(U) Theory of Radial Indications

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(U) Abstract: Computed Tomography is usually not an option when inspecting manufactured parts in a production environment. This is largely due to time constraints and allocation of resources. X-Radiographic Inspection can be enhanced by capturing multiple angular radiographs of the item. In this paper we derive a formula for the radius of rotation of a relevant indication based on the starting and ending position of the indication.

(U) Research Innovation and Objective(s): To derive a mathematical relationship between the distance between two arbitrary points, the rotation angle and the radius of rotation.

(U) Impacts on Warfighter Mission: Locating the position of relevant indications relative to the surface of the shell body allows for quality product to get to the warfighter.

(U) Keywords: Non Destructive Testing (NDT), Radius of Rotation, Relevant Indication, Radiographs

1. (U) Introduction

(U) Radiographic testing is a non-destructive testing technique which allows for the detection of subsurface flaws. It is applicable to most materials and can reveal fabrication and underlying assembly errors. [1] By way of oversampling, computed tomography gives the radiographer important information that is otherwise not available with a single radiograph. Due to budget and time constraints, computed tomography is usually not a viable option in a production setting. Sometimes, two or more radiographs can be obtained. In this setting, an object is radiographed at two angles, zero and some angle θ. If one finds a relevant indication that moves under rotation, theoretically its position within the shell body can be calculated. The goal of this paper is to identify where the relevant indication is located relative to the center of rotation.

2. (U) Theory

2.1 (U) Setup

(U) Consider a radiographic setup diagrammed in Figure 1 below. We have an x ray source, some cylindrical object placed in the beam path for nondestructive testing and a detector. Note that the geometry shown in figure 1 would yield the following radiograph shown in figure 2. The axis of rotation is depicted with the straight vertical line with the relevant indication shown as a filled in circle. Figure 3 depicts a top down view of our cylindrical object. Referring to figure 3, assume the standard Cartesian coordinate system placed at the center of the circle. Let us assume there is a relevant indication at point P_1 located a distance r from the center of a cylindrical object. We then rotate our object counter clockwise so that the relevant indication has moved to a point P_2. Referring to the above discussion, distances labelled a, d in the figure are measurable known quantities obtainable from the radiographs. We derive an equation for r via the steps below.

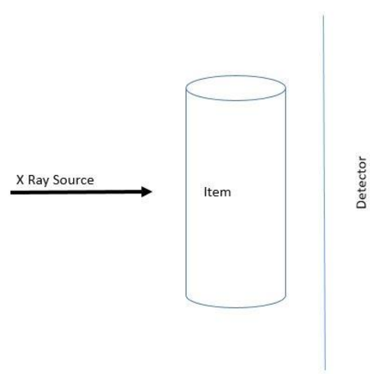


Figure 1. Radiographic Setup

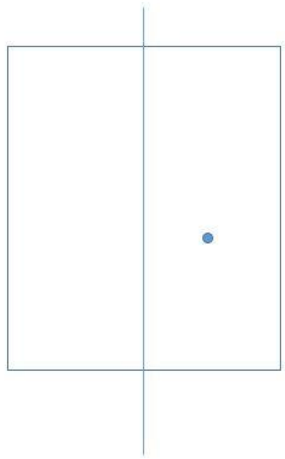


Figure 2. Not drawn to scale. A radiograph of the scenario described in section 2.1.

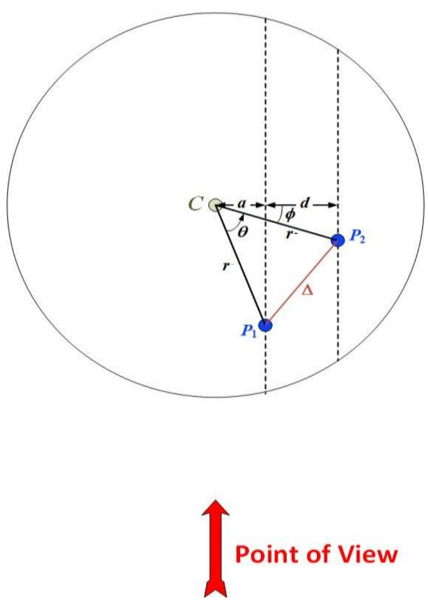


Figure 3. Geometry where points lie in the fourth quadrant with clockwise rotation

2.1.1 (U) Derivation

(U) Referring to figure 3, a is the distance from the center to the point closest to the center of rotation, and d is the distance between the two points under consideration. θ is the angle of rotation and ϕ is the angle the rotated point makes with the horizontal. We can write an equation for a using trigonometry.

$$a = r\cos(\theta + \phi) \qquad (1)$$

We expand this expression for a, and substitute $\sin\varphi$ for $\sqrt{1 - \cos^2\phi}$

$$a = r\cos(\theta + \phi) = r(\cos\theta\cos\phi - \sin\theta\sin\phi)$$

$$= rcos\theta\cos\phi - r\sin\theta\sqrt{1 - \cos^2\phi}$$

It is also evident from the figure that

$$a + d = r\cos\phi \qquad (2)$$

so that

$$\frac{a + d}{r} = \cos\phi$$

Therefore,

$$a = (a + d)\cos\theta - r\sin\theta\sqrt{1 - \left(\frac{a + d}{r}\right)^2}$$

$$= (a + d)\cos\theta - \sin\theta\sqrt{r^2 - (a + d)^2}$$

Solving for r^2 we get,

$$r^2 = \left\{\frac{a - (a + d)\cos\theta}{\sin\theta}\right\}^2 + (a + d)^2$$

$$\boldsymbol{r} = \frac{\sqrt{\boldsymbol{2a(a + d)(1 - \cos\theta) + d^2}}}{\boldsymbol{\sin\theta}} \qquad (3)$$

Let us generalize figure 3 to the next four cases outlined below. The four cases illustrated in figures 4a-4d are characterized by the fact that whatever quadrant the point is in before the rotation, it ends up in the same quadrant after rotation. This restricts us to rotation angles strictly less than 90 degrees.

Consider the case in figure 4a. This is where the initial point is at a small angle with the horizontal, and it's rotated clockwise. But the derivation doesn't actually distinguish between which point is which, or the direction of rotation, so you'd expect to get the same answer. Equations 1 and 2 are applicable to figures 4a-4d and so we can expect the previous derivation to follow culminating in equation 3.

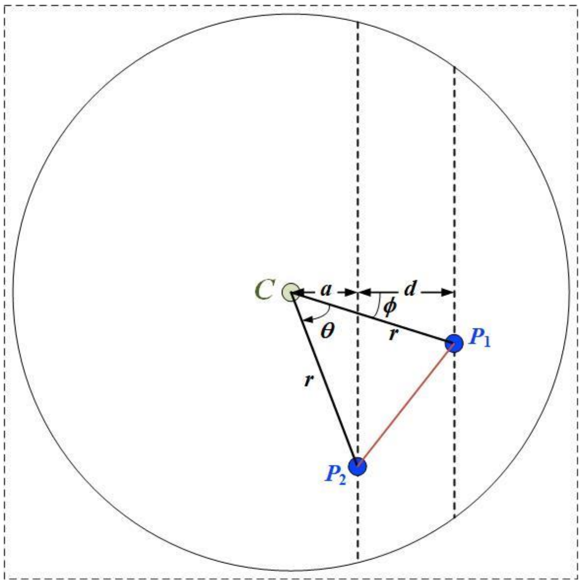
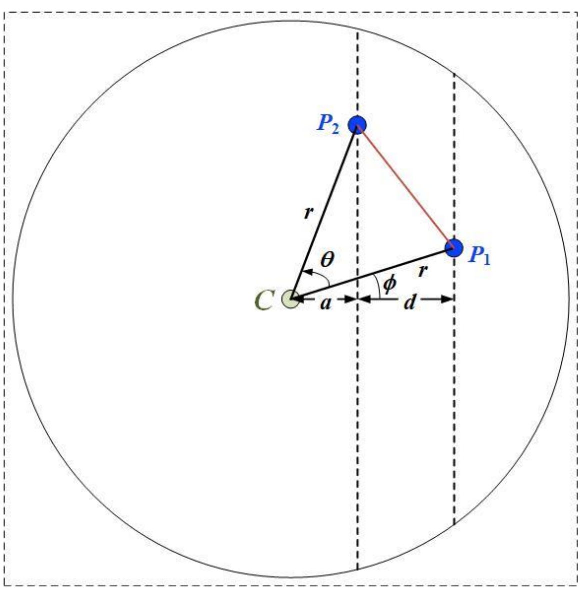


Figure 4a. Geometry where points lie in the fourth quadrant with clockwise rotation

Figure 4b. Geometry where points lie in the first quadrant with clockwise rotation



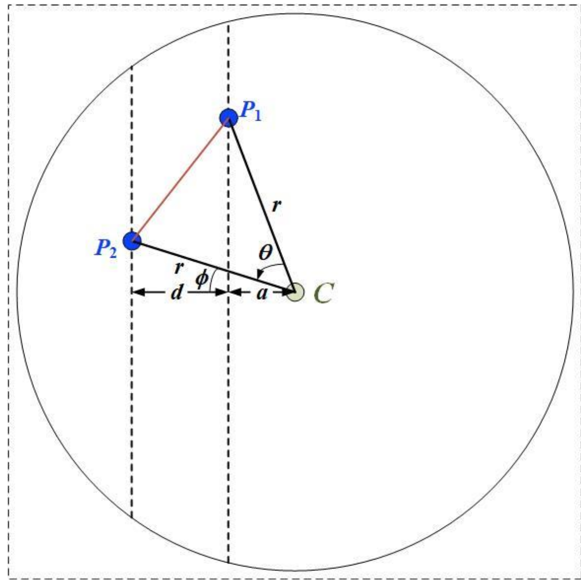


Figure 4c. Geometry where points lie in the second quadrant with clockwise rotation

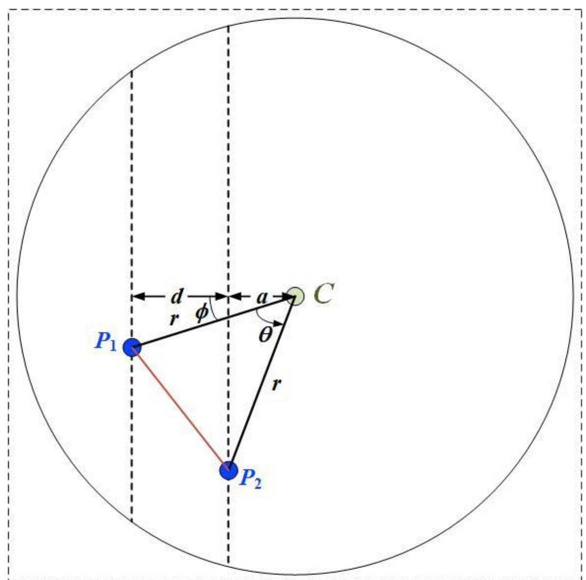


Figure 4d. Geometry where points lie in the third quadrant with clockwise rotation

We now consider a different geometry. Consider the cases diagrammed in figure 5a-5b. Here, the point moves to a different quadrant, but both points lie either to the left or to the right of the y-axis.

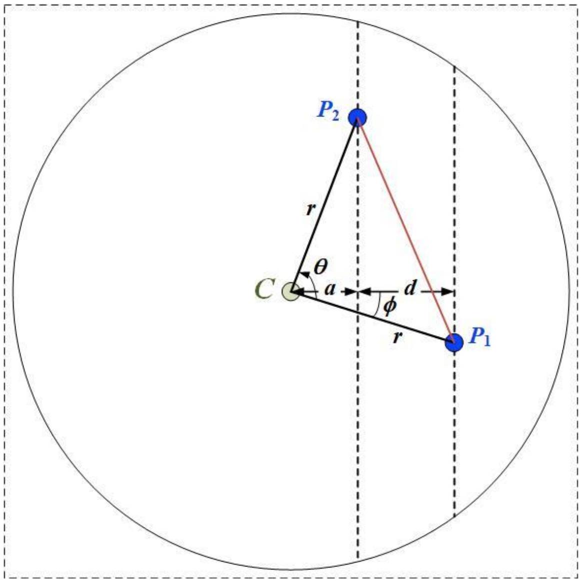


Figure 5a. Point initially in fourth quadrant ends up in first quadrant after rotation.

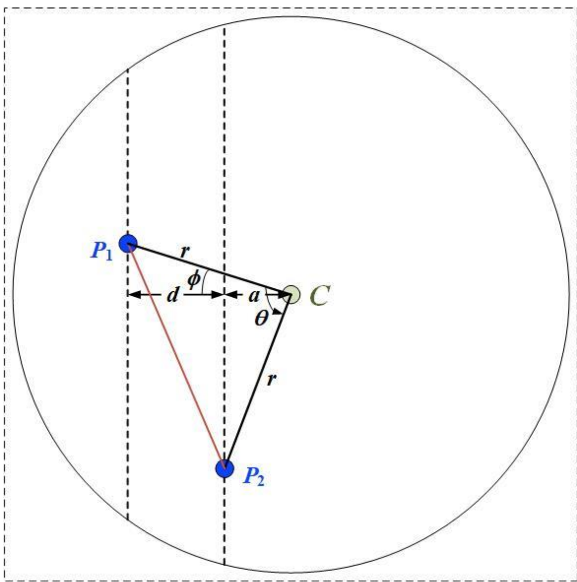


Figure 5b. Point initially in fourth quadrant ends up in first quadrant after rotation.

In both figures 5a and 5b we derive an expression for r in a similar manner.

Referring to figures 5a-5b

$$a = r\cos(\theta - \phi) = r\cos\theta\cos\phi + r\sin\theta\sqrt{1 - \cos^2\phi}$$

$$a + d = r\cos(\phi)$$

Combining these two equations yields,

$$a = (a + d)\cos\theta + r\sin\theta\sqrt{1 - \left(\frac{a + d}{r}\right)^2}$$

After simplifying and rearranging we obtain,

$$r^2 - (a + d)^2 = \left[\frac{a - (a + d)\cos\theta}{\sin\theta}\right]^2$$

This is identical to the equation obtained previously and so the rest of the derivation will be identical and we'll again obtain equation (3).

We now derive an expression for *r* for the last two cases. Consider figures 6a-6b shown below.

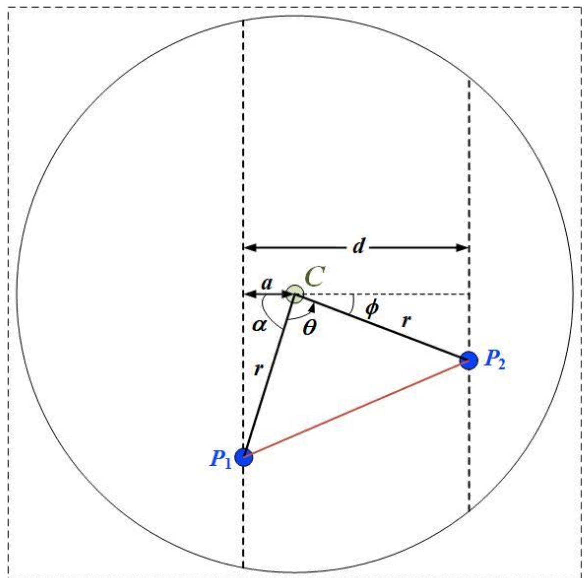


Figure 6a. Point initially in third quadrant ends up in fourth quadrant after rotation.

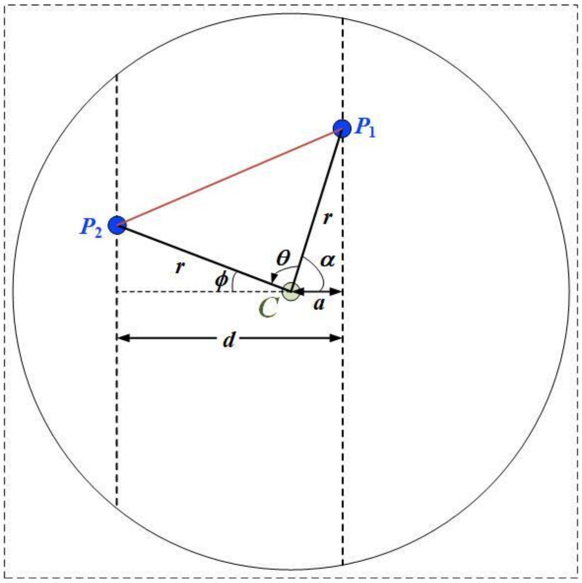


Figure 6b. Point initially in first quadrant ends up in second quadrant after rotation.

As before, a is the distance from the center to the point closest to the center of rotation, and d is the distance between the two points under consideration. θ is the angle of rotation and φ is the angle the rotated point makes with the horizontal. We also introduce the angle α, the angle the initial point makes with the horizontal.

$$\alpha = 180 - \theta - \phi$$

$$a = r\cos\alpha$$

$$d - a = r\cos\phi$$

Notice the difference between these equations to what we had before. Using these equations and simplifying we would obtain this equation for r,

$$a = r\cos(180 - \theta - \phi)$$

$$r = \frac{\sqrt{2a(a-d)(1-\cos\theta) + d^2}}{\sin\theta}$$

It is important to note that this answer makes sense in the context of how we defined a and d. That is, in all the previous cases one travels from the center of rotation a distance a, until we reach the closest point. Then one continues travelling an additional distance d in the same direction to get to the next point. However, in pictures 6a and 6b, one must travel a distance d, in the opposite direction to get to the next point. So in this sense, d is assigned a negative value.

3. (U) Results & Discussion

(U) Using equation (3) we can demonstrate the applicability of our formula to 155mm shells. Consider an example where the following parameters were measured.

$$a = 1.6\ inches$$

$$d = 0.15\ inches$$

$$\theta = 30\ degrees$$

Equation 3 yields a radius of rotation of 1.76 inches or 44.7 mm.

Intuitively, one might expect that as the distance between the points increases, the radius of rotation also increases. This is confirmed in figure 7 below. We fixed the distance a and the angle θ to the values displayed above. The x-axis in figure 7 represents increasing the distance d, in increments of .01 inches. The y axis is the radius of rotation in units of inches.

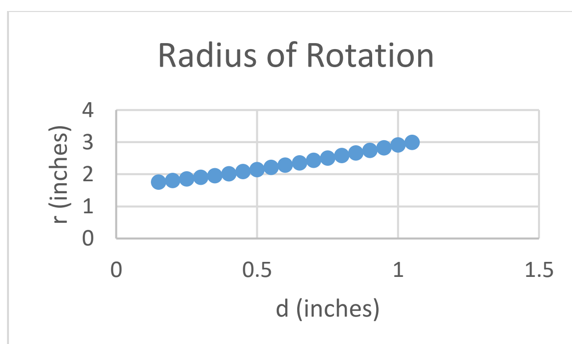


Figure 7. Plot of the radius of rotation of a relevant indication while increasing the distance between the points. θ was set to thirty degrees. Distance a was fixed at 1.6 inches.

Now, keeping the variables a and d fixed, we can increase the rotation angle θ. As the rotation angle increases, the calculated radius of rotation increases. In figure 8 below, the radius of rotation is plotted on the y axis in units of inches. The x axis is the angle θ, in degrees.

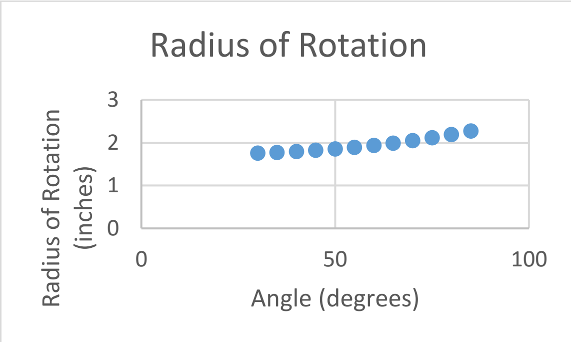


Figure 8. Plot of the radius of rotation of a relevant indication while increasing the rotation angle in increments of 5 degrees. Distance a was fixed at 1.6 inches. Distance d was fixed at 0.15 inches.

4. (U) Conclusion and Future Work

(U) The formula derived here was confirmed qualitatively in recent investigations. Further confirmation of the theory presented here using a combination of non-destructive testing and destructive methods will be explored. Currently the radiographic laboratory is obtaining a high energy computed tomography system that will aid in experimental verification of the theory presented here.