoff function: the DM receives a payoff equal to the relative rank of the selected applicant. The DM's goal is to minimize the average rank of the selected applicant.

s_{NBTB} — Optimal start position for NBTB payoff in the C2PS. This start position maximizes Pr{select rank 1}.

s_{MinERR} — Optimal start position for MinERR payoff in the C2PS. This start position achieves the smallest average rank of the selected applicant for the C2PS.

R^2 Coefficient of determination: a commonly used measure of the goodness-of-fit of a linear regression, after transformation if necessary (log, exponential, power, etc.). It describes how much of the variation in the dependent variable is explained by the regression model. R^2 values range from 0 to 1; closer to 1 is a better fit.

Advantageous position, Non-advantageous position

For a given solution, an *advantageous position* is any position in the queue where an applicant has a probability of being selected at least as great as a reasonable benchmark from the point of view of the applicant. In some cases, the benchmark is the probability of selection for that rank in the last position in the queue. In other cases, the benchmark is given as a threshold that is at least fraction T of the maximum probability of selection in any position for that rank. The complement of the set of advantageous positions is the set of *non-advantageous* positions.

$z(N, r, p)$ For the C2PS, a given value of N, and a given applicant with rank r and position p, this is the ratio of the probability of selecting that applicant when in position p of the queue to the probability of selecting that applicant when in position N (the last position in the queue). The dependence on start position s cancels out: $z(N, r, p)$ is independent of s. The relationships between $z(N, r, p)$ and any advantageous positions are as follows:

- For an applicant of rank r, $z(N, r, p) \geq 1$ for every p that is an advantageous position.
- Positions for which $z(N, r, p) \geq 1$ where $p < s$ are non-advantageous because the probability of selection is in fact zero in the observation-only phase.

Critical position: $p_c(N, r)$

For the C2PS, a given value of N, and an applicant with rank r, the critical position is the largest value of p for which every position 2, 3, … p yields $z(N, r, p) \geq 1$; otherwise, no such position exists and $p_c(N, r) = 0$. The relationships between the critical position and any advantageous positions are as follows:

- If $s \leq p_c(N, r)$, then every position s, s+1, … $p_c(N, r)$ is advantageous in addition to the last position. Otherwise, no positions are advantageous except the last position.
- Positions *greater than* the critical position but less than the last position ($p_c(N, r)$+1, $p_c(N, r)$+2, … N−1) are non-advantageous.

s_p Constants for Chow's solution for the CSP for MinERR payoff. The s_p constants are used as thresholds for comparison to apparent ranks. These constants are obtained from the c_i constants. Note that c_0 is the expected relative rank achieved by Chow's solution. This symbol is not to be confused with start position *s* or sample standard deviation *s* for test of hypothesis. (The symbol "*s*" is retained from the original source [Chow 1964].)

r_L, r_U Lower and upper ranks in a consecutive range of ranks. Rank r_L is the CA's estimate of its *best* possible rank and r_U is the CA's estimate of its *worst* possible rank.

n, m, μ, s Sample size, sample mean, true mean, and sample standard deviation, respectively, for a test of hypothesis concerning the difference of two means. This use of the symbol *s* is not to be confused with start position *s* or Chow's s_p constants. (The symbol "*s*" is retained based on common usage for the sample standard deviation.)

α Level of significance. A test of hypothesis concerning the difference of two means is designed to reject, with probability α, a sample from a population for which its mean equals the actual mean μ. That is, α is the probability of rejecting the null hypothesis when it is in fact true. The level of significance is the designed false alarm rate or probability of Type I error.

t The *t*-statistic for a test of hypothesis.

$t_{\alpha/2,df}$ The value of *t* for which the right-tail probability of Student's *t*-distribution with $df = n-1$ degrees of freedom equals $\alpha/2$.

L Threshold for apparent rank used in Bajnok's solution for MinERR payoff.

p_L Position of the applicant with apparent rank *L* after observing the first *s*-1 applicants in Bajnok's solution for MinERR payoff.

t Termination position for the observation-only phase in a version of the SP that allows recall of previously observed applicants (see Appendix C). The termination position equals the number of applicants observed by the DM. This symbol is not to be confused with the *t*-statistic used for a statistical test of hypothesis.

Solicit, solicitation

In a version of the SP that allows recall of previously observed applicants, solicitation is an attempt by the DM to select a previous applicant but, depending on the degree of recall permitted by the specific solution, that applicant may not be available for selection. Solicitation follows the termination of the observation-only phase. The DM stops at the end of the solicitation phase by making a selection.

First-Best Available (1BA)

In a version of the SP that allows recall of previously observed applicants, the applicant with apparent rank 1 is always available when solicited.

Second-Best Available (2BA)

In a version of the SP that allows recall of previously observed applicants, the applicant with apparent rank 1 is never available when solicited, but the applicant with apparent rank 2 is always available.

Constant Probability of Availability (CPA)

In a version of the SP that allows recall of previously observed applicants, applicants are solicited, in turn, starting with apparent rank 1, but are available only with a given probability.

$\lfloor x \rfloor, \lceil x \rceil$	Floor of *x* (largest integer that is less than or equal to *x*) and ceiling of *x* (smallest integer that is greater than or equal to *x*), respectively.
round (*x*)	Rounded value of *x* (nearest integer).
ln *x*	Natural logarithm of *x* (logarithm base *e*).
$\underset{x}{\operatorname{argmax}} E(x)$	The value of *x* that maximizes expression *E* which is a function of *x*.
$\underset{x}{\operatorname{argmin}} E(x)$	The value of *x* that minimizes expression *E* which is a function of *x*.
i.i.d.	Independent and identically distributed.

III. Payoff Functions and Search Times

The C2PS for NBTB payoff may have received the most attention in the literature due to its elegance but it has the disadvantage that it results in longer search times than those required by at least one other plausible payoff function. Notably, the C2PS for MinERR payoff achieves selection in far fewer steps as N increases. Two ways of considering search costs are to look at the number of *"wasted" applicants that are observed without chance of selection* in the observation-only phase of the DM's process and to look at the *total number of applicants* observed by the DM to achieve selection.

The fraction of wasted applicants observed without chance of selection is given by the ratio of the approximate theoretical optimal start positions for NBTB and MinERR payoffs which is $(N/e)/\sqrt{N} = \sqrt{N}/e$. For example, for N = 20 this ratio is about 1.6, for N = 100 it is 3.8, and for N = 1,000 it is 11.4.

A simulation of the CSP was created as a check on the findings. A random permutation of an applicant queue was created for each simulated selection process using the function "RANPER" of Nijenhuis and Wilf [Nijenhuis 1978] and was driven by the "RAN2" random number generator given by Press [Press 2002]. The simulations were run on an Intel Core i5-2390T CPU @ 2.70 GHz running Cygwin on Windows 10.

Throughout this paper, unless otherwise noted, the simulations were run with the following "standard" parameters:

N = 10	to 100	by 10	(10^6 random runs for each N)
N = 200	to 1,000	by 100	(10^5 random runs for each N)
N = 2,000	to 10,000	by 1,000	(10^4 random runs for each N)

Note that one "simulated random run" is a single complete simulated selection by a DM for a unique randomly generated queue.

In spite of well-articulated and valid arguments of Pawlikowski, Jeong, and Lee [Pawlikowski 2002] regarding the demonstration of statistical validity, error bars are not given on the simulation results. However, Section XII provides an in-depth discussion of the challenges of statistical significance in this problem area.

Simulated experiments provided estimates of the expected total number of applicants presented to the DM; this statistic is the same as the expected value of the position at which the DM selected an applicant. Results are shown in Table 1 for NBTB and Table 2 for MinERR payoffs. Different random seeds were used for each value of N but otherwise the two tables differ only in their scoring of each selection process according to the NBTB and MinERR payoff functions. For each value of N, the number of simulated random runs was as listed above.

The observed "best start" values derived from the simulations would, of course, vary slightly with repeated random runs even with such a large number of simulations. Even so, it is striking how closely these empirically derived start positions match the theoretical optimal start positions. In Table 1, the NBTB payoff's theoretical optimal start position N/e for N = 20 is lower by less than 1 from the empirical optimal start position and for N = 1,000 the theoretical start position is 2.2% lower. Similarly, in Table 2, the MinERR payoff's theoretical optimal start position $\sqrt{N}$ for N = 20 which again is lower by less than 1 from the empirical optimal start position and for N = 1,000 the theoretical start position is lower by 1.2%.

With the start position chosen for optimal NBTB payoff, a DM not only observes more applicants before starting selection but also observes many more applicants *in toto* than a DM using MinERR payoff. To the degree that these two versions of the C2PS reflect real-world optimal decision problems, one could argue that the C2PS for MinERR payoff achieves a more practical goal and, additionally, does so with fewer observations. For N = 20 the ratio of total observations for NBTB to MinERR payoff is about 1.4, for N = 100 it is 2.4, and for N = 1,000 it is 5.3.

Figure 2 shows the NBTB and MinERR payoffs as a function of start positions obtained for N = 100 and 10^6 simulated random runs.[6] The normalized expected relative rank has a sharp minimum at s = 10 so choosing even a slightly smaller start position has a large impact on the expected relative rank of the selected applicant. However, the probability of selecting the best applicant is much flatter around the maximum; the maximum probability of selecting the best applicant is 37.1% at s = 38 but when the start position is reduced arbitrarily by 30% to s = 26 the probability is still above 35%.

Search times can be reduced systematically to achieve specific goals. Krotofil et al. [Krotofil 2014] explored several approaches for reducing the frequency of instances where the DM is forced to select the last applicant by default in the C2PS for NBTB payoff. One of their approaches involved using a start position of s = N/(ln N) instead of N/e. Figure 2 shows the probability of selecting the applicant in the last position as a function of the start position. For N = 100, N/(ln N) is 21.71; a start position of 22 is about 43% shorter than the NBTB start position of 38 and reduces the probability of selecting the last applicant from 37% to 21%, yet the probability of selecting the best applicant is still above 33%.

Thus, Figure 2 indicates that a DM in a hurry to make a decision might reasonably shorten NBTB search time by using a start position less than the theoretical optimum and not suffer much penalty in terms of payoff.

Aside from impact on payoff, some researchers have also explicitly considered the *costs* of delay in making a selection, or, equivalently, the benefits of reducing the number of applications to be observed; a version of the SP involving decision delay costs was investigated by Wang et al. [Wang 2010c].

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[6] The results shown in Figure 2 for N = 100 and 10^6 simulated random runs executed in 52.155 "user" seconds according to the Cygwin Linux *time* command. The simulation also implements the exact expressions in *Glass* that were presented in Section IV; those expressions required only 0.015 "user" seconds.

Human decision-making processes are often contrary to the mathematically optimal courses of action; see, for example, articles by Sigmund et al. [Sigmund 2002] and Basu [Basu 2007] on the failure of game theory to account for actual human behavior. Several researchers have performed studies specifically on SP-related problems involving actual human subjects as the DMs. Most studies seem to indicate that humans under-search (that is, stop too early) although over-search has also been observed [Seale 1997], [Bearden 2004], [Wang 2010b], [Wang 2010c], [Mak 2014].

The results presented in this section indicate that start positions that maximize the NBTB payoff (that is, start positions that are approximately N/e) may be overly long compared to more reasonable start positions such as those that minimize the expected relative rank. Thus, this paper explores the properties of solutions for the CSP in regions that have not previously received much attention and, specifically, how the CSP is impacted by cheating behavior.

N	*N/e*	best start pos	Pr{sel rank 1} at best start pos	ave rank at best start pos	ave pos observed for best start pos	Pr{sel last pos} at best start pos
10	3.68	4	0.3984	3.02	6.98	0.3327
20	7.36	8	0.3847	4.98	14.68	0.3677
30	11.04	12	0.3782	6.99	22.36	0.3796
40	14.72	15	0.3762	8.55	29.03	0.3585
50	18.39	19	0.3739	10.52	36.72	0.3672
60	22.07	23	0.3739	12.48	44.37	0.3720
70	25.75	26	0.3724	14.04	51.06	0.3620
80	29.43	30	0.3713	16.02	58.73	0.3668
90	33.11	33	0.3714	17.55	65.41	0.3597
100	36.79	38	0.3709	20.01	74.12	0.3740
200	73.58	73	0.3704	37.45	145.74	0.3595
300	110.36	109	0.3674	55.60	218.62	0.3610
400	147.15	150	0.3698	76.21	297.02	0.3749
500	183.94	186	0.3682	94.27	369.07	0.3712
600	220.73	235	0.3679	118.99	454.89	0.3913
700	257.52	267	0.3690	134.47	524.09	0.3810
800	294.30	282	0.3672	142.29	575.02	0.3525
900	331.09	319	0.3704	159.81	649.09	0.3525
1000	367.88	376	0.3682	190.18	743.72	0.3756
2000	735.76	791	0.3650	398.43	1526.60	0.3984
3000	1103.64	1001	0.3746	494.08	2093.59	0.3283
4000	1471.52	1433	0.3741	711.92	2888.32	0.3541
5000	1839.40	1806	0.3767	882.08	3628.00	0.3552
6000	2207.28	2273	0.3745	1120.80	4462.27	0.3721
7000	2575.16	2688	0.3678	1352.08	5243.69	0.3838
8000	2943.04	3235	0.3777	1570.76	6152.42	0.3977
9000	3310.91	3625	0.3717	1757.95	6882.43	0.3933
10000	3678.79	4083	0.3604	2044.70	7727.43	0.4089

Table 1. Statistics for C2PS for NBTB. Number of simulated random runs for each value of *N* is specified in Section III.

N	sqrt (*N*)	best start pos	Pr{sel rank 1} at best start pos	ave rank at best startpos	ave pos observed for best start pos	Pr{sel last pos} at best start pos
10	3.16	3	0.3652	2.93	5.65	0.2216
20	4.47	5	0.3430	4.20	10.85	0.2098
30	5.48	5	0.2843	5.17	12.53	0.1383
40	6.32	6	0.2715	5.97	15.85	0.1280
50	7.07	7	0.2632	6.71	19.20	0.1230
60	7.75	8	0.2580	7.37	22.49	0.1184
70	8.37	8	0.2370	7.98	23.53	0.1010
80	8.94	9	0.2353	8.55	26.86	0.1012
90	9.49	10	0.2362	9.08	30.18	0.1010
100	10.00	10	0.2222	9.56	31.15	0.0907
200	14.14	14	0.1805	13.73	49.07	0.0653
300	17.32	17	0.1577	16.89	63.56	0.0538
400	20.00	20	0.1480	19.48	77.51	0.0474
500	22.36	22	0.1333	21.80	87.88	0.0420
600	24.49	24	0.1268	23.72	98.40	0.0373
700	26.46	27	0.1228	26.03	112.67	0.0375
800	28.28	27	0.1109	28.24	115.96	0.0335
900	30.00	29	0.1090	29.37	125.24	0.0306
1000	31.62	32	0.1086	30.90	139.53	0.0307
2000	44.72	47	0.0889	44.13	216.90	0.0220
3000	54.77	58	0.0716	51.03	275.24	0.0179
4000	63.25	63	0.0641	58.64	315.91	0.0144
5000	70.71	61	0.0491	70.11	329.22	0.0121
6000	77.46	78	0.0544	76.50	396.91	0.0128
7000	83.67	75	0.0460	82.09	401.41	0.0098
8000	89.44	88	0.0447	90.08	476.18	0.0114
9000	94.87	86	0.0488	90.00	486.04	0.0100
10000	100.00	95	0.0416	96.63	524.89	0.0092

Table 2. Statistics for C2PS for MinERR. Number of simulated random runs for each value of N is specified in Section III.

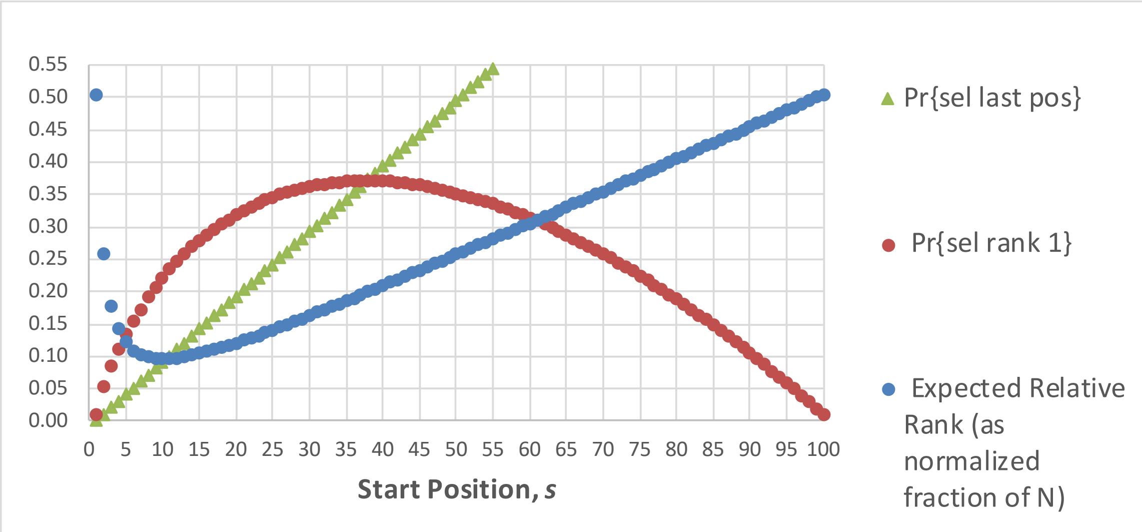

Figure 2. C2PS for MinERR payoff has sharp minimum but NBTB payoff has wider range around maximum. Probability of selecting last applicant increases linearly with start position. $N = 100$ and results are based on 10^6 simulated random runs.

IV. Selection Probabilities

As noted in Section I, the investigation in *Glass* may be unique in the literature by addressing the CSP from the applicant's point of view. That work presented a comprehensive set of expressions for selection probabilities and applied those expressions to the C2PS for NBTB payoff. In this paper, the expressions derived in *Glass* are used to generalize those results and extend them to start positions smaller than those that optimize NBTB payoff.

Glass Theorem 1 gives the probability that the applicant in position p is selected, regardless of rank, when the DM starts selection in position s. Expressed in the notation of this paper:

$$\Pr\{\text{select position } p\} = \begin{cases} 0 & \text{for} \quad p < s \text{ or } p > N \\ \frac{s-1}{p(p-1)} & \text{for} \quad s \le p < N \\ \frac{s-1}{N-1} & \text{for} \quad p = N \end{cases} \tag{1}$$

Glass Theorem 3 provides the conditional probability that the applicant with rank r is selected, given the applicant is in position p, when the DM starts selection in position s:

$$\Pr\{\text{select rank } r \mid \text{position } p\} = \begin{cases} 0 & \text{for} \quad p < s \text{ or } p > N \\ \frac{\binom{N-r}{p-1}}{\binom{N-1}{p-1}} \times \frac{s-1}{p-1} & \text{for} \quad s \le p < N \\ \frac{s-1}{N-1} & \text{for} \quad p = N \end{cases} \tag{2}$$

These expressions are valid for $s \ge 2$. Clearly, for $s = 1$ the very first applicant in the queue would always be selected regardless of rank.

Note that the numerator of expression (2) is zero for $N-r < p-1$ so Pr{select rank r | position p} = 0 for $N-r+1 < p < N$. As will be shown to be crucial later, however, expression (2) shows that Pr{select rank r | position N} is always non-zero.

According to the definition of the CSP, any of the N possible positions is equally likely, so expression (2) can be unconditioned by multiplying it by $1/N$:

$$\Pr\{\text{select rank } r \cap \text{position } p\} = \Pr\{\text{select rank } r \mid \text{position } p\} \,/\, N \tag{3}$$

Armed with these expressions, the statistics estimated and summarized in Tables 1 and 2 resulting from random experiments can be calculated directly. For a given value of N and NBTB payoff the best start position s_{NBTB} is given by:

$$s_{\text{NBTB}} = \underset{s}{\operatorname{argmax}} \sum_{p=s}^{N} \Pr\{\text{select rank } 1 \cap \text{ position } p\} \tag{4}$$

For a given value of N and MinERR payoff the best start position s_{MinERR} is given by:

$$s_{\text{MinERR}} = \underset{s}{\operatorname{argmin}} \sum_{p=s}^{N} \sum_{r=2}^{N} r \times \Pr\{\text{select rank } r \cap \text{ position } p\} \tag{5}$$

For NBTB or MinERR payoff, use $s = s_{\text{NBTB}}$ or s_{MinERR}, respectively, in expressions (6-8):

$$\Pr\{\text{select rank 1}\} = \sum_{p=s}^{N} \Pr\{\text{select rank } 1 \cap \text{ position } p\} \tag{6}$$

$$\text{Expected selected rank} = \sum_{p=s}^{N} \sum_{r=2}^{N} r \times \Pr\{\text{select rank } r \cap \text{ position } p\} \tag{7}$$

$$\text{Expected number of applicants observed by DM to make selection} = \sum_{p=s}^{N} p \times \Pr\{\text{select position } p\} \tag{8}$$

These expressions were calculated for the same values of N as were used in the simulated random runs summarized in Table 1 and Table 2 and these results are provided in Table 3 and Table 4 for comparison. As would be expected with a large number of simulated results, the values obtained were very close to the results of the exact expressions.[7] For NBTB payoff, the largest difference between the results obtained from the simulations and the exact expressions for the probability of selecting the best applicant was 2.66% and most of the other differences were less than 1%. Similarly, for MinERR payoff, the largest difference was –6.5% for the expected relative rank and most of the other differences again were less than 1%.

[7] This qualitative analysis of statistical validity is made in spite of the good advice given by Pawlikowski, Jeong, and Lee [Pawlikowski 2002].

N	*N/e*	Best start pos for rank 1	Expected Relative Rank selected	Pr{select rank 1}	Ave pos selected when using best start pos for rank 1
10	3.68	4	0.3987	3.03	6.99
20	7.36	8	0.3842	4.99	14.68
30	11.04	12	0.3787	6.98	22.36
40	14.72	16	0.3757	8.97	30.03
50	18.39	19	0.3743	10.52	36.71
60	22.07	23	0.3732	12.51	44.39
70	25.75	27	0.3724	14.50	52.07
80	29.43	30	0.3719	16.03	58.75
90	33.11	34	0.3714	18.02	66.43
100	36.79	38	0.3710	20.01	74.10
200	73.58	74	0.3695	38.04	146.89
300	110.36	111	0.3689	56.54	220.68
400	147.15	148	0.3687	75.04	294.47
500	183.94	185	0.3685	93.54	368.26
600	220.73	222	0.3684	112.04	442.04
700	257.52	258	0.3683	130.04	514.83
800	294.30	295	0.3683	148.54	588.62
900	331.09	332	0.3682	167.04	662.41
1000	367.88	369	0.3682	185.54	736.20
2000	735.76	737	0.3680	369.54	1472.07
3000	1103.64	1104	0.3680	553.04	2206.95
4000	1471.52	1472	0.3680	737.04	2942.83
5000	1839.40	1840	0.3679	921.04	3678.71
6000	2207.28	2208	0.3679	1105.04	4414.59
7000	2575.16	2576	0.3679	1289.04	5150.47
8000	2943.04	2944	0.3679	1473.04	5886.35
9000	3310.91	3312	0.3679	1657.04	6622.23
10000	3678.79	3680	0.3679	1841.04	7358.11

Table 3. Statistics for C2PS for NBTB payoff based on exact equations in *Glass*.

N	sqrt(*N*)	Best start pos for Expected Relative Rank	Expected Relative Rank selected	Pr{select rank 1}	Ave pos selected when using best start pos for Expected Relative Rank
10	3.16	3	2.93	0.3658	5.66
20	4.47	5	4.20	0.3429	10.86
30	5.48	6	5.17	0.3131	14.39
40	6.32	6	5.98	0.2713	15.85
50	7.07	7	6.70	0.2635	19.18
60	7.75	8	7.37	0.2582	22.49
70	8.37	8	7.99	0.2369	23.58
80	8.94	9	8.55	0.2360	26.88
90	9.49	9	9.10	0.2203	27.83
100	10.00	10	9.60	0.2214	31.14
200	14.14	14	13.71	0.1800	49.01
300	17.32	17	16.88	0.1579	63.38
400	20.00	20	19.55	0.1459	77.37
500	22.36	22	21.91	0.1341	88.05
600	24.49	24	24.04	0.1258	98.50
700	26.46	26	26.00	0.1197	108.79
800	28.28	28	27.82	0.1150	118.98
900	30.00	30	29.53	0.1112	129.10
1000	31.62	32	31.16	0.1082	139.17
2000	44.72	45	44.24	0.0842	212.43
3000	54.77	55	54.29	0.0725	271.43
4000	63.25	63	62.76	0.0647	320.84
5000	70.71	71	70.23	0.0599	369.30
6000	77.46	77	76.97	0.0554	408.52
7000	83.67	84	83.18	0.0527	451.59
8000	89.44	89	88.95	0.0497	485.36
9000	94.87	95	94.38	0.0477	523.29
10000	100.00	100	99.51	0.0457	556.40

Table 4. Statistics for C2PS for MinERR payoff based on exact equations in *Glass*.

V. A Cheating Applicant

The approach in *Glass* addresses the C2PS for NBTB payoff from a direction that appears to be unique in the SP literature: the selection process is viewed from the vantage point of a single applicant that can choose its position in the queue.

D.B. Glass referred to the applicant applying "optimal strategy" but the term "cheating applicant (CA)" seems to be more appropriate due to a single applicant being able to break a rule while the other applicants are assumed not to have the same opportunity.

Even at a superficial level of analysis, it can be seen that position in the queue has a great impact on an applicant's probability of being selected. Regardless of their ranks, by the rules of the CSP the hapless applicants presented to the DM in the observation-only phase have zero probability of being selected; therefore, the first goal of the CA seeking to improve its chances of selection would be to avoid being placed in any position 1, 2, … $s-1$.

For positions s and greater, *Glass* Theorem 1 (restated in this paper as expression (1)) yields the probability that the applicant in position p is selected when averaged over all ranks. This is shown in Figure 3 for $N = 20$ and optimal NBTB start position $s = \lceil 20/e \rceil + 1 = 9$.

For NBTB payoff, *Glass* Theorem 2 states that, for $N \geq 9$, the applicant in position N has the highest probability of being selected.[8]

The use of this information or the more sophisticated approaches presented in Section VI by a CA is a "deception" technique in the sense that the DM expects every applicant to be in a randomly assigned position but the CA violates that expectation by arranging to be placed in a position that is advantageous[9] to its chances for selection.

Even if the applicant in question cannot cheat by maneuvering into an advantageous position in the queue, this information could still be used to inform an action such as "opt-out" (that is, withdraw) from the selection process. This could be of value to an applicant when participation in the selection process involves the expenditure of resources.

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[8] The high probability of selecting the last applicant, regardless of its rank, is a disadvantage to the DM under any payoff function. This issue has prompted the study of solutions for the SP that reduce the probability of selecting the last applicant [Krotofil 2014] as discussed in Section III and in the version of the SP introduced in Section VIII.

[9] The term "advantageous position" is given a specific definition in Section VI. Briefly, an advantageous position is one where an applicant's probability of selection is at or above an appropriate threshold. For the C2PS, that threshold is the probability of selection in the last position in the queue.

When the DM uses the C2PS, a CA that has no information about its rank relative to the other applicants has its greatest opportunity for selection in the last position in the queue. Even at this initial level of analysis, it can be seen that the probability of selection in the last position serves as a reasonable benchmark for advantageous placement.

The next section explores how a CA can use knowledge of its own rank to either increase its chances for selection when *seeking selection* or decrease its chances for selection when *seeking to evade selection*.

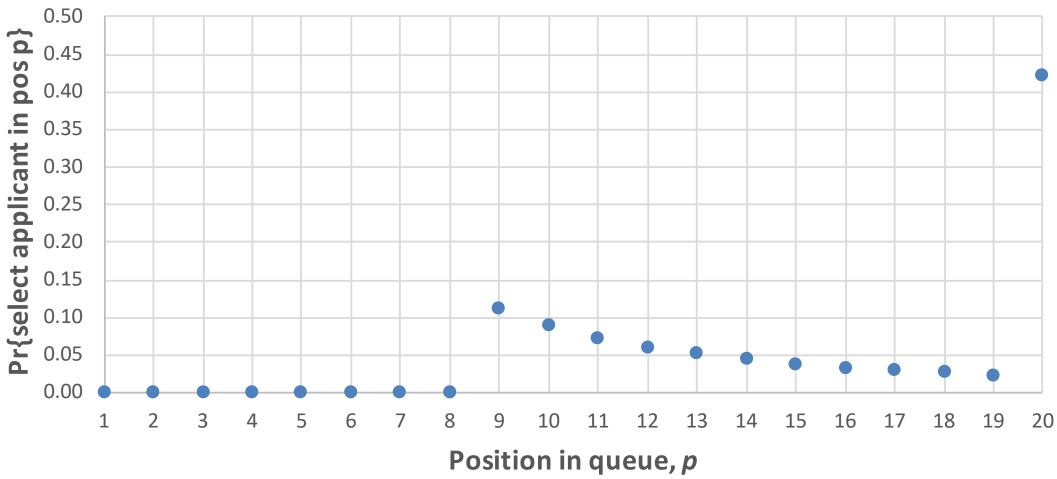

Figure 3. Conditional probability of selecting the applicant in position p, regardless of rank, for $N = 20$ and start position $s = 9$.

VI. *Strategy for Cheating the Classical Two-Phase Solutions*

Although the last position is seen to be best when averaged over *all* ranks, the analysis in *Glass* goes deeper into the preferred course of action for a CA by considering the factor of its *specific* rank.

The following assumptions and adjustments to the definition of the C2PS for NBTB payoff are made in *Glass*:

- The DM uses the optimal NBTB start position $s = \lceil N/e \rceil + 1$.
- There is a single CA.
- The CA knows N and s.
- The CA knows (or can closely approximate[10]) its own rank relative to the other applicants.
- The CA knows and has control over its position in the queue. Other than that one violation of "randomness," every other ordering of the remaining applicants is equiprobable.
- The CA does not know the ordering of the other applicants in the queue.[11]

A critical finding in *Glass* was its Theorem 4 which stated that, with the optimal start position $s = \lceil N/e \rceil + 1$, applicants with rank 1, 2, or 3 have the highest probability of being selected in position s and all other applicants have the highest probability of selection in position N if N is "sufficiently large."[12] It is this result that is explored and generalized in this paper.

Figures 4a-4d show the conditional probabilities of selection of applicants with ranks 1, 2, 3, and 4 with $N = 20$ and $s = \lceil 20/e \rceil + 1 = 9$. The applicants with ranks 1 and 2 have their best chances in position 9, but applicants with ranks 3 and 4 have their best chances in position 20; this value of N happens to be below the threshold for *Glass* Theorem 4 to hold.

---

[10] Section X deals with the case where the applicant does not know its exact rank in the queue but can estimate it within a range.

[11] This assumption is not stated but is implicit in *Glass*. If the CA were to know the ordering of the other applicants in the queue then predicting selection or non-selection of the CA would be a certainty rather than a probability calculation.

[12] For *Glass* Theorem 4 to hold, "sufficiently large" N can be found to be $N \geq 99$. For $2 \leq N \leq 99$ there are 69 values of N such that only rank 1 or ranks 1 and 2 have higher probabilities of being selected in position s than in position N. For $N \geq 99$, ranks 1, 2, and 3 have higher probabilities of being selected in position s than in position N and those are the only such ranks.

Advantageous Positions for an Applicant Seeking Selection

The observations above can be generalized as follows. Other than the two applicants with ranks 1 and N, the applicants in positions s, s+1, … N−1 initially have monotonically decreasing probabilities of being selected and then a sharp spike in position N due to the CSP rule that the last applicant in the queue must be selected if the DM has not selected an applicant prior to that point. The applicant with rank 1 has a monotonically decreasing probability of being selected in positions s, s+1, … N. The applicant with rank N has zero probability of being selected in any position except the last position.

Due to the high probability of selection for an applicant in the last position, that position serves as a good benchmark for the effectiveness of selection and thus also for the effectiveness of cheating.

Consequently, for the C2PS **an *advantageous position* is defined to be any position that provides a probability of selection for an applicant *at least as great as that for the last position* in the queue. Any other position is *non-advantageous*.**[13] More formally, p is an *advantageous position* for an applicant of rank r if and only p satisfies the condition

$$\Pr\{\text{select rank } r \mid \text{position } p\} \geq \Pr\{\text{select rank } r \mid \text{position } N\} \tag{9}$$

It follows from this definition that the last position is *always* an advantageous position. Of course, no position before the start position s that begins the observation-and-selection phase can be advantageous.

Generalizing the Choice of Start Position

While every advantageous position is considered desirable for a CA, in the C2PS the CA's probability of selection is improved the most by maneuvering into the *earliest* available advantageous position.

As discussed in Section III, DMs may not always use the optimal NBTB start position. That start position results in search times that may be explicitly penalized in versions of the SP that consider cost or implicitly by human DMs who may tend to under-search. The optimal start position for NBTB payoff also performs poorly when measured by the expected value of the rank of the selected applicant. *Glass* Theorem 4 does not consider start positions smaller than the optimal NBTB start position and thus does not address the advantages of those positions to the CA. Therefore, it is productive to generalize the model in *Glass* to account for other start positions, specifically to explore the benefits to the CA created by start positions smaller than the optimal value for NBTB payoff.

[13] For an applicant with rank 1, the last position is still considered advantageous although it is the *least* advantageous for that applicant.

From this point on, the model in *Glass* is relaxed in that the DM may choose *any* start position $s \geq 2$, rather than fixing it at the NBTB optimal value of $\lceil N/e \rceil + 1$.[14] Other than removing the restriction on the DM's start position, the rest of the assumptions in *Glass* (as stated in Section V) are retained.[15,16]

Finding the Advantageous Positions

This generalization of *Glass* Theorem 4 is demonstrated in Figure 5 by choosing the smallest possible start position. This figure shows the conditional probability Pr{select rank r | position p}, given by expression (2), plotted for $N = 20$, $s = 2$, and for an applicant with rank 5. It can be seen that the probability of selection of this applicant is higher at positions 2 through 5 than it is in position 20; all other positions provide a lower probability of this applicant being selected than in the last position.

In line with the arguments in *Glass*, the question is to determine which positions (if any) in the queue provide equal or better probabilities of selection for the CA than the final position; these are the *advantageous positions* that satisfy the condition in expression (9). This question is answered using expression (2) (restated from *Glass* Theorem 3) by determining the range of positions p for which Pr{select rank r | position p} in the range $s \leq p < N$ is greater than or equal to the selection probability at position $p = N$.

For the C2PS and given N, s, and r, **define $z(N, r, p)$ to be the ratio of the probability of selecting an applicant with rank r in position p to the probability of selecting that applicant if it were in the last position**, for the moment ignoring the fact that the probability of selection is zero for $p < s$:

$$z(N, r, p) = \frac{\Pr\{\text{select rank } r \mid \text{position } p\}}{\Pr\{\text{select rank } r \mid \text{position } N\}}, \quad \text{for } s \leq p \leq N \text{ and } s \geq 2 \qquad (10)$$

As noted earlier, expression (2) establishes that the probability of selection for any rank is non-zero in the last position, so the denominator of expression (10) is guaranteed to be non-zero.

[14] As noted earlier, the constraint that $s \geq 2$ is needed because $s = 1$ results in the first applicant being selected regardless of rank.

[15] Briefly, the assumptions are as follows. There is a single CA that knows N, s, and its rank. The CA has control of or has influence over its position in the queue, and all other orderings of applicants are equiprobable.

[16] In this paper it is assumed the DM's choice of s is known with certainty to the CA, perhaps due to inside information. However, the value of s might instead be known only approximately because of rounding issues (taking the ceiling, floor, rounding to nearest, ceiling plus 1, etc.) or because of human factors relating to personality or deadlines. Where there is uncertainty in the value of s, a margin of safety would have to be included to guarantee the CA does not inadvertently choose a position prior to the DM's start position and thus have zero chance of selection.

Expanding the numerator and denominator in expression (10) it is seen that, remarkably, in the C2PS the dependence on s cancels out: the ratio $z(N, r, p)$ is invariant with s. Due to the constraint that $s \geq 2$ in expression (2), however, it is still required that $p \geq 2$:

$$z(N, r, p) \; = \begin{cases} 0 & \text{for } p < 2 \text{ or } p > N \\ \dfrac{\binom{N-r}{p-1}}{\binom{N-1}{p-1}} \times \dfrac{N-1}{p-1} & \text{for } \; 2 \; \leq \; p < \; N \\ 1 & \text{for } \; p = N \end{cases} \tag{11}$$

As noted earlier in regard to expression (2), the numerator of expression (11) is zero for $N-r < p-1$ so $z(N, r, p) = 0$ for $N-r+1 < p < N$.

Because the dependence on s cancels out in expression (11), the value of $z(N, r, p)$ can also be calculated directly from expression (10) for any value of s for which Pr{select rank r | position p} is nonzero; using $s = 2$ is the most general choice.

The Critical Position

The question posed by D.B. Glass and answered by *Glass* Theorem 4 can now be generalized by finding the ***critical position*** in the queue, denoted by $p_c(N, r)$, that delimits the range of positions for which the probability of selection is at least as great as in the last position and thus satisfy the condition of expression (9):

$$p_c(N, r) = \begin{cases} \text{largest value of } p \text{ for which } z(N, r, p) \geq 1 \\ \qquad\qquad\qquad \text{for } every \text{ position in the range } 2, 3, \ldots \; p \\ 0 \;\; \text{if no such value of } p \text{ exists} \end{cases} \tag{12}$$

Due to the fact that the critical position is defined in terms of $z(N, r, p)$, which is independent of s, the critical position $p_c(N, r)$ is also independent of s in the C2PS. The invariance of the critical position with respect to the start position appears to be a special property of the CSP and does not necessarily hold for other versions of the SP or related problems. This invariance property makes possible the method presented in Section VII for approximating the critical position.

Figure 5 is annotated with a description of the advantageous positions and the critical position. The probability of selection at position 6 is just slightly below that of the last position so it is not an advantageous position (because $z(N = 20, r = 5, p = 6) = 0.9814$ which is less than one). The probability of selection at position 5 is well above that of the last position ($z(N = 20, r = 5, p = 5) = 1.6728$) so it is the advantageous position that satisfies expression (12) and thus is the critical position.

Finding the critical value for small ranks is not difficult. For rank 1 the algebraic solution for expression (12) is trivial: $p_c(N, r = 1) = N$.

For rank 2 the solution for $p_c(N, r = 2)$ is $(N+1)/2$, or, in practical terms, it is the floor of that value.

For rank 3 the solution for $p_c(N, r = 3)$, for large N and after neglecting insignificant terms, is the one feasible solution for the quadratic equation $p^2-3Np+N^2 = 0$ which is $(3-\sqrt{5})N/2 \approx 0.382N$.

Since the degrees of the algebraic equations resulting from expression (12) increase with rank and numerical solutions are quickly required it is more straightforward to use a simple linear search to find the critical positions.

The critical position can be found using nothing more than spreadsheet functions or by writing a simple program. For given values of queue length N and rank r, the pseudocode for such a program consists of these steps:

```
pc = 0; /* remains zero if no advantageous position found */
p = 2;  /* first position to test */

/* Test increasing positions until probability of selection
   falls below that of last position (i.e., z(N,r,p) < 1) */

while (p ≤ N and z(N,r,p) ≥ 1−ε) {
   pc = p;   /* save largest p so far where z(N,r,p) ≥ 1 */
   p  = p+1; /* advance to test next position */
}

/* On exit from the loop, the value of pc is pc(N, r) */
```

Note that, due to the practical issue of the unreliability of a floating-point equality check, a small adjustment factor is used: $\varepsilon = 10^{-6}$ was found to be adequate.[17]

Figures 6a-6d show Pr{select rank r | position p} and $z(N, r, p)$ graphed on the same axes. In each case, $N = 20$ and $s = 2$:

- Figure 6a shows the case where the applicant has rank 1. Every position $p = 2, 3, \ldots 20$ has $z(N = 20, r = 1, p) \geq 1$, so the critical position for this applicant is 20. Thus, the critical position for rank 1 is always N.

- Figure 6b shows the case where the applicant has rank 4. Every position $p = 2, 3, \ldots 6$ has $z(N = 20, r = 4, p) \geq 1$ but at subsequent positions that ratio is less than 1 until position 20, so the critical position for this applicant is 6.

[17] When factorials are calculated using the Gamma Function and the post-increment of p is by fractional values instead of 1, it can be seen that $p_c(N, r)$ is not necessarily an integer. For example, the largest integer value of $p_c(N=50, r=3)$ is 19, but its fractional value is 19.6. In this paper, however, the floor of $p_c(N, r)$ is used.

- Figure 6c shows the case where the applicant has rank 15. Only position $p = 2$ has $z(N = 20, r = 15, p) \geq 1$ until position 20, so the critical position for this applicant is 2.

- Figure 6d shows the case where the applicant has rank 20. Every position has $z(N = 20, r = 20, p) = 0$ until position 20, so the critical position for this applicant is 0. Thus, the critical position for rank N is always 0.

As noted earlier, expression (11) shows that the value of the critical position is independent of the start position in the C2PS so Figures 6a-6b used the smallest possible start position ($s = 2$) to make the critical positions visible. For an applicant of a given rank, if the critical position is greater than or equal to the start position there exists a range of positions that would have a probability of selection of that applicant that are at least as favorable as the last position. If the start position is larger than the critical position, then the last position is to be preferred, of course.

The Critical Position and Cheating

The CA uses knowledge of the critical position to increase its probability of selection using the following procedure. Assumed to know N, its own rank r, and the DM's choice of start position s, the CA calculates the critical position $p_c(N, r)$. If $p_c(N, r) < s$ then no available position in the queue is better than the last position. Otherwise, the CA has a higher probability of selection in any position in the range s, s+1, … $p_c(N, r)$ than in the last position, and the last position is preferred over the remaining positions in the range $p_c(N, r)+1$, $p_c(N, r)+2$, … $N-1$. Furthermore, the probability of selection is improved by maneuvering into the earliest possible advantageous position.

Consider a case where $N = 20$ and the CA knows its rank to be 3; thus, the critical position is 8. Knowing where the DM set the start position s, the CA should maneuver into one of the advantageous positions s, s+1, s+2, … 8; otherwise, the last position is best. From Table 1 or Table 3, it is seen that the optimal start position for NBTB payoff is $s_{\text{NBTB}} = 8$, so only position 8 and the last position are advantageous for the CA. From Table 2 or Table 4, it is seen that the optimal start position in the C2PS for MinERR payoff is $s_{\text{MinERR}} = 5$, which means positions 5, 6, 7, and 8 are better than the last position for the CA.

The fact that the critical position has been shown to be invariant with respect to the DM's start position in the C2PS reduces the uncertainty of the CA's range of choices for selection of an advantageous position. It has been assumed in this paper that the CA knows the DM's choice of s, but the CA might anticipate some variation in s (due to rounding, the DM's personality, etc.) and thus include some margin of safety in the "left-hand side" of the range. However, the "right-hand side" of the range is delimited by the critical position that is independent of the DM's start position.

Based on the start position being chosen using NBTB payoff, *Glass* Theorem 4 shows the desirability of applicants of rank 1, 2, or 3 being placed exactly at $s = \lceil N/e \rceil + 1$. Indeed, this position s does provide the maximum probability of selection for those applicants. However, that theorem does not address the advantageous positions that are larger than s for those applicants:

- For rank 1, $p_c(N, r = 1)$ is N. For $N = 1{,}000$, every position from the start position of 369 to the critical position 1,000 is advantageous to this applicant.

- For rank 2, $p_c(N, r = 2)$ is $\lfloor (N+1)/2 \rfloor$. For $N = 1{,}000$, every position from the start position of 369 to the critical position 500 is advantageous to this applicant.

- For rank 3, $p_c(N, r = 3)$ is $\lfloor 0.382N \rfloor$. For $N = 1{,}000$, every position from the start position of 369 to the critical position 382 is advantageous to this applicant.

- For rank 4, $p_c(N = 1000, r = 4)$ is 318, which is less than the start position of 369. As predicted by *Glass* Theorem 4 there is no advantageous position for this applicant other than the last position in the queue.

Figure 7 and Figure 8 demonstrate, for $N = 100$, the benefits to the CA obtained by using the cheating strategy of being put in the earliest possible advantageous position. Those figures show the ratios of the probability of selection for that CA to the probability of selection of an applicant randomly placed anywhere in the queue as well as to that of an applicant randomly placed but fortunate enough to be positioned at or beyond the DM's start position.

For the optimal NBTB start position $s = 38$, Figure 7 shows the ratios for CAs with ranks 1, 2, and 3. For the optimal MinERR start position $s = 10$, Figure 8 shows the ratios for the 23 CAs that have critical positions greater than or equal to s. The benefits of the cheating strategy of being placed immediately at the start of the observation-and-selection phase increase dramatically with the relative rank of the CA.

Even if the CA is not able to achieve the maximum benefit of being placed in the *first* advantageous position, being in *any* advantageous position greatly increase the CA's probability of selection. Figure 9 and Figure 10, again for $N = 100$, show the ratios of the probability of selection for that CA *averaged over all advantageous positions* to the probability of selection of an applicant randomly placed anywhere in the queue as well as to that of an applicant randomly placed at or beyond the DM's start position. Figure 9 gives the results for the optimal NBTB start position $s = 38$ and Figure 10 gives the results for the optimal MinERR start position $s = 10$. While the improvements are less than those for the best cheating strategy, the ratios are still greatly to the CA's benefit.

A CA seeking to evade selection could choose a *non-advantageous position* (that is, a position that is not in the set of advantageous positions) and in that way deceive the DM. A CA can evade selection with certainty by being placed earlier in the queue than the start position. Failing that, the CA in the observation-and-selection phase should choose a position in the queue that substantially reduces its probability of selection.

Unless otherwise noted, the default assumption is that CAs are seeking selection. It is explicitly stated when CAs are seeking to evade selection.

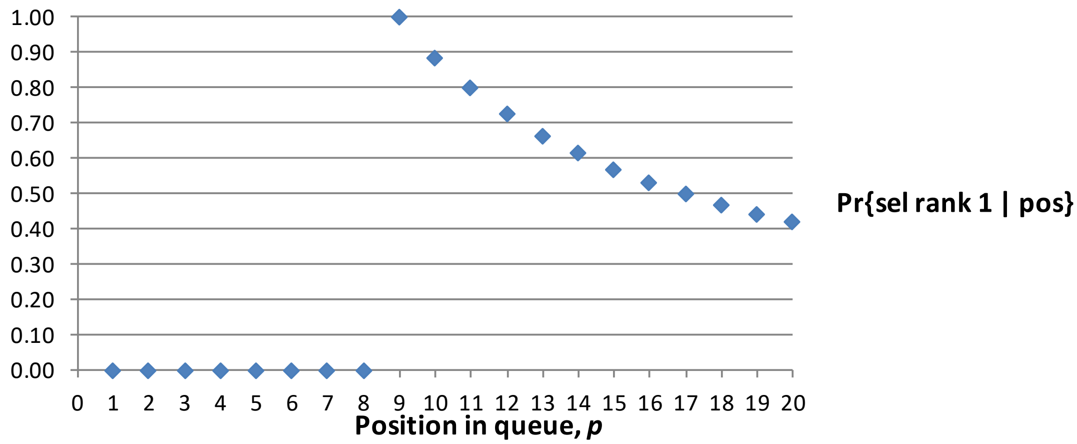

Figure 4a. Conditional probability of selecting applicant with rank 1, given applicant is in position p in queue. Probability of selection is higher in every position than in last position. N = 20 and start position s = 9.

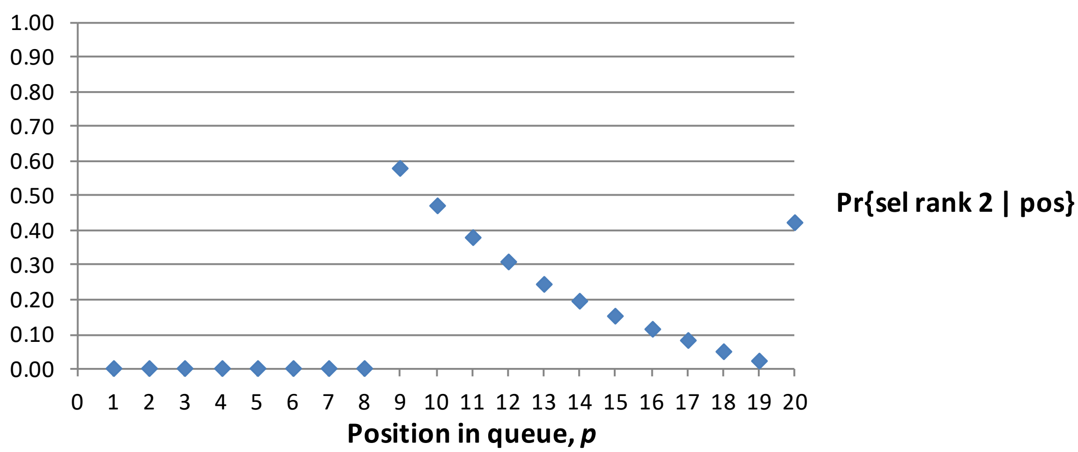

Figure 4b. Conditional probability of selecting applicant with rank 2, given applicant is in position p in queue. Probability of selection is higher at p = 9 and 10 than in last position. N = 20 and start position s = 9.

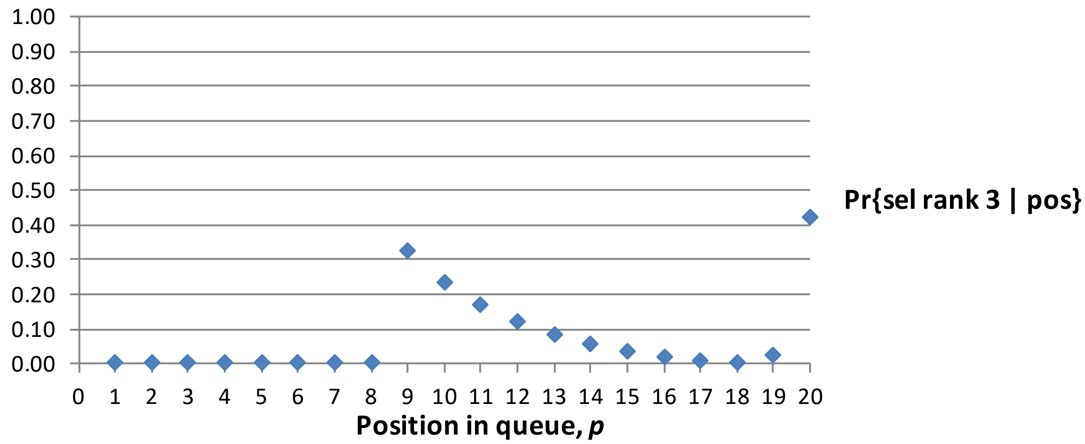

Figure 4c. Conditional probability of selecting applicant with rank 3, given applicant is in position p in queue. Probability of selection is lower in every position than in last position. $N = 20$ and start position $s = 9$.

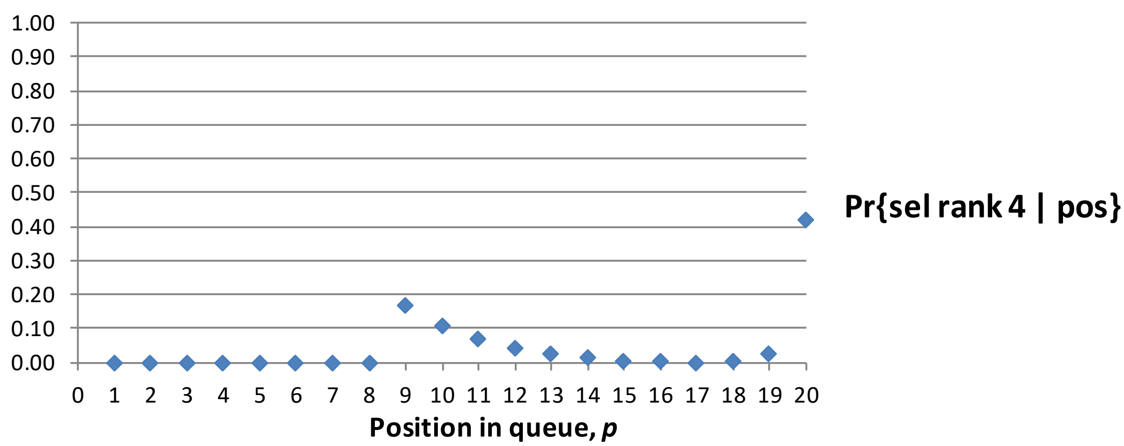

Figure 4d. Conditional probability of selecting applicant with rank 4, given applicant is in position p in queue. Probability of selection is lower in every position than in last position. $N = 20$ and start position $s = 9$.

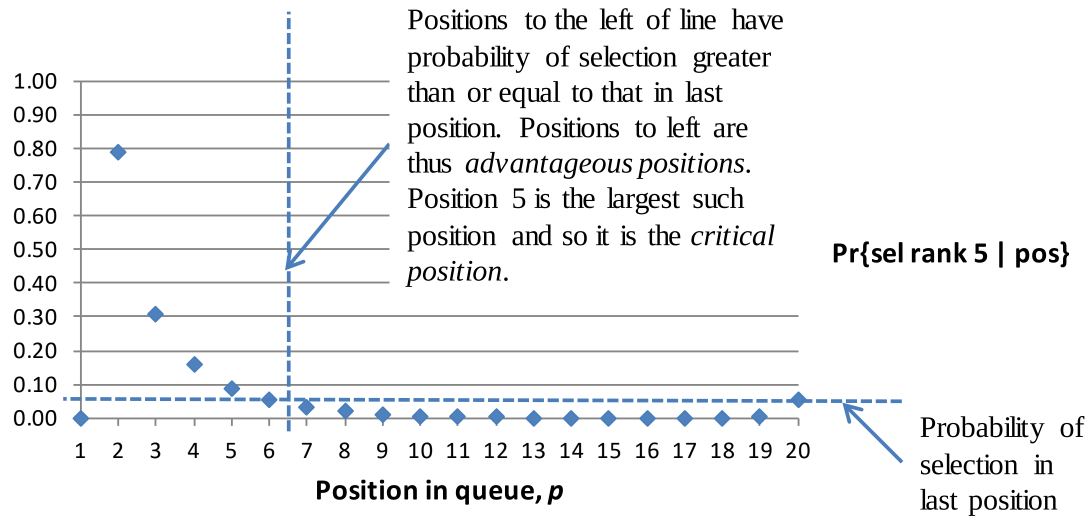

Figure 5. Conditional probability of selecting applicant with rank 5, given applicant is in position p in queue. Probability of selection is higher in positions p = 2, 3, 4, and 5 than in last position. N = 20 and start position s = 2.

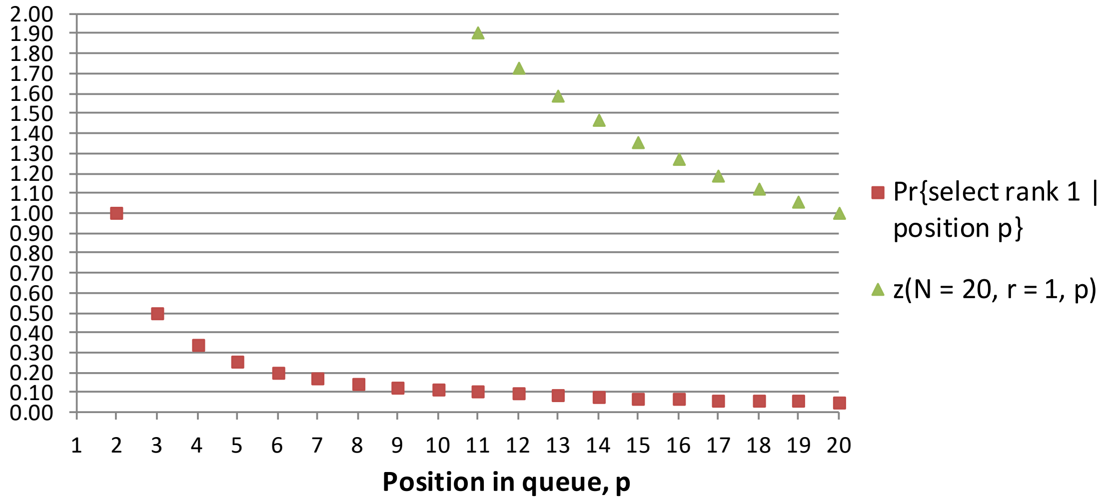

Figure 6a. Conditional probability of selecting applicant with rank 1, given applicant is in position p, and $z(N, r, p)$. $N = 20$ and start position $s = 2$. Ratio $z(N, r, p) \geq 1$ in every position $2 \leq p \leq 20$, so $p_c(N, r) = 20$.

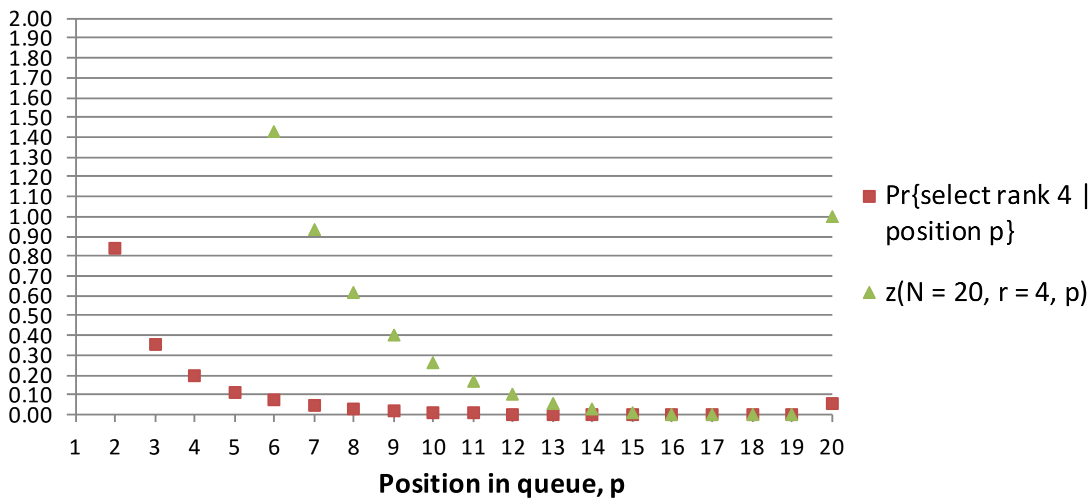

Figure 6b. Conditional probability of selecting applicant with rank 4, given applicant is in position p in queue and $z(N, r, p)$. $N = 20$ and start position $s = 2$.
Ratio $z(N, r, p) \geq 1$ in every position $2 \leq p \leq 6$ but less than 1 in subsequent positions, so $p_c(N, r) = 6$.

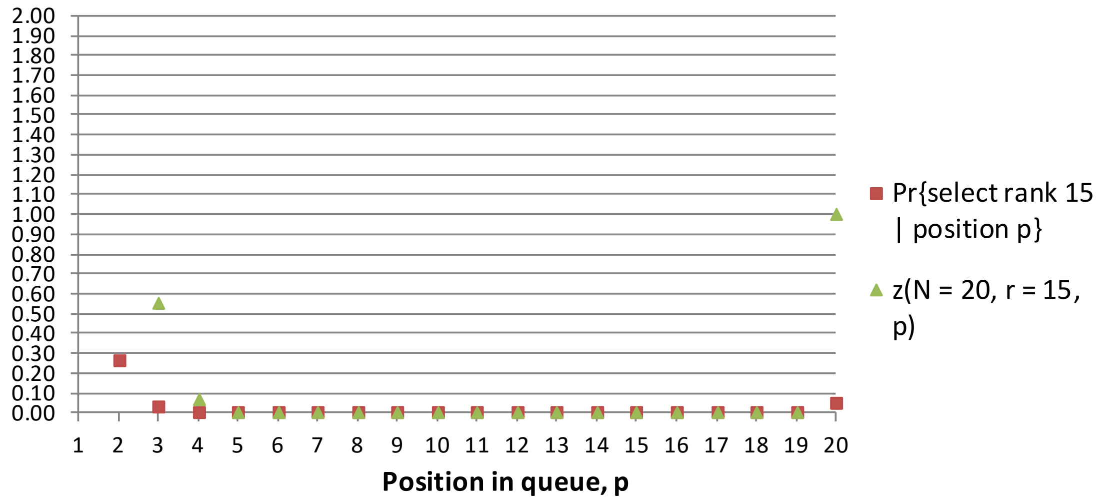

Figure 6c. Conditional probability of selecting applicant with rank 15, given applicant is in position p in queue and $z(N, r, p)$. $N = 20$ and start position $s = 2$.
Ratio $z(N, r, p) \geq 1$ in position 2 but less than 1 in subsequent positions, so $p_c(N, r) = 2$.

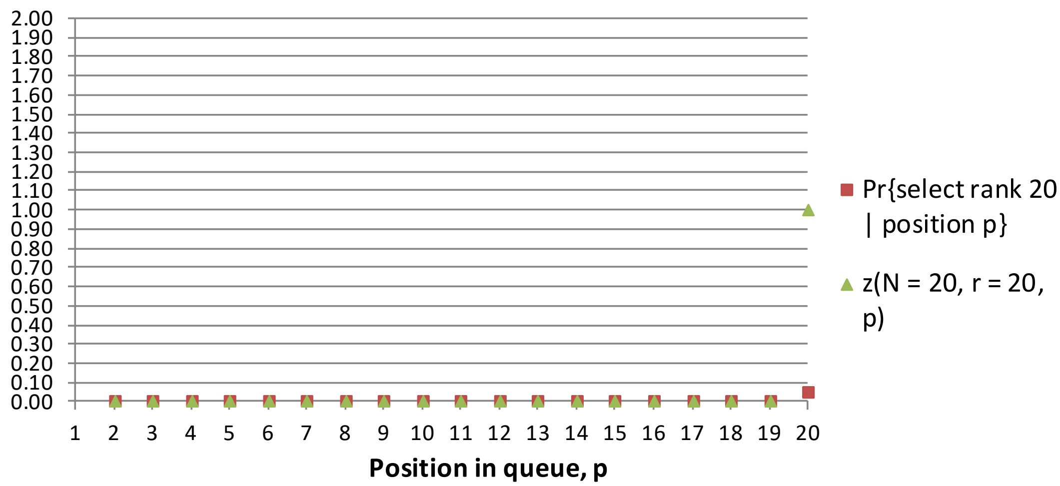

Figure 6d. Conditional probability of selecting applicant with rank 20, given applicant is in position p in queue and $z(N, r, p)$. $N = 20$ and start position $s = 2$.
Ratio $z(N, r, p) < 1$ in position 2, so $p_c(N, r) = 2$.

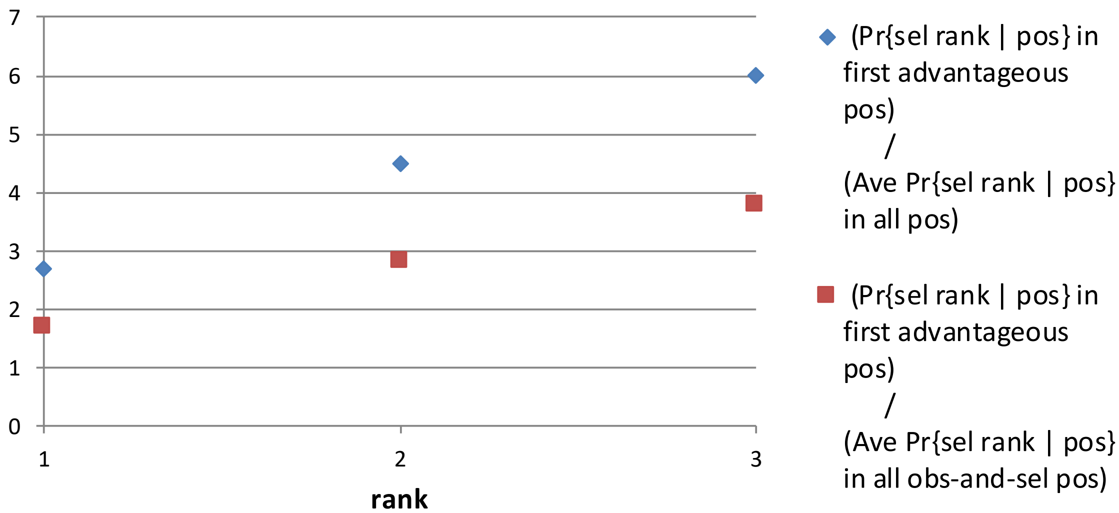

Figure 7. Blue points: Ratio of the probability of selection in the *first advantageous position* to the average probability of selection *in any position*.
Red points: Ratio of probability of selection in the *first advantageous position* to the average probability of selection to *any position in observation-and-selection phase*.
N = 100 and NBTB start position s = 38.

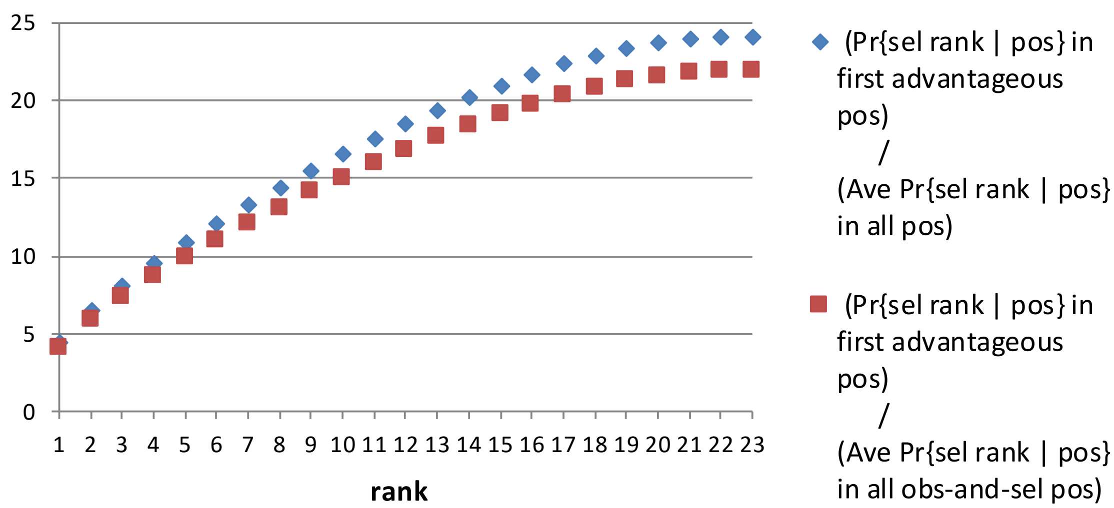

Figure 8. Blue points: Ratio of the probability of selection in the *first advantageous position* to the average probability of selection *in any position*.
Red points: Ratio of probability of selection in the *first advantageous position* to the average probability of selection to *any position in observation-and-selection phase*.
N = 100 and MinERR start position s = 10.

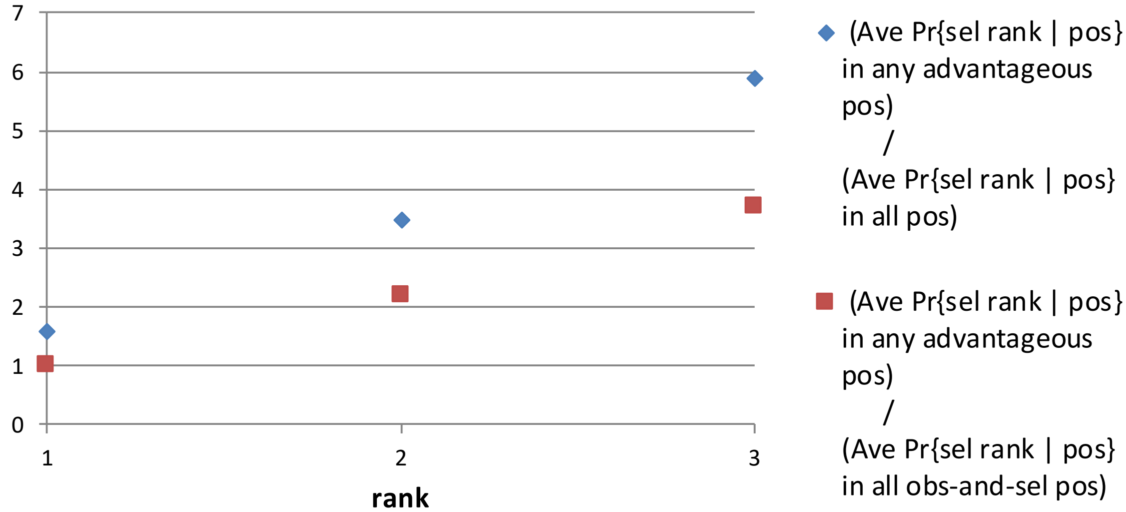

Figure 9. Blue points: Ratio of the probability of selection in *any advantageous position* to the average probability of selection *in any position*.
Red points: Ratio of probability of selection in the *first advantageous position* to the average probability of selection to *any position in observation-and-selection phase*.
N = 100 and NBTB start position s = 38.

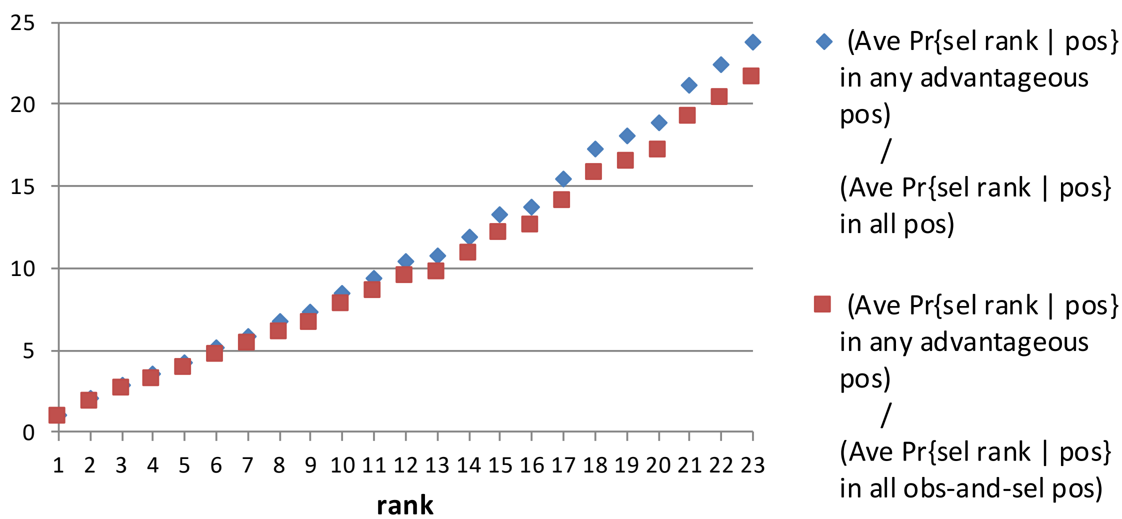

Figure 10. Blue points: Ratio of the probability of selection in *any advantageous position* to the average probability of selection *in any position*.
Red points: Ratio of probability of selection in the *first advantageous position* to the average probability of selection to *any position in observation-and-selection phase*.
N = 100 and MinERR start position s = 10.

VII. Estimating the Critical Position

Although the exact values for the critical position for the C2PS can be obtained with only a small amount of programming, it is now shown that $p_c(N, r)$ appears to have a limiting behavior that makes it straightforward to closely estimate its value from tabulated data. This is simplified by the fact shown in this paper that the critical position for the C2PS is invariant with respect to the DM's start position.

Figure 11 shows the behavior of the ratio $p_c(N, r)/N$ for several values of r and values of N up to 1,000. At $N = 1{,}000$ these values (to a few decimal places) are:[18]

Rank 1: $p_c(N = 1000, r = 1)/1000 = 1.000$
Rank 2: $p_c(N = 1000, r = 2)/1000 = 0.500$
Rank 3: $p_c(N = 1000, r = 3)/1000 = 0.382$
Rank 4: $p_c(N = 1000, r = 4)/1000 = 0.318$
Rank 5: $p_c(N = 1000, r = 5)/1000 = 0.275$

Observing how quickly and closely these ratios converge to their apparent limits, it appears that tabulated values of $p_c(N', r)/N'$ for some large but otherwise arbitrary number of applicants N' can be used to approximate the critical position for another queue of length N. An *initial* hypothesis is proposed:

$$p_c(N, r) \approx N \times p_c(N', r)/N', \quad \text{where } N' \text{ is large (say, } N' = 1000) \tag{13}$$

This hypothesis can be tested with the critical position values provided above with $N' = 1{,}000$ to see how well it predicts the actual critical positions for, say, $N = 20$:

$p_c(N = 20, r = 1) = 20,\ 20 \times p_c(N' = 1000, r = 1)/1000 = 20.00$
$p_c(N = 20, r = 2) = 10,\ 20 \times p_c(N' = 1000, r = 2)/1000 = 10.00$
$p_c(N = 20, r = 3) = 8,\ 20 \times p_c(N' = 1000, r = 3)/1000 = 7.64$
$p_c(N = 20, r = 4) = 6,\ 20 \times p_c(N' = 1000, r = 4)/1000 = 6.36$
$p_c(N = 20, r = 5) = 5,\ 20 \times p_c(N' = 1000, r = 5)/1000 = 5.50$

For these particular values of N and r, the approximation given by expression (13) is seen to differ by less than 1 from the actual value for the critical positions.

[18] As stated in Section I, the NBTB start position is approximately N/e or about $0.368N$. Note that the ratio for rank 3 is greater than 0.368 while the ratio for rank 4 is below that value; this means that for large N the critical position for rank 3 is greater than the NBTB start position, but the critical position for rank 4 is less. This demonstrates the limiting behavior predicted by *Glass* Theorem 4 that only ranks 1, 2, and 3 have early positions in the queue that are better than the last position.

The hypothesis given as expression (13) was based on simple observations rather than rigorous proof but it is easily tested exhaustively for reasonable problem sizes. The approximation for $p_c(N, r)$ was tested using $N' = 1{,}000$ for all values of N from 2 to 1,000 and every r from 1 to N. Figure 12 shows a histogram of the differences between the value obtained using expression (13) and the actual critical position for each of the 500,499 possible combinations of N and r. It is observed that almost all of the differences were greater than −1 and less than 2.5.

Empirically, it was found that rounding expression (13) and subtracting 2 yielded estimates for $p_c(N, r)$ that were never larger than the actual value of $p_c(N, r)$ and, with the understanding that $p_c(N, r = N) = 0$, only 1.37% of the values were less than $p_c(N, r)$ by more than 2. Underestimation of the value of the critical position still provides a valid range of advantageous positions for the CA. Thus, expression (13) provides a close estimate of the range of $p_c(N, r)$.

Based on these observations, the initial hypothesis in expression (13) is revised as follows. With the understanding that $p_c(N, r = N) = 0$:

$$pc(N,r) \approx \max \begin{cases} \text{round}\left(N \times \dfrac{pc(N', r)}{N'} \right) - 2\,, \\ \qquad \text{where } N' \text{ is large } (\text{say}, N' = 1000) \\ 0 \end{cases} \tag{14}$$

Values for $p_c(N', r)/N'$ for $N' = 1{,}000$ and ranks up to 50 are given in Table 5.

Whether approximated using expression (14) or calculated directly from expression (11), the critical position for any value of N can be used by a CA of rank r to choose a preferential position in the queue or take another action such as withdraw from the selection process. A CA seeking to evade selection can use this information to reduce its selection probability.

This information could also be used by a wary DM to watch for and potentially *counter* the actions of a deceptive CA to prevent the compromise of the selection process. Outside of the rules of the CSP, a DM could observe applicants' activities that appear to facilitate their being placed in advantageous positions or avoiding those positions.

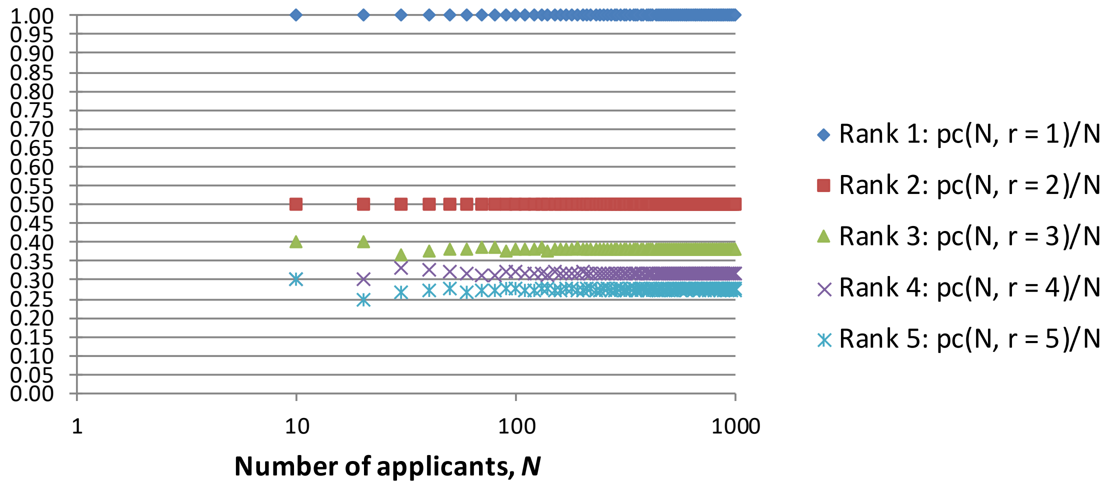

Figure 11. Critical position $p_c(N,r)$ divided by N appears to approach constant value even for small N.

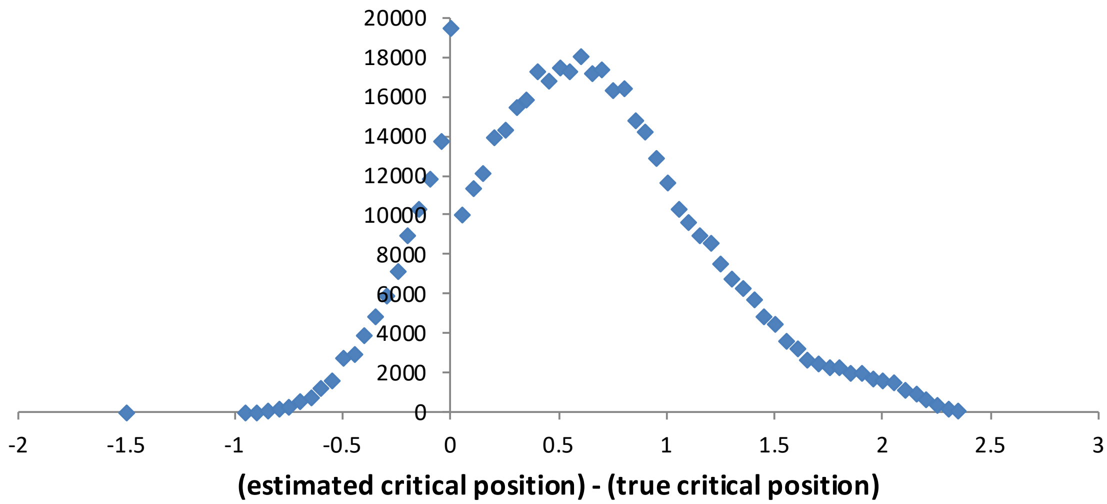

Figure 12. Histogram of differences between estimated $p_c(N, r)$ given by $N \times p_c(N', r)/N'$ (for N'= 1,000) and true $p_c(N, r)$ for all combinations of N = 2, 3, … 1,000 and all ranks 1, 2, … N.

rank r	$p_c(N=1000, r)$ / 1000	rank r	$p_c(N=1000, r)$ / 1000	rank r	$p_c(N=1000, r)$ / 1000	rank r	$p_c(N=1000, r)$ / 1000	rank r	$p_c(N=1000, r)$ / 1000
1	1.000	11	0.165	21	0.106	31	0.080	41	0.065
2	0.500	12	0.155	22	0.102	32	0.078	42	0.064
3	0.382	13	0.147	23	0.099	33	0.077	43	0.063
4	0.318	14	0.140	24	0.096	34	0.075	44	0.062
5	0.275	15	0.134	25	0.094	35	0.073	45	0.061
6	0.245	16	0.128	26	0.091	36	0.072	46	0.060
7	0.222	17	0.123	27	0.088	37	0.070	47	0.059
8	0.203	18	0.118	28	0.086	38	0.069	48	0.058
9	0.188	19	0.113	29	0.084	39	0.068	49	0.057
10	0.176	20	0.109	30	0.082	40	0.066	50	0.056

Table 5. Values of $p_c(N', r)/N'$ for N'=1,000 and ranks r = 1, 2, … 50.

VIII. *Cheating a Variant of the Secretary Problem Allowing Non-Selection (SPANS)*

This section introduces and explores a new variation on the CSP to further demonstrate how to develop a cheating strategy.

This variant of the SP was introduced primarily to demonstrate how the methods used in this paper could be applied to a situation more complex than the CSP. This variant may have value in its own right, however, as it introduces the option of *non-selection* of the last applicant and thus matches some practical situations better than the CSP. It reduces the CSP's disproportionate probability of forcing the DM to select the applicant in the last position by default.

The option of non-selection appears to have been almost unexplored in the literature.[19] There may be situations where a DM would prefer to make no selection at all rather than be forced to accept a poor applicant that happens to be in the last position. Truong [Truong 2013] considered the possibility of non-selection under several models but concluded that the DM should always select an applicant. This seems to be unreasonable, however, because a DM in practice (perhaps knowing the true values of the applicants) should certainly refuse to make a selection if that selection would result in a negative net reward.

This variant of the SP and its two-phase solution are introduced and discussed initially from the point of view of the DM. Note that, as in the CSP, the solution for this variant of the SP stops when the DM makes a decision; the procedure is generalized in that the decision may be *not* to select an applicant. Following that introduction, the problem is reversed and addressed from the viewpoint of a CA.

For brevity the results in this section are obtained solely through simulation and only for $N = 100$.

Benefits to the DM of Non-Selection

Two examples are presented to demonstrate situations where non-selection may be the preferred outcome.

For example, consider the case where a homebuyer (the DM) is viewing potential purchases (the applicants). In a strong seller's market, the decision must often be made practically on the spot so there is no opportunity to "recall a previous applicant." As mentioned in the Introduction, a brief Internet search of postings by real estate agents and buyers indicated that prospective buyers might look at 10, 25, or even 70 properties – a

[19] Krotofil et al. [Krotofil 2014] used the term "non-selection" to mean the case where the DM is forced to accept the last applicant by default if no selection is made before that point. In this paper "non-selection" means the DM can decline to make any selection at all if the last applicant does not meet certain criteria.

common lament by agents and buyers was that the search often ends with a "non-selection" because a homebuyer who is not desperate for housing would rather make no purchase rather than a purchase that has poor standing in the offerings. The buyer may choose to restart a search at a later date or in a different area.

As a second example, consider the case where the pilot or crew (the DM) of a military attack aircraft flies sorties in a zone where the rules of engagement allow legal targets (the applicants) to be engaged on sight. In this scenario, due to constraints such as terrain or airspace control the aircraft is unable to revisit and engage previously observed potential targets. It is reasonable that, weighing mission and weapon costs against target values, the DM would choose in some cases to return to base with unexpended ordnance rather than be forced by default to waste it on a target that ranks poorly relative to other targets observed during that mission.

Model for Non-Selection

The variation of the SP introduced in this section addresses problems similar to these two scenarios by following all the rules of the CSP as described in the Introduction but with one modification. Rather than defaulting to selecting the last of N applicants if no selection is made before that point, in this version of the SP **the DM selects the last applicant only if the rank of the last applicant is less than or equal to $\sqrt{N}$ *and otherwise makes no selection***. That is, the DM does not accept the last applicant if that applicant's rank is greater than the minimum expected relative rank achieved by the classical two-phase solution for the CSP. This variation of the SP is referred to in this paper as the **Secretary Problem Allowing Non-Selection (SPANS)**.

It is assumed in this section that the DM employs the same two-phase solution[20] outlined in the Introduction (modified to include the "non-selection" rule change) and selects the start position s to optimize a given payoff function: either maximize the probability of selecting NBTB or achieve MinERR. An additional consideration for the DM that uses SPANS is that the non-selection probability cannot be too high, as a decision process that only rarely makes a selection is of little value.

Performance of the Solution with Non-Selection

The simulation process for the C2PS described in Section III was modified to incorporate the rule change where non-selection of the last applicant occurs if its rank is greater than $\sqrt{N}$. For N = 100 and 10^6 simulated random runs, Figure 13 shows, as a function of s: the probability of making a selection (the complement of the probability of non-selection), the

[20] While SPANS is a two-phase solution for an SP, this paper does not refer to it as a "C2PS" because it is not a "classical" solution and, *a fortiori*, the problem solved is not the "classical" SP.

probability of selecting the best applicant, and the normalized expected relative rank (that is, the expected relative rank divided by N).[21]

For this value of N, the probability of making a selection decreases linearly from nearly unity (0.991) at $s = 2$ to 0.100 at $s = 100$.

The maximum probability of selecting the best applicant occurs at $s = 69$ for SPANS with $N = 100$. That is quite late in the applicant queue compared to the CSP's theoretical optimal start position at about $N/e \approx 0.368N \approx 37$. However, the peak probability of selecting the best applicant for SPANS is 0.691 which is considerably greater than the CSP's peak of about $1/e \approx 0.368$ although this high NBTB payoff comes at the price of a non-selection probability of 61.8%.

For $N = 100$, MinERR occurs at $s = 53$ which is also far later in the sequence than the two-phase solution for the CSP theoretical optimal start position $s = 10$. Again, the rule change in SPANS provides an advantage in that the expected relative rank with $s = 53$ is 1.85 which is considerably better than the CSP's MinERR of about 10. This incurs a non-selection probability of 47.3%.[22]

From the viewpoint of a DM, these late start positions have the disadvantage that they imply long search times. This runs counter to the tendencies for human DMs to shorten searches as noted in Section III. For $N = 100$, the optimal value for the start position for NBTB payoff requires an average of 85.3 total observations and the optimal value for the start position for MinERR payoff requires an average of 73.9 total observations.

Reducing the DM's Search Time

It can be seen in Figure 13 that the curve for the normalized expected relative rank for SPANS is quite flat. The flatness of this curve suggests an approach to shorten the search. Suppose that the DM would be satisfied with an expected relative rank no worse than 10% more than the minimum achievable with SPANS. A series of simulations were carried out for SPANS to determine the smallest start position that yielded expected relative rank no greater than 110% of MinERR. The simulations were run with the "standard" parameters given in Section III.

These optimal start positions obtained for each value of N and their linear trendline are plotted in Figure 14. The slope of the regression for the start position was 0.3689 (R^2 = 0.9959). Remarkably, in spite of what amounts to an arbitrary modification to the CSP to

[21] The *normalized* expected relative rank is shown so it can be plotted on the same scale as the two probabilities. Multiplying the normalized value by $N = 100$ recovers the actual expected relative rank.

[22] The MinERR payoff for SPANS was about 1.5 for $N = 10$ and appeared to reach a limiting value of approximately 2 based on experiments with value of N up to 10,000. The knee of the curve was at about $N = 400$.

create SPANS and the arbitrary choice of a 10% threshold over MinERR, an approximation of the fundamental constant e appears again. The reciprocal of the regression slope is 2.711 which has an “error” of –0.277% with respect to the value of e. Thus, it is reasonable to use a start position of $s = N/e$ to obtain an expected relative rank within 10% of the minimum achievable using the two-phase solution for SPANS.[23,24]

For $N = 100$, therefore, an acceptable approximation for the SPANS start position to obtain an expected relative rank approximately within 10% of the minimum is $N/e \approx 36.788$ which is rounded to 37.[25] This start position achieves an expected relative rank of 2.05. The probability of non-selection is reduced to 32.8% whereas, as noted above, it is 47.3% at the optimal start position $s = 53$. With non-selection reduced from nearly one-half to less than a third, there is a great benefit to the DM with little penalty in terms of payoff accrued by using the earlier start position that achieves an expected relative rank within about 10% of the minimum of 1.85.

Cheating the Non-Selection Solution

With the description of SPANS complete at this point from the viewpoint of the DM, the problem is reversed and approached from the viewpoint of a CA. As in the C2PS, cheating in SPANS requires the definition of a set of *advantageous positions* and their determination.

The goal of cheating in the SP is to find positions that gain the CA an advantage. That benefit is obtained by increasing the probability of selection or, alternatively, by decreasing that probability so as to evade selection. In either case, it is necessary to determine a baseline or threshold that constitutes being “advantageous” or “non-advantageous.”

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[23] The small “error” of –0.277% with respect to e for the reciprocal of the regression slope for SPANS to achieve expected relative rank within 10% of the minimum is merely happenstance, but it is notable nonetheless. The same set of random experiments for those values of N were run to obtain the regression slope for the optimal CSP start positions for NBTB payoff and it was found to be 0.3751; the “error” of the reciprocal of that slope is –1.925% with respect to e and the approximation $1/e$ is universally accepted. Thus, based on the results from simulation, N/e appears to be as good an estimate of the optimal start position for SPANS to achieve expected relative rank within 10% of the minimum as it is for the optimal start position for the two-phase solution for the CSP for NBTB payoff.

[24] Applying the same linear regression for those values of N to the CSP start positions for NBTB payoff obtained using the *exact* expressions obtained in *Glass* found the regression slope to be 0.3680 ($R^2 = 1$). The “error” of the reciprocal of that slope is –0.033% with respect to e.

[25] In the interests of full disclosure, it must be acknowledged that the actual optimal start position obtained for $N = 100$ from the 10^6 runs reported above for SPANS was $s = 38$. Due to this serendipitous opportunity to follow customary practice in the SP literature, however, the estimated value $s = 37$ based on N/e is used in this section.

The probability that an applicant of rank r is selected, given the applicant is in position p *and* the DM makes a selection, is denoted by Pr{select rank r | position $p \cap$ selection occurs}. Figures 15a-15e show this probability for $N = 100$ and $r = 1, 2, 3, 5$, and 11. In the case of SPANS, the probability of an applicant of rank r being selected is conditioned on the probability of the joint event of the applicant being in position p *and* the DM making a selection. Figures 15a-15d resemble the selection probabilities for the two-phase solution for the CSP: Figures 15a-15c (ranks 1, 2, and 3) show the selection probability at the start position and subsequent positions is higher than in the last position while Figure 15d (rank 5) shows a case where the selection probability is maximized at the last position.

Because SPANS treats applicants with ranks that are at or below the minimum expected relative rank of $\sqrt{N}$ differently from those above that rank, separate definitions are needed for these two types of advantageous positions.

Cheating by Applicants with Low Ranks

First, consider applicants in the last position with low ranks: those that are at or below the minimum expected relative rank achieved by the C2PS for the CSP. For $N = 100$, the selection probabilities as a function of position in the queue for ranks $\boldsymbol{r} \leq \sqrt{N}$ (that is, up to 10 in this case) behave similarly to those obtained for the CSP. For those ranks, a reasonable approach is to use the probability of selection in the last position as the benchmark as was done in the C2PS. Thus, in SPANS **an *advantageous position* for an applicant with rank $\boldsymbol{r} \leq \sqrt{N}$ is defined to be any position that provides a probability of selection for an applicant that is *at least as great as that for the last position* in the queue**. Expressed in the same manner as expression (9), p is an *advantageous position* for an applicant of rank r if and only p satisfies the condition

$$\begin{aligned} &\Pr\{\text{select rank } r \mid \text{position } p \cap \text{selection occurs}\} \\ &\quad \geq \Pr\{\text{select rank } r \mid \text{position } N \cap \text{selection occurs}\} \end{aligned} \tag{15}$$

As was defined in Section VI for the CSP, in SPANS the ranks $\boldsymbol{r} \leq \sqrt{N}$ have a ***critical position*** in the queue, denoted by $p_c(N, r)$, that delimits the range of positions for which the probability of selection is at least as great as in the last position.

For $N = 100$, the advantageous positions and critical positions for a CA with rank r are, for example:

$r = 1$: advantageous positions are 37-100,	$p_c(N = 100, r = 1) = 100$
$r = 2$: advantageous positions are 37-61, and 100,	$p_c(N = 100, r = 2) = 61$
$r = 3$: advantageous positions are 37-45, and 100,	$p_c(N = 100, r = 3) = 45$
$r = 5$: advantageous position is 100,	$p_c(N = 100, r = 5) = 100$

To evade selection, a CA should maneuver into any position that is non-advantageous for selection. Of course, the most effective course of action would be to maneuver into a position where there is zero chance of selection such as in the early segment of the queue in the observation-only phase.

Cheating by Applicants with High Ranks

Next, consider applicants in the last position with high ranks: those that are above the minimum expected relative rank achieved by the C2PS for the CSP. For N = 100 and ranks $\boldsymbol{r} > \sqrt{N}$ (that is, ranks greater than 10 for this value of N), the distinction between the CSP and SPANS comes into play: an applicant with rank greater than $\sqrt{N}$ has *zero* probability of selection in the last position because the DM makes no selection rather than select an applicant with rank worse than the achievable expected relative rank for the C2PS. This is shown in Figure 15e for an applicant with rank 11.

For a CA having the goal of being selected, there is a substantial penalty in SPANS for having a rank greater than $\sqrt{N}$ due to the rule change from the CSP. For N = 100, when averaged over all positions, an applicant with rank 10 has an overall selection probability that is 0.91 times that of an applicant with rank 9 but 10.8 times that of an applicant with rank 11. In spite of this penalty, however, a CA with rank greater than $\sqrt{N}$ can still improve its probability of selection by maneuvering to be as early in the observation-and-selection phase as possible. While an applicant with rank 11 has only 0.0608% chance of being selected on average (given that the DM makes a selection), that applicant's chances for selection are multiplied by 19.4 if it is at the start position s = 37.

Since the situation of an applicant having a rank greater than $\sqrt{N}$ lacks the natural benchmark served in expression (15) by the last position, it is necessary to use a different approach: an applicant-defined threshold, T. Thus, for SPANS, a useful definition for "**advantageous positions**" would be those **positions that have probabilities of selection that are at least fraction *T* of the maximum for that rank**. Expressed in the same manner as expressions (9) and (15), p is an *advantageous position* for an applicant of rank r for threshold T if and only p satisfies the condition

$$\Pr\{\text{select rank } r \mid \text{position } p \cap \text{selection occurs}\} \geq T * \Pr\{\text{select rank } r \mid \text{position } p_{max} \cap \text{selection occurs}\} \quad (16)$$

where

$$p_{max} = \underset{p}{\text{argmax}} \Pr\{\text{select rank } r \mid \text{position } p \cap \text{selection occurs}\} \quad (17)$$

Alternatively, to evade selection, a CA with high rank could maneuver into a position that has a probability of selection that is less than a threshold of T percent of the maximum for that rank. The best action would be to maneuver into a position where there is zero chance of selection such as the last position or in the early segment of the queue that is in the observation-only phase.

Thus, for N = 100 and T = 50%, a CA with rank 11 could improve its probability of selection by choosing one of the advantageous positions 37-41. A CA with that rank seeking to evade selection should choose any other position. In this example, a single threshold was used for the goals of both selection and evasion, but a CA could set a higher threshold T to improve its probability of selection or set a lower T to increase its chances of evasion.

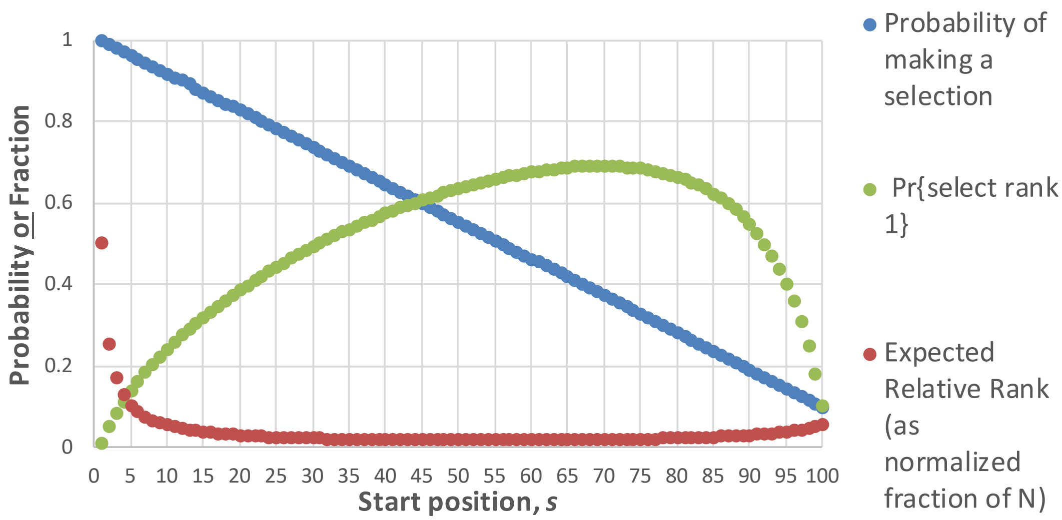


Figure 13. Probability of making a selection (the complement of the probability of non-selection), probability of selecting the best applicant, and normalized expected relative rank (expected relative rank divided by N). Plotted as a function of start position s. $N = 100$ and results are based on 10^6 simulated random runs.

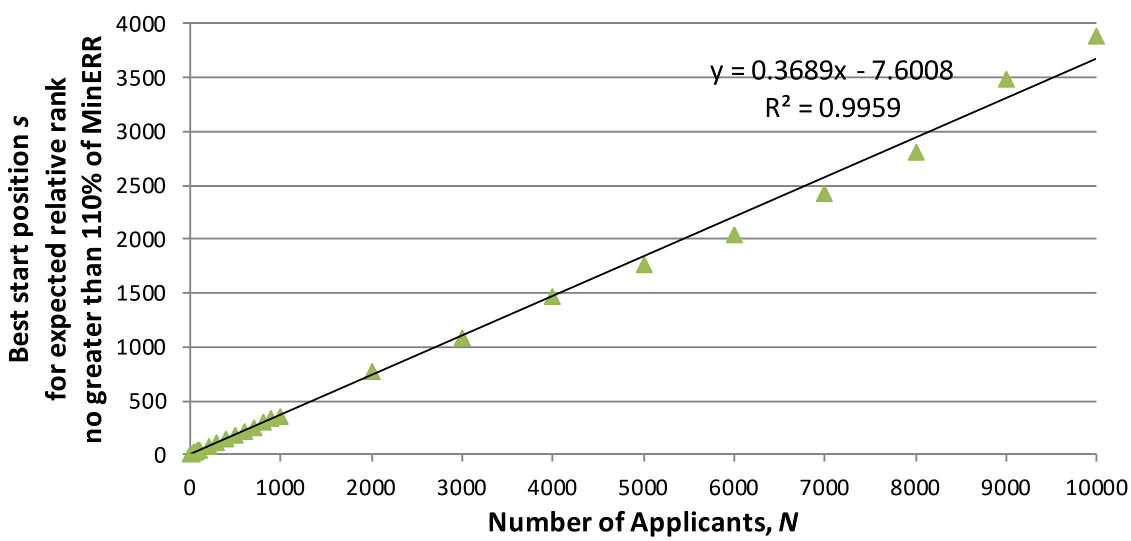


Figure 14. Linear trend line for smallest start positions that yield expected relative rank less than or equal to MinERR. Slope of the regression for these start positions was 0.3689.

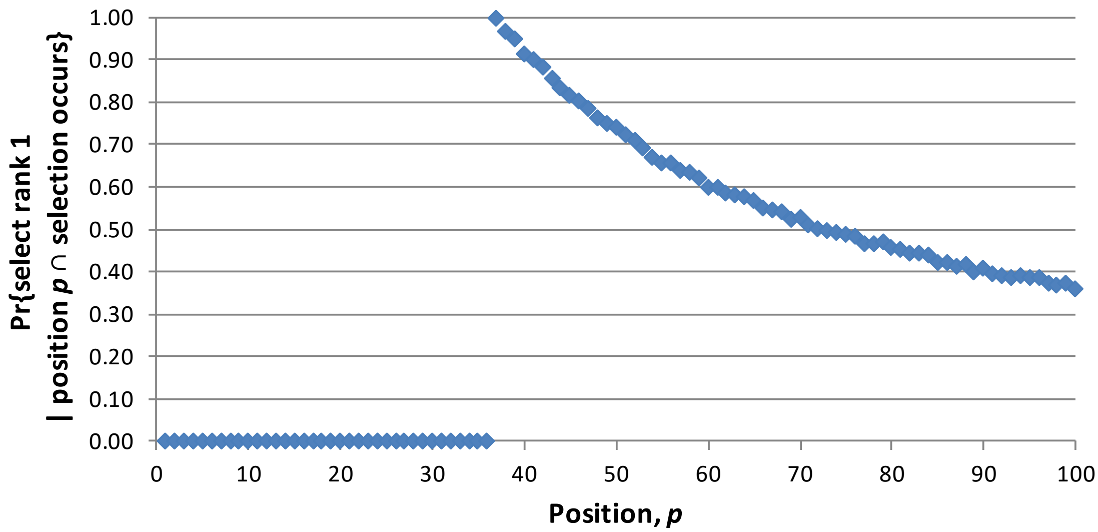


Figure 15a. For SPANS with $N = 100$, probability that an applicant of rank $r = 1$ is selected, given the applicant is in position p and the DM makes a selection. Note that Pr{select rank 1 | position p ∩ selection occurs} ≥ Pr{select rank 1 | last position ∩ selection occurs} for p = 37, 38, ... 100.

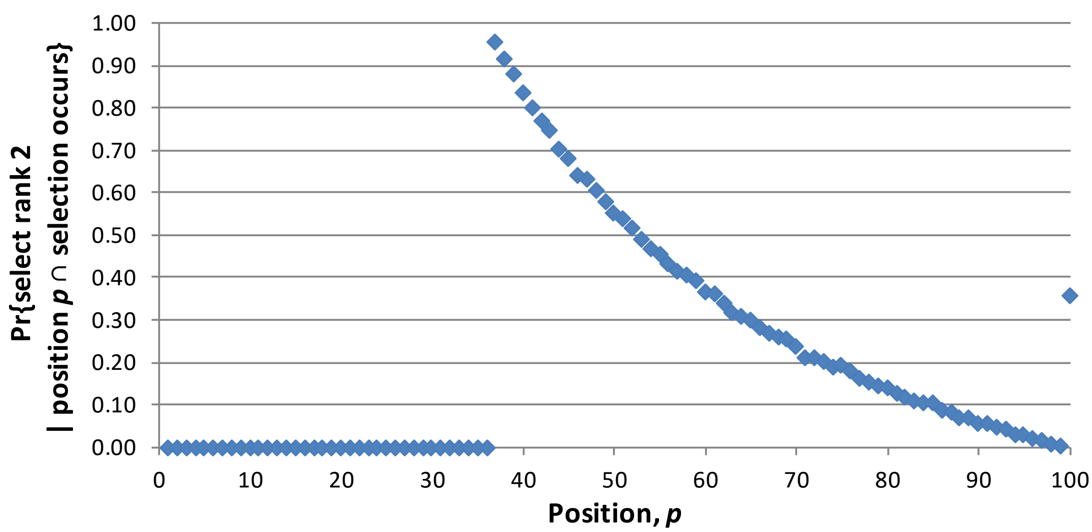


Figure 15b. For SPANS with $N = 100$, probability that an applicant of rank $r = 2$ is selected, given the applicant is in position p and the DM makes a selection. Note that Pr{select rank 2 | position p ∩ selection occurs} ≥ Pr{select rank 2 | last position ∩ selection occurs} for p = 37, 38, ... 61, and 100.

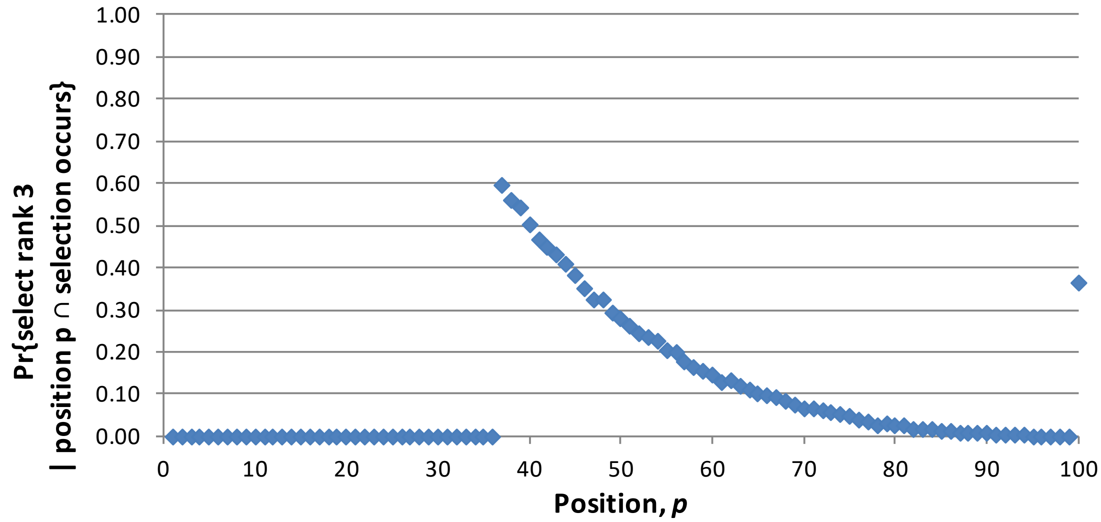


Figure 15c. For SPANS with $N = 100$, probability that an applicant of rank $r = 3$ is selected, given the applicant is in position p and the DM makes a selection. Note that Pr{select rank 3 | position p ∩ selection occurs} ≥ Pr{select rank 3 | last position ∩ selection occurs} for p = 37, 38, ... 45, and 100.

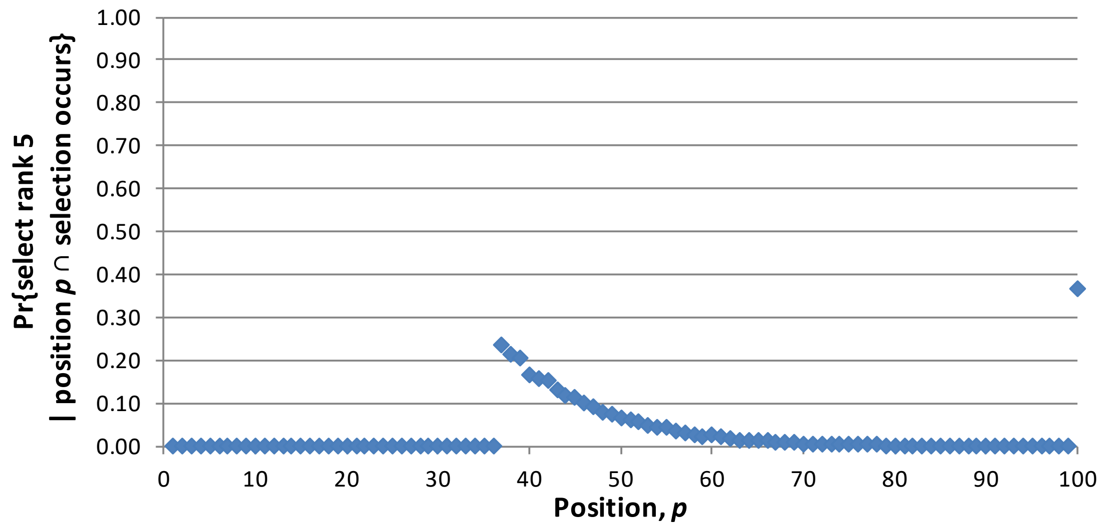


Figure 15d. For SPANS with $N = 100$, probability that an applicant of rank $r = 5$ is selected, given the applicant is in position p and the DM makes a selection . Note that Pr{select rank 5 | position p ∩ selection occurs} < Pr{select rank 5 | last position ∩ selection occurs} for all $p < 100$.

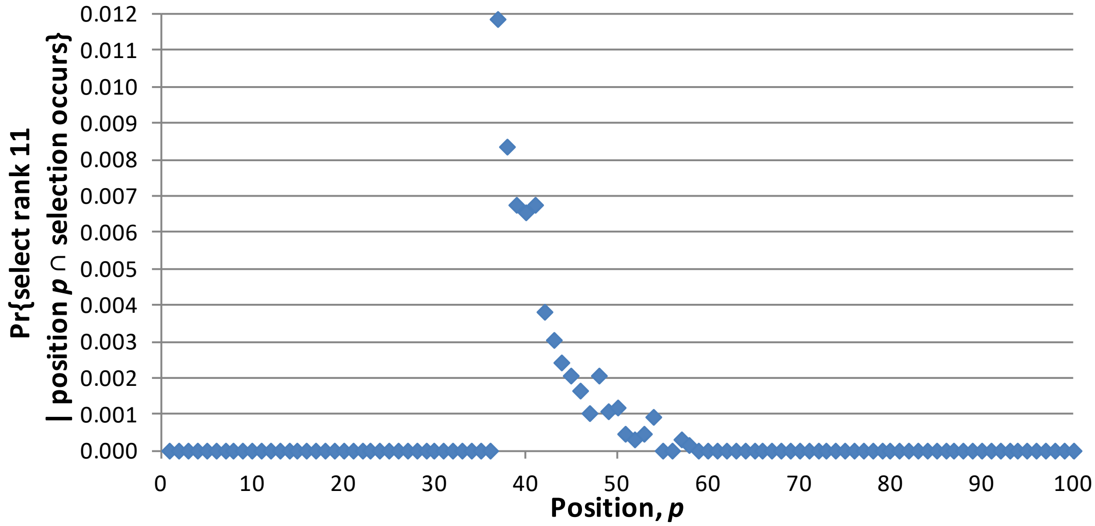


Figure 15e. For SPANS with $N = 100$, probability that an applicant of rank $r = 11$ is selected, given the applicant is in position p and the DM makes a selection. Note that Pr{select rank 11 | last position ∩ selection occurs} = 0, but Pr{select rank 11 | position p ∩ selection occurs} is non-zero in positions up to at least $p = 58$. Plot is noisy because only 409 out of 10^6 simulated random runs resulted in applicant of rank 11 being selected.

IX. *Cheating Chow's Solution for the Classical Secretary Problem*

The C2PS is optimal for NBTB payoff because it provides the actual maximum probability of selecting the best applicant when the start position is chosen appropriately. As pointed out in the Introduction, however, the C2PS does not provide the actual minimum expected relative rank. Previously, this paper has framed the discussion of MinERR payoff in terms of the best that is achievable by the C2PS; its start position and the resulting expected relative rank are called optimal only in the context of the C2PS that uses the rule that the *first* applicant observed to have lower rank than any previously observed candidate in the queue is chosen.

Based on work by Lindley [Lindley 1961], Chow, Moriguti, Robbins, and Samuels [Chow 1964] presented a solution for the CSP that achieves the *actual* MinERR. Incredibly, the expected relative rank achieved by Chow's solution for the CSP for MinERR payoff is bounded by 3.8695 regardless of the size of N.[26] This section describes Chow's solution and then demonstrates a strategy for an applicant to cheat. The approach for cheating resembles the way a CA with rank $\boldsymbol{r} > \sqrt{N}$ can select advantageous or non-advantageous positions in SPANS.

As was done in Section VIII, for brevity the results in this section are obtained solely through simulation.

Chow's Solution for the CSP for MinERR Payoff

In contrast to the C2PS, Chow's solution has a single phase in that every position is treated the same way. Instead of using a single start position that begins an observation-and-selection phase, the DM uses an individual precomputed threshold for each position in a queue of N applicants to decide either to stop with the selection of an applicant or to continue and observe the next applicant.

For N applicants, Chow's solution first requires a set of constants c_i ($i = 0, 1, \ldots N–1$) and constants s_p ($p = 1, 2, \ldots N$) to be computed. Note that Chow's s_p constants have no direct relationship to the start position s used in the C2PS. These c_i and s_p constants are easily calculated for any N using expressions (18), (19), (20), and (21) as follows.

[26] Although the SPANS solution introduced in Section VIII (with the optimal choice of start position) appears to achieve an expected relative rank of about 2, it does so only at the cost of failing to choose any applicant rather than choose a poor applicant. Using Chow's solution, the DM always selects an applicant yet achieves an expected relative rank less than 4.

The floating-point c_i constants are computed sequentially in reverse order using expression (18) and then (19):

$$c_{N-1} = \frac{N+1}{2} \tag{18}$$

$$c_{i-1} = \frac{1}{i} \sum_{j=1}^{i} \min\left(\frac{N+1}{i+1} j, c_i\right) \quad \text{for } i = N\text{–}1, N\text{–}2, \dots 1 \tag{19}$$

The integer s_p constants are then computed using expressions (20) and (21):

$$s_p = \left\lfloor \frac{p+1}{N+1} c_p \right\rfloor \quad \text{for } p = 1, 2, \dots N\text{–}1 \tag{20}$$

$$s_N = N \tag{21}$$

Appendix D lists Chow's s_p constants for values of N from 3 to 100. The table has over 100 columns of data so it is printed in size 1 font in a format that can be copied and pasted into a plain text file and opened as Comma Separated Values (CSV) using a spreadsheet program.

One more quantity must be defined before outlining Chow's solution. As mentioned in the Introduction, D.V. Lindley [Lindley 1961] categorized the relative rank of an applicant as *true rank* with respect to the entire queue and *apparent rank* with respect to only the applicants observed up to a given point. Chow's solution employs the **apparent rank** of the applicant in position p, denoted by y_p, which is determined as:

y_p = 1 + (the number of applicants in positions 1, 2, … p–1
that have smaller rank than the applicant in position p) (22)

Recall the two applicant queues used in the Introduction's Examples 1 and 2. In Example 1, the true ranks of the applicants were 8, 1, 5, 3, 2, 4, 6, 10, 9, 7 so the apparent ranks of the applicants, in the order they would be encountered by the DM, would be $y_{1\text{-}10}$ = 1, 1, 2, 2, 2, 4, 6, 8, 8, 7. In Example 2, the true ranks of the applicants were 8, 10, 5, 6, 9, 2, 1, 4, 7, 3 so the apparent ranks of the applicants, in the order they would be encountered by the DM, would be $y_{1\text{-}10}$ = 1, 2, 1, 2, 4, 1, 1, 3, 6, 3.

Chow's solution for the CSP for MinERR payoff consists of the DM executing these two steps beginning with the applicant at position p = 1:

- Observe the applicant in position p and determine the apparent rank y_p of that applicant with respect to the previously observed applicants.

- If $y_p \le s_p$, then select the applicant in position p and stop; otherwise, repeat these two steps with the applicant in position p+1.

Note that expression (21) causes the last applicant to be selected if the DM reaches position N, so Chow's solution is guaranteed to stop with the selection of an applicant.

Chow's solution does not explicitly state the notion of a "start position" in the same sense as it is used in the C2PS. Expression (20) implicitly creates such a position, however. For $N = 20$, for example, the $s_1, s_2, \ldots s_5$ are zero and $s_6 = 1$ so effectively Chow's solution in this case has a start position of 6.

Thus, in this section, the term **start position** refers to the first position p for which the constant s_p is non-zero. Just as with the C2PS, knowledge of the DM's start position for Chow's solution is useful to a CA seeking to evade selection; this is explored later in this section.

Figures 16a-16c demonstrate some properties of Chow's solution that are independent of the ranks of specific applicants. Note that many of the plots in this section and others that are concerned with Chow's solution use line charts or point-and-line charts; this is necessary for readability due to the jagged nature of the plots.

For any N, the value of c_0 is the expected relative rank of the applicant selected using Chow's solution; it is this value that Chow et al. showed to have an upper bound of approximately 3.8695. The calculated c_0 obtained using expressions (18) and (19) is plotted as a function of N in Figure 16a; the plot ends at $N = 955$ where c_0 is within 1% of its limit. This convergence of the expected relative rank of the selected applicant to a constant value less than 4 is in sharp contrast to the performance of the C2PS for NBTB payoff where the expected relative rank grows approximately as N/e and the C2PS for MinERR payoff where the expected relative rank grows approximately as $\sqrt{N}$.

In Section V, Figure 3 demonstrated for the C2PS that, when averaged over all applicants, the probability of an applicant being selected varies greatly as a function of position in the queue. In this section, Figures 16b and 16c demonstrate similar behavior for Chow's solution as well.[27] As for the C2PS, this observation offers the opportunity for a CA to affect its probability of selection by maneuvering in the queue.

Demonstrating Chow's Solution

Two examples were given of the C2PS with $N = 10$ in the Introduction. The same two applicant queues are used again in this section to demonstrate Chow's solution.

Prior to beginning the selection process, the DM calculates the c_i constants for the given value of N and, from them, the s_p constants. Alternatively, the DM can simply look up the necessary s_p constants in a table such as the one given in Appendix D of this paper.

For $N = 10$, the constants c_0 through c_9 obtained from expressions (18) and (19) are 2.558, 2.558, 2.558, 2.558, 2.677, 2.888, 3.154, 3.590, 4.278, and 5.500. Using the c_i constants,

[27] Figure 16b shows the conditional probability of selection for an applicant given its position p in the queue for $N = 20$ for comparison to Figure 3. Figure 16c shows the same for $N = 100$ to accentuate the jagged nature of the probability plot.

s_1 through s_{10} are obtained from expressions (20) and (21) as 0, 0, 0, 1, 1, 2, 2, 3, 5, and 10; note that the first non-zero constant in this list is s_4 so the start position for $N = 10$ is 4.[28]

Example 3:

Suppose $N = 10$ applicants are presented to the DM in this relative rank order: 8, 1, 5, 3, 2, 4, 6, 10, 9, 7.

The first applicant has no previous applicants so its apparent rank $y_1 = 1$. Since $y_1 > s_1$, the first applicant is not selected.

Since the start position is 4, the example can jump to that point. The apparent rank of the applicant in position $p = 4$ is $y_4 = 2$. Since $y_4 > s_4$, this applicant is not selected.

The apparent rank of the applicant in position $p = 5$ is $y_5 = 2$. Since $y_5 > s_5$, this applicant is not selected.

This process continues through the queue until the last applicant (position $p = 10$) is reached. The apparent rank of the last applicant is $y_{10} = 7$ and, since $y_{10} \leq s_{10}$, the DM selects the last applicant for a payoff equal to the true rank of the last applicant: 7.

(End of example)

Example 4:

Suppose $N = 10$ applicants are presented to the DM in this relative rank order: 8, 10, 5, 6, 9, 2, 1, 4, 7, 3.

Since the start position is 4, the example can jump to that point. The apparent rank of applicant in position $p = 4$ is $y_4 = 2$. Since $y_4 > s_4$, this applicant is not selected.

The apparent rank of applicant in position $p = 5$ is $y_5 = 4$. Since $y_5 > s_5$, this applicant is not selected.

The apparent rank of applicant in position $p = 6$ is $y_6 = 1$. Since $y_6 \leq s_6$, the DM selects the applicant in this position for a payoff equal to the true rank of this applicant: 2.

(End of example)

Chow's solution received a poor payoff for the applicant queue used in Example 3 but scored well for the applicant queue used in Example 4.

[28] These s_p values are found in Appendix D and are shown in the sample text in normal font.

Performance of Chow's Solution

Table 6 provides the results of simulated random runs for Chow's solution for a range of applicant queue lengths. As was done in Section VIII, some observations about the performance of Chow's solution and comparisons to the C2PS are given for $N = 100$ and 10^6 simulated random runs.[29]

Figure 17 shows the probability of selection for each rank (across all possible positions) using the C2PS for NBTB and MinERR payoffs and Chow's solution for MinERR payoff. For an apples-to-apples comparison, the results are derived from simulations using the same random seeds for the three solutions. The probability of selecting an applicant falls off so quickly with rank that it has to be shown on a log scale. This plot shows how effective these solutions are at rejecting high-rank applicants, although the issue of selecting the applicant in the last position causes the C2PS for both payoffs to converge to a constant value. Chow's solution, on the other hand, did not select any applicant with rank higher than 52 in any of the million simulated random runs.

For Chow's solution, most significantly the expected relative rank obtained was 3.6034, which has about –0.005% error with respect to the predicted value of $c_0 = 3.6032$. Clearly, the MinERR payoff achieved by Chow's solution is considerably better than the C2PS optimal MinERR payoff of about $\sqrt{N} = \sqrt{100} = 10$.

While Chow's solution is designed to achieve MinERR payoff, it does not perform well for NBTB payoff. For $N = 100$ and 10^6 simulated random runs, Pr{select rank 1} ≈ 0.2631 for Chow's solution, compared to about 0.3709 for the C2PS for NBTB payoff as shown in Table 1. However, this probability of selecting the best applicant for Chow's solution exceeded the probability of about 0.2222 obtained for the C2PS for MinERR payoff as shown in Table 2.

Interestingly, Pr{select rank 1} resulting from Chow's solution decreases as N increases and seems to approach a limiting value of roughly 0.25.

The average number of observations required for Chow's solution for $N = 100$ was 53.34. This value was almost at the midpoint between the average number of observations for the C2PS for NBTB payoff (74.12 observations) and MinERR payoff (31.15 observations) as shown in Table 1 and Table 2.

Finally, another property of Chow's solution is the reduced probability that the applicant in the last position is selected by default, even though that rule is still implicitly implemented by expression (21). For $N = 100$, the simulation of Chow's solution found the probability that the applicant in the last position to be about 0.0018. This is a much smaller probability than that resulting from the C2PS for NBTB payoff (0.3740) or for MinERR payoff (0.0907).

[29] The results shown in Figure 2 for $N = 100$ and 10^6 simulated random runs executed in 52.155 "user" seconds according to the Cygwin Linux *time* command.

Appendix A applies Chow's solution for MinERR payoff and the C2PS solutions for NBTB and MinERR payoffs to a notional physical system where the operator is the DM and sensor readings are sampled to generate the "applicant queue." The system is subjected to various types of drift in the statistics of the samples to explore how robust the various solutions are to slight violations of the assumption of random ordering of the applicant queue.

The effectiveness of Chow's solution for reaching the minimum expected relative rank is so profound that it is worth comparing its performance to that of models of SPs where the DM is allowed full or partial recall of previously observed applicants. Appendix C gives the results of such a comparison. These models implemented an observation-only phase followed by a phase that solicited (that is, attempted to recall) previously observed applicants. Three models involving recall were used under two separate conditions: either they terminated the observation-only phase at the point where Chow's solution stopped on average or they terminated the observation-only phase at the point where their expected relative rank equaled that of Chow's solution. Upon termination of the observation-only phase, one model was allowed to recall the *best* applicant observed up to that point, a second model was allowed to recall the *second-best* applicant observed at that point, and a third model successively solicited previously observed applicants (beginning with the best) but each those applicants *was available for selection with probability 0.5*. The solutions for these models stopped upon successful solicitation and thus selection of an applicant. The first model performed far better than Chow's solution, of course, but an unexpected result was that Chow's solution performed almost identically to the second and third models.[30]

Cheating Chow's Solution

Having introduced Chow's solution and demonstrated some of its properties from the viewpoint of the DM, the problem is now reversed and approached from the viewpoint of a CA. As before, this requires the definition and then the determination of a set of *advantageous positions*. The critical factor needed for a CA to determine its advantageous positions is the conditional probability of selection for its rank in a given position: Pr{select rank r | position p}.

Figure 18 shows Pr{select rank r | position p} for N = 100 and r = 1 through 10. Figures 19a-19e break out Pr{select rank r | position p} for N = 100 and r = 1, 2, 3, 4, and 8 to show some important features in more detail:

- The last position appears to have the *smallest* non-zero probability of selection for the applicants with small values of r. This is in sharp contrast with the C2PS

[30] For the models that allow recall, note the distinction between *terminating* the observation-only phase (which begins the solicitation phase) and *stopping* with a selection decision. The models that do not allow recall stop with a selection decision at the same time they terminate observation of applicants.

where the spike in the probability of selection in the last position serves as a natural benchmark for advantageous positions.

- Instead of the C2PS's monotonically decreasing probability of selection from the start position to the last position (for rank 1) or to the next-to-last position (for other ranks), the probability of selection for Chow's solution is a jagged distribution with peaks and valleys. If the last position is taken to be a valley, then it appears, at least for small r, that Pr{select rank r | position p} has r peaks and r valleys. To complicate the situation, the heights of the peaks do not necessarily decrease as shown in Figure 19e for rank 8.

Since the probability of selection lacks a natural benchmark (served by the last position in the C2PS), it is necessary to choose a threshold as was done in SPANS for applicants with ranks greater than $\sqrt{N}$.

Given the observations above, a reasonable definition for "**advantageous positions**" for Chow's solution would be those **positions that have probabilities of selection that are at or above a threshold given by a fraction *T* of the maximum for that rank**. Expressed in the same manner as expressions (9), (15), and (16), p is an *advantageous position* for an applicant of rank r for threshold T if and only p satisfies the condition

$$\begin{aligned}&\Pr\{\text{select rank } r \mid \text{position } p\ \} \\ &\quad \geq T * \Pr\{\text{select rank } r \mid \text{position } p_{max}\ \}\end{aligned} \tag{23}$$

where

$$p_{max} = \underset{p}{\text{argmax}} \Pr\{\text{select rank } r \mid \text{position } p\} \tag{24}$$

Alternatively, to evade selection, a CA could maneuver into a position that has a probability of selection that is less than fraction T of the maximum for that rank. The best such action would be to maneuver into a position where there is zero chance of selection such as a position prior to the start position established by the first non-zero s_p constant.[31]

Due to the jagged shape of the plots it is clear that the concept of a "critical position" does not apply to Chow's solution in the sense that it did for the C2PS.

For N = 100 and T = 50%, a CA with rank 1 could improve its probability of selection by choosing one of the advantageous positions 28-51. A CA with that rank seeking to evade selection should choose any other position in the queue. In this example, a single threshold was used for the goals of both selection and evasion, but a CA could set a higher threshold T to improve its probability of selection or a lower threshold T to increase its chances of evasion. Consider the following examples.

[31] The exact start position for any N is calculated easily from expressions (16) and (17). However, a close estimate of the first non-zero s_p value is given by the linear trendline $0.2585N+2.2172$ (R^2 = 1). The difference between this estimate and the actual start value is between −0.6253 and +1.5097 over every value of N from 3 to 10,000.

Figures 19a-19e are labeled to show the thresholds for N = 100 and T = 50%:

r = 1: advantageous positions are 28-51
r = 2: advantageous positions are 28-44 and 48-59
r = 3: advantageous positions are 28-39, 48-57, and 60-66
r = 4: advantageous positions are 28-37, 48-55, 60-66, and 68-73
r = 8: advantageous positions are 28-32, 78-79, and 81-87

A CA that requires a more advantageous set of positions for selection might set the threshold higher, such as T = 75%:

r = 1: advantageous positions are 28-37
r = 2: advantageous positions are 28-33 and 48-49
r = 3: advantageous positions are 28-32 and 48-50
r = 4: advantageous positions are 28-31, 48-49, and 60
r = 8: advantageous positions are 28-30, 81, and 83-84

A CA seeking to evade selection might set the threshold lower to identify *non-advantageous* positions for selection, such as T = 25%:

r = 1: non-advantageous positions are 1-27 and 68-100
r = 2: non-advantageous positions are 1-27, 73-100
r = 3: non-advantageous positions are 1-27, 78-100
r = 4: non-advantageous positions are 1-27, 47, and 83-100
r = 8: non-advantageous positions are 1-27, 38-47, 51-59, 61, 63-67, 71-73, and 92-100

Appendix E lists the advantageous positions for Chow's solution for values of N from 3 to 100. The table lists the advantageous positions for the thresholds T = 25%, 50%, and 75% resulting from simulation of 10^6 simulated random runs. The table is printed in size 1 font in a format that can be copied and pasted into a plain text file and opened as Comma Separated Values (CSV) using a spreadsheet program.

The improvement in the probability of selection using the advantageous positions listed in Appendix E are shown in Figure 20a for N = 20, T = 50%, and ranks 1 through 10. Overall, it can be seen that the probability of selection is roughly doubled as intended using this choice of T. Figure 20b shows similar improvements under the same conditions but for N = 50.

A CA seeking to evade selection can use the complement of the set of advantageous positions listed in Appendix E. Figure 20c shows that, for N = 20, T = 50%, and ranks 1 through 10, the probability of selection can be cut roughly in half as intended with that value of T. Figure 20d shows similar reductions in the probability of selection under the same conditions but for N = 50.

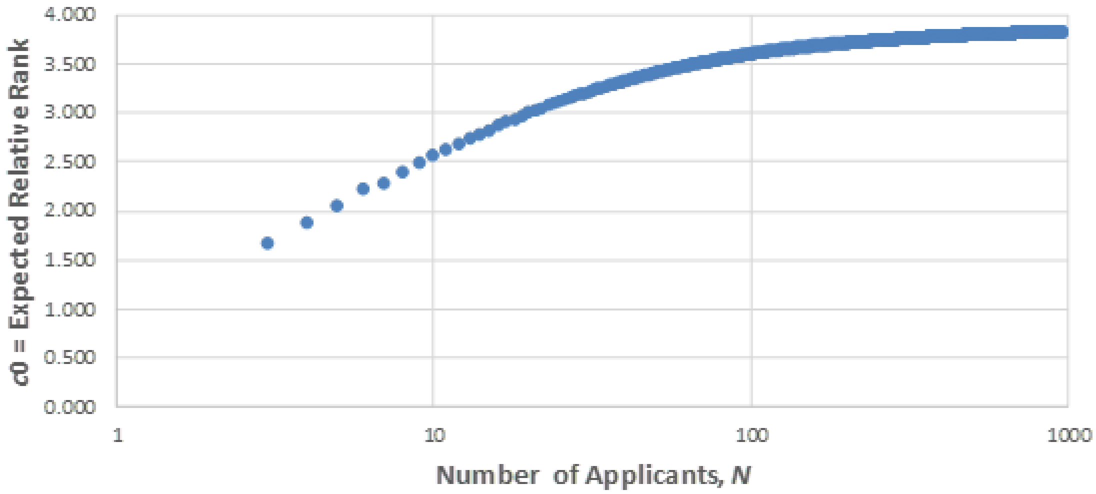


Figure 16a. The value of c_0, the expected relative rank of the applicant selected using Chow's solution for MinERR payoff, as a function of N. Plot ends at $N = 955$ where c_0 is within 1% of its upper bound of approximately 3.8695.

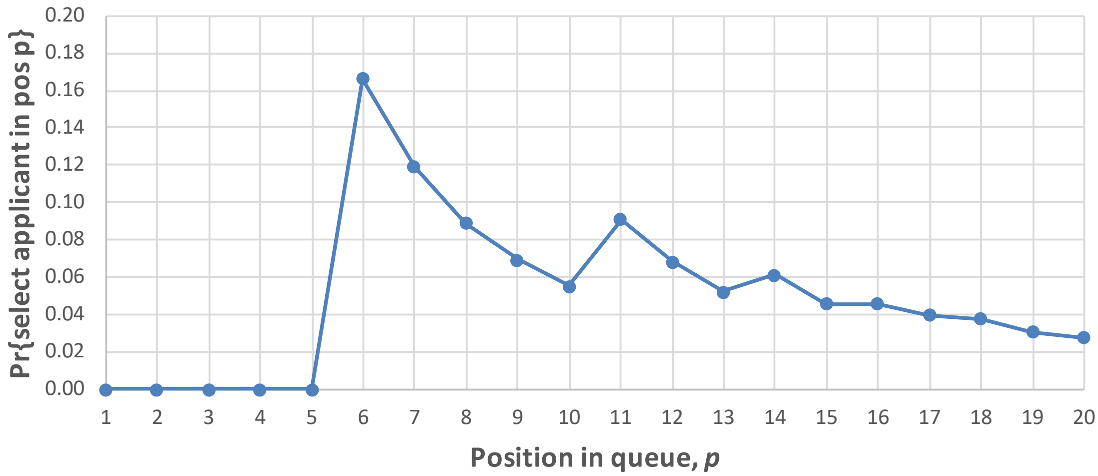


Figure 16b. Conditional probability of selecting the applicant in position *p* using Chow's solution for MinERR, regardless of rank. $N = 20$ and results are based on 10^6 simulated random runs.

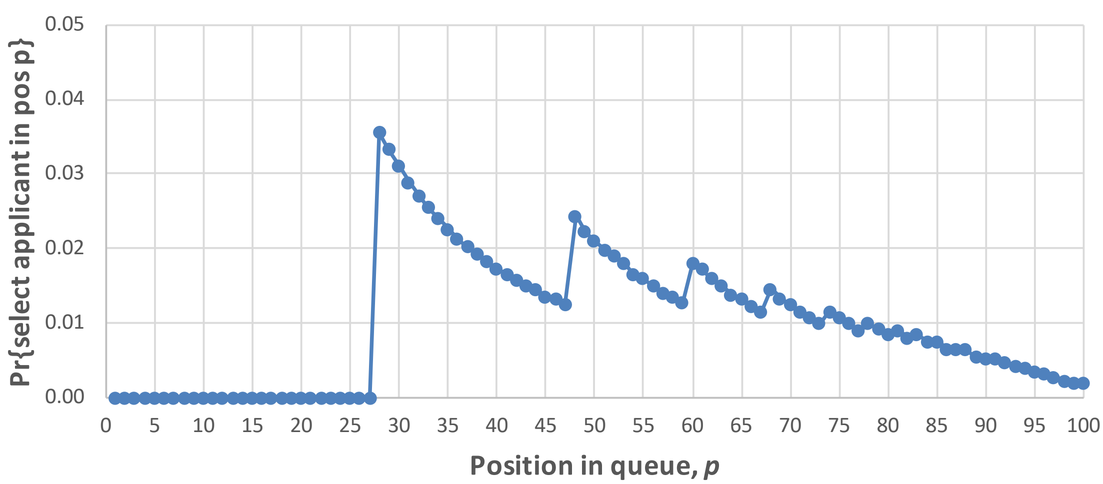


Figure 16c. Conditional probability of selecting the applicant in position *p* using Chow's solution for MinERR, regardless of rank. $N = 100$ and results are based on 10^6 simulated random runs.

N	c_0	Chow ave true rank	Chow Pr{sel rank 1}	Chow Pr{sel last pos}	Chow ave pos sel
10	2.5579	2.5577	0.3295	0.0792	6.29
20	3.0017	3.0037	0.2997	0.0281	11.00
30	3.2036	3.2039	0.2876	0.0144	16.64
40	3.3282	3.3271	0.2766	0.0099	22.08
50	3.4121	3.4089	0.2725	0.0060	26.63
60	3.4704	3.4738	0.2717	0.0048	32.31
70	3.5157	3.5123	0.2677	0.0036	37.70
80	3.5510	3.5518	0.2671	0.0028	42.39
90	3.5791	3.5872	0.2628	0.0023	47.67
100	3.6032	3.6034	0.2631	0.0018	53.34
200	3.7192	3.7265	0.2523	0.0004	104.44
300	3.7631	3.7936	0.2524	0.0003	154.34
400	3.7864	3.7916	0.2532	0.0001	205.23
500	3.8011	3.7915	0.2519	0.0001	255.79
600	3.8112	3.8050	0.2515	0.0001	306.74
700	3.8186	3.8107	0.2515	0.0000	357.27
800	3.8242	3.8269	0.2511	0.0000	408.95
900	3.8287	3.8020	0.2513	0.0000	459.94
1000	3.8324	3.8389	0.2491	0.0000	510.58
2000	3.8495	3.8565	0.2521	0.0000	1021.00
3000	3.8557	3.8724	0.2487	0.0000	1519.91
4000	3.8588	3.8870	0.2518	0.0000	2024.77
5000	3.8608	3.9102	0.2494	0.0000	2530.11
6000	3.8621	3.9004	0.2466	0.0000	3039.77
7000	3.8631	3.8594	0.2516	0.0000	3539.89
8000	3.8638	3.9197	0.2424	0.0000	4069.70
9000	3.8644	3.9114	0.2488	0.0000	4570.24
10000	3.8649	3.8965	0.2436	0.0000	5060.60

Table 6. Statistics for Chow's solution for MinERR payoff. Number of simulated random runs for each value of N is specified in Section III.

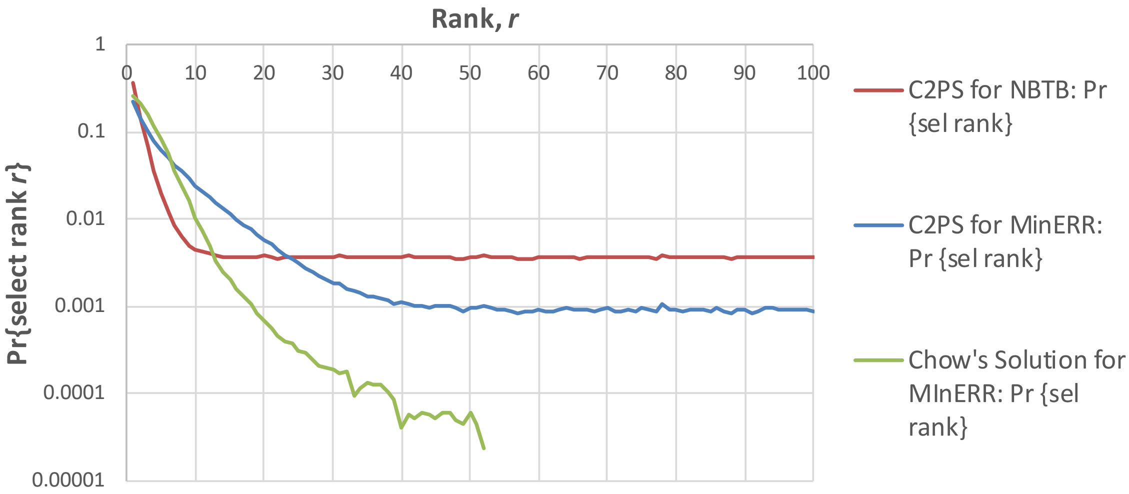


Figure 17. Probability of selecting given rank, r, for C2PS for NBTB payoff, C2PS for MinERR, and Chow's solution for MinERR. $N = 100$ and results are based on 10^6 simulated random runs.

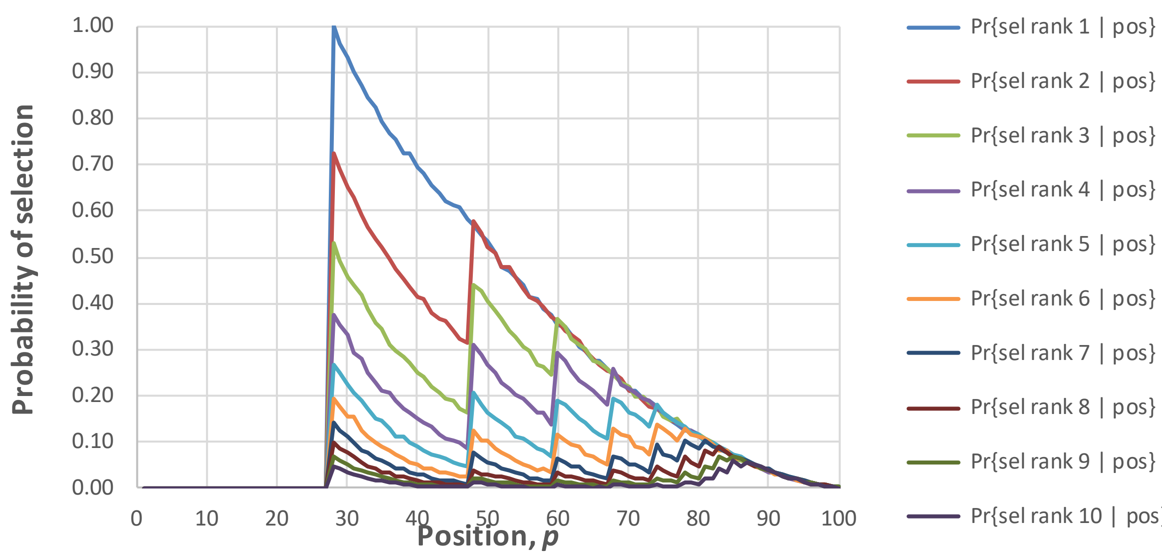


Figure 18. For Chow's solution for MinERR, individual probabilities that applicants of rank $r = 1$ through 10 are selected, given the applicant is in position p. $N = 100$ and results are based on 10^6 simulated random runs.

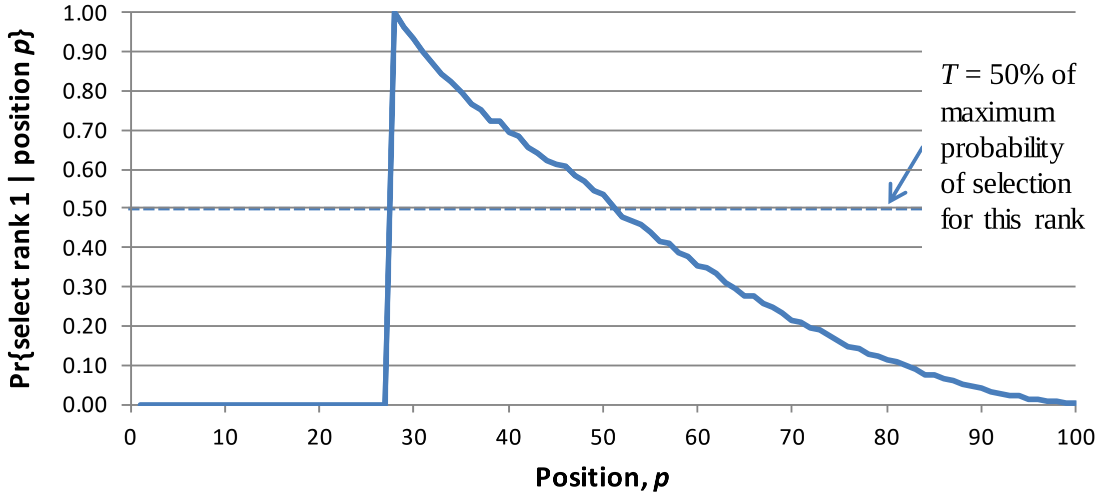


Figure 19a. For Chow's solution with $N = 100$, probability that an applicant of rank $r = 1$ is selected, given the applicant is in position p.

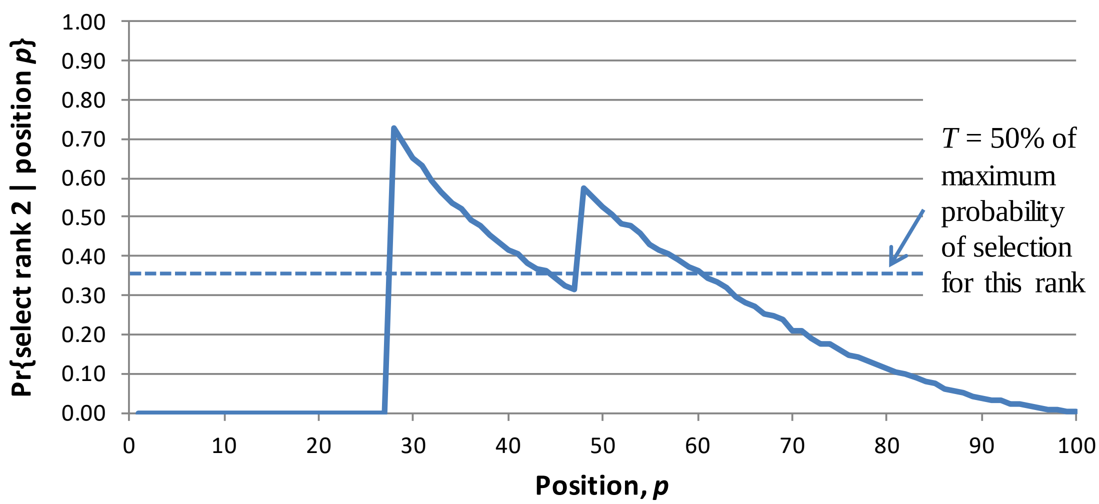


Figure 19b. For Chow's solution with $N = 100$, probability that an applicant of rank $r = 2$ is selected, given the applicant is in position p.

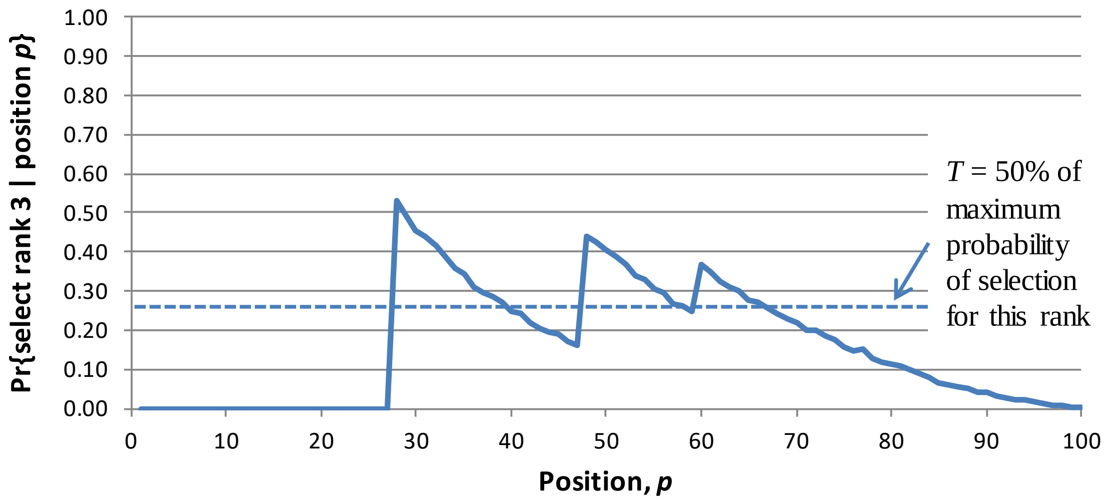


Figure 19c. For Chow's solution with $N = 100$, probability that an applicant of rank $r = 3$ is selected, given the applicant is in position p.

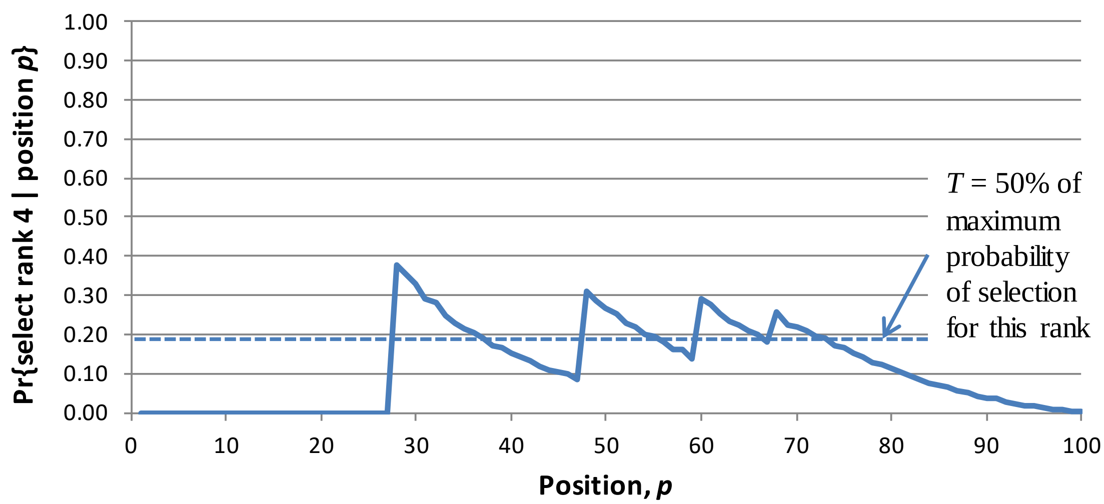


Figure 19d. For Chow's solution with $N = 100$, probability that an applicant of rank $r = 4$ is selected, given the applicant is in position p.

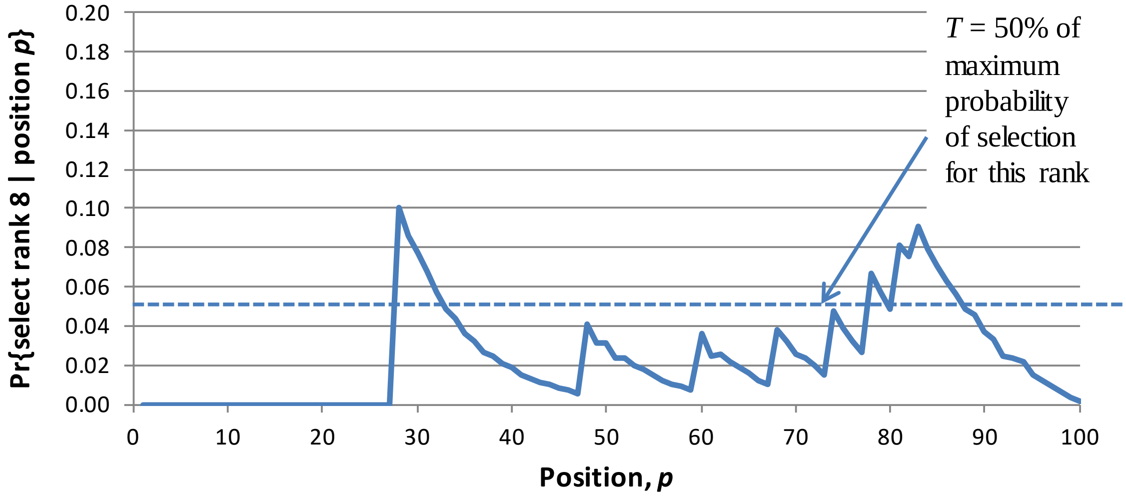


Figure 19e. For Chow's solution with $N = 100$, probability that an applicant of rank $r = 8$ is selected, given the applicant is in position p.

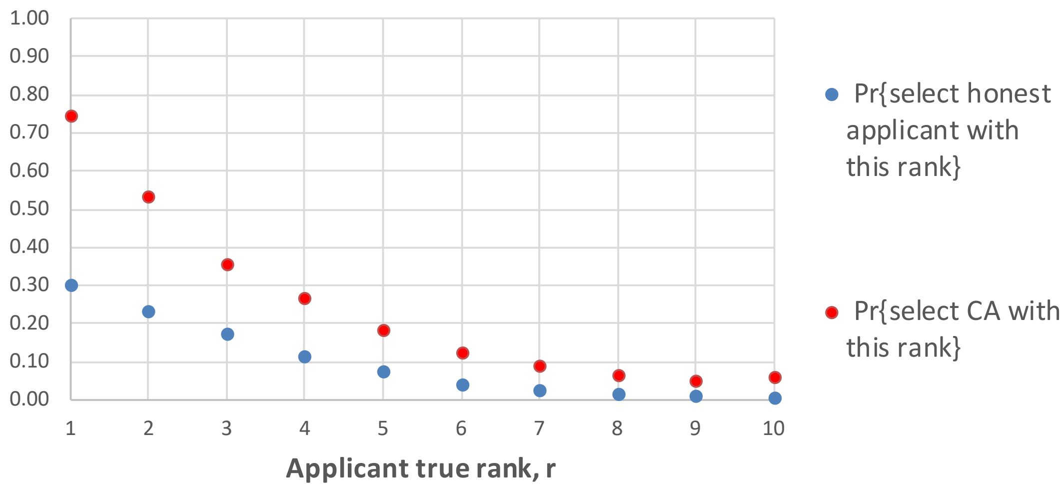


Figure 20a. Probability of selection for CAs seeking selection by choosing an advantageous position as given in Appendix D for T = 50%. N = 20 and results are based on 10^6 simulated random runs.

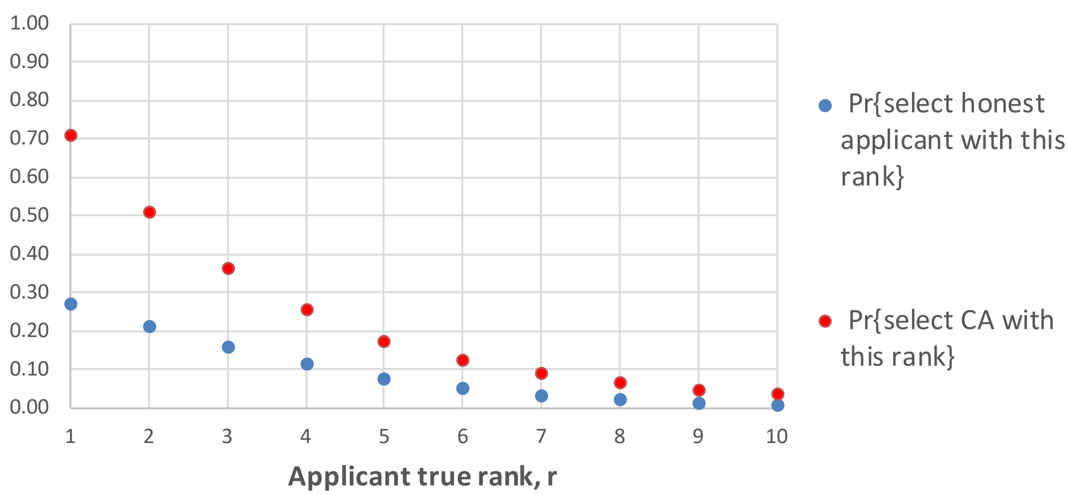


Figure 20b. Probability of selection for CAs seeking selection by choosing an advantageous position as given in Appendix D for T = 50%. N = 50 and results are based on 10^6 simulated random runs.

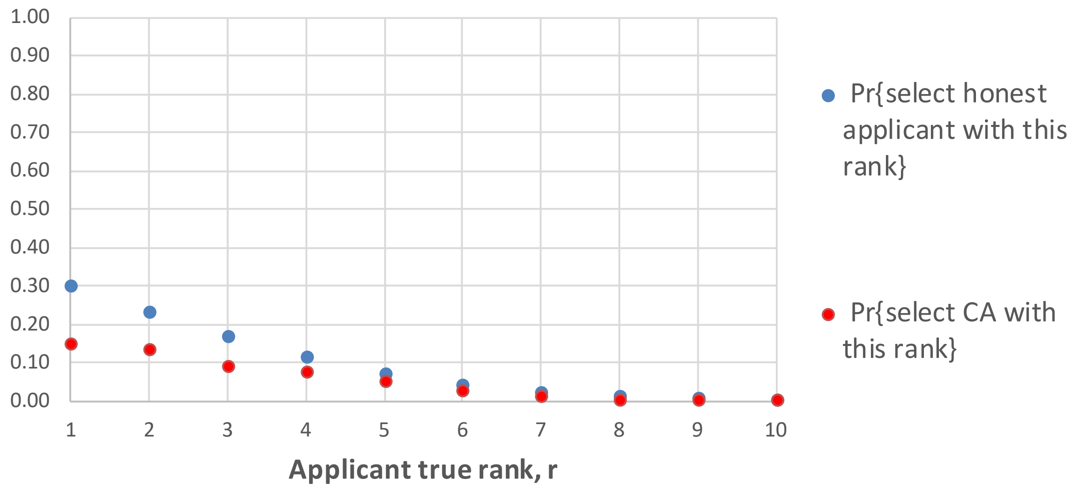


Figure 20c. Probability of selection for CAs seeking to avoid selection by choosing a non-advantageous position using Appendix D for T = 50%.
N = 20 and results are based on 10^6 simulated random runs.

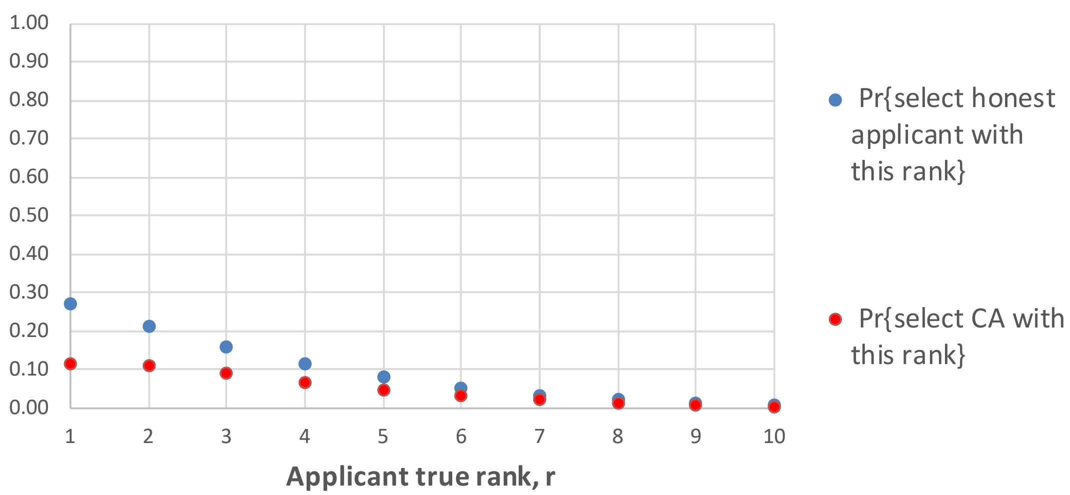


Figure 20d. Probability of selection for CAs seeking to avoid selection by choosing a non-advantageous position using Appendix D for T = 50%.
N = 50 and results are based on 10^6 simulated random runs.

X. Cheating with only Approximate Knowledge of Own Rank

An assumption given at the start of Section VI was that the CA "knows (or can closely approximate) its own rank relative to the other applicants." It may be that the CA does not know its own rank to be exactly r, but the CA may have a high level of confidence that its rank is bounded by lower and upper values r_L and r_U.

This situation may be reasonable when a CA has historical knowledge, personal acquaintanceships, illicit inside information, or other situational awareness about the other applicants, perhaps even more than does the DM. A CA might have knowledge about the distribution of true values (SAT scores, IQ scores, etc.) that would lead to an accurate estimation of the CA's rank in the queue.

There are theories regarding human ability to make quick quality judgements based on limited observations, such as Ambady and Rosenthal's "thin slices of behavior" [Ambady 1992]. Mathematical approaches for pattern recognition such as Principal Component Analysis (PCA) and factor analysis are used to reduce the number of features or the dimensionality of feature space needed to make good decisions [Duda 1973]. While this area is usually addressed from the point of view of the DM, a human or autonomous CA could similarly make use of rapid (even "quick-and-dirty") decision processes and exploit that knowledge to maneuver deceptively within the applicant queue.

Two distinct approaches are explored in this section for a CA seeking selection to choose a set of **jointly advantageous positions** that would be valid for a range of ranks r_L through r_U.[32] The first ensures that every position in that range is advantageous. The second weights the desirability of the positions for the possible ranks of the CA by the probability of selection of that rank. A hybrid of these two methods is discussed also.

Method Based on Set Intersection

The first approach for a CA seeking selection is to take the *intersection of the sets of advantageous positions* for each rank in the range r_L through r_U as the set of *jointly advantageous positions*. Every position in this set is advantageous for every rank in this range.

A CA seeking to evade selection can take the *intersection of the complements of the sets of advantageous positions* for each rank in the range r_L through r_U as the set of *jointly non-advantageous positions*. Every position in this set is non-advantageous for every rank in this range.

For the C2PS, it is easy to obtain the intersection of the sets of advantageous positions. A CA seeking selection can simply use the set of advantageous positions obtained for rank

[32] To be clear: r_L corresponds to the CA's estimate of its *best* possible rank and r_U and corresponds to its estimate of its *worst* possible rank.

r_U as the set of jointly advantageous positions, because that rank has the smallest critical position of the ranks in that range. For example, consider the list of critical positions given in Section VII for $N = 20$: for ranks 1 through 5 the critical positions are 20, 10, 8, 6, and 5. A CA seeking selection that knows only that it is one of the top three applicants would choose as the set of jointly advantageous positions all of the positions from the start position to position 8 (if they exist) plus the last position. A CA that knows only that it has rank 4 or 5 would choose as the jointly advantageous positions all of the positions from the start position to position 5 (if they exist) plus the last position.

Reversing the problem, a CA seeking to evade selection can take the complement of the set of advantageous positions for rank r_L as the set of jointly non-advantageous positions, because that rank has the largest critical position of those ranks.

For $N = 20$ again, a CA seeking to evade selection that knows only that it has rank 4 or 5 would choose as the jointly advantageous positions all of the positions that are *not* in the set of positions from the start position s to position 6 (if they exist) plus the last position. Thus, the set of jointly non-advantageous positions would be 1 through $s–1$ and 7 through $N–1$.

A special case should be made if the range includes rank 1 because that would eliminate all positions beyond the start position; an optimistic CA could use the critical position for rank 2 or a pessimistic CA might accept only the positions before the start position. For $N = 20$, an optimistic CA seeking to evade selection that knows only that it is one of the top three applicants would choose as the set of jointly non-advantageous positions all of the positions that are *not* from the start position to position 10 and *not* the last position; a pessimistic CA would choose only a position before the start position.

In contrast to the C2PS, cheating Chow's solution for the CSP for MinERR payoff requires explicit intersections of the sets of advantageous positions of the ranks in the range r_L through r_U because the peaks and valleys create non-contiguous ranges of advantageous positions.

As an example of cheating Chow's solution using the sets of advantageous positions given in Section IX for $N = 100$ and $T = 50\%$, a CA seeking selection that knows only that it is one of the top three applicants could take the intersection of the sets of advantageous positions for ranks 1, 2, and 3 to obtain the set of jointly advantageous positions 28-39 and 48-51. A CA knowing only that it has one of those three ranks would have at least half of its maximum probability of selection (because $T = 50\%$) in any one of those positions.

A CA seeking to evade selection in this instance would take the intersection of the complements of those three sets of advantageous positions (that is, the three sets of non-advantageous positions) to obtain the jointly non-advantageous positions 1-27 and 67-100.

Again for $N = 100$ and $T = 50\%$, a CA seeking selection but able to estimate only that it has a rank in the range 6 through 10 could take the intersection of the sets of advantageous positions for those ranks to obtain the jointly advantageous positions 28-31. If seeking to evade selection, the CA would take the intersection of the non-advantageous positions for those ranks to obtain the jointly non-advantageous positions 1-27, 35-47, 51-59, 62-67, 71-73, and 92-100.

Method Based on Weighted Probability of Selection

The second approach for choosing a set of jointly advantageous positions takes into account the fact that the higher-ranked applicants have a lower probability of selection than those with lower ranks. Therefore, an overall weighted probability of selection is determined for each position.

This method is demonstrated only for Chow's solution as it is the most interesting. In this approach, the CA must estimate not only that its rank is in the range r_L through r_U but it must also guess at the probability mass function for those ranks, that is, the probability of the event that the CA has rank r_L, or rank r_L+1, … or rank r_U. As this would likely be unknowable, here it is assumed that ranks are equiprobable (uniformly distributed) over that range; the events are mutually exclusive, of course. Therefore, it is reasonable that a figure of merit for choosing a position in the queue would be the conditional probability of selection at that position *averaged* over the CA's estimated range of ranks.

For example, suppose a CA is seeking selection but knows only that it is one of the top three applicants. Figure 21 shows, for the same simulations as in Figure 18, the three conditional probabilities of selection for those ranks, given their position in the queue. Figure 22 shows the average of the three conditional probabilities which, again, assumes that the CA's best estimate of its probability of having one of the ranks 1, 2, or 3 is 1/3. The maximum value of that average is 0.564; a good threshold for the jointly advantageous positions using this approach would be those positions where the average probability is at least T = 50% of the maximum value. In this example, those positions would be 28-45 and 48-56.

Under the same conditions, a CA seeking selection but able to estimate only that it has a rank in the range 6 through 10 would have the conditional probabilities of selection shown in Figure 23 and would average those probabilities at each position to obtain the profile shown in Figure 24. The maximum value of that average is 0.111 and a threshold of at least T = 50% of that value would yield the jointly advantageous positions 28-33, 74, 78-79, and 81-87.

A CA seeking to evade selection would select a jointly non-advantageous position in the complement of the set of jointly advantageous positions defined by the average probability of selection.

Method Based on Hybrid of Two Previous Methods

A conservative CA could use a hybrid of the two approaches and consider the jointly advantageous positions to be those that are in the intersection of the advantageous positions for each possible rank *and* have an average probability of selection that is above a given threshold; effectively, these positions would be the intersection of the two approaches. For a CA that knows it has rank 1, 2, or 3, the hybrid of the two approaches yields the jointly advantageous positions 28-39 and 48-51. For a CA that knows it has a rank in the range 6-10 the hybrid approach yields the jointly advantageous positions 28-31. For these particular cases, it happens that the hybrid approach yields simply the result of the first approach.

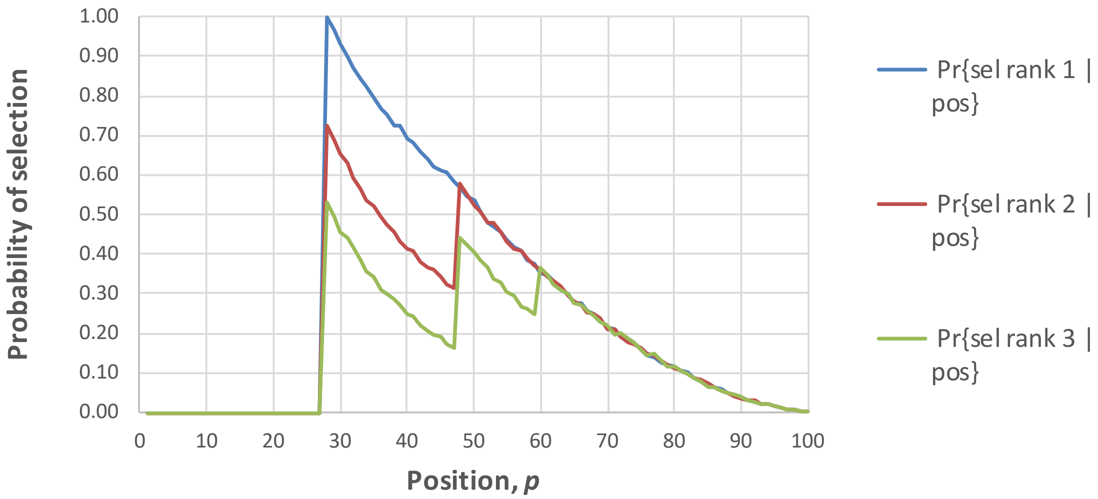


Figure 21. For Chow's solution for MinERR, ranks 1, 2, and 3: individual probabilities of selecting those ranks, given position p. N = 100 and results are based on 10^6 simulated random runs.

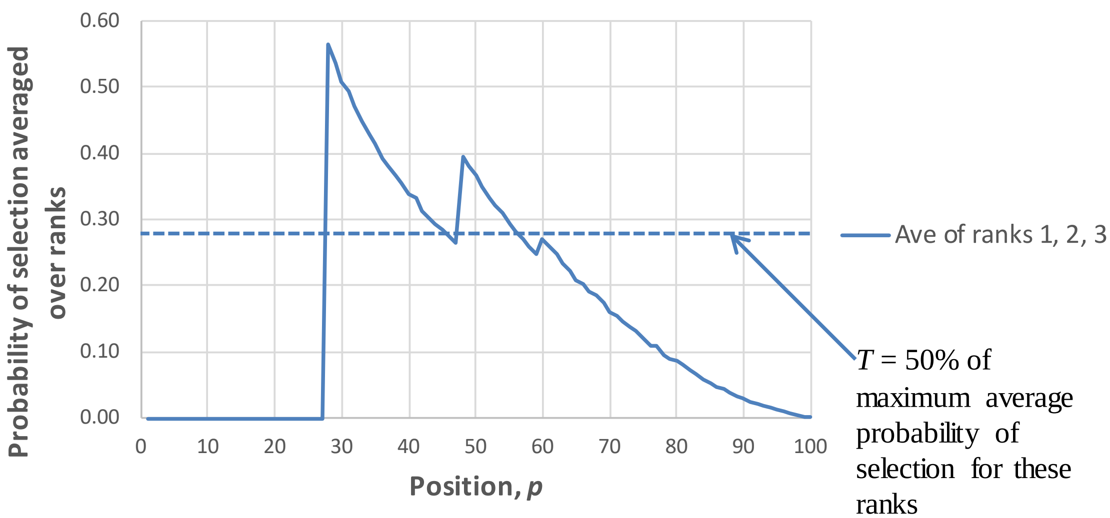


Figure 22. For Chow's solution for MinERR, ranks 1, 2, and 3: average probability of selecting those ranks, given position p. N = 100 and results are based on 10^6 simulated random runs.

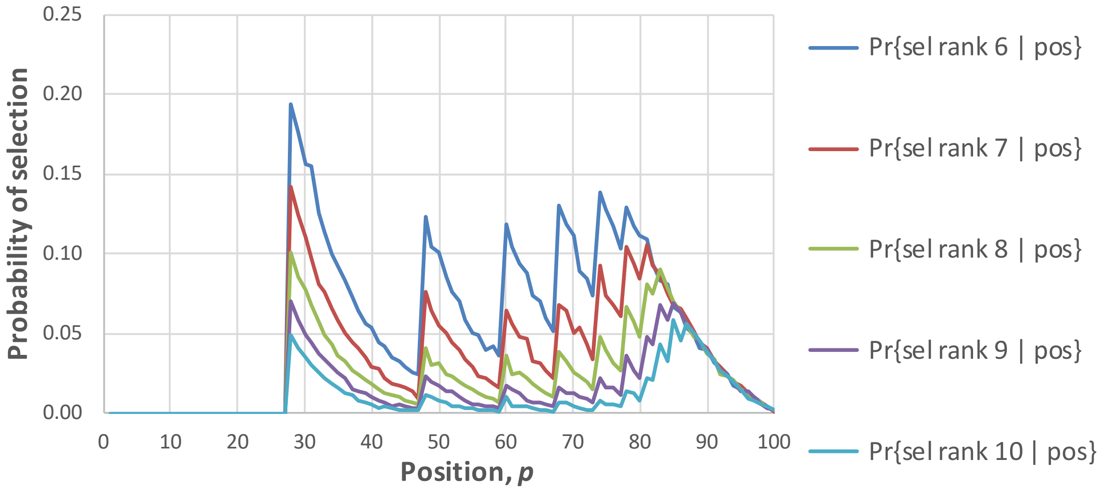


Figure 23. For Chow's solution for MinERR, ranks 6 through 10: individual probabilities of selecting those ranks, given position p. N = 100 and results are based on 10^6 simulated random runs.

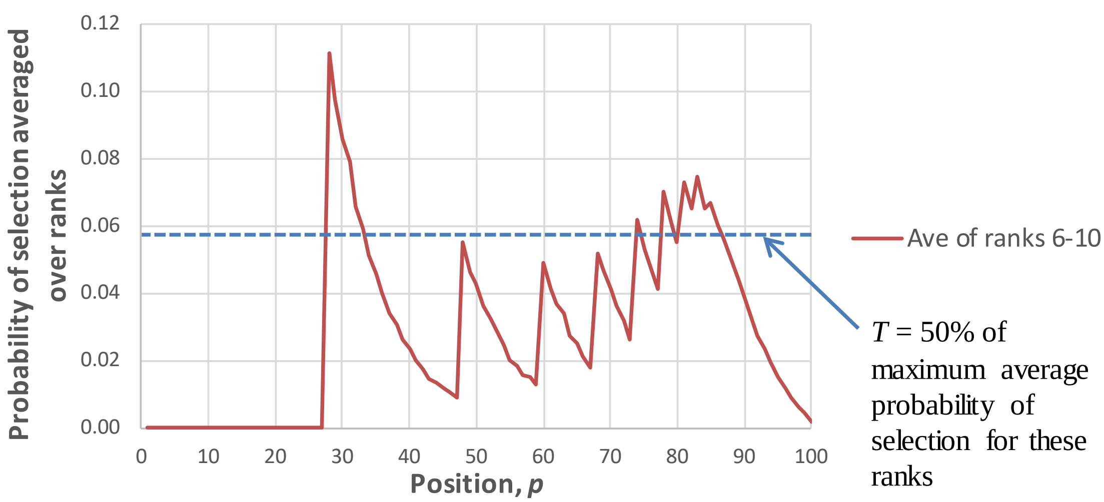


Figure 24. For Chow's solution for MinERR, ranks 1, 2, and 3: average probability of selecting those ranks, given position p. N = 100 and results are based on 10^6 simulated random runs.

XI. Apparent Rank vs. True Rank

It is important for the DM to know, ahead of selection, what the probability of selecting only the truly best applicant or what the average true rank of the selected applicant will be out of the queue of N applicants. According to the rules of the CSP, however, while considering an applicant at any given position the DM knows nothing about applicants yet to be observed. Thus, selection decisions are based on the *apparent rank* of an applicant rather than its *true rank* as defined by Lindley [Lindley 1961] and discussed in the Introduction.[33]

Unless the selection process reaches the last applicant or the nature of the problem abstracted as a CSP is such that the DM would still (for reasons unknown) observe and rank the applicants in the queue *even after the selection has taken place*, the DM cannot know the true rank of the selected applicant. Instead, at the end of the selection process the DM generally knows only the apparent rank of the selected applicant based on the applicants observed up to the position of the selected applicant.

At the end of the selection process, the apparent rank of the selected applicant is given by expression (22). Clearly, the apparent rank is always less than or equal to the real rank. Experiments were run to determine the degree to which they differ.

Simulations were run for the C2PS and Chow's solution for the CSP for honest-only queues with the "standard" parameters given in Section III.

As outlined in the Introduction, the C2PS for NBTB payoff has a predicted probability of about 37% of selecting the applicant with true rank 1. The simulation results reported in Table 7 demonstrate that fact and show that the probability of selecting the applicant with apparent rank of 1 (that is, not necessarily the overall best applicant but only the best applicant observed up to the point of selection) tends toward about 60%. The trends for these two probabilities are plotted in Figure 25.

Table 7 also shows the results of the simulations for average true rank and average apparent rank. As outlined in the Introduction, when the start position is set at about N/e to maximize NBTB payoff, the expected true rank of the selected applicant is about $N/(2e)$ [Bajnok 2015]. Remarkably, the expected apparent rank turns out to be nearly identical; these two results are plotted in Figure 26 but look like a single set of data. The linear trendline for those data has a slope of 0.1956, which yields an error of about –6% from the theoretical coefficient of $1/2e$.

The simulation results for the C2PS for MinERR payoff listed in Table 8 and plotted in Figure 27 demonstrate that the probability of selecting the applicant with true rank 1 falls off with N while the probability of selecting the applicant with apparent rank 1 approaches 1. The expected relative true and apparent ranks are shown in Figure 28 and behave quite

[33] Note that the true ranks and apparent ranks discussed in this section are relative ranks rather than true values.

differently from the situation with the C2PS for NBTB payoff. For the C2PS for MinERR payoff, the expected relative true rank has a trendline of $0.9324N^{0.5055}$ which closely matches the predicted value of about $\sqrt{N}$ as discussed in the Introduction. However, the expected relative apparent rank has a trendline of $0.6905N^{0.453}$ which differs from the expected relative true rank in both scale and the exponent. Notably, the expected relative apparent rank appears to grow slightly slower than $\sqrt{N}$.

The simulation results for Chow's solution for MinERR payoff given in Table 9 and shown in Figure 29 appear to indicate that the probability of selecting the applicant with true rank 1 approaches a value of about 0.25 while the probability of selecting the applicant with apparent rank 1 seems to approach a value of about 0.6. Figure 30 shows the average true rank, as expected, rises to its predicted limiting value of a little less than 4 while the average apparent rank seems to have a limiting value of about 2.2.

The results for the C2PS and Chow's solution show great discrepancies between the predicted true ranks and the measurable apparent ranks of the selected applicants. Thus, any post-selection analyses must be made on the basis of the statistics for apparent ranks.

The next section utilizes the average apparent rank to explore the potential ability of a DM to detect systematic and repeated cheating by applicants.

N	best start pos	Pr{sel true rank 1} at best start pos	Pr{sel apparent rank 1} at best start pos	ave true rank at best start pos	ave apparent rank at best start pos	ave pos observed for best start pos
10	4	0.3984	0.7008	3.02	2.50	6.98
20	8	0.3847	0.6507	4.98	4.49	14.68
30	12	0.3782	0.6331	6.99	6.51	22.36
40	15	0.3762	0.6503	8.55	8.01	29.03
50	19	0.3739	0.6401	10.52	9.99	36.72
60	23	0.3739	0.6342	12.48	11.97	44.37
70	26	0.3724	0.6432	14.04	13.50	51.06
80	30	0.3713	0.6378	16.02	15.48	58.73
90	33	0.3714	0.6441	17.55	17.00	65.41
100	38	0.3709	0.6297	20.01	19.50	74.12
200	73	0.3704	0.6421	37.45	36.89	145.74
300	109	0.3674	0.6403	55.60	55.04	218.62
400	150	0.3698	0.6259	76.21	75.70	297.02
500	186	0.3682	0.6294	94.27	93.73	369.07
600	235	0.3679	0.6094	118.99	118.52	454.89
700	267	0.3690	0.6196	134.47	133.98	524.09
800	282	0.3672	0.6480	142.29	141.69	575.02
900	319	0.3704	0.6480	159.81	159.22	649.09
1000	376	0.3682	0.6248	190.18	189.66	743.72
2000	791	0.3650	0.6016	398.43	397.97	1526.60
3000	1001	0.3746	0.6717	494.08	493.42	2093.59
4000	1433	0.3741	0.6460	711.92	711.36	2888.32
5000	1806	0.3767	0.6449	882.08	881.53	3628.00
6000	2273	0.3745	0.6279	1120.80	1120.30	4462.27
7000	2688	0.3678	0.6162	1352.08	1351.58	5243.69
8000	3235	0.3777	0.6023	1570.76	1570.33	6152.42
9000	3625	0.3717	0.6067	1757.95	1757.51	6882.43
10000	4083	0.3604	0.5911	2044.70	2044.26	7727.43

Table 7. Results for C2PS for NBTB payoff showing true rank and apparent rank of selected applicants. Number of simulated random runs for each value of *N* is specified in Section III. These data were generated by the same simulations that produced Table 1.

N	best start pos	Pr{sel true rank 1} at best start pos	Pr{sel apparent rank 1} at best start pos	ave true rank at best start pos	ave apparent rank at best start pos	ave pos observed for best start pos
10	3	0.3652	0.8006	2.93	2.00	5.65
20	5	0.3430	0.8008	4.20	2.99	10.85
30	5	0.2843	0.8663	5.17	3.00	12.53
40	6	0.2715	0.8752	5.97	3.50	15.85
50	7	0.2632	0.8795	6.71	4.01	19.20
60	8	0.2580	0.8834	7.37	4.50	22.49
70	8	0.2370	0.9005	7.98	4.49	23.53
80	9	0.2353	0.9000	8.55	4.99	26.86
90	10	0.2362	0.9001	9.08	5.49	30.18
100	10	0.2222	0.9102	9.56	5.48	31.15
200	14	0.1805	0.9349	13.73	7.53	49.07
300	17	0.1577	0.9464	16.89	9.06	63.56
400	20	0.1480	0.9527	19.48	10.45	77.51
500	22	0.1333	0.9581	21.80	11.45	87.88
600	24	0.1268	0.9628	23.72	12.22	98.40
700	27	0.1228	0.9626	26.03	14.08	112.67
800	27	0.1109	0.9666	28.24	14.37	115.96
900	29	0.1090	0.9695	29.37	14.68	125.24
1000	32	0.1086	0.9694	30.90	16.14	139.53
2000	47	0.0889	0.9780	44.13	23.59	216.90
3000	58	0.0716	0.9821	51.03	26.03	275.24
4000	63	0.0641	0.9856	58.64	27.40	315.91
5000	61	0.0491	0.9879	70.11	30.12	329.22
6000	78	0.0544	0.9872	76.50	39.21	396.91
7000	75	0.0460	0.9902	82.09	36.36	401.41
8000	88	0.0447	0.9886	90.08	44.32	476.18
9000	86	0.0488	0.9900	90.00	39.89	486.04
10000	95	0.0416	0.9908	96.63	45.64	524.89

Table 8. Results for C2PS for MinERR payoff showing true rank and apparent rank of selected applicants. Number of simulated random runs for each value of *N* is specified in Section III. These data were generated by the same simulations that produced Table 2.

N	Pr{sel true rank 1}	Pr{sel apparent rank 1}	ave true rank	ave apparent rank	ave pos observed
10	0.3293	0.6215	2.5594	1.8197	6.29
20	0.3002	0.6712	2.9994	1.8730	10.99
30	0.2881	0.6399	3.2021	2.0214	16.64
40	0.2769	0.6162	3.3251	2.0937	22.09
50	0.2719	0.6335	3.4193	2.0678	26.62
60	0.2718	0.6252	3.4724	2.1316	32.31
70	0.2676	0.6134	3.5180	2.1636	37.71
80	0.2670	0.6267	3.5564	2.1484	42.37
90	0.2636	0.6146	3.5786	2.1616	47.69
100	0.2640	0.6112	3.6018	2.1968	53.35
200	0.2554	0.6105	3.7361	2.2304	104.30
300	0.2518	0.6143	3.7725	2.2042	154.24
400	0.2553	0.6121	3.7887	2.2210	205.08
500	0.2521	0.6114	3.7980	2.2108	255.57
600	0.2518	0.6111	3.7986	2.2105	306.34
700	0.2493	0.6087	3.8138	2.2184	357.51
800	0.2506	0.6096	3.8362	2.2336	408.37
900	0.2516	0.6089	3.8416	2.2357	459.90
1000	0.2482	0.6093	3.8190	2.2169	510.13
2000	0.2444	0.6056	3.9022	2.2588	1022.03
3000	0.2537	0.6193	3.8053	2.1935	1517.61
4000	0.2578	0.6165	3.8271	2.2318	2028.11
5000	0.2527	0.6172	3.8026	2.1997	2533.01
6000	0.2463	0.6080	3.9647	2.3327	3048.07
7000	0.2476	0.6095	3.8610	2.2348	3547.55
8000	0.2516	0.6108	3.8602	2.2489	4055.87
9000	0.2446	0.6082	3.8293	2.2127	4568.16
10000	0.2385	0.6124	3.8249	2.1599	5019.92

Table 9. Results for Chow's solution for MinERR payoff showing true rank and apparent rank of selected applicants. Number of simulated random runs for each value of *N* is specified in Section III. These data are slightly different from those given in Table 6 because they are from an independent set of simulations.

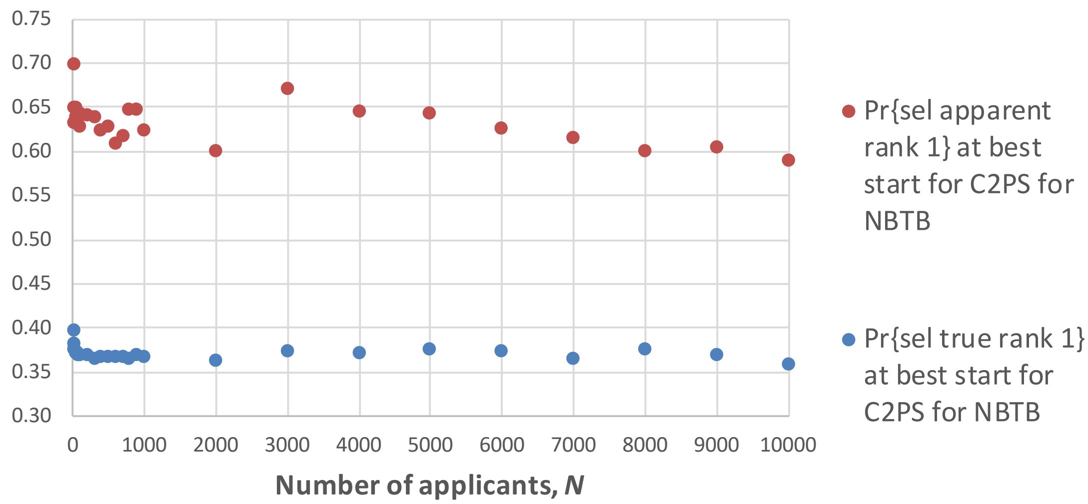


Figure 25. Probabilities of selection for true rank 1 and apparent rank 1 for C2PS for NBTB.

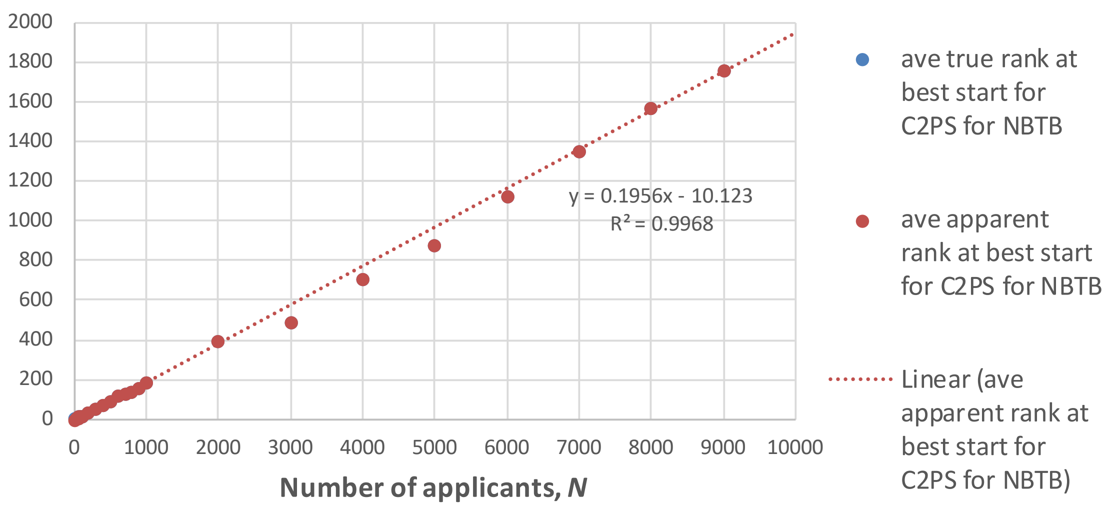


Figure 26. Expected relative true rank and apparent rank for C2PS for NBTB. The plots of the two sets of points essentially coincide at this scale.

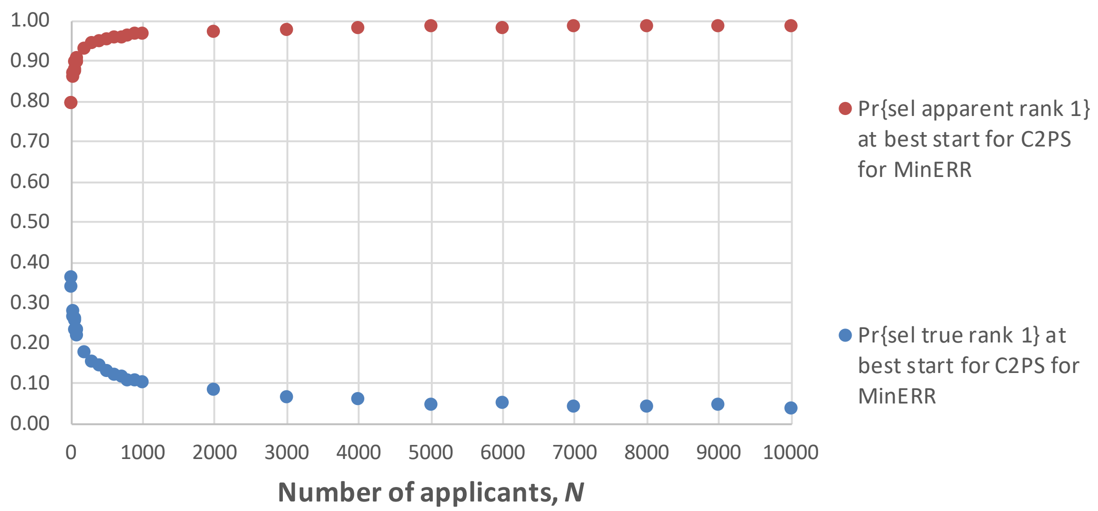


Figure 27. Probabilities of selection for true rank 1 and apparent rank 1 for C2PS for MinERR.

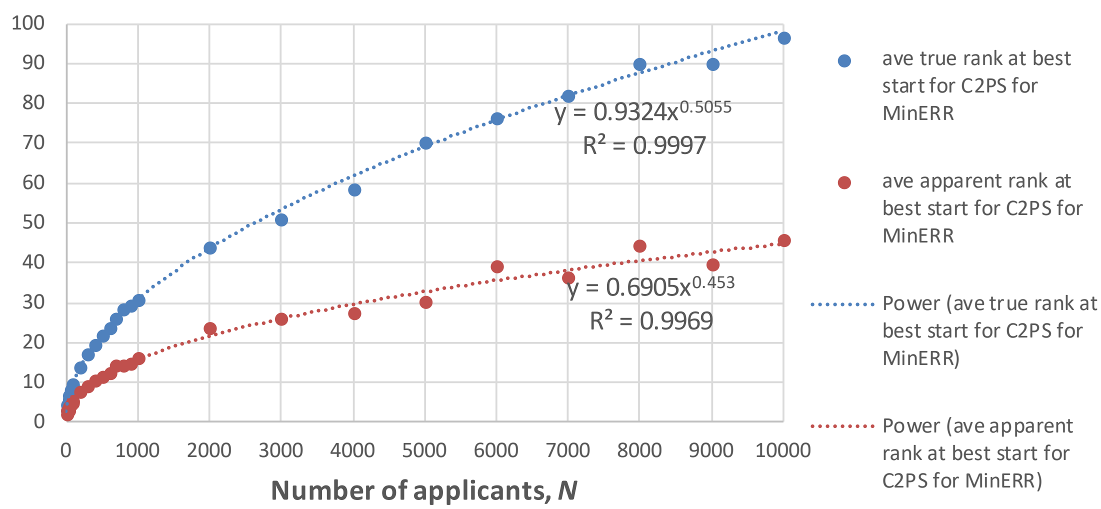


Figure 28. Expected relative true rank and apparent rank for C2PS for MinERR.

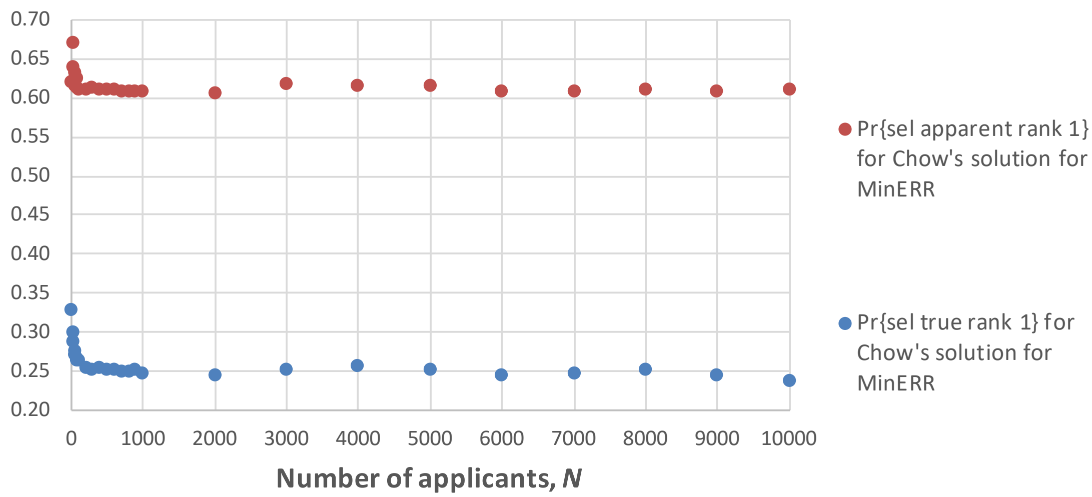


Figure 29. Probabilities of selection for true rank 1 and apparent rank 1 for Chow's solution for MinERR.

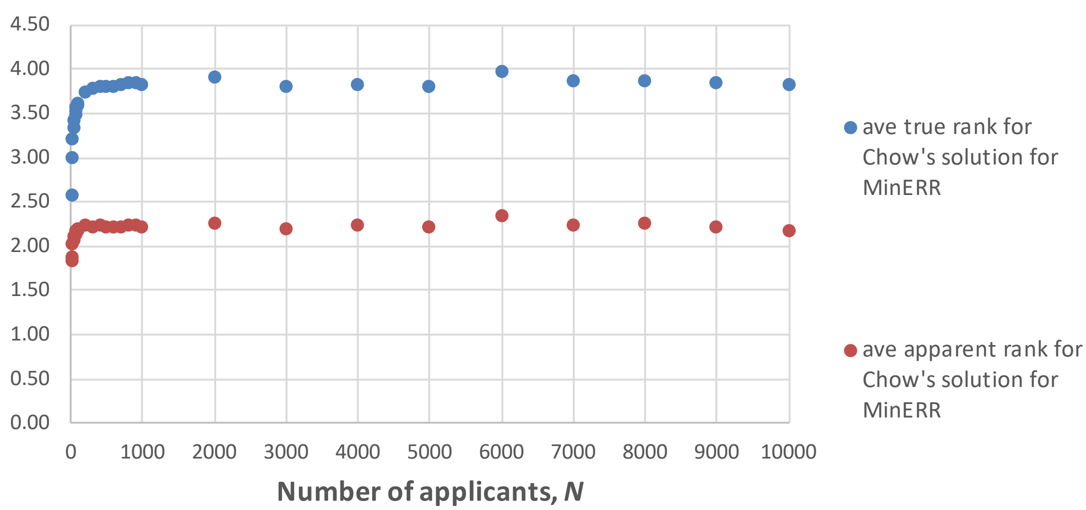


Figure 30. Expected relative true rank and apparent rank for Chow's solution for MinERR.

XII. Detecting Cheating

The previous section explored the statistical properties of the apparent rank of the selected applicant; in most situations the true rank of that applicant is unknown unless the DM reaches the last applicant. This section investigates the ability of the DM to detect cheating based on apparent rank. Only Chow's solution for the CSP for MinERR payoff is considered here because it is the most challenging approach of the CSP solutions considered in this paper.

The apparent rank is calculated using expression (22).

Figures 20a-20b in Section IX demonstrate that a CA can, roughly speaking, more than double its chances of being selected as intended when a position is selected from the "T = 50%" section of the lists given in Appendix E. This section investigates whether repeated cheating by applicants changes the statistics of the apparent ranks of the selected applicants sufficiently for the DM to detect that cheating behavior. The cases of cheating by applicants *seeking selection* and *seeking to evade selection* are handled separately.

The only method considered in this paper is the test of hypothesis concerning the difference of two means [Mendenhall 1979]. This paper does not, for example, address observations of the specific behavior of applicants (such as "cutting in line") or correlating ranks of applicants across multiple queues to identify suspicious behavior.

This analysis is simplified in that it considers *only a single CA rank at a time* that is systematically and repeatedly cheating. It would be worth considering a more general model for detecting a variety of cheating ranks only if the simpler case of detecting a single cheating rank were shown to be practical.

Simulation Statistics

The *test of hypothesis concerning the difference of two means* used in this paper is a two-tailed test that a sampled average apparent rank is either less than or greater than the actual average apparent rank. For a given number of applicants N, the actual average applicant rank can be closely estimated by simulation as shown in previous sections for several CSP solutions. These "actual" average applicant ranks are summarized for a range of values of N in Tables 7, 8, and 9 in Section XI; this section uses the statistics for Chow's solution given in Table 9. For the values of N used in this section, these values are based on 10^6 simulated random runs and thus should be an accurate baseline for the "actual" average applicant ranks.[34]

Figure 31a shows for N = 20 how the average applicant rank varies for queues with CAs with ranks 1 through 10 compared to queues with all-honest applicants. These simulation results are based on 10^6 random runs. Each random run is comprised of a simulation of a

[34] This qualitative analysis of statistical validity is made in spite of the good advice given by Pawlikowski, Jeong, and Lee [Pawlikowski 2002].

randomly ordered queue with honest applicants and then a simulation with a CA with the given rank. Note that all of the blue dots representing the all-honest simulations hover closely around the "actual" average applicant rank of 1.8730 given in Table 9.

Each simulation of a CA begins with a randomly ordered queue. If the CA is not already in an advantageous position, the CA selects at random an advantageous position (using the "T = 50%" section of Appendix E) and swaps positions with the applicant already in that position. Figure 31a shows that, when the CA with rank 1 cheats, that applicant effectively is always selected. For CAs with greater rank, the average apparent rank is practically the same or only slightly higher than that for the all-honest queues of applicants.

Figure 31b shows the same scenario for $N = 50$. Again, a CA with rank 1 is almost always selected, but cheating by other CAs changes the average apparent rank only slightly.

These results, based on a large number (10^6) of random runs, foreshadow that statistical tests based on small samples may not be effective in detecting cheating.

Hypothesis Testing

The test of hypothesis concerning the difference of two means is conducted as follows for sets of queues of N applicants. A sample size n is chosen. For either an honest applicant or a CA of a given rank, n random queues are generated and the sample mean m and sample standard deviation s are calculated for the applicant ranks of the n selected applicants.[35] The "actual" average rank for selected applicants for this value of N is denoted by μ, and, as mentioned earlier, this average is based on 10^6 random simulated runs for the values of N used in this section.

Following common procedure [Mendenhall 1979], the **null hypothesis** is that the mean m of the n samples of applicant ranks is the same as the actual average applicant rank μ for N applicants; the **alternative hypothesis** is that these two means are different. Both the z-test and the t-test were performed.

The z-test can be used when the underlying distribution satisfies certain criteria and the sample mean can be assumed to be normally distributed for sufficiently large sample size n. For m and s obtained from n samples, the z-statistic is given by:

$$z = \frac{m-\mu}{s/\sqrt{n}} \tag{25}$$

The **level of significance** was chosen to be $\alpha = 0.05$, which is to say this test of hypothesis was designed to reject, with only 5% probability, a sample from a population for which its mean equals the actual mean μ and so the null hypothesis is in fact true. The level of significance is the designed false alarm rate or probability of Type I error. Based on the

[35] The sample standard deviation is calculated as the square root of the unbiased sample variance. Note that the symbol s used in this section is not to be confused with the start position s used in other sections in connection with the C2PS or Chow's s_p constants.

normal distribution, a two-tailed z-test rejects the null hypothesis if $z < -1.96$ or $z > 1.96$. "Rejecting the null hypothesis" is equivalent to declaring that cheating is detected.

The assumptions for the z-test to hold are rather strong, and the test is invalid if the underlying distribution has, for instance, outliers or heavy asymmetric tails. Figure 32a for $N = 20$ and Figure 32b for $N = 50$ show histograms for the apparent ranks for queues with honest-only applicants and CAs with ranks 1 through 4. Clearly, the underlying distributions have heavy tails.

The critical factor for the z-test to be valid is whether the distribution of the sample mean for sample size n closely approximates a normal distribution. The Central Limit Theorem assures that this holds for large enough n [Mendenhall 1979], but a "bad" underlying distribution may require n to be very large indeed. For $N = 20$, Figures 33a-33f show the distribution of the mean of n samples for $n = 30$ and Figures 34a-34f show the same for $n = 100$; the plots show the results for honest-only queues and queues with CAs with ranks 1 through 5. CAs of rank 1 are so successful at being selected that the distribution of the mean is a spike at average apparent rank 1. Other than for CAs of rank 1, the distributions of the means for both the queues with honest-only applicants and CAs seeking selection are seen to be right-skewed. Such asymmetry is an issue for the z-test.

The t-test is more robust to the violation of some assumptions required for the z-test to be valid, and, in general, requires smaller samples sizes. For m and s derived from n samples, the t-statistic is given by:

$$t = \frac{m - \mu}{s/\sqrt{n}} \tag{26}$$

The level of significance for the t-test was again chosen to be $\alpha = 0.05$. For sample size n and known actual mean μ, the t-test uses Student's t-distribution rather than the normal distribution. Let $t_{\alpha/2,df}$ denote the value of t for which the right-tail probability of Student's t-distribution with $df = n-1$ degrees of freedom equals $\alpha/2$. Because this distribution is symmetric, a two-tailed t-test rejects the null hypothesis if $t < -t_{\alpha/2,df}$ or $t > t_{\alpha/2,df}$. With $\alpha = 0.05$, the values of these thresholds for various sample sizes n are:

$n = 30$:	$t_{\alpha/2,29} = 2.0452$
$n = 50$:	$t_{\alpha/2,49} = 2.0086$
$n = 100$:	$t_{\alpha/2,99} = 1.9840$
$n = 1{,}000$:	$t_{\alpha/2,999} = 1.9623$

Sample Size Needed to Detect Cheating Applicants Seeking Selection

The first question to be addressed in this section is whether repeated cheating by applicants seeking selection changes the statistics of the apparent ranks of the selected applicants sufficiently for the DM to detect that cheating behavior.

The first sample size to be tested in this study was $n = 30$, as this is customarily considered to be roughly the smallest sample size where the sample mean may be expected to be normally distributed. Figure 35a shows the reject rates for the test of hypothesis concerning the difference of two means for *honest-only applicants* for applicant queue lengths N ranging from 20 to 100.

As the level of significance $\alpha = 0.05$ was used, the test is designed so that the fraction of honest-only queues rejected as "cheating" should be only 5%. However, both the z-test and the t-test are shown to have reject rates that are roughly 15% to 20% for this range of N values; these false alarm rates are far higher than the design value of 5%. Figure 35b shows that a CA of rank 1 is always detected. Figure 35c shows that a CA of rank 2 is detected with rates that are for the most part higher than the false alarm rates of queues with honest-only applicants. Unfortunately for the DM, Figures 35d-35f show that CAs with ranks greater than 2 have reject rates comparable to the false alarm rates of honest-only applicants. Clearly, this sample size is too small for the Central Limit Theorem to hold for this underlying distribution.

Next, sample size $n = 50$ was tested. Figure 36 shows that the false alarm rates for queues with honest-only applicants are still unacceptably high (approximately 13% to 18%). For sample size $n = 100$, Figure 37 shows that the false alarm rates for queues with honest-only applicants are still double-to-triple the design rate of 5%. With false alarm rates so high, there is little point in looking at how the statistics for queues with CAs played out for these sample sizes.

Finally, sample size $n = 1{,}000$ was tested. Figure 38a shows the false alarm rate for queues with honest-only applicants at $N = 20$ was close to the design rate of 5%, but climbed to slightly over 8% at $N = 100$. Figure 38b shows that a CA with rank 1 is almost always detected. Figures 38c-38f show detection rates that for the most part are higher than the false alarm rates for honest-only applicant queues, but the results are inconsistent.

It is clear that, based on these results, the t-test provided little improvement in the quality of the test of hypothesis concerning the difference of two means over those of the z-test based on the distribution of apparent ranks obtained using Chow's solution for the CSP.

These results are summarized for queues with $N = 20$ applicants and z-test sample sizes $n = 30$, 50, 100, and 1,000 in Figures 39a-39d, respectively. Results for ranks 1 through 10 are shown, but the results for ranks higher than, say, 5, are unreliable because of the small numbers of instances when those ranks were selected; as noted in Section IX, Chow's solution for the CSP is extremely efficient in selecting applicants with lower ranks.

Perhaps the large sample sizes required to get the false alarm rate close to the design value of 5% would be practical in a situation where samples of a random physical or cyber process with consistently repeatable statistics serves as the "applicant queue," but it is hard to envision any situation where this might be achievable when biological entities are involved.

A physical system such as the notional example given in Appendix A might be such a candidate. A system that is continually monitored and operates under essentially constant conditions might be amenable to generating the samples sizes needed to support such a statistical analysis.

The experiments demonstrate that only a CA of rank 1 seeking selection has a detection rate substantially higher than that of honest applicants. One can reasonably argue that a DM would not care that a CA of rank 1 cheats to improve its probability of selection; no moral judgement is made in this paper. It has been noted repeatedly in this paper, however, that the goal of a CA (especially the applicant with rank 1!) may instead be to evade selection.

Sample Size Needed to Detect Cheating Applicants Seeking to Evade Selection

The second question to be addressed in this section is whether repeated cheating by applicants seeking to evade selection changes the statistics of their apparent rank enough for the DM to detect that cheating behavior. Figures 20c-20d in Section IX demonstrate that a CA seeking to evade selection can cut its chances of being selected roughly in half when a non-advantageous position is selected from *complement* of the advantageous positions in the "T = 50%" section of the lists given in Appendix E. The simulations summarized above were rerun with evasive behavior by the CAs.

Figures 40a-40b show that, for N = 20 and N = 50, respectively, the average apparent rank of the selected applicants does not change significantly except when the CA has rank 1. In those cases, the apparent rank is seen to increase, albeit not as dramatically as the decrease observed when the CA cheats to improve its chances of selection.

As discussed previously in this section, the test of hypothesis concerning the difference of two means is unreliable if the distribution of the mean of n samples fails to be at least roughly normal. For N = 20, Figures 41a-41f show the distribution of the mean of n samples for n = 30 and Figures 42a-42f for n = 100. The results for the honest-only applicants are essentially the same as shown in Figure 33a and Figure 34a as they were produced by the same simulation procedure but with different random seeds. The results for CAs with rank 1 seeking to evade detection are quite different from those for CAs seeking selection as the distribution of the sample mean is mound-shaped rather than exhibiting a spike at apparent rank 1. As shown previously for applicants seeking selection, it can be seen that the distributions of the means for the queues with both honest-only applicants and CAs seeking to evade selection are right-skewed and can be expected to be a challenge for a statistical test of hypothesis to distinguish between the mean apparent ranks resulting from honest and cheating behavior by the applicants.

Analyses of intermediate results are omitted here and the discussion jumps to the summaries of the reject rates. These results are summarized for queues with N = 20 applicants and z-test sample sizes n = 30, 50, 100, and 1,000 in Figures 43a-43d. Results for ranks 1 through 10 are shown. The results for CAs seeking to evade selection are comparable to those for CAs seeking selection, which is to say that the test of hypothesis concerning the difference of two means implemented in this section is ineffective for accepting honest-only queues and rejecting queues with CAs. The only point of difference is the situation with CAs of rank 1. For n = 30 and n = 50, the reject rates are close to 7% while the reject rates for honest-only queues are about 17% and 13%, respectively, which is not useful from the DM's point of view. At n = 100, the reject rate for CAs with rank 1 climbs to about 15% while the reject rate for honest-only queues drops to about 10%. At n = 1,000 the reject rate for CAs with rank 1 was measured at 98.8% (virtual certainty) while the reject rate for honest-only applicants is close to the design value of 5%; however, this sample size is unlikely to be practical in any but the rarest situations.

The Cheating Applicant Has the Advantage

The findings of this section indicate that the DM is at a severe disadvantage when applying a statistical test of hypothesis concerning the difference of two means based on the apparent ranks of the selected applicants to detect cheating by applicants whether they are *seeking selection* or *seeking to evade selection*. These experiments assumed a simplified scenario in which only a single rank at a time was assumed to cheat and in each of the n samples the CA randomly chose an advantageous position as listed in Appendix E (with T = 50%) with equal probability. This approach was observed to fail from the viewpoint of the DM because:

- The false alarm rates for queues with honest-only applicants are far above the design value of 5% for what could conceivably be considered practical sample sizes. The nominal statistical behavior is so variable that even n = 1,000 appears to be only marginally sufficient to reduce the false alarm rate to a reasonable limit.

- Even worse than the false alarm rates being so high for queues with honest-only applicants, a more serious problem is that those false alarm rates are the same or higher than the actual detection rates for queues with CAs with ranks greater than 1 except for extremely large sample sizes.

The major finding of this section is that cheating behavior by a CA is, for all practical purposes, undetectable by the DM based on the indirect method of a test of hypothesis concerning the difference of two means involving the only observable quantity by the DM: the apparent ranks of the selected applicants. Statistically, the advantage is overwhelmingly on the side of the CA.

A DM would have to use other more direct methods of observation that are outside the bounds of the CSP to reliably detect and counter cheating behavior. Such "out of band" methods might include observing the specific actions of applicants that result in their being placed in advantageous positions or avoiding those positions.

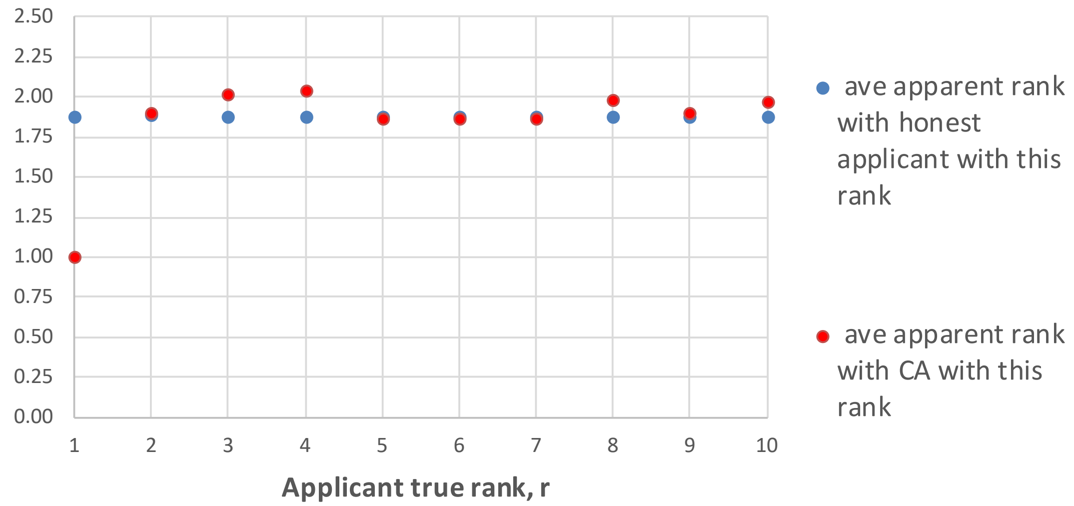


Figure 31a. Average apparent ranks for honest-only queues and CAs with ranks 1 through 10. ***N*** **= 20** and results are based on 10^6 simulated random runs.

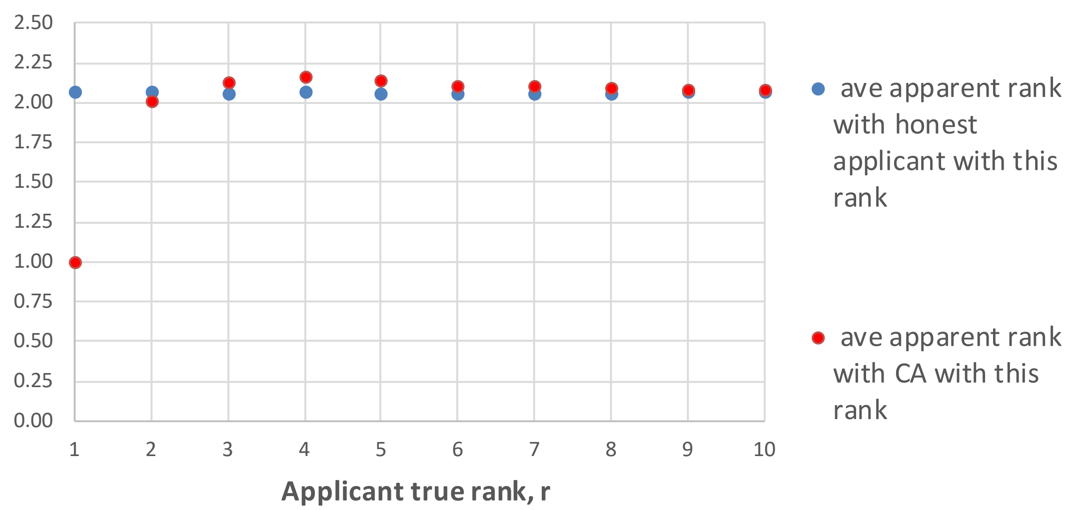


Figure 31b. Average apparent ranks for honest-only queues and CAs with ranks 1 through 10. ***N*** **= 50** and results are based on 10^6 simulated random runs.

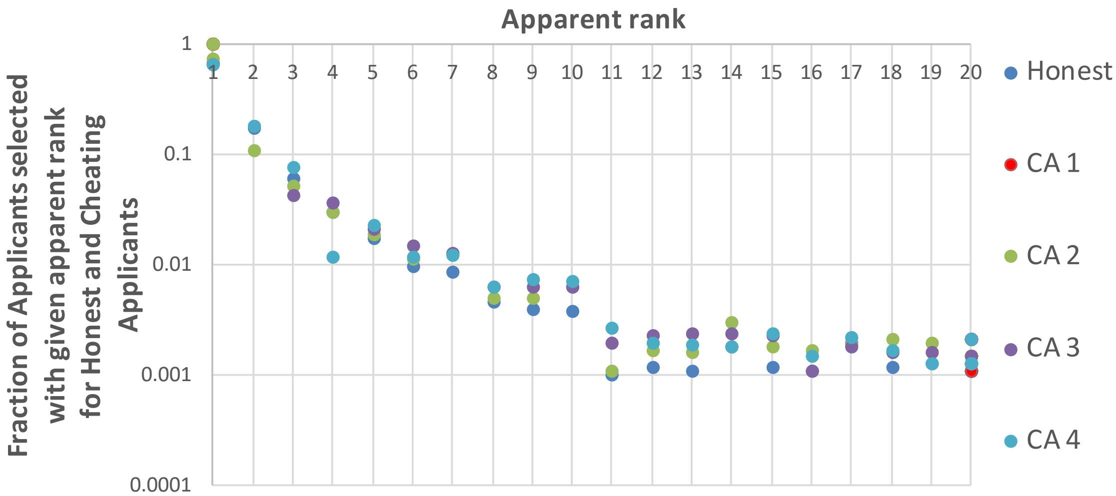


Figure 32a. Histograms for the apparent ranks for queues with honest-only applicants and CAs with ranks 1 through 4. N = **20** and results are plotted for 10^4 simulated random runs.

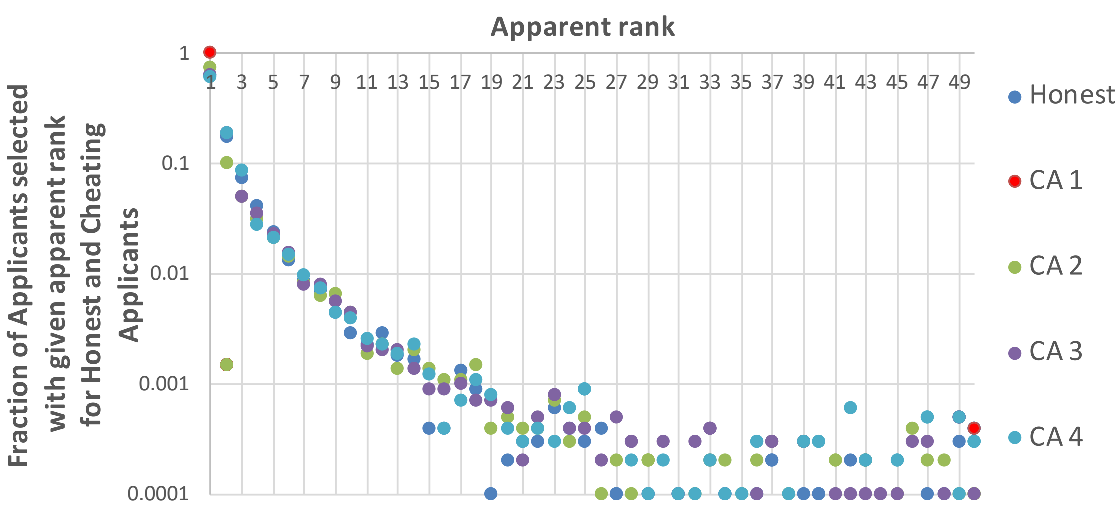


Figure 32b. Histograms for the apparent ranks for queues with honest-only applicants and CAs with ranks 1 through 4. N = **50** and results are plotted for 10^4 simulated random runs.

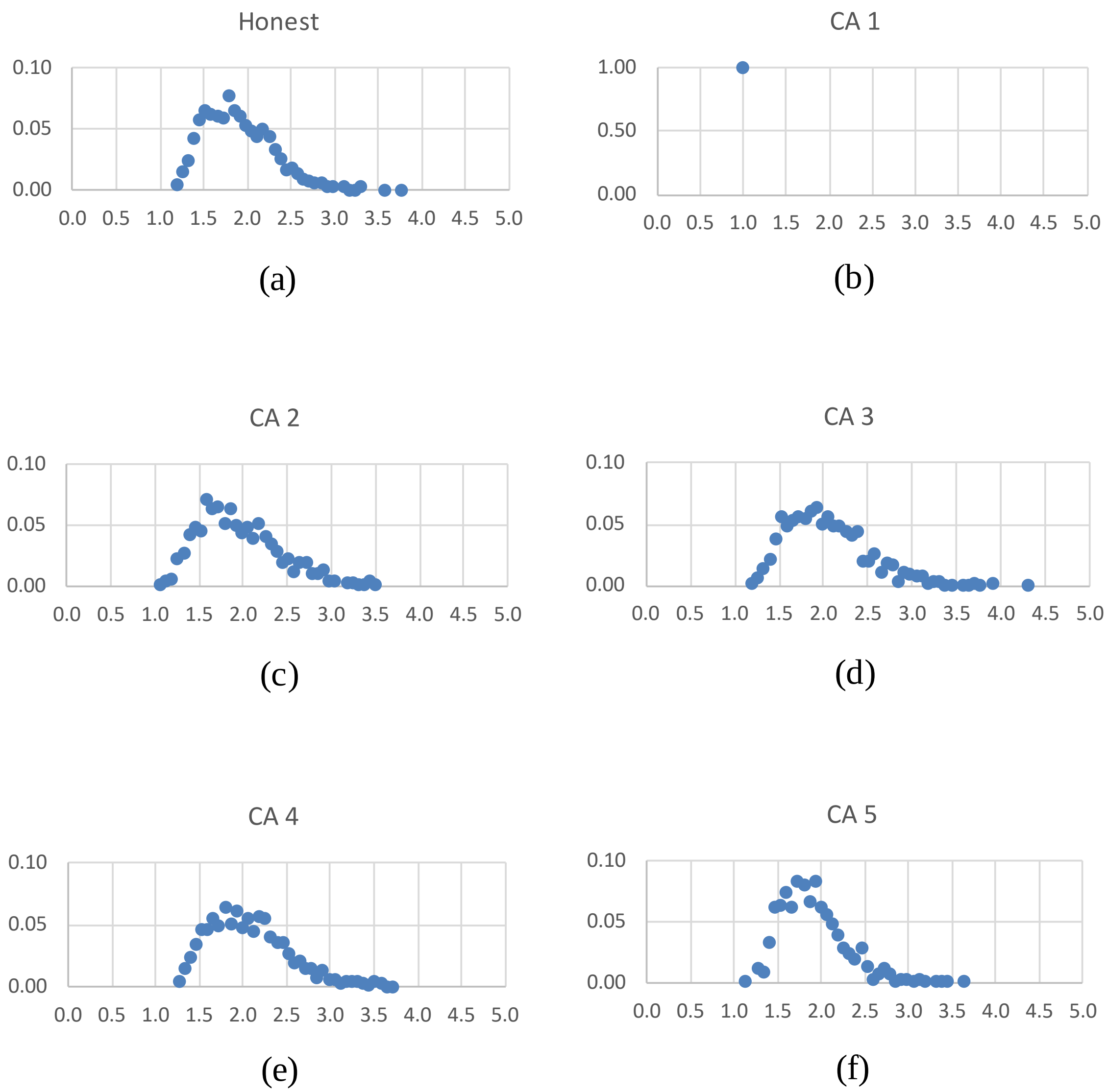


Figure 33. Distribution of the mean of ***n* = 30 samples** of apparent rank selected for honest-only queues and CAs with ranks 1 through 5. $N = 20$ and results are plotted for 10^3 sets of n simulated random runs.

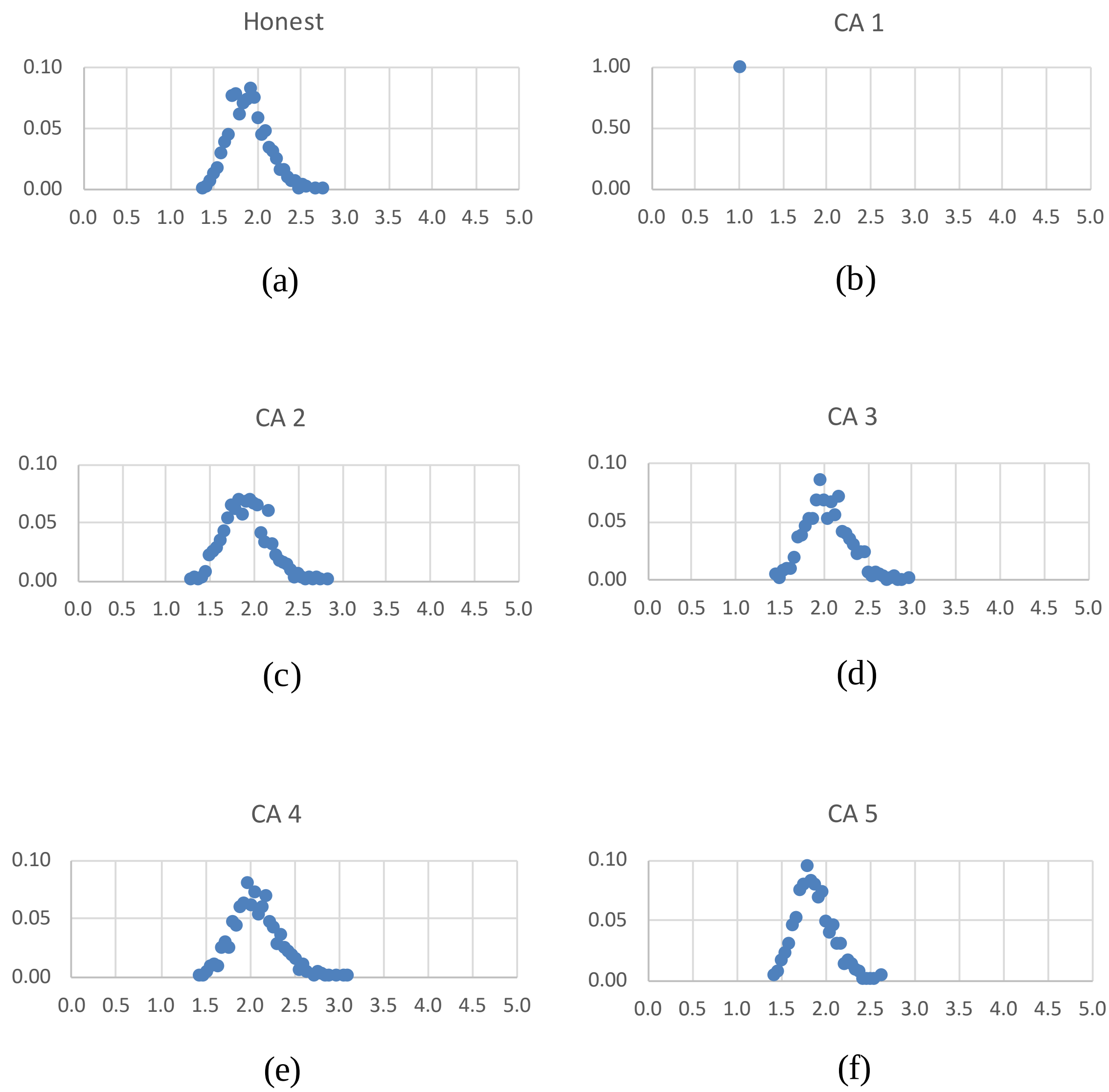


Figure 34. Distribution of the mean of ***n* = 100 samples** of apparent rank selected for honest-only queues and CAs with ranks 1 through 5. $N = 20$ and results are plotted for 10^3 sets of n simulated random runs.

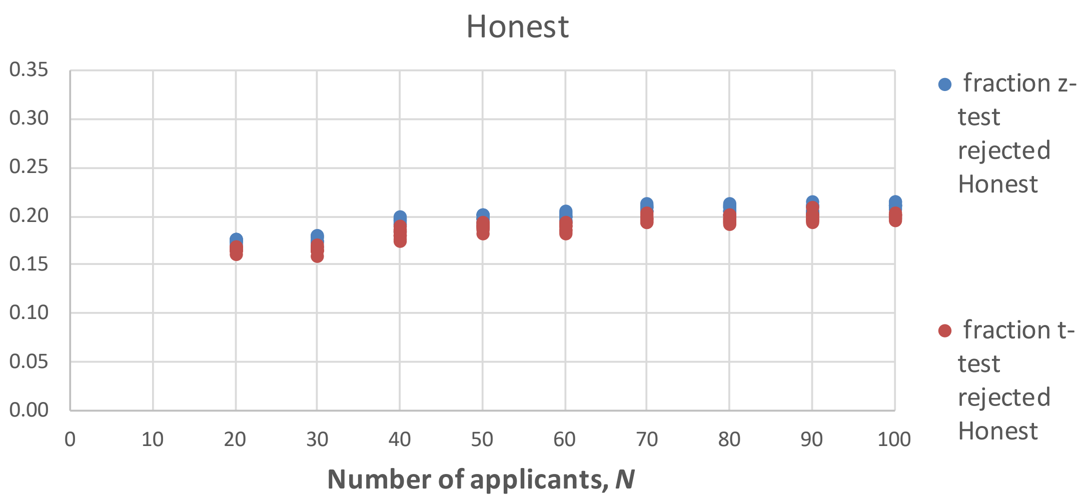


Figure 35a. Test of hypothesis reject rates for a range of N values for honest-only queues. **Sample size n = 30** for z-test and t-test. For each value of N, points are plotted for five sets of tests, where each set obtained a reject rate based on 10^4 tests of hypothesis.

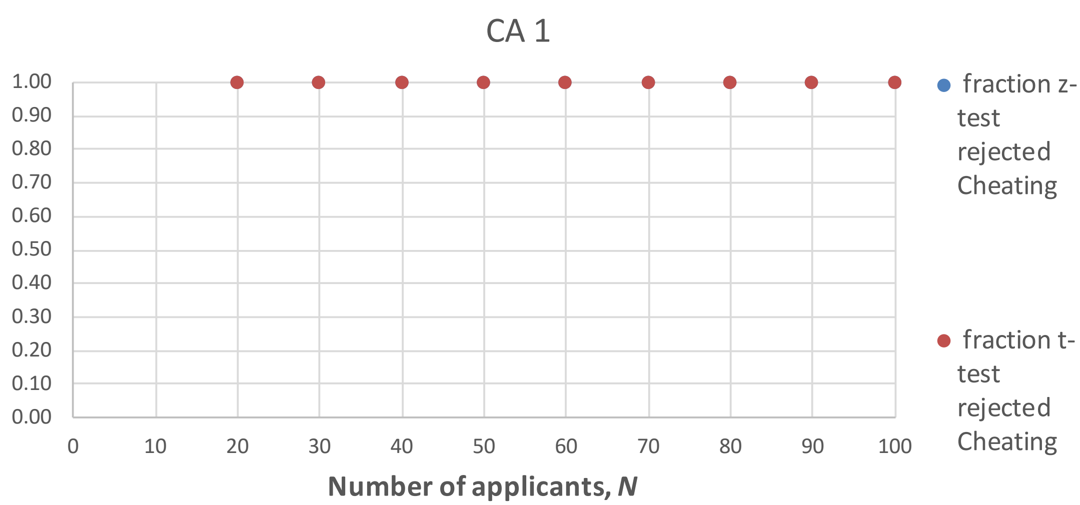


Figure 35b. Test of hypothesis reject rates for a range of N values for CAs with rank 1. Same conditions as in Figure 35a.

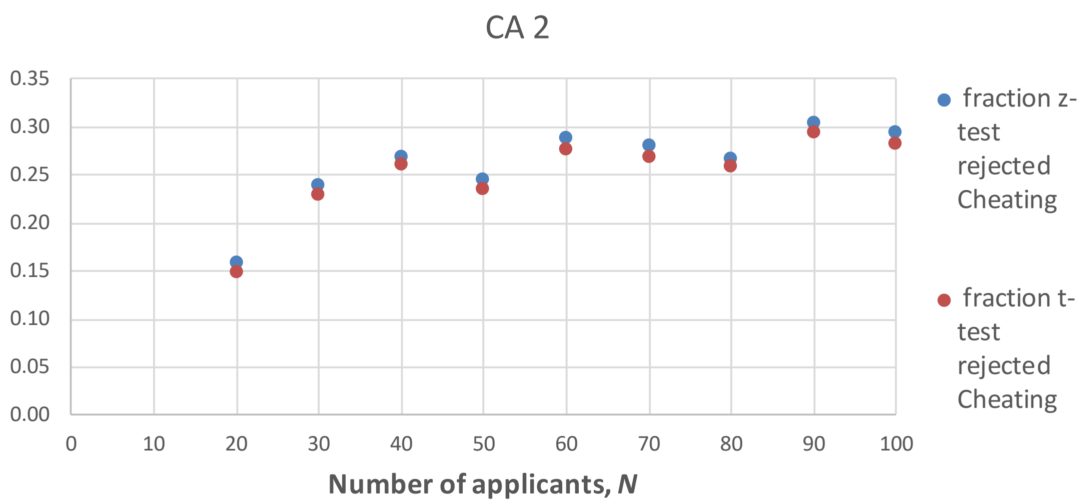


Figure 35c. Test of hypothesis reject rates for a range of *N* values for CAs with rank 2. Same conditions as in Figure 35a.

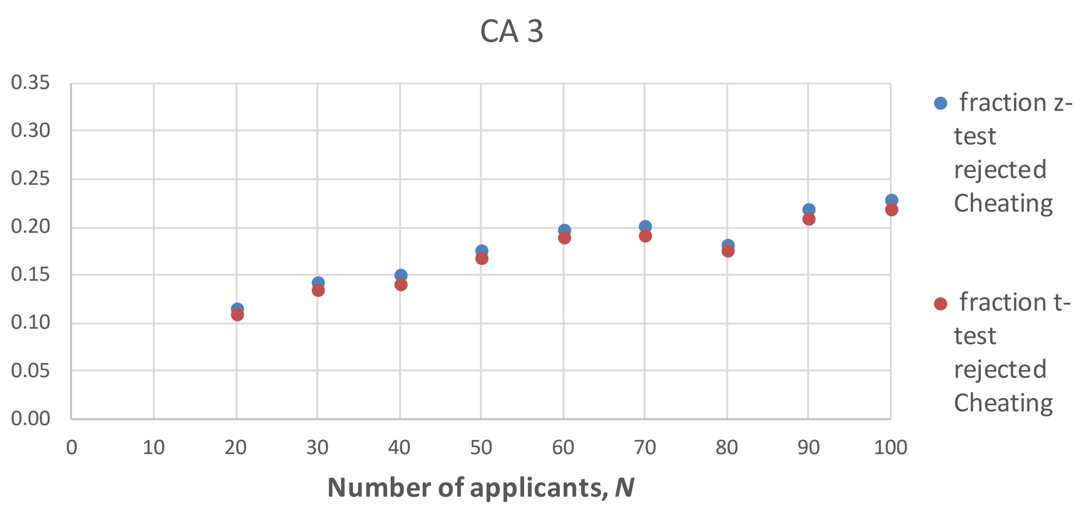


Figure 35d. Test of hypothesis reject rates for a range of *N* values for CAs with rank 3. Same conditions as in Figure 35a.

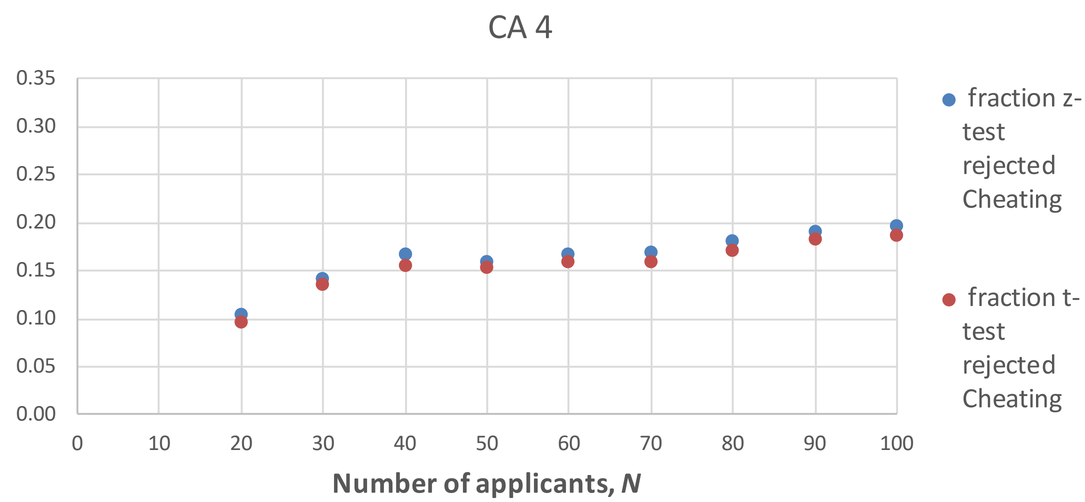


Figure 35e. Test of hypothesis reject rates for a range of *N* values for CAs with rank 4. Same conditions as in Figure 35a.

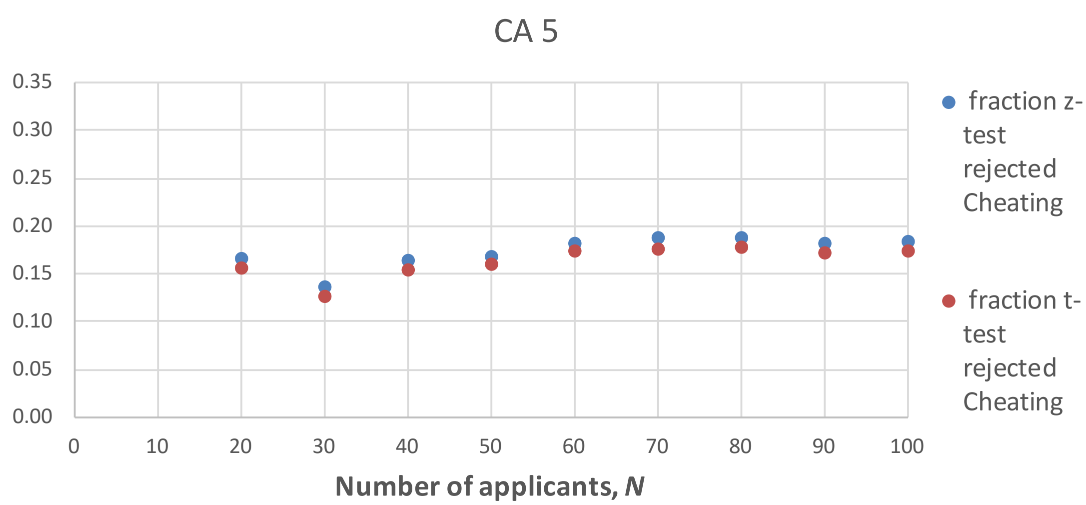


Figure 35f. Test of hypothesis reject rates for a range of *N* values for CAs with rank 5. Same conditions as in Figure 35a.

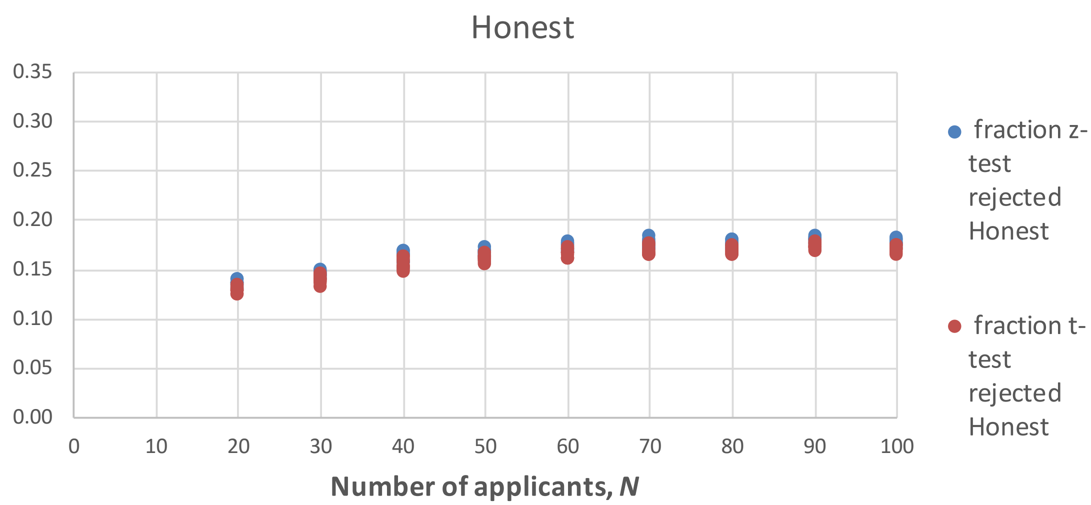


Figure 36. Test of hypothesis reject rates for a range of N values for honest-only queues. **Sample size n = 50** for z-test and t-test. For each value of N, points are plotted for five sets of tests, where each set obtained a reject rate based on 10^4 tests of hypothesis.

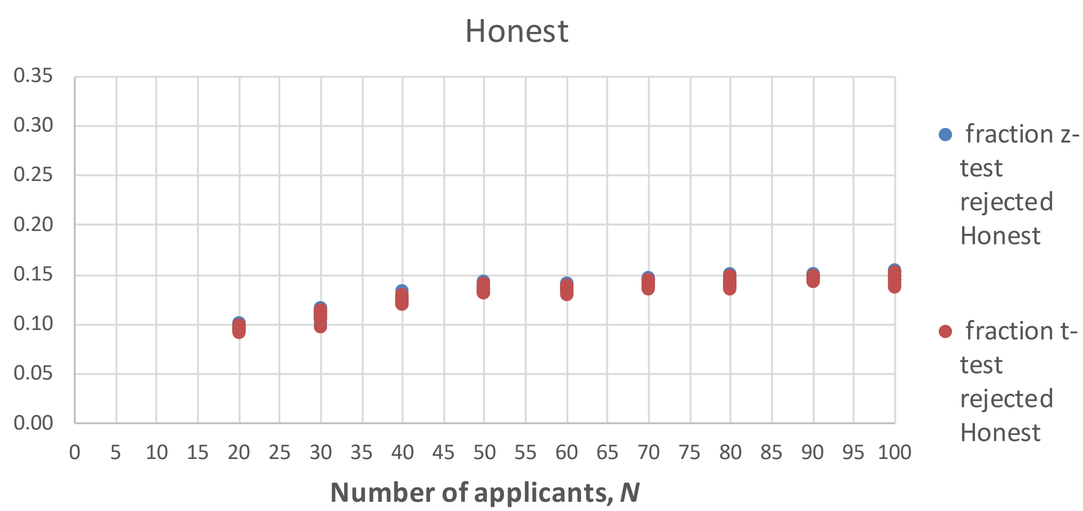


Figure 37. Test of hypothesis reject rates for a range of N values for honest-only queues. **Sample size n = 100** for z-test and t-test. For each value of N, points are plotted for five sets of tests, where each set obtained a reject rate based on 10^4 tests of hypothesis.

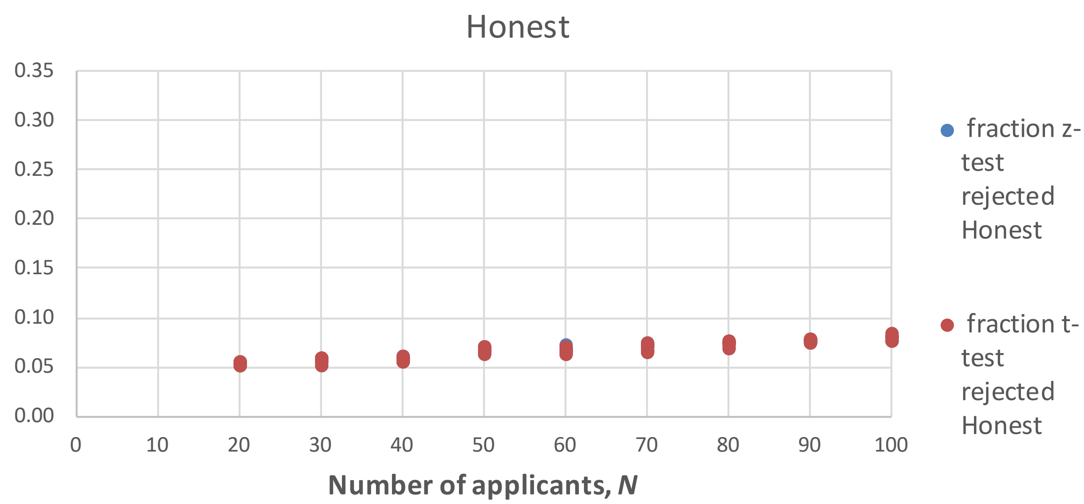


Figure 38a. Test of hypothesis reject rates for a range of N values for honest-only queues. **Sample size n = 1,000** for z-test and t-test. For each value of N, points are plotted for five sets of tests, where each set obtained a reject rate based on 10^4 tests of hypothesis.

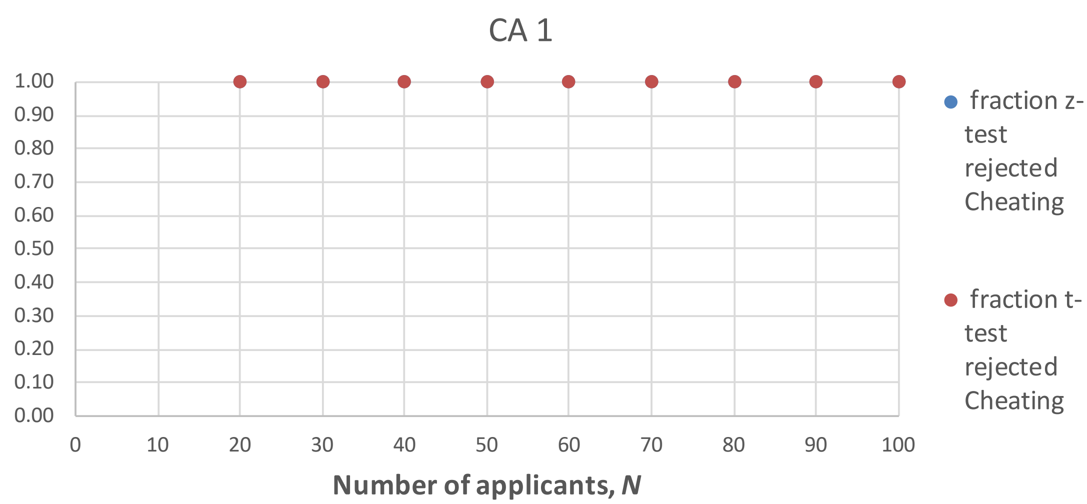


Figure 38b. Test of hypothesis reject rates for a range of N values for CAs with rank 1. Same conditions as in Figure 38a.

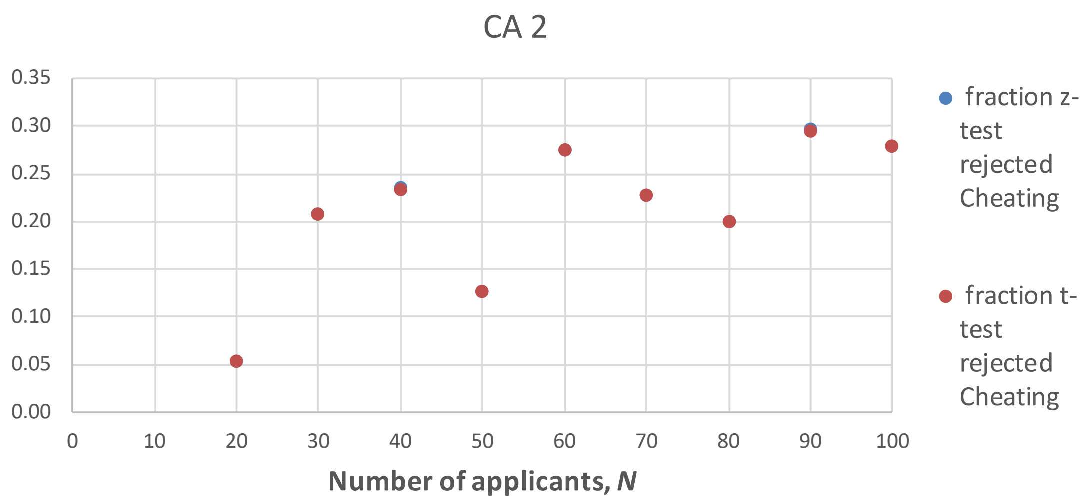


Figure 38c. Test of hypothesis reject rates for a range of N values for CAs with rank 2. Same conditions as in Figure 38a.

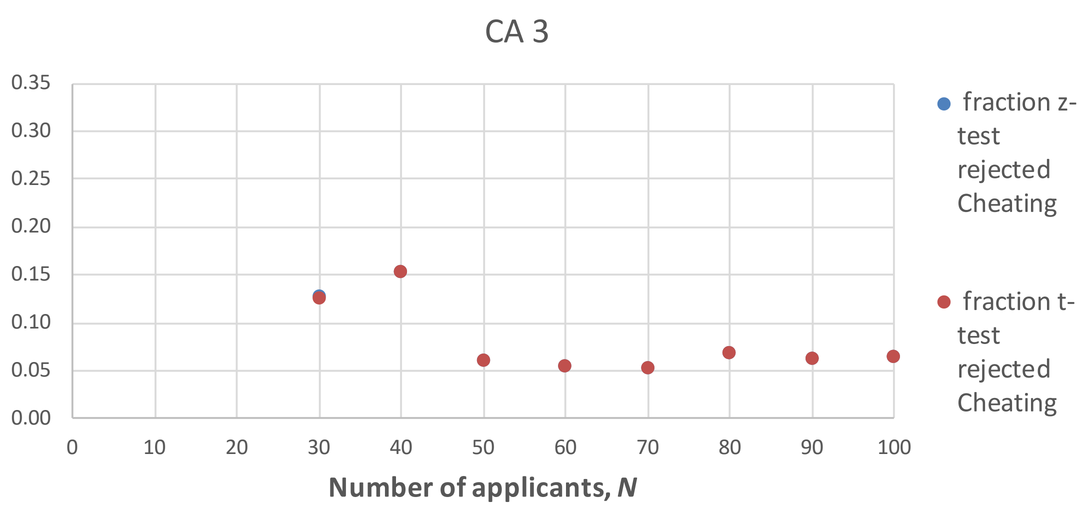


Figure 38d. Test of hypothesis reject rates for a range of N values for CAs with rank 3. Same conditions as in Figure 38a.

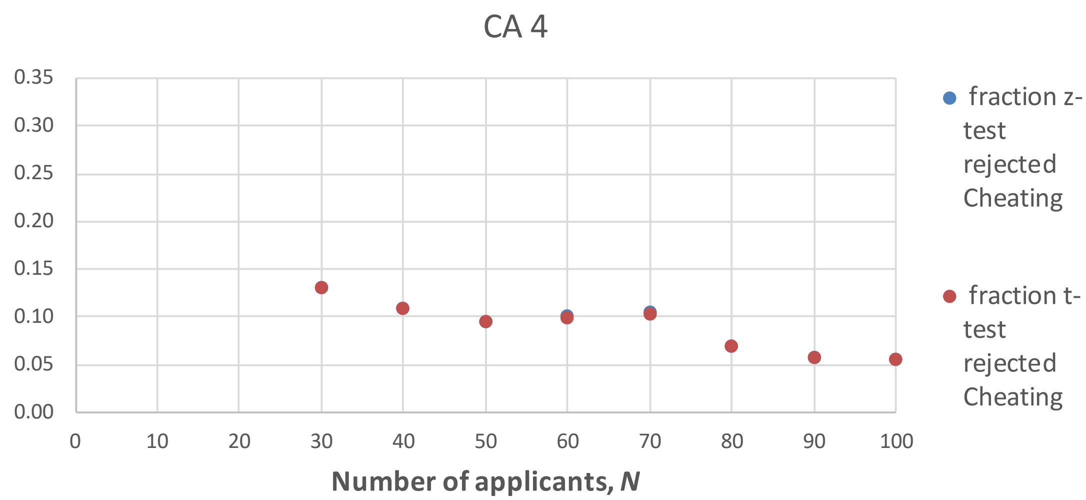


Figure 38e. Test of hypothesis reject rates for a range of N values for CAs with rank 4. Same conditions as in Figure 38a.

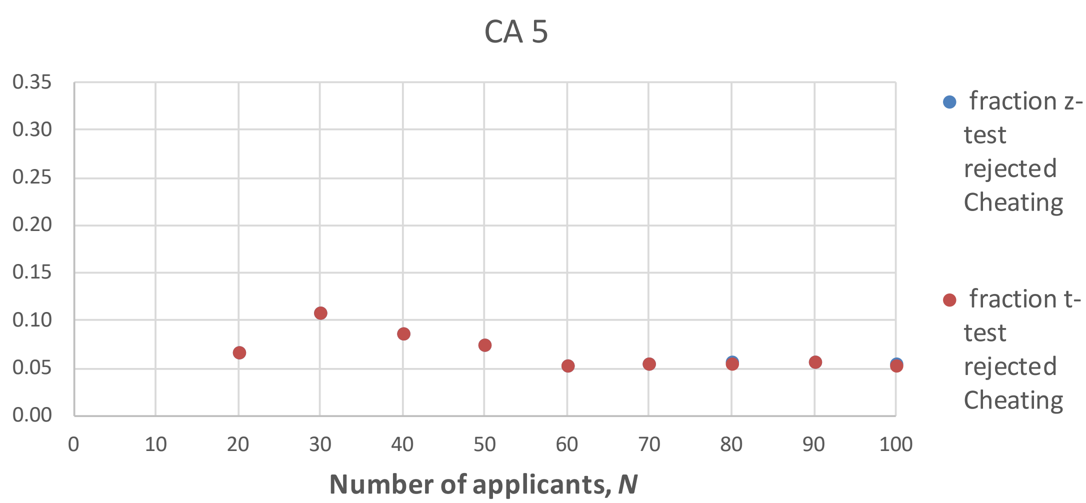


Figure 38f. Test of hypothesis reject rates for a range of N values for CAs with rank 5. Same conditions as in Figure 38a.

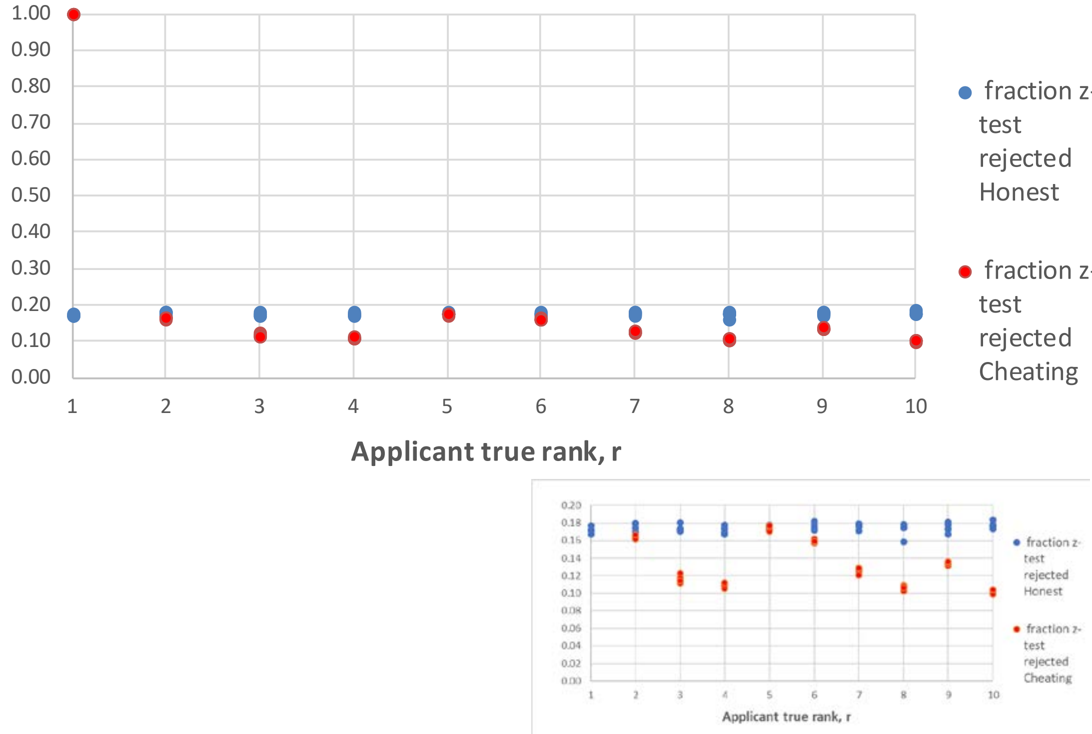


Figure 39a. Summary of reject rates for tests of hypothesis for honest-only queues and queues with CAs with ranks 1 through 10. $N = 20$ applicants and z-test **sample size n = 30**. For each CA rank, points are plotted for five sets of tests, where each set obtained a reject rate based on 10^4 tests of hypothesis. An independent simulation of the honest-only queue was performed with each CA simulation. Inset is scaled to show more detail for CAs 2 through 10.

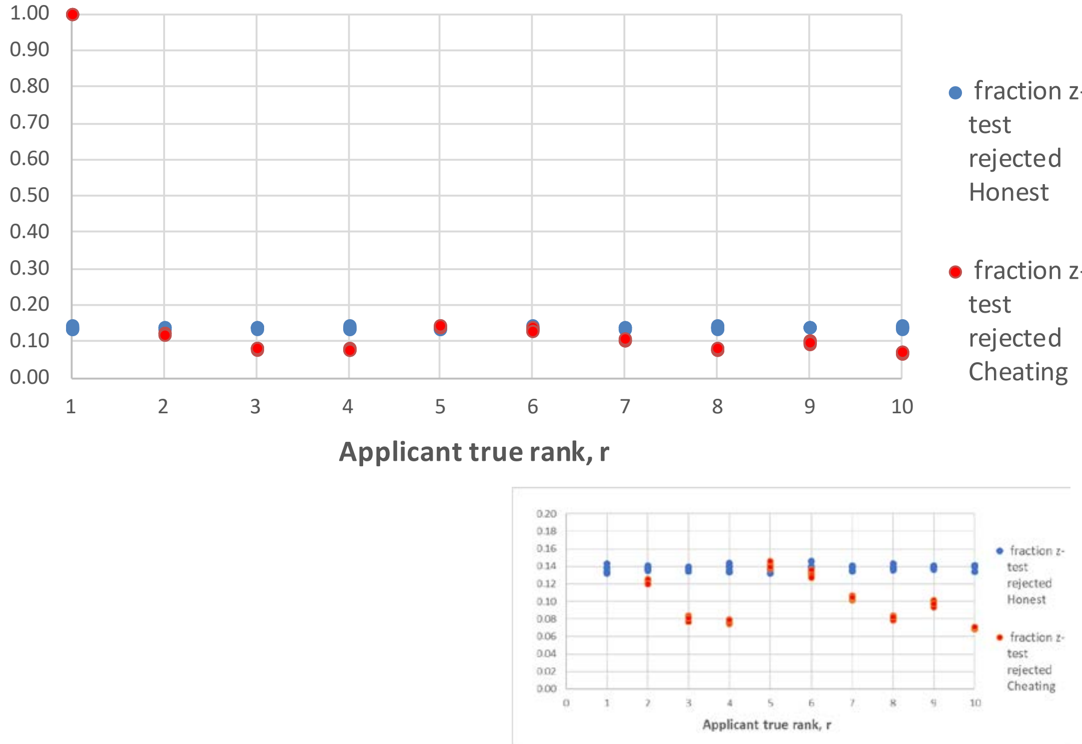


Figure 39b. Summary of reject rates for tests of hypothesis for honest-only queues and queues with CAs with ranks 1 through 10. $N = 20$ applicants and z-test **sample size $n = 50$**. For each CA rank, points are plotted for five sets of tests, where each set obtained a reject rate based on 10^4 tests of hypothesis. An independent simulation of the honest-only queue was performed with each CA simulation. Inset is scaled to show more detail for CAs 2 through 10.

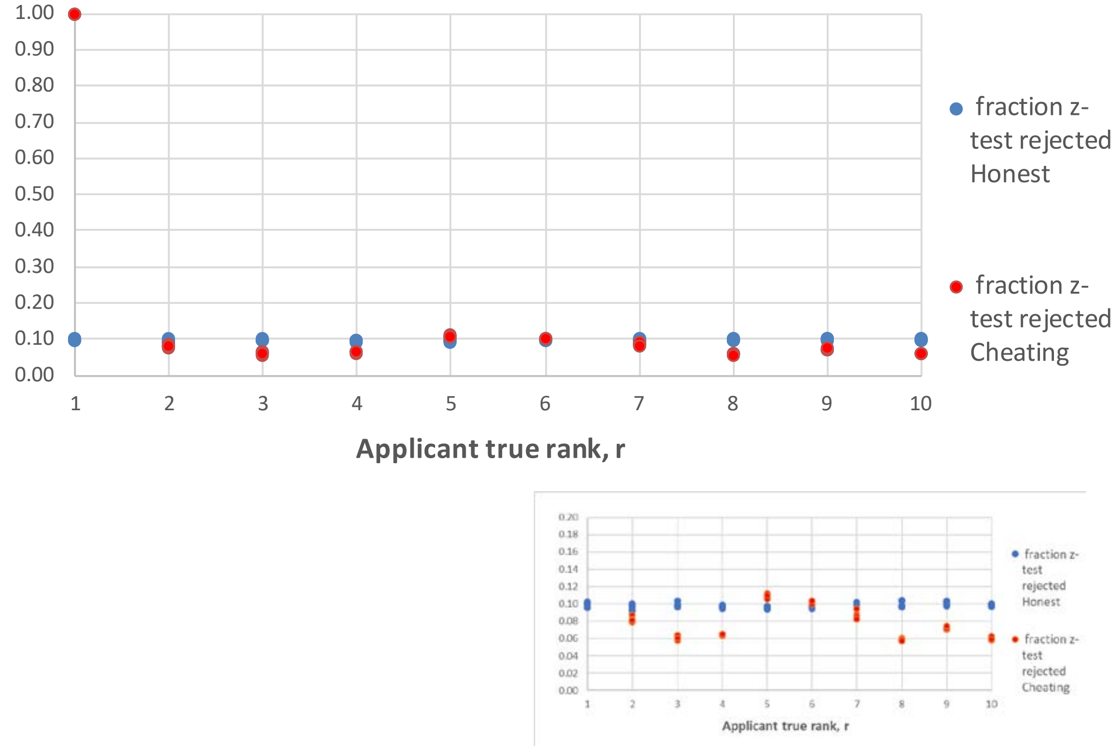
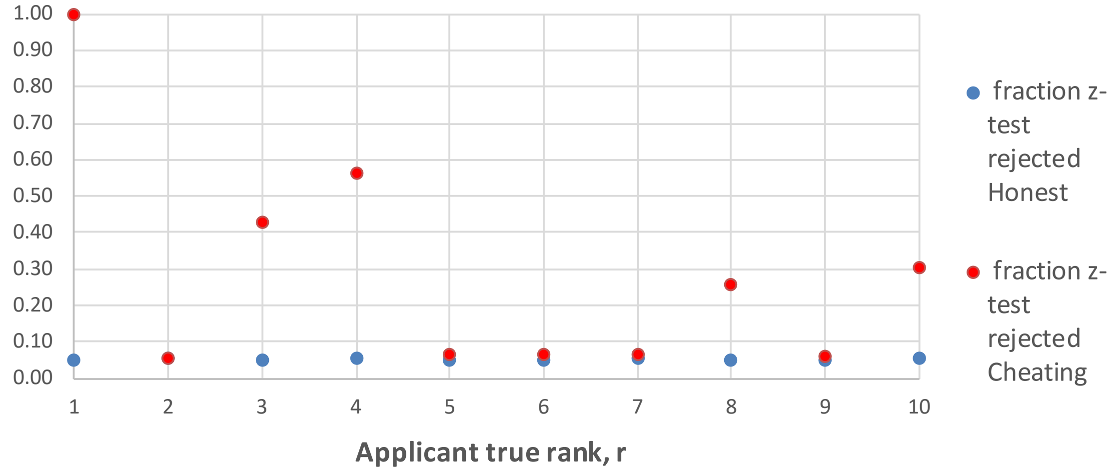


Figure 39c. Summary of reject rates for tests of hypothesis for honest-only queues and queues with CAs with ranks 1 through 10. $N = 20$ applicants and z-test **sample size n = 100**. For each CA rank, points are plotted for five sets of tests, where each set obtained a reject rate based on 10^4 tests of hypothesis. An independent simulation of the honest-only queue was performed with each CA simulation. Inset is scaled to show more detail for CAs 2 through 10.



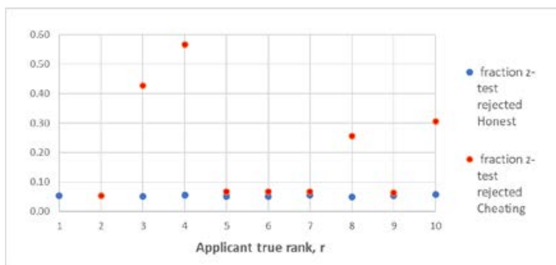


Figure 39d. Summary of reject rates for tests of hypothesis for honest-only queues and queues with CAs with ranks 1 through 10. $N = 20$ applicants and z-test **sample size n = 1,000**. For each CA rank, points are plotted for one set of 10^4 tests of hypothesis. An independent simulation of the honest-only queue was performed with each CA simulation. Inset is scaled to show more detail for CAs 2 through 10.

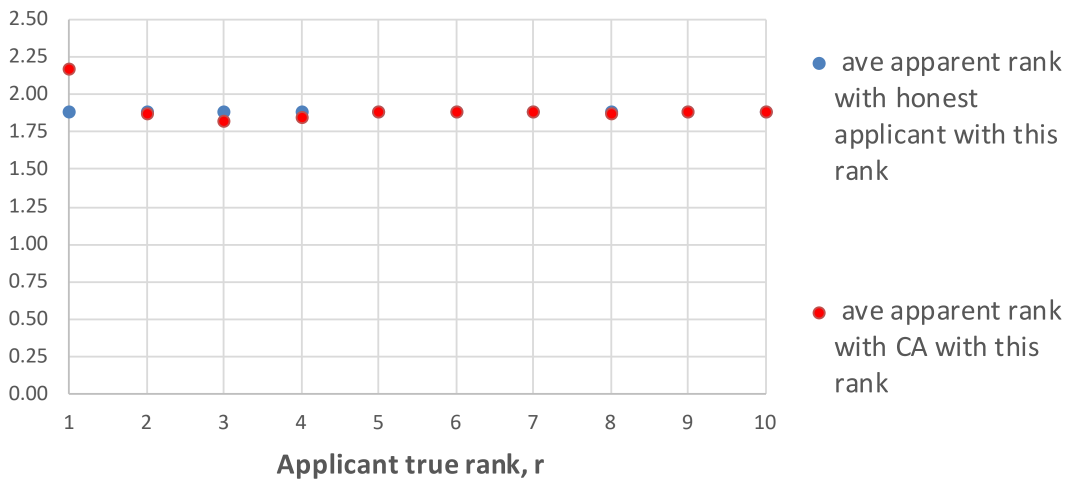


Figure 40a. Average apparent ranks for honest-only queues and CAs with ranks 1 through 10 seeking to evade selection. N = **20** and results are based on 10^6 simulated random runs.

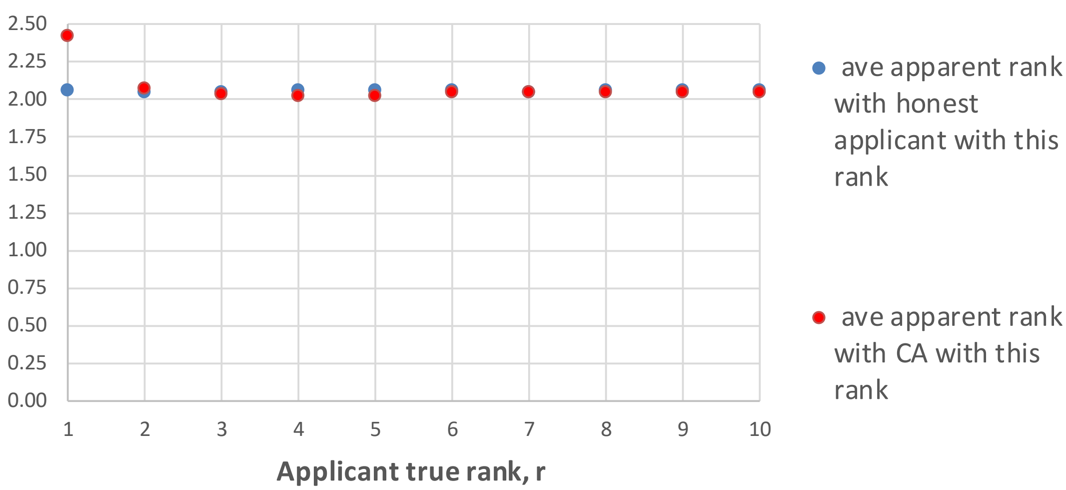


Figure 40b. Average apparent ranks for honest-only queues and CAs with ranks 1 through 10 seeking to evade selection. N = **50** and results are based on 10^6 simulated random runs.

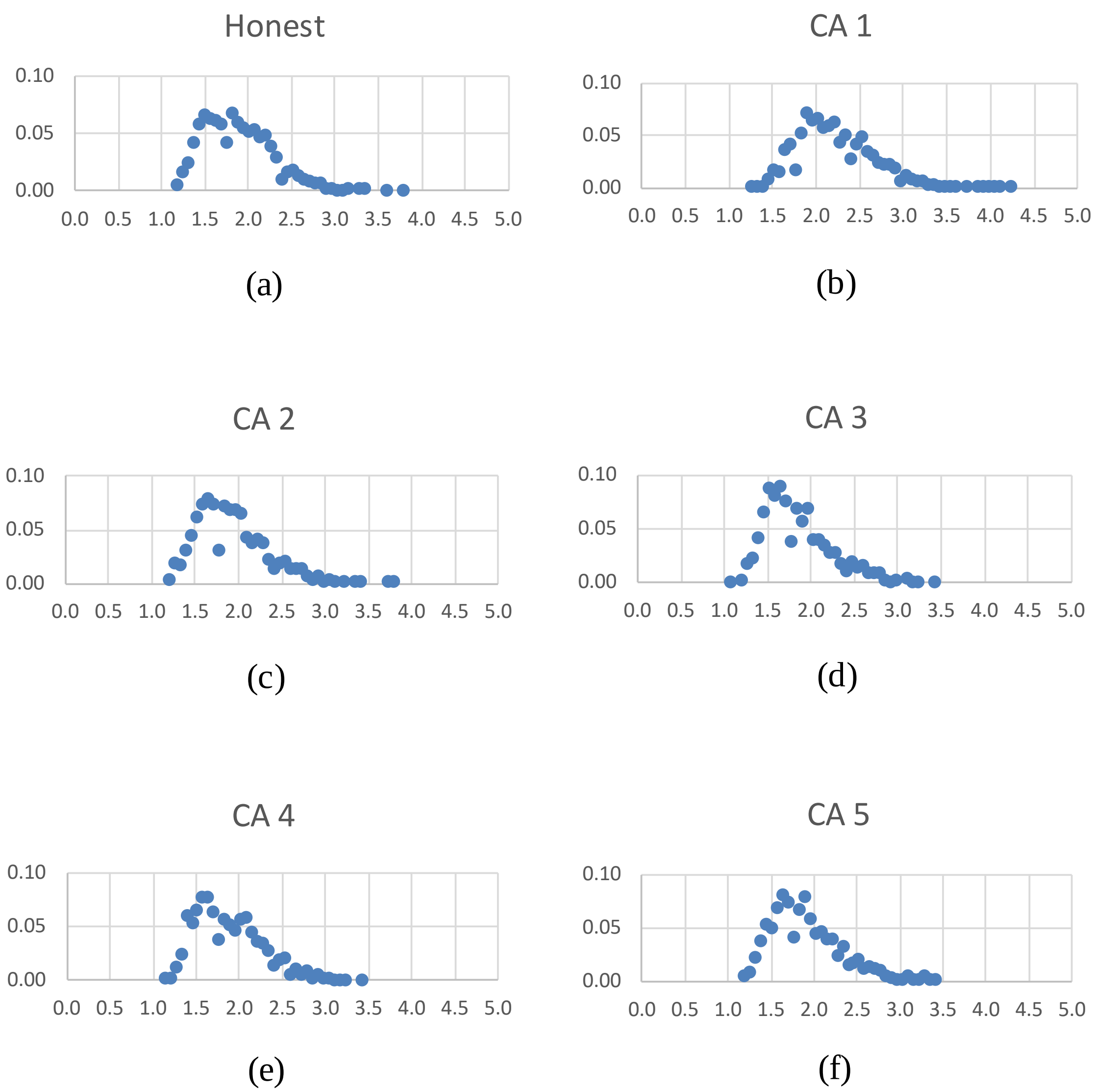


Figure 41. Distribution of the mean of ***n* = 30 samples** of apparent rank selected for honest-only queues and CAs with ranks 1 through 5 seeking to evade selection. $N = 20$ and results are plotted for 10^3 sets of n simulated random runs.

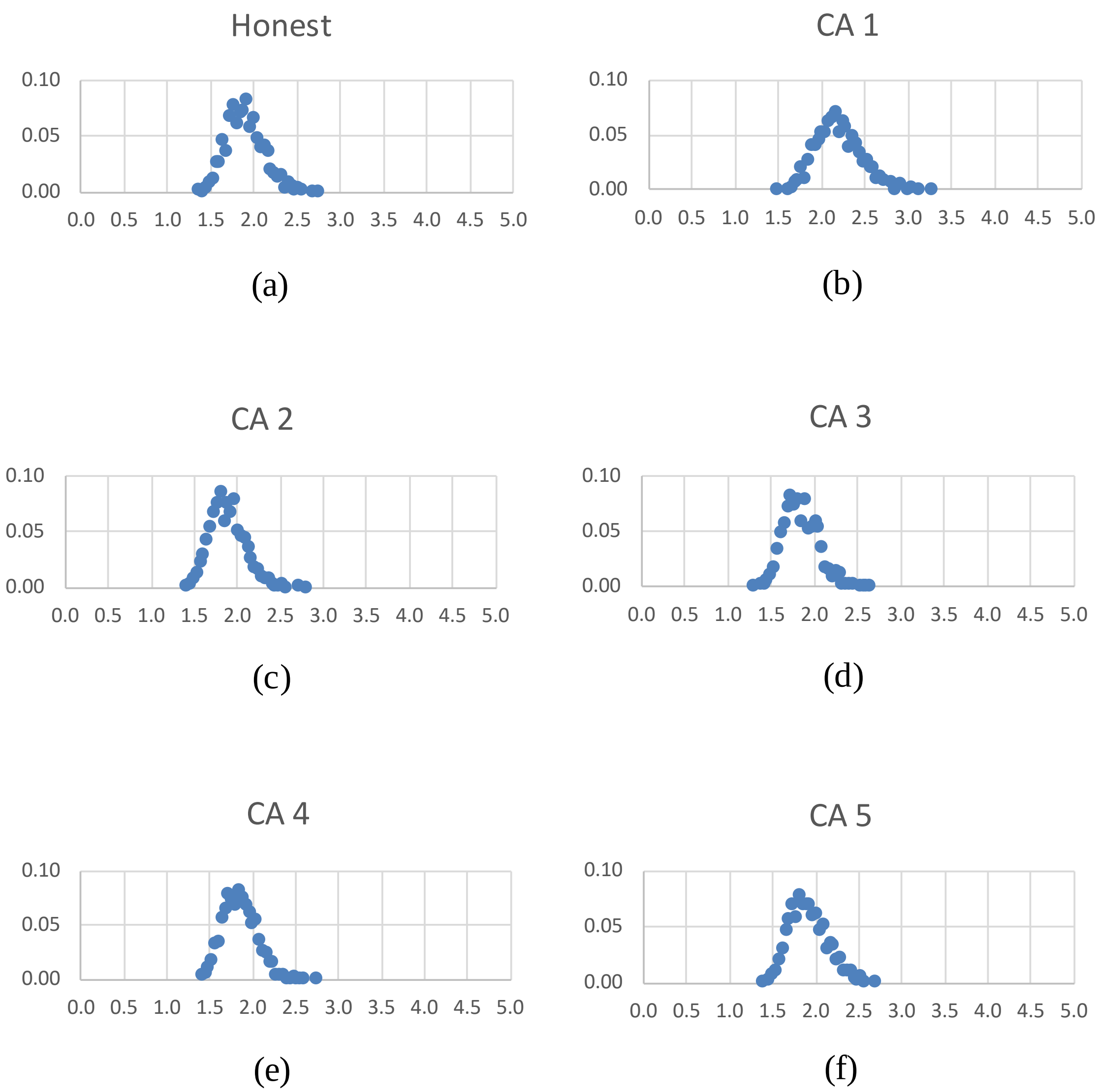


Figure 42. Distribution of the mean of $\boldsymbol{n} = \mathbf{100}$ samples of apparent rank selected for honest-only queues and CAs with ranks 1 through 5 seeking to evade selection. $N = 20$ and results are plotted for 10^3 sets of n simulated random runs.

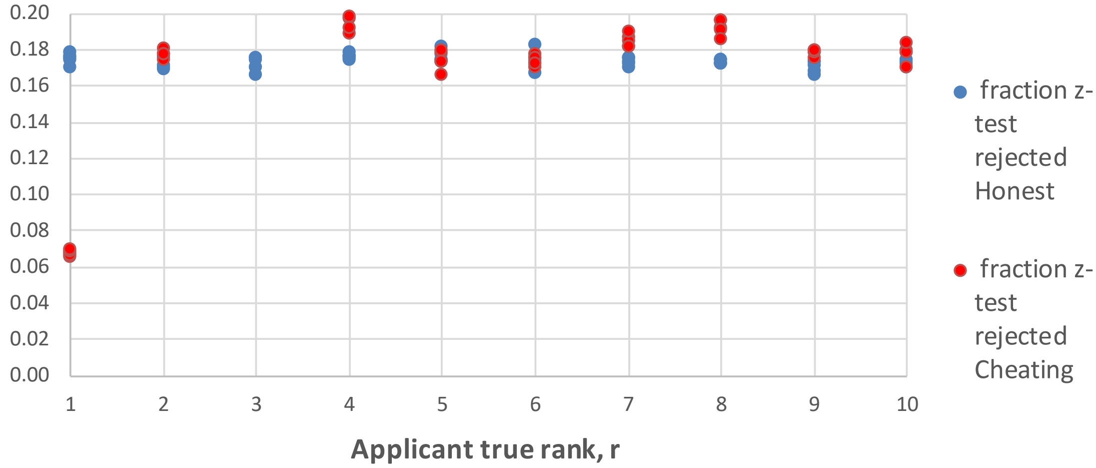


Figure 43a. Summary of reject rates for tests of hypothesis for honest-only queues and queues with CAs with ranks 1 through 10 seeking to evade selection. $N = 20$ applicants and z-test **sample size *n* = 30**. For each CA rank, points are plotted for five sets of tests, where each set obtained a reject rate based on 10^4 tests of hypothesis. An independent simulation of the honest-only queue was performed with each CA simulation.

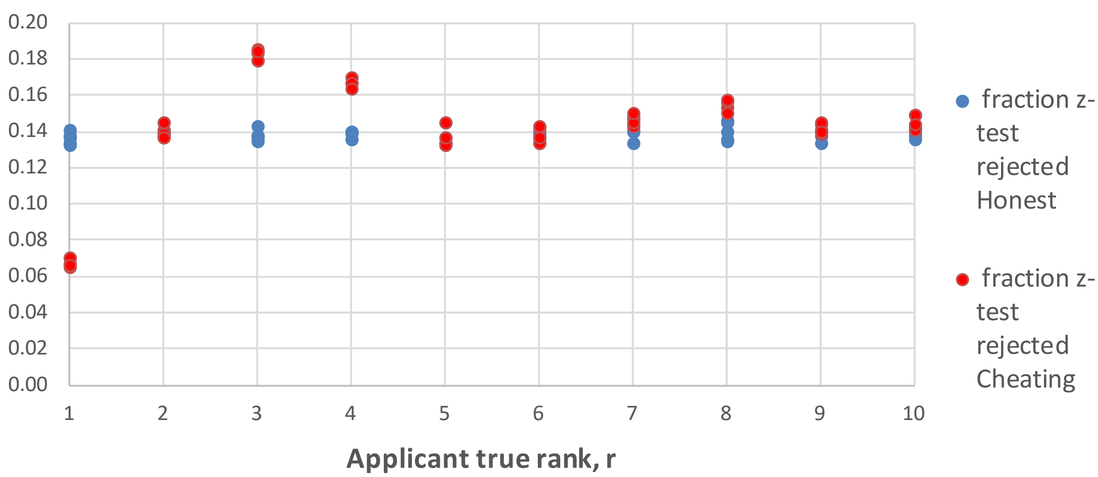


Figure 43b. Summary of reject rates for tests of hypothesis for honest-only queues and queues with CAs with ranks 1 through 10 seeking to evade selection. $N = 20$ applicants and z-test **sample size *n* = 50**. For each CA rank, points are plotted for five sets of tests, where each set obtained a reject rate based on 10^4 tests of hypothesis. An independent simulation of the honest-only queue was performed with each CA simulation.

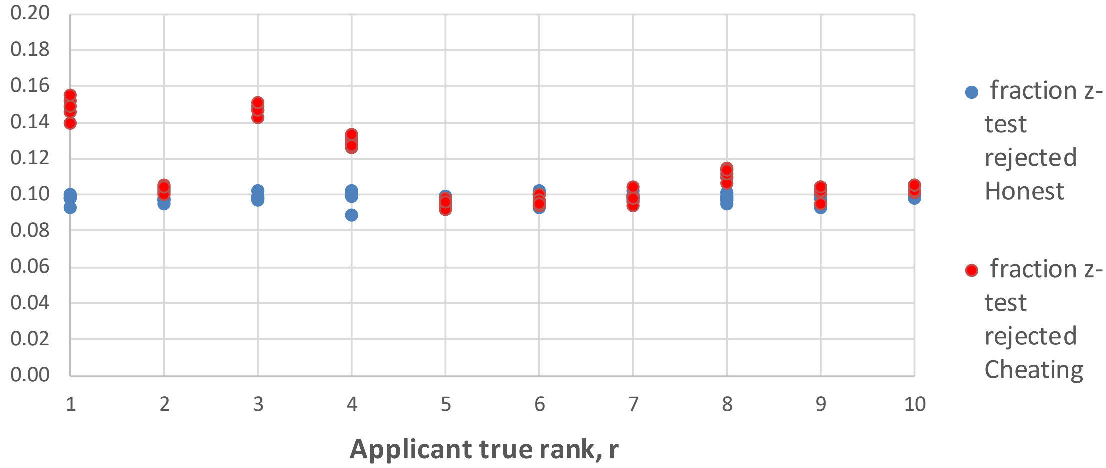
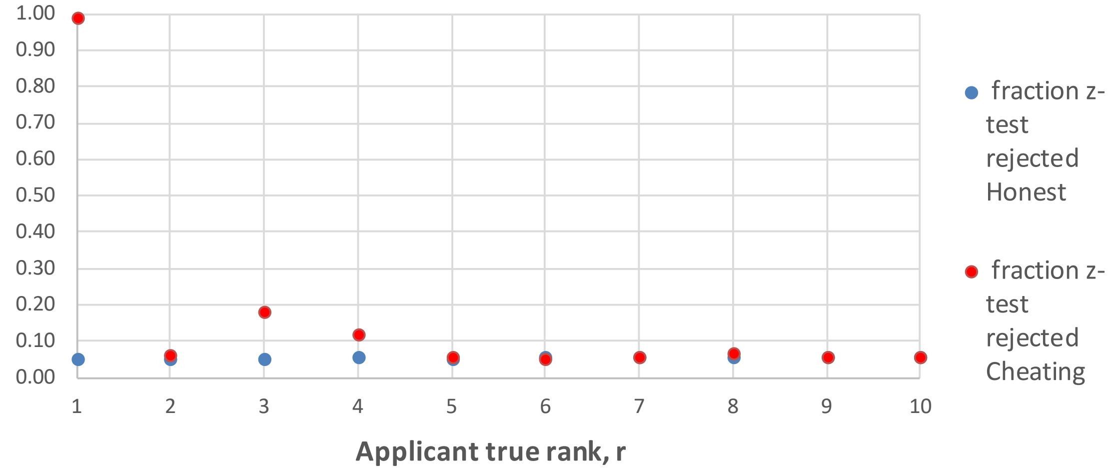


Figure 43c. Summary of reject rates for tests of hypothesis for honest-only queues and queues with CAs with ranks 1 through 10 seeking to evade selection. $N = 20$ applicants and z-test **sample size $n = 100$**. For each CA rank, points are plotted for five sets of tests, where each set obtained a reject rate based on 10^4 tests of hypothesis. An independent simulation of the honest-only queue was performed with each CA simulation.



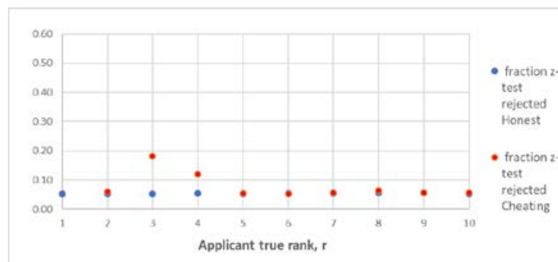


Figure 43d. Summary of reject rates for tests of hypothesis for honest-only queues and queues with CAs with ranks 1 through 10 seeking to evade selection. $N = 20$ applicants and z-test **sample size $n = 1{,}000$**. For each CA rank, points are plotted for one set of 10^4 tests of hypothesis. An independent simulation of the honest-only queue was performed with each CA simulation. Inset is scaled to show more detail for CAs 2 through 10.

XIII. Conclusion

The Classical Secretary Problem (CSP) has been studied almost exclusively from the point of view of the decision maker (DM) except, it appears, for a work by D.B. Glass (referred to in this paper as *Glass*) who addressed the problem from the applicant's side. Using the Classical Two-Phase Solution (C2PS) for the CSP, the last position in the queue provides the greatest probability of selection when averaged over all applicants, but, based on the Nothing But The Best (NBTB) payoff function, Glass showed that an applicant of rank 1, 2, or 3 (that is, one of the three best applicants) could gain an advantage over the last position in the queue by being placed immediately at the start of the observation-and-selection phase.

The C2PS for NBTB payoff requires many more applicants to be observed than the C2PS for Minimum Expected Relative Rank (MinERR) payoff which may provide more desirable results in practice. Cost-based models may favor shorter search times. Furthermore, several studies have indicated that human DMs may tend to under-search compared to mathematically optimal thresholds.

This work has generalized the results presented by D.B. Glass to apply to decision thresholds involving shorter observation-only phases than the optimal length for C2PS for NBTB payoff. In addition, the main result in *Glass* is generalized to show that there are advantageous positions beyond the immediate start of the observation-and-selection phase for up to the top three applicants.

For the C2PS, it was found that there exists a *critical position* for each rank such that the probability of selection for an applicant with that rank is at least as great as the probability of selection in the last position; the probability of selection is lower or even zero for all positions between the critical position and the last position. A cheating applicant (CA) can exploit knowledge of the critical position by maneuvering into one of those advantageous positions.

For a queue of applicants of length *N*, the critical position for a given rank grows linearly with *N* whereas the start of the observation-and-selection phase for MinERR payoff grows only as the square root of *N*. Therefore, even for small problem sizes there may be many advantageous positions for a CA when the DM uses the MinERR payoff function.

The notion of the critical position generalizes the main result in *Glass* in two directions. First, it determines the set of advantageous positions that may be exploited when start positions smaller than those required for NBTB payoff are used by the DM, which allows ranks other than the top three to benefit from cheating. Second, it extends the results outlined in *Glass* Theorem 4 to show there are advantageous positions for up to the top three ranks beyond the optimal NBTB start position $s = \lceil N/e \rceil + 1$.

The critical position can be calculated exactly from the expressions for the conditional probability of selection and can be done so with a tool as simple as a spreadsheet. The observation that, for a given rank, the ratio of the critical position to the problem size *N* rapidly approaches a limiting value even for small problem sizes leads to a simple method for estimating the critical position for any N from tabulated values. This is facilitated by the remarkable property of the CSP that the critical position is invariant with respect to the DM's choice of start position.

An applicant with knowledge of which positions are advantageous, but no influence over its position, might choose to conserve resources by opting out of the selection process.

While the CSP is normally framed in terms of applicants that have the goal of being selected, this framework can also be applied to the complementary problem where the CA is seeking to *evade* selection by choosing a position that affords a lower rather than higher probability of selection.

Most published work on the subject of the CSP has focused on the C2PS where the probability of selecting the best applicant reaches a theoretical lower bound of about 37% when the start position is chosen to optimize NBTB payoff or where the expected relative rank of the selected applicant grows as the square root of the number of applicants when the start position is chosen to optimize MinERR payoff. However, Chow et al. developed a solution that achieves an expected relative rank of the selected applicant that is less than 4 for a queue with any number of applicants. Although it is mentioned in numerous CSP-related publications, there has apparently been little analysis or application of Chow's solution for the CSP for MinERR payoff. This paper applies and analyzes Chow's solution through simulation and shows how a CA can choose or avoid advantageous positions to cheat that solution.

For any solution, the list of advantageous positions depends on the rank of a CA. Thus, the CA must know its own rank in the queue in order to select advantageous positions. This paper addresses the situation where the CA does not know its exact rank, but can only estimate its rank within lower and upper bounds, and this approach is applied in the context of Chow's solution.

In general, after a DM makes a selection, unless the last applicant is reached the only known quantity is not the true rank but rather the apparent rank of the applicant. This paper addresses the properties of the apparent ranks of selected applicants. A statistical test of hypothesis concerning the difference of two means was developed to distinguish between honest-only queues and queues with CAs for Chow's solution. Due to the heavy-tailed nature of the distribution of selected applicants, it was found that excessively large sample sizes are required to reduce the false alarm rate to the designed level of significance and so the test of hypothesis appears to be impractical. Worse, the probability of rejecting an honest-only queue as having a CA is often higher than the probability of detecting true cheating. Thus, it appears that the CA has a great advantage against the application of statistical methods of detection by the DM. The DM's best approach for the detection or countering of cheating behavior would seem to require direct observation of applicants' actions to influence their position in the queue to either be placed in advantageous positions when seeking selection or avoid those positions when seeking to evade selection.

An appendix exercises an example of a notional physical system that is sampled to provide an “applicant queue.” The various CSP solutions discussed in this paper are shown to work well in some cases even with drift of the statistical characteristics of the samples that slightly violate the assumption of randomness in the order of the applicants. Cheating is not addressed in the example.

Another appendix describes a solution by Bajnok and Semov for minimizing the expected relative rank of the selected applicant. This solution is considerably simpler to administer than Chow’s solution; although it does not perform as well as Chow’s solution it provides an expected relative rank that grows only logarithmically with the length of the applicant queue instead of as the square root as with the C2PS. Cheating with that solution is not addressed. This appendix also provides a summary of the performances of the various CSP solutions discussed in this paper from the viewpoint of the DM.

Chow’s solution requires a set of constants to be derived for a given length of the applicant queue. Even though they are easily obtained, for convenience an appendix is given that tabulates these values for queue lengths up to 100.

Advantageous positions for an applicant seeking to cheat Chow’s solution can be derived by simulation for a given threshold for improvement in the probability of selection. Lists of advantageous positions for queue lengths up to 100 have been precomputed and given in an appendix.

A fundamental assumption of the CSP is that previously observed applicants cannot be recalled. An appendix compares the performance of Chow’s solution for the CSP to the performance of some solutions for more general SP models that allow various degrees of recall; the observation phases for these more general models are terminated either at the average number of observations required by Chow’s solution or at the average number of observations that achieve the same expected relative rank as Chow’s solution. It was found that Chow’s solution performed almost identically to two models that allow recall where one model always selects the applicant with *exactly* apparent rank 2 and the other selects an applicant with an *expected* apparent rank of 2. Thus, unless the best applicant is guaranteed to be available to be recalled, Chow’s solution which lacks the ability to recall previously observed applicants is shown to be competitive with some solutions that allow recall.

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Appendix A: Example of Modeling a Sampled Physical System

This section demonstrates the application of three CSP solutions to a model of a physical system. The three CSP solutions applied to this model are the C2PS for MinERR payoff, the C2PS for NBTB payoff, and Chow's solution for MinERR payoff.

As cheating is not addressed, this material is relegated to an appendix.

Imagine that the physical system modeled here is a piece of machinery that an operator wishes to shut down. The critical component in this system is a motor that runs at highly variable speeds that are, from the viewpoint of the operator, apparently random. The operator is able to sample a signal that represents the motor speed at ten-second intervals and would prefer to turn off the motor at its lowest possible speed to minimize damage to the system. In this simplified model, it is assumed there is no significant lag between a motor speed measurement and the command to shut down when it occurs. There is a window of 1,000 seconds in which to make a decision, before the end of which deadline the system must be shut down regardless of motor speed.

Framed as a CSP, the operator is the DM and the series of signal samples represents the observations of the queue of "applicants." With the window length of 1,000 seconds and rate of one sample per ten seconds at t = 0, 10, 20, … 990 seconds, the applicant queue has length N = 100. Clearly, in a physical system such as this, the DM cannot "recall an applicant." Any given sample can be ranked relative to the signals observed up to that time. Sampling a highly variable system where the DM has no knowledge of any underlying structure or statistics of the system effectively creates a random ordering of the applicant queue from the point of view of the DM.

The model was designed to elicit knowledge of how the three CSP solutions mentioned above perform with constant characteristics from the beginning of the queue to the end as well as several cases deliberately intended to violate the CSP assumption that applicants arrive "in random order." This violation is accomplished by allowing slightly drifting characteristics over the time period of 1,000 seconds.

As this exploration did not involve cheating, it was an abbreviated study based on a small number of experiments.

The results of the experiments are summarized in Table 10. The conditions of the experiments are outlined and then the findings in the table are discussed.

The sampled signal from the hypothetical system was constructed as the superposition (sum) of three components consisting of sinusoidal signals. Five cases were considered: four with slight drift in characteristics over time and a fifth with no drift. Five random runs were simulated for each case. The various types of drift were introduced to explore how robust various CSP solutions are to slight violations of the assumption of random ordering of the applicant queue.

Figure 44a shows Signal 1. This component has a mean value or "d.c. offset" of 20 units. To introduce gentle drift in the characteristics, Signal 1 was a sine wave that had a half-period over 1,000 seconds and amplitude 5. The first four cases were generated by varying phase shifts: 0 radians to slowly drift from low to high and low again, $\pi/2$ radians to drift from high to low, π radians to drift from high to low and high again, and $3\pi/2$ radians to drift from low to high. In the fifth case, Signal 1 had zero amplitude to make phase shift not applicable ("N/A") so there was no drift in characteristics.

Signal 2 (shown in Figure 44b) had zero mean, amplitude 4, and 15 periods over 1,000 seconds. Signal 3 (shown in Figure 44c) had zero mean, amplitude 10, and 41 periods over 1,000 seconds. Signals 2 and 3 each had a randomly selected phase shift in the range 0 to 2π radians for each random run.[36]

Signal 1+2+3 represents the superposition of the three Signals 1, 2, and 3 for each case.

Signal 1+2+3 for the first case with drift (phase shift of 0 radians to slowly drift from low to high and low again) is shown in Figure 45a; the result of sampling this analog signal at ten-second intervals is shown in Figure 45b. Again, from the point of view of the DM, the values of the samples and thus the "applicants" apparently arrive randomly.

The sampled values are the true values of the applicants and, because "lower is better" in this situation, the relative ranks are assigned to the true values in ascending order. For the values represented in Figure 45b, the five smallest sampled values are 6.92, 9.58, 9.83, 10.58, and 11.47 which correspond to relative ranks 1, 2, 3, 4 and 5, respectively.[37]

Signal 1+2+3 for the second case with drift (phase shift of $\pi/2$ radians to drift from high to low) is shown in Figures 46a-46b. The third case (phase shift of π radians to drift from high to low and high again) is shown in Figures 47a-47b. The fourth case (phase shift of $3\pi/2$ radians to drift from low to high) is shown in Figures 48a-48b. The fifth case (no drift in characteristics) is shown in Figures 49a-49b.

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[36] The number of periods of Signals 2 and 3 over the 1,000 second sampling time (15 and 41 periods, respectively) were chosen to be relatively prime so that the periodicities of their superposition did not create duplicate signal values over the sampling time. This was done to ensure there was no violation of the CSP assumption that the relative ranks of the "applicants" (in this case, the sampled signal values) did not contain ties.

[37] Figures 44 and 45 show the results of the first random run for the first case of drift. The smallest sample value and thus the best ranked "applicant" occurred in the last position. This happened entirely by chance – the simulated random runs were generated and tabulated without regard for their outcomes.

Table 10 gives the results for the three CSP solutions for the four cases with gentle drift of the characteristics over time and a fifth case with no drift. For each case, the results of only five random runs are given just to get an indication of the performance of the CSP solutions because this section does not address the primary subject of cheating.

The first impression is that the ranks of the selected applicants are either very good or very bad, which suggests that the distribution of ranks is heavy-tailed. All three of the solutions had their best performance in the third case where Signal 1+2+3 was low in the middle.

It is tempting to calculate statistics and compare them to the theoretical performance of the three CSP solutions, but sets of five random runs are too small to allow this. A few general observations are clear, however.

The C2PS for NBTB payoff indeed provided the greatest number of "best" applicants by nearly a factor of two. The average ranks of the applicants it selected were much higher than the two solutions for MinERR payoff, however; Chow's solution yielded an average rank that was approximately half the average obtained using the C2PS for NBTB payoff. The C2PS for MinERR payoff performed almost as well as Chow's solution.

All three solutions performed well for the third case (low in the middle); the C2PS for NBTB payoff performed extraordinarily well. The worst performance for all three solutions was the fourth case (high at the end).

The most surprising result was that for the fifth case (flat – no drift in characteristics) the C2PS for NBTB payoff yielded a slightly better average relative rank than did the C2PS for MinERR payoff. Chow's solution was the far-and-away winner in this case.

These results indicate that the various CSP solutions can still perform well under certain violations of the assumption of randomness in the order of the applicant queue.

Case	Phase of sinusoidal offset	Trend of signal samples due to phase offset	random run	C2PS for NBTB: position selected	C2PS for NBTB: true rank of selected applicant	C2PS for MinERR: position selected	C2PS for MinERR: true rank of selected applicant	Chow: position selected	Chow: true rank of selected applicant
1	0 radians	high in the middle	1	100	1	20	2	80	3
			2	100	42	100	42	62	5
			3	96	1	96	1	76	6
			4	93	1	13	2	86	9
			5	98	1	13	5	78	4
2	π/2 radians	high at the beginning	1	53	5	13	17	33	10
			2	54	5	14	16	34	11
			3	43	12	28	14	28	14
			4	42	15	22	22	35	18
			5	37	9	12	32	37	9
	π radians	low in the middle	1	41	1	21	4	41	1
			2	44	1	24	4	44	1
			3	42	1	22	3	42	1
			4	49	1	29	3	29	3
			5	40	2	15	14	40	2
4	3π/2 radians	high at the end	1	100	92	16	1	96	22
			2	100	75	12	1	99	27
			3	100	92	100	92	100	92
			4	100	100	11	1	99	48
			5	100	94	100	94	96	24
5	N/A	flat	1	62	2	23	3	62	2
			2	38	3	26	4	38	3
			3	67	1	67	1	67	1
			4	100	62	100	62	64	3
			5	40	1	40	1	40	1

Table 10. Results of simulations of physical system. Five cases are shown - four with drift over time and one without drift. Five random runs are simulated for each of the three CSP solutions considered. For each solution, the position of the selected applicant and the rank of the selected applicant are given.

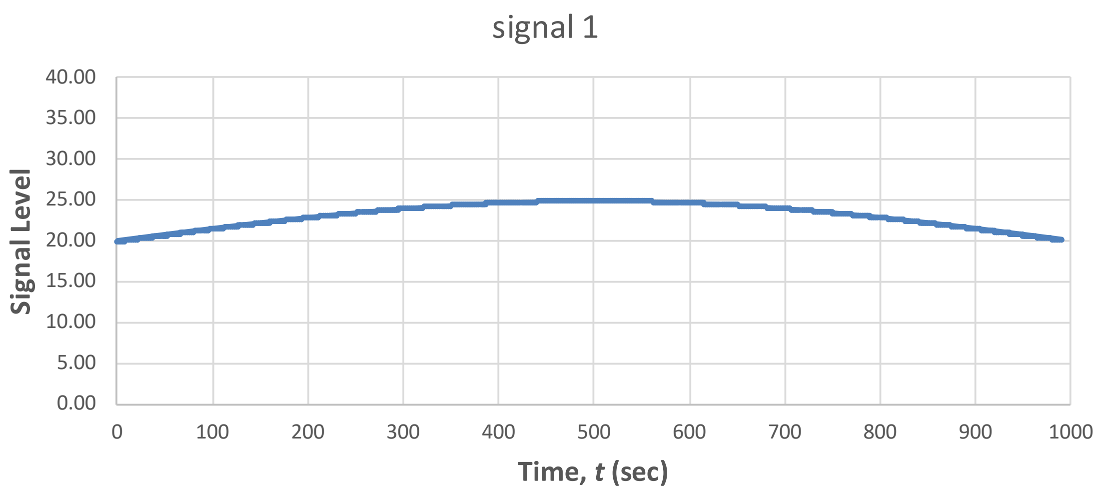

Figure 44a. Signal 1 provides constant “d.c. offset” of 20. It also provides drift over time for the first four out of five cases. The first case is shown in this figure with amplitude 5 and phase shift of 0 radians over half a period to introduce slow drift from low to high and low again.

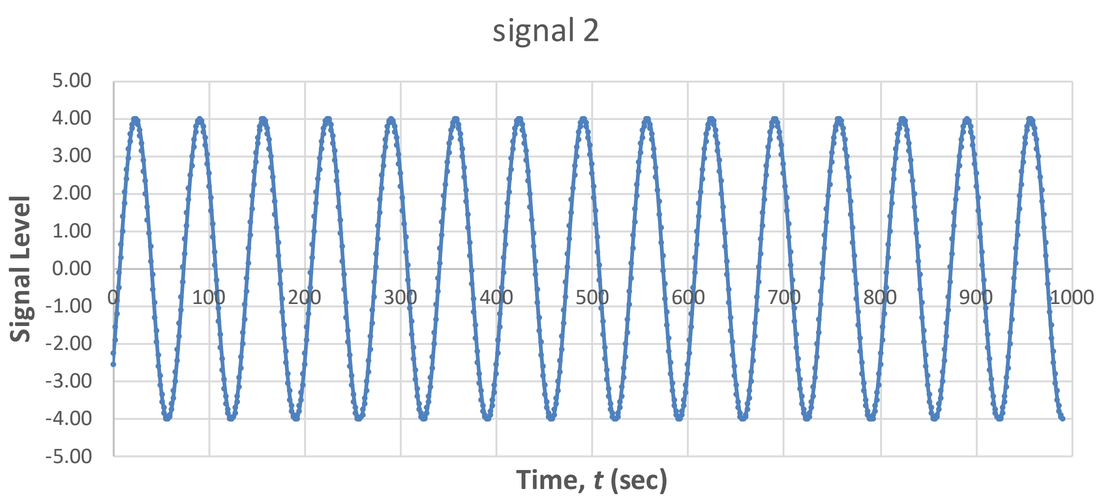

Figure 44b. Signal 2 has amplitude 4 and 15 periods over 1,000 seconds. Each random run uses a random phase shift in the range 0 to 2π radians.

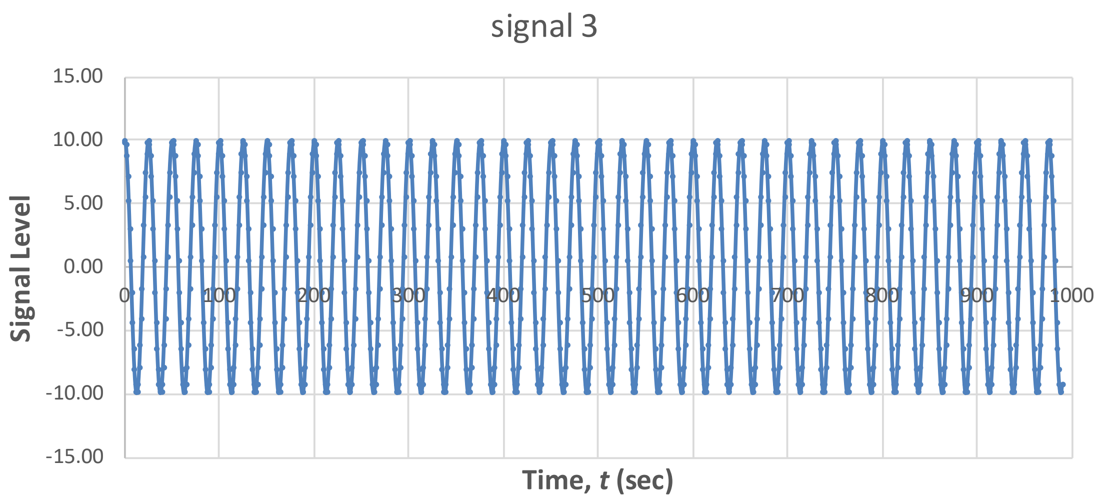

Figure 44c. Signal 3 has amplitude 4 and 41 periods over 1,000 seconds. Each random run uses a random phase shift in the range 0 to 2π radians. As noted in the text, the number of periods over the sampling time was chosen to be relatively prime with the number of periods of Signal 2.

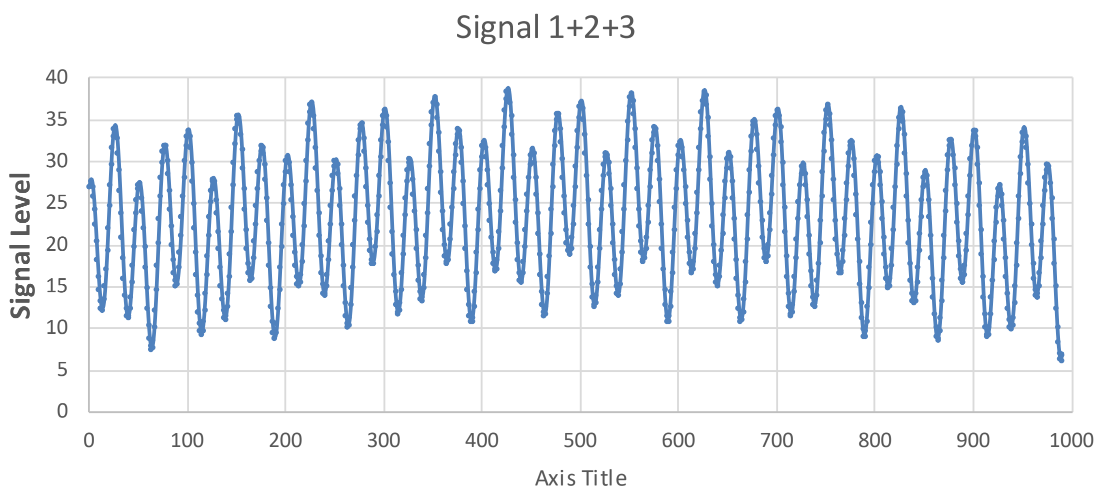

Figure 45a. Signal 1+2+3 is the sum of Signals 1, 2, and 3. This figure shows the first case (with drift) where Signal 1 has amplitude 5 and phase shift of 0 radians over half a period to introduce slow drift from low to high and low again. It is the superposition of the signals shown in Figures 44a, 44b, and 44c.

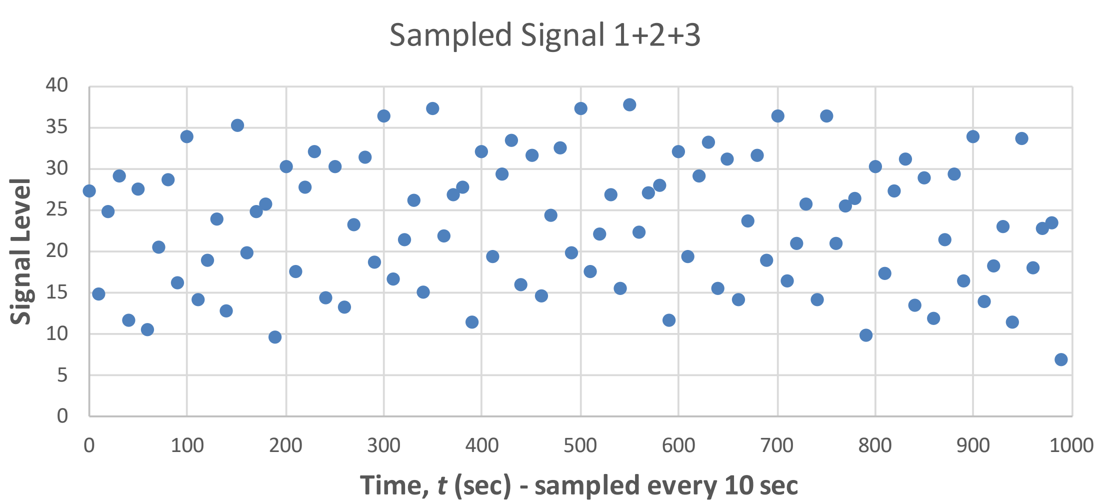

Figure 45b. Result of sampling at ten-second intervals the analog Signal 1+2+3 shown in Figure 45a.

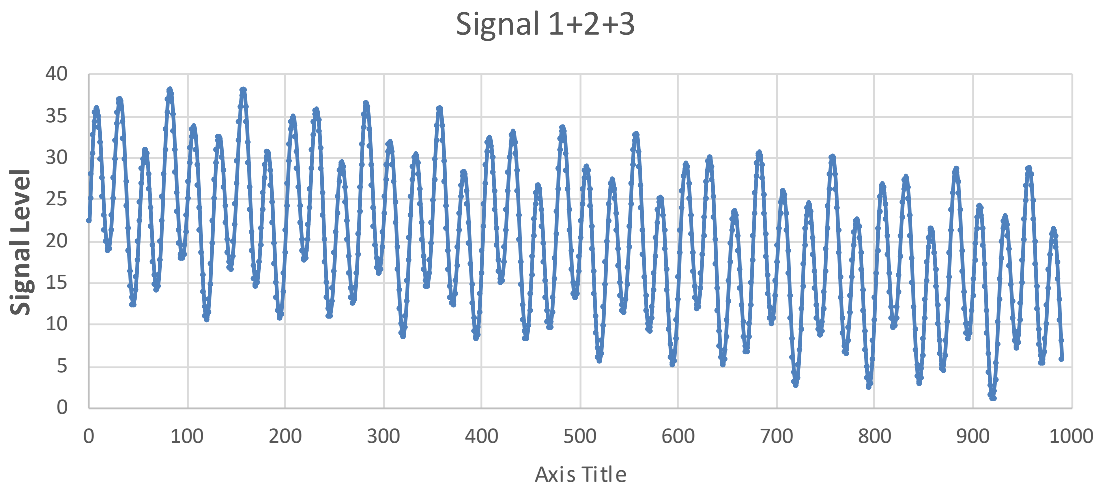

Figure 46a. Signal 1+2+3 for the second case (with drift) where Signal 1 has amplitude 5 and phase shift of $\pi/2$ radians over half a period to introduce slow drift from to drift from high to low.

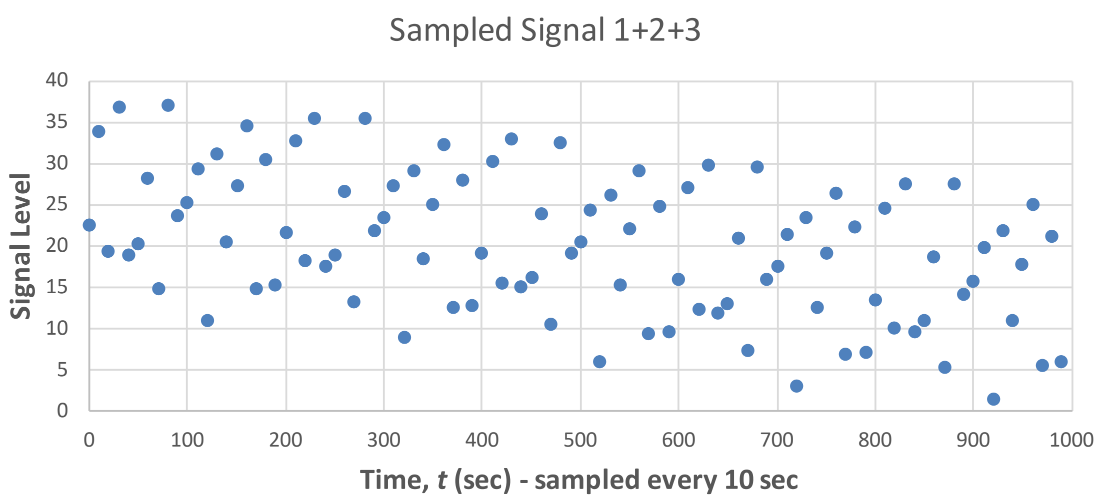

Figure 46b. Result of sampling at ten-second intervals the analog Signal 1+2+3 shown in Figure 46a.

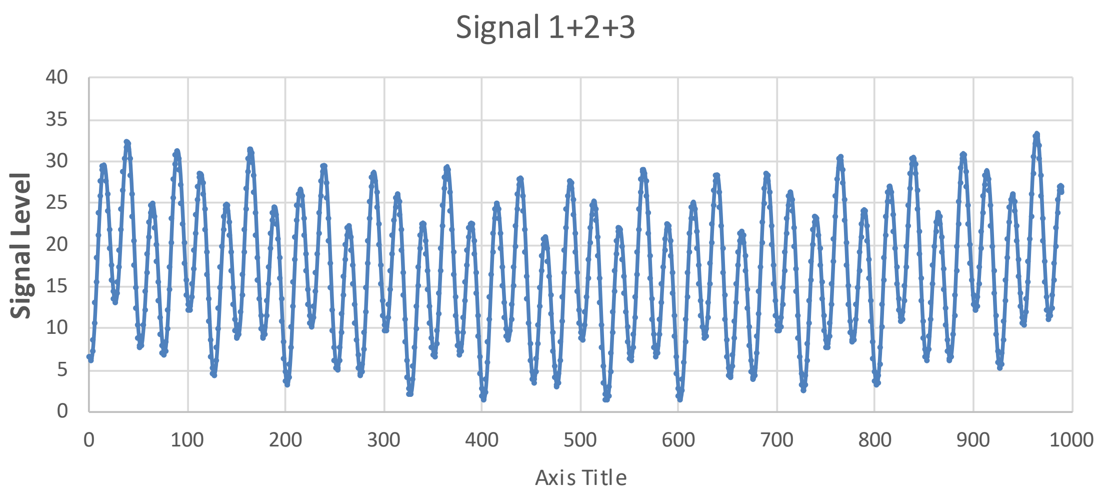

Figure 47a. Signal 1+2+3 for the third case (with drift) where Signal 1 has amplitude 5 and phase shift of π radians over half a period to drift from high to low and high again.

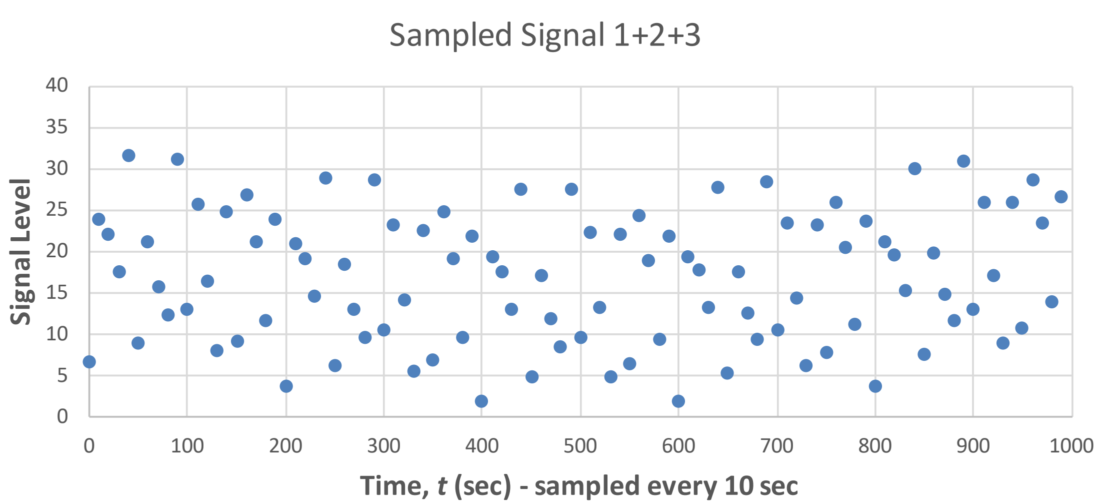

Figure 47b. Result of sampling at ten-second intervals the analog Signal 1+2+3 shown in Figure 47a.

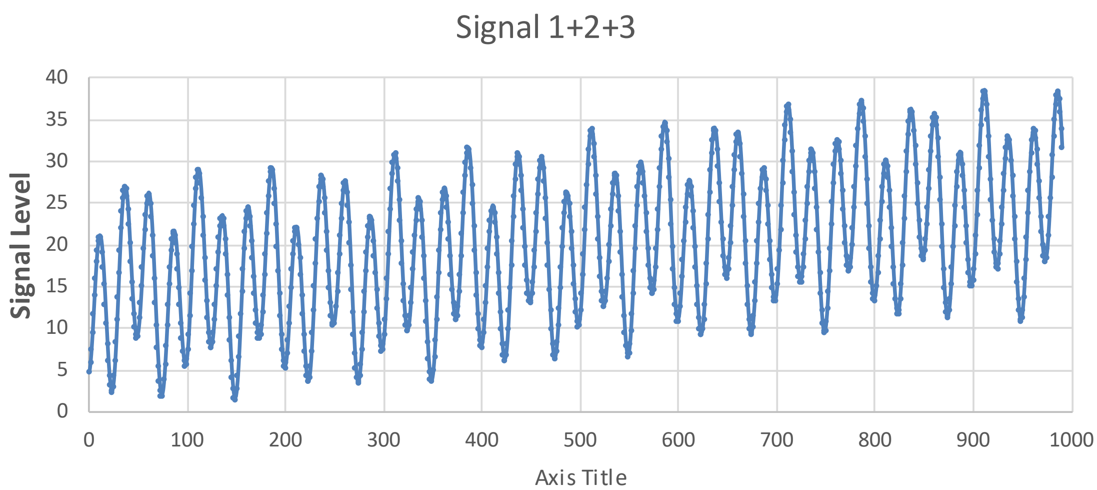

Figure 48a. Signal 1+2+3 for the fourth case (with drift) where Signal 1 has amplitude 5 and phase shift of $3\pi/2$ radians over half a period to drift from low to high.

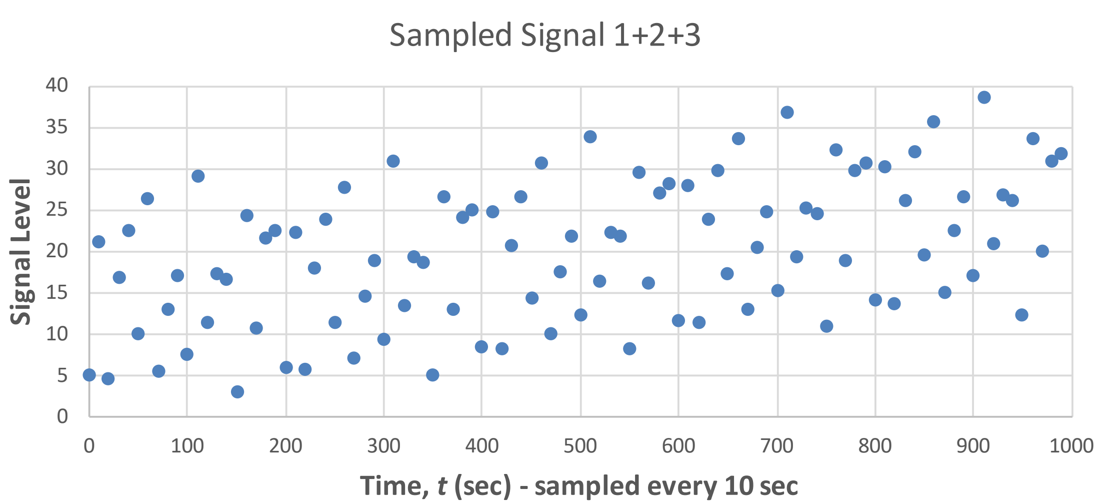

Figure 48b. Result of sampling at ten-second intervals the analog Signal 1+2+3 shown in Figure 48a.

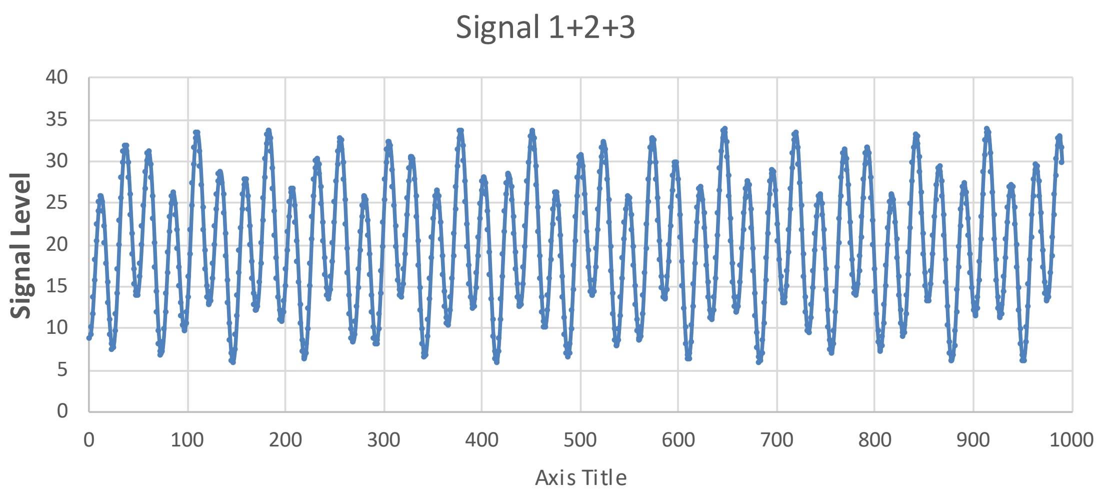

Figure 49a. Signal 1+2+3 for the fifth case (without drift) where Signal 1 has amplitude 0 and provides only a "d.c. offset" of 20.

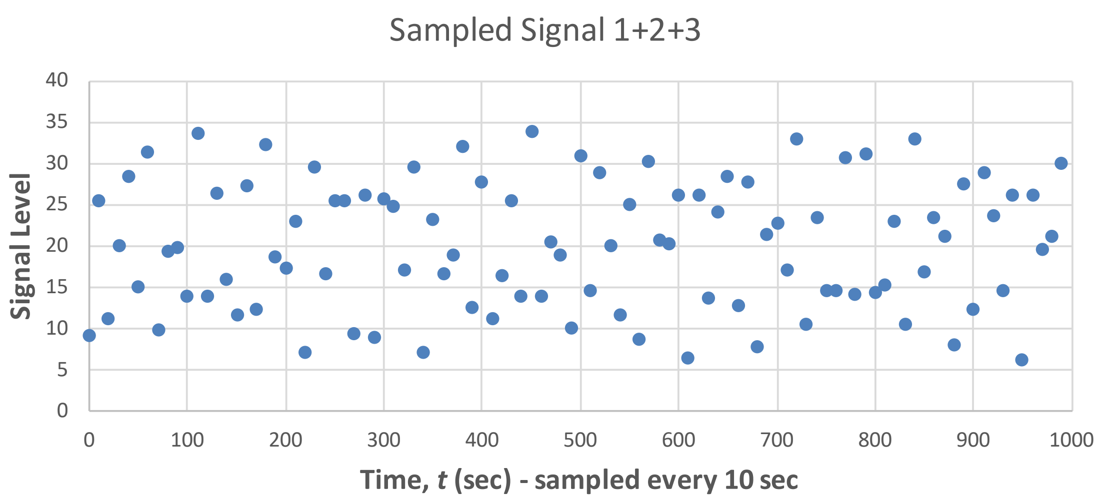

Figure 49b. Result of sampling at ten-second intervals the analog Signal 1+2+3 shown in Figure 49a.

Appendix B: Bajnok's Solution for the Classical Secretary Problem

This section describes a method by Bajnok and Semov [Bajnok 2015] that is a two-phase solution for the CSP for MinERR payoff. This solution may be of interest in some situations because it achieves it achieves a smaller expected rank (approximately $\frac{e}{2}\ln N$) than does the C2PS for MinERR payoff (approximately $\sqrt{N}$) and it is simpler in its application than Chow's solution for the CSP for MinERR payoff. While this solution does not provide the actual minimum expected relative rank as does Chow's, the procedure finds the thresholds that achieve the minimum possible with this approach.

As cheating is not addressed, this material is relegated to an appendix.

Since this section completes the descriptions of the CSP solutions addressed in this paper, it includes a summary of the performances of those solutions from the point of view of the DM.[38]

Bajnok's solution for MinERR payoff resembles the C2PS for NBTB payoff in that it has an observation-only phase of approximately length N/e followed by an observation-and-selection phase.[39] In the experiments described in this section, the start position for the second phase was chosen as $s = \lceil N/e \rceil$.

Bajnok's solution is described as follows. For a given queue length N, the DM chooses start position s in the queue as above. The DM chooses a threshold L (to be described shortly). Then two phases are executed:

- *Observation-only phase*: The DM only observes the relative ranks of the first $s-1$ applicants presented to the DM *without selecting any of them*. Let p_L denote the position of the applicant with the L^{th} smallest rank observed.

- *Observation-and-selection phase*: The DM continues to observe the applicants as they are presented but, starting with the applicant in position s, the DM *selects the first applicant observed to have lower rank* than that of the applicant in position p_L. If no candidate has been selected before position N is reached, then, per the rules of the CSP, the DM *must select the last candidate* regardless of its rank.

---

38 SPANS (described in Section VIII) is not included in the summary because it is not a CSP solution but instead addresses a modified version of the SP.

39 While Bajnok's solution is a two-phase solution for the CSP, this paper does not refer to it as a "C2PS" because it is not a "classical" solution.

Bajnok and Semov showed that, for any value of L, the *expected relative rank* of the selected applicant is:

$$\frac{N+1}{2}\left(\frac{L}{s}+\frac{\binom{N-L}{s-1-L}}{\binom{N}{s-1}}\right) \tag{27}$$

Note that when $L = 1$ this procedure is the same as the C2PS for NBTB payoff and expression (27) yields an expected relative rank of $N/(2e)$.

Bajnok and Semov showed that expression (27) is minimized when the DM chooses threshold L that is approximately $\ln N - 1$. In the experiments described in this paper, the value used was:

$$L = \text{round}\,(\ln N - 1) \tag{28}$$

This value of L was used because, for most values of N tested, the rounded value gave slightly better minimum expected relative rank than did the floor or ceiling. With L given by expression (28), expression (27) provides an *expected relative rank* for the selected applicant of about:

$$\frac{e}{2}\ln N \tag{29}$$

Examples of the C2PS were given in the Introduction and for Chow's solution in Section IX. Longer applicant queues must be used in this section to demonstrate Bajnok's solution that involved values of L greater than 1.

Example 5:

Suppose $N = 13$ applicants are presented to the DM in this relative rank order: 7, 5, 2, 9, 10, 11, 6, 13, 4, 12, 8, 3, 1.

The DM sets the start position at $s = \lceil N/e \rceil = \lceil 13/e \rceil = \lceil 4.78 \rceil = 5$. The DM sets threshold $L = \text{round}\,(\ln 13 - 1) = \text{round}\,(2.56 - 1) = 2$.

In the observation-only phase, the DM observes the first $s-1 = 4$ applicants. Because $L = 2$, the DM searches for the applicant with the second lowest apparent relative rank of those four applicants and finds it is the applicant with true rank 5 in position $p_L = p_2 = 2$ (thus, applicant #2). The DM continues into the observation-and-selection phase beginning with the applicant in position $s = 5$ and would select the first applicant with relative rank lower than that of applicant #2.

The DM observes applicants #5-#9; applicant #9 has lower relative rank than applicant #2 and is selected with true rank 4 or apparent rank 2.

(End of example)

Example 6:

Suppose N = 34 applicants are presented to the DM in this rank order: 8, 30, 25, 9, 4, 13, 24, 20, 7, 28, 1, 2, 33, 12, 21, 17, 23, 31, 26, 15, 11, 16, 34, 19, 27, 5, 29, 6, 22, 3, 18, 10, 32, 14

The DM sets the start position at $s = \lceil N/e \rceil = \lceil 34/e \rceil = \lceil 12.51 \rceil = 13$. The DM sets threshold L = round (ln 34 – 1) = round (3.53 – 1) = 3.

In the observation-only phase, the DM observes the first $s-1 = 12$ applicants. Because $L = 3$, the DM searches for the applicant with the third lowest relative rank of those 12 applicants and finds that it is the applicant with true rank 4 in position $p_L = p_3 = 5$ (thus, applicant #5). The DM continues into the observation-and-selection phase beginning with the applicant in position 13 and would select the first applicant with relative rank lower than that of applicant #5.

The DM observes applicants #13-#30; applicant #30 is found to have a lower relative rank than applicant #5 and is selected with true rank 3 as well as apparent rank 3.

(End of example)

A series of simulations were carried out for Bajnok's solution to gather statistics. The simulations were run with the "standard" parameters given in Section III.

The results are summarized in Table 11. Inspection of the table shows the expected relative rank obtained by simulation of Bajnok's solution closely matches the exact and theoretical averages predicted by Bajnok and Semov in expressions (27) and (29). Figure 50 plots the expected relative ranks resulting from simulation of four of the solutions considered in this paper. Note that the rank scale is logarithmic. The expected relative rank achieved by the various solutions is demonstrated to grow in the following ways:

- *Linearly with N* for the C2PS for NBTB payoff,
- *As the square root of N* for the C2PS for MinERR payoff,
- *Logarithmically with N* for Bajnok's solution for MinERR payoff,
- *Toward a constant less than* 4 for Chow's solution for MinERR payoff.

Figure 51 plots the average number of positions observed before making a selection using each of these four solutions. The longest search time is required by the C2PS for NBTB payoff. Next are Chow's solution and Bajnok's solution. The C2PS for MinERR payoff requires by far the fewest number of observations to make a selection out of these four solution.

While not shown in the tables, simulation results based on 10^6 random runs indicate that the logarithmic growth of the expected relative rank of Bajnok's solution always improves on the square root growth of the C2PS for MinERR payoff for $N \geq 14$.

Chow's solution for MinERR payoff achieves a far smaller expected relative rank than Bajnok's solution but would be difficult for a human DM to implement as a purely mental calculation. The cognitive effort demanded by Bajnok's solution, on the other hand, is almost as small as that required by the C2PS for NBTB or MinERR payoff; one would have only to keep track of a few top applicants rather than only the best. Thus, for reasonable queue lengths that a human DM would attempt to optimize manually or mentally, Bajnok's solution is quite practical.

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N	Start = $\lceil N/e \rceil$	Threshold L	Bajnok theo ave rank (exact)	Bajnok theo ave rank (approx) = $\frac{e}{2} \ln N$	ave true rank	Pr{sel true rank 1}	Pr{sel last pos}	ave pos observed
10	4	1	2.7500	3.1295	3.0198	0.3989	0.3331	6.99
20	8	2	3.6053	4.0716	3.7833	0.2401	0.1228	11.79
30	12	2	4.3966	4.6227	4.5484	0.2397	0.1354	18.21
40	15	3	4.7368	5.0137	4.8461	0.1571	0.0397	20.26
50	19	3	4.9882	5.3170	5.1086	0.1590	0.0449	25.97
60	23	3	5.2631	5.5648	5.3550	0.1604	0.0475	31.64
70	26	3	5.5091	5.7743	5.6008	0.1569	0.0444	36.04
80	30	3	5.7790	5.9558	5.8549	0.1588	0.0462	41.72
90	34	3	6.0605	6.1159	6.1238	0.1601	0.0479	47.40
100	37	4	6.1565	6.2591	6.2074	0.1149	0.0156	47.49
200	74	4	7.0881	7.2012	7.1788	0.1159	0.0171	96.12
300	111	5	7.6943	7.7522	7.7253	0.0909	0.0065	137.01
400	148	5	8.0402	8.1432	7.9676	0.0916	0.0060	182.99
500	184	5	8.3788	8.4465	8.2805	0.0896	0.0059	227.71
600	221	5	8.7187	8.6943	8.6248	0.0893	0.0061	273.54
700	258	6	8.9653	8.9038	8.8803	0.0733	0.0021	308.19
800	295	6	9.0894	9.0853	9.2017	0.0721	0.0027	352.57
900	332	6	9.2144	9.2454	9.2055	0.0733	0.0023	396.31
1000	368	6	9.3425	9.3886	9.2589	0.0737	0.0022	439.75
2000	736	7	10.3999	10.3307	9.8126	0.0645	0.0004	856.57
3000	1104	7	10.8568	10.8818	11.0728	0.0625	0.0011	1284.19
4000	1472	7	11.3138	11.2728	11.7572	0.0637	0.0012	1716.11
5000	1840	8	11.6987	11.5761	11.2894	0.0513	0.0001	2101.59
6000	2208	8	11.8669	11.8239	12.0582	0.0564	0.0004	2521.97
7000	2576	8	12.0351	12.0334	11.4451	0.0530	0.0003	2945.06
8000	2944	8	12.2032	12.2149	12.6027	0.0516	0.0004	3367.51
9000	3311	8	12.3711	12.3750	12.0993	0.0515	0.0004	3780.60
10000	3679	8	12.5389	12.5182	13.8464	0.0525	0.0005	4207.08

Table 11. Results for Bajnok's solution for the CSP. Number of simulated random runs for each value of N is specified in Section III.

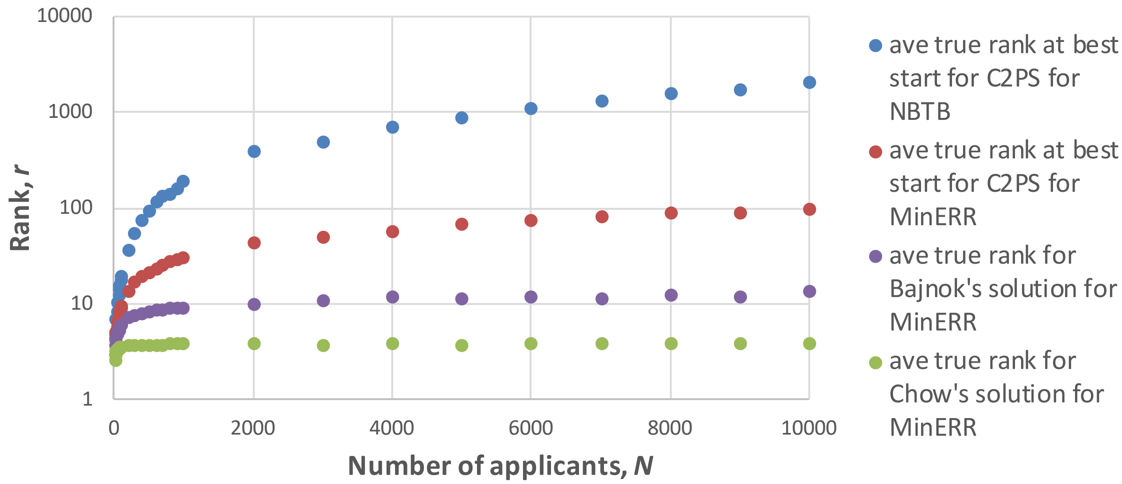

Figure 50. Expected relative ranks achieved by four solutions for the CSP. Values are taken from Tables 7, 8, 9, and 11. Note that rank axis is logarithmic.

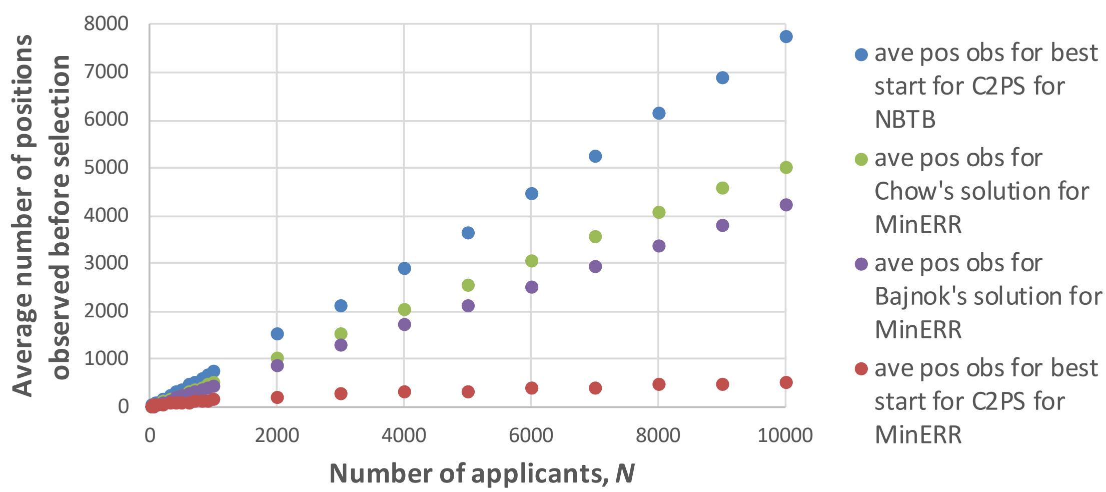

Figure 51. Average number of positions observed before selection is made by four solutions for the CSP. Values are taken from Tables 7, 8, 9, and 11.

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Appendix C: Comparing Chow's Solution to Solutions That Allow Recall

The statement of the CSP, as given in the Introduction, does not allow the DM to recall rejected applicants. Even when recall is not in fact impossible, it may be that the CSP is still a useful model because recall is expensive, time-consuming, or socially embarrassing.

This section investigates how a CSP solution performs relative to some variations of the SP that allow more liberal selection procedures where the DM has full or partial ability to recall previously observed applicants. The only payoff considered here is the expected relative rank of the selected applicant. Also, only Chow's solution for MinERR payoff is discussed because the other CSP solutions cannot compete with the almost unbelievably small limiting value of 3.8695 achieved by Chow, Moriguti, Robbins, and Samuels [Chow 1964].

As cheating is not addressed, this material is relegated to an appendix.

The first major work on the use of recall appears to be that of Yang [Yang 1974]. His approach was a three-phase algorithm in which the first two phases operated similarly to the C2PS but it included a third phase that allowed recall of applicants observed prior to the start position of the second phase. Several variations on that approach by other authors were summarized in a survey by Freeman [Freeman 1983].

An assumption in the CSP is that, at the time of observation, when a DM selects an applicant that applicant is always available for selection. This section uses Yang's term "**solicit**" [Yang 1974] to refer the DM attempting to select a previous applicant but, depending on the degree of recall permitted by the specific solution, that applicant may not be available for selection. As noted above, it is assumed that the DM always begins by soliciting the applicant with the lowest apparent rank first, and in the event that applicant is unavailable solicits the applicant with the next lowest rank, and so on.

The recall solutions considered in this paper are simplified from those described in Yang's work and that of subsequent researchers so as to provide a better apples-to-apples comparison to the operation of Chow's solution. This approach is a two-phase solution that consists of an *observation-only phase* and a *solicitation phase*. During the observation-only phase, the DM observes applicants in a queue of length N up to a **termination position**, t ($1 \leq t \leq N$), and then, without further observation, solicits the best available applicant observed in the queue up to position t; the termination position t equals the number of applicants observed by the DM.[40] The solution **stops** at the end of the solicitation phase.

[40] Note that the symbol t used in this section is not to be confused with the t statistic used in a statistical test of hypothesis.

For each recall solution and given value of N, two measures are used for comparison to Chow's solution based on alternative termination rules:

- **Termination Rule 1:** Determine the *termination position t* where the expected relative rank falls below that achieved by Chow's solution.

- **Termination Rule 2:** Determine the *expected relative rank* achieved when the termination position t is set at the average number of observations required by Chow's solution.

Three recall solutions are used in this study to explore how the expected relative rank yielded by Chow's solution compares to that achieved by various degrees of complete and partial recall.

The first degree of recall is the "gold standard" of performance: the DM is able to select the applicant with apparent rank 1 regardless of where that applicant was in the queue in the positions up to the termination position t. That is, solicitation of the best applicant is always successful. This model is referred to as **First-Best Available (1BA)**.

It is worth noting the work by Moriguti [Moriguti 1992] who showed that, for given values of N and t, the *expected apparent rank* of the selected applicant under the 1BA model is:[41]

$$\frac{N+1}{t+1} \tag{30}$$

The second degree of recall is where the best applicant observed up to position t is never available but the *second-best* applicant (that is, the applicant with apparent rank 2) is always available. That is, solicitation of the first-best applicant is never successful but solicitation of the second-best is always successful. This model is referred to as **Second-Best Available (2BA)**.

The third degree of recall is where previously observed applicants are available with a constant probability, set here at 0.5. That is, after having observed the applicant at position t, the DM, beginning with the first-best applicant and working upward in apparent rank, solicits applicants in turn with a 50% success rate and continues until successful. To guarantee that the procedure will stop the rule is used that, if reached, the applicant with the highest apparent rank observed (that is, the t^{th} highest rank) is assumed to be available and is selected. Clearly, the probability of this rule being invoked is only 2^{-t}. This model is referred to as **Constant Probability of Availability (CPA)**.

[41] Moriguti's result, given here as expression (30), was used to verify the results of the simulations for the 1BA model. It would be interesting to know if simple closed-form solutions could also be obtained for the 2BA and CPA models. In any case, adding the simulations for these three models to the C language code that implemented Chow's solution required only a half-dozen or so lines each of executable code.

In the simplest case for each of these three models, the DM observes all N of the applicants (thus, the termination position t is N) and then attempts to make a selection. Obviously, for 1BA the expected relative rank achieved is 1 and for 2BA (when $N > 1$) it is 2. The CPA model yields an expected relative rank that is very nearly 2 when the probability of availability is set to 0.5.[42]

The situation becomes more interesting when the DM does not observe all N of the applicants in the queue but either terminates at a position that would equal or improve on Chow's solution's expected relative rank (Termination Rule 1) or terminates the observation phase at the average position where Chow's solution ends with a selection (Termination Rule 2).

A series of simulations were carried out to gather statistics to compare the statistics for these three degrees of recall to Chow's solution. The simulations were run with the "standard" parameters given in Section III.

The results are summarized in Table 12. The first column is the number of applicants in a queue, N. The next three data columns show the *number of positions observed* that are required to achieve an expected relative rank less than or equal to that obtained using Chow's solution (Termination Rule 1). The last three data columns in the table show the *expected relative rank* obtained when DM terminates the observation phase at the average number of positions required by Chow's solution (Termination Rule 2). For brevity, Table 12 does not show the data for Chow's solution but those data are given in Table 6.

These values are interpreted as follows using $N = 100$ as an example.

Consider first the position columns given in Table 12 and plotted in Figure 52 (Termination Rule 1). From Table 6, the average true rank selected using Chow's solution (resulting from simulation) is 3.6034 and (from Table 6) the average number of positions observed using Chow's solution before an applicant is selected is 53.35. For each of the recall models, the termination position is given where the expected relative rank would be less than or equal to that obtained using Chow's solution.

Using the 1BA model (where the applicant observed to have apparent rank 1 is always available), Table 12 shows that, on average, Chow's solution can be matched by observing 28 applicants before terminating and soliciting the best observed applicant; this number is considerably less than the average of 53.35 applicants observed using Chow's solution before it makes a selection.

A surprising result is that the 2BA model (which always selects the applicant with *exactly* apparent rank 2) requires 56 applicants to be observed and the CPA model (which selects applicants with an *average* apparent rank 2 when the probability of availability is set to

[42] For the termination position $t = N$, the 2BA and CPA models both effectively yield an expected relative rank of 2, but the distributions of the ranks selected are quite different. For 2BA and $N \geq 2$, the selected applicant always has exactly rank 2. For CPA, the selected applicants may have any rank but their average rank is very close to 2.

0.5) requires 55 applicants to be observed. These values are so strikingly close to the average number of positions required by Chow's solution that they are nearly indistinguishable in Figure 52.

Consider next the average ranks given in Table 12 and which are plotted in Figure 53 (Termination Rule 2). From Table 6, the average number of positions observed using Chow's solution before an applicant is selected is 53.35. For each of the recall models, the DM chooses the termination position $t = \lceil 53.34 \rceil = 54$.

Using the 1BA model, Table 12 shows that the expected relative rank of the selected applicant is 1.8326 which is much better than the value of 3.6034 obtained using Chow's solution. Once again, however, the expected relative ranks of 3.6701 and 3.6725 for 2BA and CPA, respectively, are so close to the value resulting from Chow's solution that they are nearly indistinguishable in Figure 53.

These results indicate that, under these models, when either the number of observations or the expected relative rank are held to those achievable by Chow's solution, the results under the "no recall" condition of the CSP may be no worse than where recall is allowed unless the best observed candidate is always available when solicited.

N	First-Best Available (1BA): pos observed to achieve Chow ave rank	Second-Best Available (2BA): pos observed to achieve Chow ave rank	Constant Probability of Availability (CPA) of 0.5: pos observed to achieve Chow ave rank	First-Best Available (1BA): ave rank at Chow ave pos	Second-Best Available (2BA): ave rank at Chow ave pos	Constant Probability of Availability (CPA) of 0.5: ave rank at Chow ave pos
10	4	8	8	1.3749	2.7499	2.7268
20	6	13	13	1.7499	3.5013	3.5016
30	9	19	19	1.7234	3.4451	3.4467
40	12	24	24	1.7088	3.4172	3.4172
50	14	29	29	1.8200	3.6393	3.6375
60	17	35	35	1.7930	3.5881	3.5891
70	20	40	40	1.8182	3.6370	3.6374
80	22	45	45	1.8393	3.6805	3.6778
90	25	50	50	1.8556	3.7128	3.7107
100	28	56	55	1.8326	3.6701	3.6725
200	54	107	107	1.9001	3.7929	3.8188
300	79	158	159	1.9318	3.8635	3.8470
400	105	211	211	1.9344	3.8689	3.8814
500	132	264	263	1.9504	3.8990	3.9004
600	158	316	317	1.9586	3.9077	3.9002
700	184	367	368	1.9551	3.9089	3.9156
800	208	418	419	1.9555	3.9068	3.9175
900	235	474	472	1.9537	3.9108	3.8932
1000	260	520	519	1.9501	3.9069	3.9009
2000	514	1033	1022	1.9373	3.8958	3.8342
3000	788	1550	1536	1.9862	3.9502	3.9401
4000	1034	2065	2059	1.9916	3.9749	3.9628
5000	1308	2573	2519	1.9840	3.9708	3.9829
6000	1549	3090	3048	1.9844	3.9641	3.9424
7000	1822	3626	3578	1.9744	3.9575	3.9520
8000	2016	4089	4036	1.9762	3.9374	3.9725
9000	2308	4614	4551	1.9777	3.9490	3.8923
10000	2578	5107	5030	1.9630	3.9361	3.9353

Table 12. Comparing the performance of three models for unlimited recall to Chow's solution for MinERR. First three data columns show average number of positions observed to achieve the expected relative rank obtained using Chow's solution (Termination Rule 1). Last three data columns show expected relative rank obtained when DM stops at average number of positions required by Chow's solution (Termination Rule 2). Values for Chow's solution are given in Table 6. Number of simulated random runs for each value of *N* is specified in Section III.

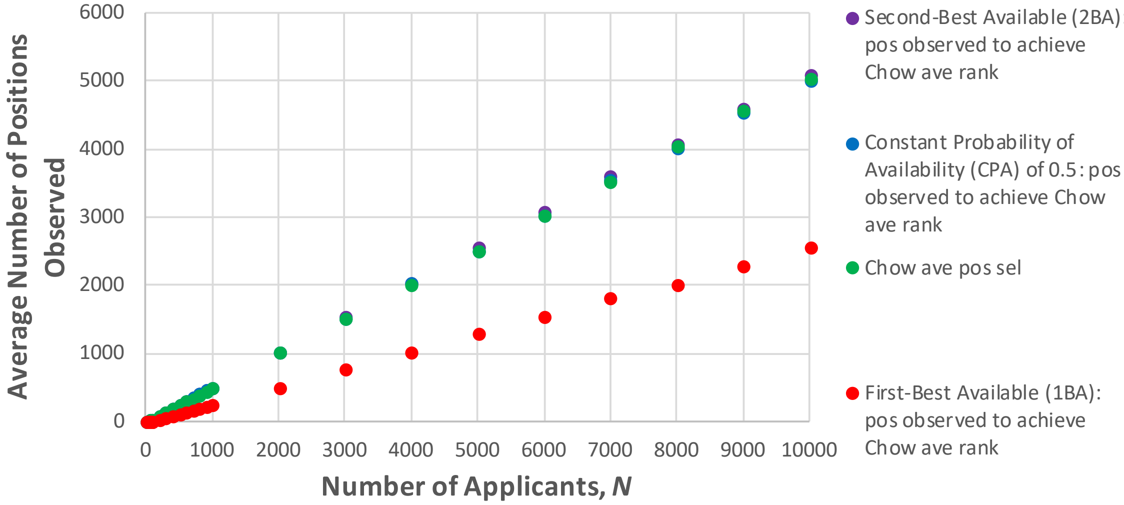

Figure 52. Plot of columns in Table 12 showing average number of positions observed to achieve the expected relative rank obtained using Chow's solution (Termination Rule 1) with the inclusion of the number of positions required by Chow's solution. Note that all except 1BA essentially coincide.

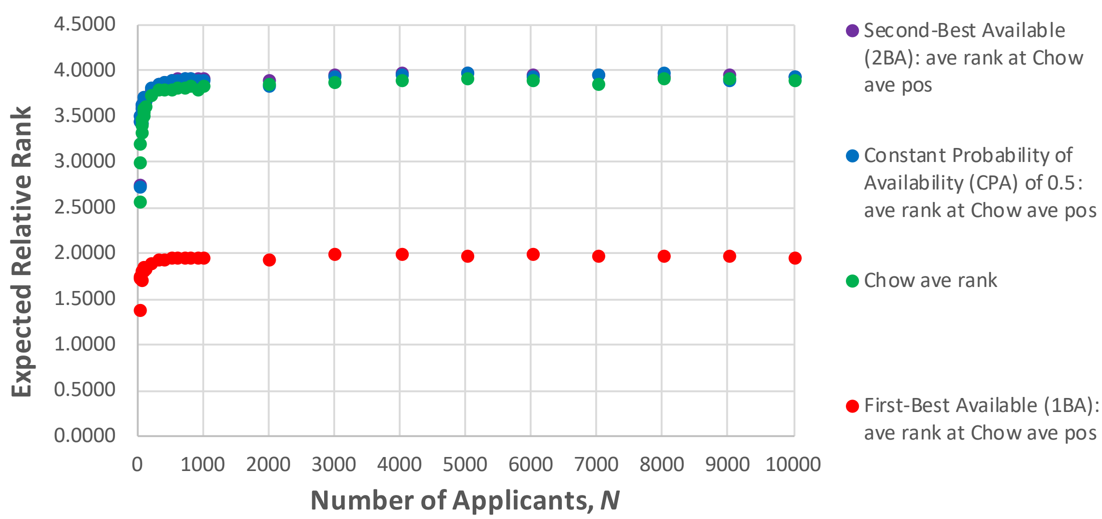

Figure 53. Plot of columns in Table 12 showing expected relative rank obtained when DM stops at average number of positions required by Chow's solution (Termination Rule 2) with the inclusion of the expected relative rank achieved by Chow's solution. Note that all except 1BA essentially coincide.

Appendix D: Constants for Chow's Solution

This appendix lists the s_p constants used by Chow's solution for MinERR payoff for values of *N* from 3 to 100. The table has over 100 columns of data so it is printed in size 1 font in a format that can be copied and pasted into a plain text file and opened using a spreadsheet program as Comma Separated Values (CSV).

No code or spreadsheet macros are contained in these data.

Each line gives the value of *N* and the value of Chow's c_0 constant which is the expected relative rank achieved by Chow's solution for that value of *N*. Those data are then followed by the *N* individual s_p constants that correspond to positions $p = 1, 2, \ldots N$ in the queue.

A sample of the first few lines of the data set below is given here in readable font:

```
   ,    ,       ,      , pos p:,   1,   2,   3,   4,   5,   6,   7,   8,   9,  10
 N:,   3, c[0]:, 1.667,  s[p]:,   0,   1,   3
 N:,   4, c[0]:, 1.875,  s[p]:,   0,   1,   2,   4
 N:,   5, c[0]:, 2.050,  s[p]:,   0,   1,   1,   2,   5
 N:,   6, c[0]:, 2.217,  s[p]:,   0,   0,   1,   2,   3,   6
 N:,   7, c[0]:, 2.276,  s[p]:,   0,   0,   1,   1,   2,   3,   7
 N:,   8, c[0]:, 2.400,  s[p]:,   0,   0,   1,   1,   2,   2,   4,   8
 N:,   9, c[0]:, 2.496,  s[p]:,   0,   0,   0,   1,   1,   2,   3,   4,   9
 N:,  10, c[0]:, 2.558,  s[p]:,   0,   0,   0,   1,   1,   2,   2,   3,   5,  10
```

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Appendix E: Advantageous Positions for Chow's Solution

This appendix lists the advantageous positions for Chow's solution for values of *N* from 3 to 100 as described in Section IX. The table lists the advantageous positions for the thresholds $T = 25\%$, 50%, and 75% resulting from 10^6 simulated random runs. The table is printed in size 1 font in a format that can be copied and pasted into a plain text file and opened as Comma Separated Values (CSV) using a spreadsheet program. The lines should be sorted and any extraneous lines of text deleted, such as the delimiters between sections for the three values of *T*, page numbers, etc.

No code or spreadsheet macros are contained in these data.

Each line gives the value of the threshold *T*, number of applicants *N*, and a rank in the range 1 through *N*. There are *N* such lines for each value of *T* and *N*.

A sample of a few lines of the data set below is given here in readable font:

```
T:, 0.50, N:,  10, rank:,   1, adv pos:,   4,   5,   6
T:, 0.50, N:,  10, rank:,   2, adv pos:,   4,   5,   6,   7
T:, 0.50, N:,  10, rank:,   3, adv pos:,   4,   6,   7,   8
T:, 0.50, N:,  10, rank:,   4, adv pos:,   4,   6,   8,   9
T:, 0.50, N:,  10, rank:,   5, adv pos:,   4,   6,   9
T:, 0.50, N:,  10, rank:,   6, adv pos:,   9,  10
T:, 0.50, N:,  10, rank:,   7, adv pos:,  10
T:, 0.50, N:,  10, rank:,   8, adv pos:,  10
T:, 0.50, N:,  10, rank:,   9, adv pos:,  10
T:, 0.50, N:,  10, rank:,  10, adv pos:,  10
T:, 0.50, N:,  11, rank:,   1, adv pos:,   4,   5,   6,   7
T:, 0.50, N:,  11, rank:,   2, adv pos:,   4,   5,   7,   8
T:, 0.50, N:,  11, rank:,   3, adv pos:,   4,   5,   7,   9
T:, 0.50, N:,  11, rank:,   4, adv pos:,   4,   7,  10
T:, 0.50, N:,  11, rank:,   5, adv pos:,   4,  10
T:, 0.50, N:,  11, rank:,   6, adv pos:,   4,  10,  11
T:, 0.50, N:,  11, rank:,   7, adv pos:,  11
T:, 0.50, N:,  11, rank:,   8, adv pos:,  11
T:, 0.50, N:,  11, rank:,   9, adv pos:,  11
T:, 0.50, N:,  11, rank:,  10, adv pos:,  11
T:, 0.50, N:,  11, rank:,  11, adv pos:,  11
T:, 0.50, N:,  12, rank:,   1, adv pos:,   4,   5,   6
T:, 0.50, N:,  12, rank:,   2, adv pos:,   4,   5,   7
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End copy text for T = 50% section

Begin copy text for T = 75% section:

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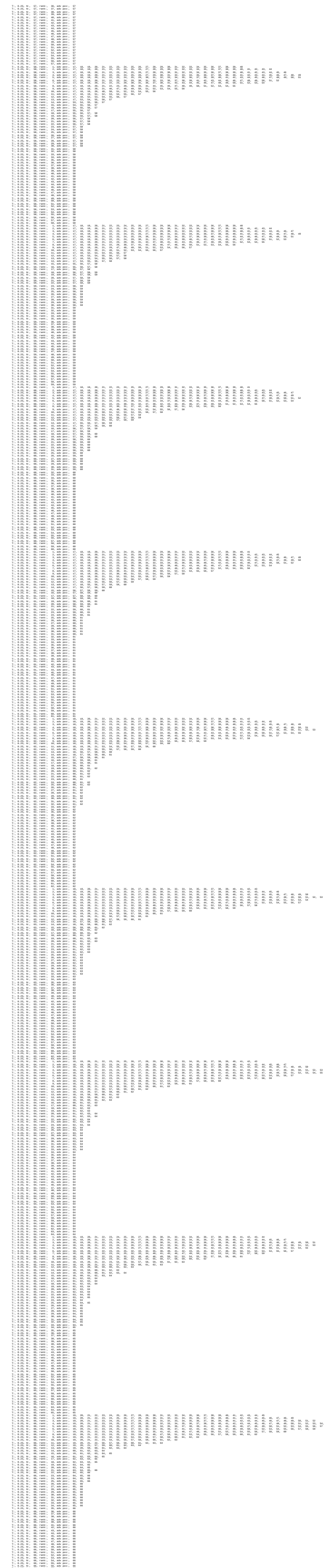

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