

## TIME SERIES FORECASTING OF TANKER TRAINING DEMAND

## GRADUATE RESEARCH PAPER

Steven R. Hawkins, Major, USAF

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Wright-Patterson Air Force Base, Ohio

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## TIME SERIES FORECASTING OF TANKER TRAINING DEMAND

GRADUATE RESEARCH PAPER

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Steven R. Hawkins, BS, MBA

Major, USAF

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#### Abstract

Since 1998 the USAF tanker fleet has decreased by 155 aircraft, with the additional anticipated retirement of the equivalent of 201 KC-135 tankers by 2029. Meanwhile, the new KC-46A is scheduled to replace only 179 boom-equipped tankers by 2029, resulting in a total decrease of 131 aircraft which corresponds to a reduction of 24.5% in total air refueling fuel capacity in just over three decades. This research examines historical tanker training requests, drawn from the 618th AOC's Air Refueling Scheduling Tool (ARST), and uses multiple forecasting techniques, including autoregressive integrated moving average (ARIMA) models, in order to create a model for predicting future air refueling training demand and communicating that demand in terms of aircraft flight hours. Air refueling remains a supply and demand problem in which there will always be more demand than the ability to supply with any realistically sized tanker fleet. The ability to understand, predict, and prepare for increased air refueling demand holds real value to planners, tanker units, and receiver units. This research is a first step in more clearly understanding unsupported air refueling training demand in terms of tanker aircraft flight hours.

To that receiver low on gas, may a tanker not be far away.

#### Acknowledgments

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## Steven "Bandit" Hawkins

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## TIME SERIES FORECASTING OF TANKER TRAINING DEMAND

## **I. Introduction**

"There is nothing we do ... without tankers," – Former United States Air Force Chief of Staff, General T. Michael Mosely

## **General Issue**

The United States Air Force (USAF) is facing a dramatic decrease in its projected air refueling (AR) tanker fleet. In 1998 the USAF operated 610 boom-equipped air refueling tankers (HQ AMC Office of History, 2018); twenty-one years later that number is down to 455 (U.S. Air Force, 2014b, 2018), the smallest fleet since the early 1960s (Boeing, 2018). The USAF total force currently operates 396 KC-135R/T and 59 KC-10A boom aircraft as the preponderance of the land-based air refueling fleet for the Department of Defense (DoD) (U.S. Air Force, 2014b, 2018). This equates to 455 booms and 501 KC-135 equivalent tankers in terms of overall fuel carrying capacity at the end of the 2018 fiscal year. The USAF is currently in the process of acquiring the new KC-46A, which is a similar size to the older KC-135, as it plans to retire all of the much larger KC-10As and some older KC-135R/Ts (Gertler, 2013, p. 1). The exact future tanker force structure remains uncertain because of a range of factors, including delays in the acquisition of the KC-46A, undetermined divestment timing of the KC-10A, and an aging fleet of KC-135 R/Ts with unknown reliability in the coming decades. What is known is that the target air refueling tanker mix includes a purchase of 179 KC-46A with a 6% fuel capacity increase over the KC-135 R/T and the sustainment of about 300 of the KC-135 R/T aircraft by 2029 (U.S. Air Force, 2016). This understanding is important because it means that tanker availability or the actual number of booms in the air will

remain roughly the same as the current status quo while decreasing in fuel capacity of up to 42 KC-135 equivalent aircraft in the next ten years, depicted in Figure 1.



Figure 1: Historical and Projected USAF Tanker Capacity

The above estimates for projected tanker capacity by the researcher are strikingly different from USAF projections for tanker recapitalization released by the Air Force Association just ten years ago as shown in Figure 2 (Knight & Bolkcom, 2008). This shortfall was primarily a result of years of failed recapitalization plans for the tanker fleet. These failed plans therefore have reduced the overall air refueling capacity and that has global implications for the Joint Force's lethality, capability, and mobility as there will certainly be an increase in future air refueling requirements, not a decrease. Therefore, an analysis and time series forecast of the United States Transportation Command (USTRANSCOM) validated tanker training demand would inform decision making on future tanker force structure and mission readiness.



Figure 2: Air Force 2008 Tanker Recapitalization Plan

#### Source: (Knight & Bolkcom, 2008, p. 22)

This research focuses exclusively on tanker training data versus global operational missions because the training data was readily available over multiple years via the 618 Air Operations Center's (AOC) Air Refueling Scheduling Tool (ARST), "the USAF system of record for matching receiver training air refueling needs to tanker capacity" (HQ AMC/A3O, 2016). In particular, this computer-based system was designed to match lower priority air refueling requests to tanker availability. The database of air refueling requests provides an important demand signal as this lower level of training requests often represents the line between what can and cannot be supported due to several factors including aircraft and crew availability, higher priority taskings, and competing training opportunities. The most valuable aspect of the data set is that the requests are grouped by

five different statuses, thus allowing some understanding for the researcher as to how air refueling requests were supported.

Using the currently available data the researcher utilized the definitions below (Table 1) to identify whether an air refueling request was supported by the tanker unit and then modeled all five request statuses in order to assist in future analysis and research.

G (* 1	
Confirmed	All parties have been notified and have accepted the displayed event's
	terms. Supported AR by both tanker and receiver units
Bought	The tanker unit has agreed to support the request and is awaiting receiver
Dougin	The tanker unit has agreed to support the request and is awaring receiver
	unit to confirm AR details to confirm the transaction. Supported AR
Filled	The receiver unit may have pre-coordinated with the tanker unit and
	places some tanker information in the request. Potentially Supported AR
Published	Air Refueling requests that have not been supported and have been
I uonsneu	An Keruening requests that have not been supported and have been
	entered into the system without tanker details.
Cancelled	Requests that have been cancelled or not supported by tanker units.
1	

**Table 1: Air Refueling Scheduling Tool Request Statuses** 

The way that the ARST system works is that an air refueling receiver unit enters details of their air refueling request in the computer database. If the user includes specific details of their requested tanker unit, the status is recorded as Filled. If no tanker details are entered the request is entered in the Published status. Once a tanker unit agrees to support the request, the former clicks a "buy" button which enters their tanker unit details and changes the status to Bought. The last step is for the receiver unit to confirm all the entered or updated air refueling details to include air refueling track location, rendezvous time, altitude, etc. by clicking the "confirm" button, thus changing the request status to Confirmed. At any time either the tanker or receiver unit can cancel the request, for any reason, by clicking "cancel," changing the status to Cancelled.

The Confirmed and Bought statuses most likely indicate supported requests that the tanker unit was capable of supporting. In contrast, the Filled and Cancelled statuses currently have no way of determining if they were or were not supported by the tanker unit in the ARST system. This is because a request can be cancelled by either the receiver unit or the tanker unit for a multitude of reasons or never even supported. Currently, the Published category is the clearest representation of unsupported air refueling requests that were never supported by a tanker unit in the ARST. The time series forecasting presented here focused on all five request statuses in order to build representative models in the hopes that future updates to the ARST will provide more fidelity on unsupported requests and reasoning for more detailed future analysis. For example, if either the tanker or receiver unit could annotate why a request was being unsupported or why it was cancelled, it would assist in future capacity and bottleneck analysis. Sorties could be unsupported for common reasons like crew or aircraft availability, but also because the request has insufficient training value, a low designated priority, or any number of other reasons. Such a modification to the ARST would not alter this research but would assist future models and provide relevant and timely feedback to receiver units as to why their request was not supported while providing the receiver units the details to better communicate the impact of a lack of support back to the tanker units, 618<sup>th</sup> AOC, and Air Mobility Command. An aggregated analysis of

supported and unsupported unit types, aircraft types, and request priority was also analyzed and is further discussed in Chapter 5.

Forecasting of air refueling demand and, in particular, training demand allows tanker units and planners to use assets more efficiently and to prepare for periods of increased/decreased tanker capacity. This represents a new way of thinking about tanker scheduling and is proactive rather than reactive; it is a new way to think about the data the 618<sup>th</sup> AOC is already capturing as part of the ARST program. Moreover, a reliable forecast has second-order effects on unit maintenance and support activities by allowing for more effective operations and an increase in overall readiness. In the end, the whole point of this system is to provide the most training available to requested receivers, thus enabling combat readiness. Air refueling remains a supply and demand problem in which there will always be more demand than the ability to supply with any realistically sized tanker fleet. Therefore, being able to understand better, predict, and prepare for increased air refueling demand holds real value to planners, tanker units, and receiver units.

Focused research on this problem is warranted now because the tanker fleet is not expected to increase in the near future while increased joint and international air refueling requirements are being added, thus increasing both the mission and training demand with the same or fewer air refueling assets for the near future. A specific example includes the next generation fighters that use two to two and a half times as much fuel for much shorter ranges than older legacy aircraft (U.S. Air Force, 2014a, 2015b, 2015a). Therefore, it is critical that planners gain a better understanding of what the training demand data is telling us with regard to request frequency in order to better understand the overall demand market for air refueling.

As previously stated, the current air refueling demand vastly outpaces the USAF's supply. One of the most promising solutions to reduce a segment of the demand remains the inclusion of commercial air refueling tankers which could reduce a segment of the training demand. This would likely include routine receiver training, some test and evaluation support, and non-combat related flight extension. The concept has been studied multiple times starting in 1998 by the US Transportation Command in its "Concept Development Report on Contracted Aerial Refueling" (USTRANSCOM TCJ5, 1999). This report led to research including the Defense Science Board Task Force Report on "Aerial Refueling Requirements" released in 2004 and a RAND Project Air Force report in 2006 entitled "Analysis of Alternatives (AoA) for KC-135 Recapitalization (Defense Science Board, 2004; Kennedy et al., 2006). The barriers to market entry range from policy issues such as no FAA certification for commercial air refueling to a significant financial burden for any organization without a known or guaranteed market size. Therefore, due to numerous technical and bureaucratic challenges, no commercial enterprise currently offers boom air refueling service. Omega Air Refueling Services has operated a small fleet of probe and drogue only aircraft (utilizing a basket at the end of a hose that the receiver connects to versus the USAF's mainstay of a flying boom) since 2001, primarily servicing the US Navy along with some international partners (Omega Air, 2018). The root cause of the problems highlighted above all focus on not truly understanding the DoD air refueling market demand. If air refueling demand could be forecasted, units could plan more efficiently longer into the future and, consequently, more training could be accomplished. Furthermore, if commercial augmentation was, at some future time, supported by the DoD as a viable

option to moderate some training demand by supporting less desirable training opportunities, planners could make informed decisions based on a forecast from historical employment. It is this question that this research seeks to answer. Utilizing historical tanker training requests, this research will focus on multiple forecasting techniques in order to create a model for predicting future air refueling training demand.

### **Problem Statement**

Currently, no known forecast method exists for predicting tanker training demand.

## **Purpose Statement**

This research will consider multiple forecasting techniques utilizing historical tanker training requests in order to model future training demand. The future demand will then be estimated using the metric of air refueling flight hours.

## **Research Questions/Objective**

RQ1: Which forecasting technique best predicts the various air refueling training requests for each designated status in the air refueling scheduling tool?

RQ2: Can the number of requested air refueling flight hours be predicted, with up to 90% accuracy annually, for each designated status in the air refueling scheduling tool?

### Methodology

This research utilizes time series forecasting to analyze the available ARST data from Jun 2010 – March 2018. The initial data collection only included Published and Confirmed statuses and, because there is no way to identify the other three statuses from

that earlier data, the analysis started with the April 2012 data, when all five statuses started to be recorded. The data included several air refueling details about each event which were further aggregated in order to learn more about patterns within the data set. The focus of this time series analysis was on the ARST system's five designated statuses of Confirmed, Bought, Filled, Published, and Cancelled over a regular time period. Annual analysis was rejected due to a limited number of data points; daily time periods were problematic as some statuses had zero requests on a given day resulting in errors in the computations for variance. Following a careful review, the researcher settled on weekly time periods for analysis, the reasoning for which will be discussed in Chapter 4. After running multiple time series analysis techniques for each of the five statuses, the researcher used validation techniques against one year of reserved data to verify the best model for each request status. Once the best forecast model was determined, the researcher applied multiple computational techniques to estimate total annual flight hours by each designated status.

## **Assumptions/Limitations**

As discussed above, the ARST data used for forecasting was restricted to April 2012 through March 2018 due to the limited statuses recorded Between June 2010 and April 2012. The database management of the ARST was changed in March of 2018; which resulted in the collected data from that time forward being unavailable to this researcher. The data that was inputted into the ARST system also limited this research; there is a high probability that not all air refueling training requests were inputted into the system. Additionally, air refueling requests were most likely both supported and

unsupported that were never entered into the ARST. However, the system shows stability over time, with a similar number of requests (an average of 21350 per year from 16 Mar 2014 – 10 Mar 2018 with the sums each year remaining within 2-4% of the mean) which leads to the conclusion that it represents a consistent proportion of the air refueling demand over time. Furthermore, future data can be entered into the proposed models thereby improving the model accuracy over time. Lastly, commercial air refueling costing structures were not attainable by the researcher because a boom capable commercial air refueling aircraft does not exist and, therefore, this research does not include a potential cost comparison for future commercial tankers. For this reason, all cost comparisons will use DoD cost structures for current USAF tanker aircraft. When commercial tankers do come to the market, the cost will likely be on a per flight hour basis. Therefore, this research methodology is designed to predict air refueling flight hours to easily apply to future cost comparisons between different tanker aircraft types for future research endeavors and decision making.

#### Implications

Air refueling demand will always outpace the available supply. This research is designed to find the best model for forecasting future air refueling requests as defined by the ARST status. The analysis outlined here can then use the forecast data to estimate the flight hour duration requested annually in terms of the same ARST statuses. These research questions are a first step to a better understanding of unsupported air refueling training demand in terms of aircraft flight hours. While several metrics are important, this research focused on the use of flight hours in the final analysis because they have a defined value to the DoD. Furthermore, research based on using flight hours can be used in the future in order to either justify increased expenditures to execute unsupported air refueling sorties or to compare USAF flight hour costs to future commercial tanker flight hour costs. When commercial air refueling service is available to the public market, the research and methods outlined here can provide an actual cost comparison given the quantitative forecasting background of the models. Such a comparison is likely to be far more accurate when the commercial tanker costs are available and this research, with its outlined mathematical rigor, is applied to known costs and more detailed needs rather than on educated assumptions about the unsupported air refueling training market using current ARST data.

## **II.** Literature Review

"Aerial refueling will be the biggest shortfall in our Mobility Air Forces" - Secretary of the United States Air Force, Heather Wilson

### **Chapter Overview**

This research began with a focus on understanding the air refueling challenges and, specifically, on exploring how the USAF fleet capacity has changed and is expected to change in the future. The air refueling fleet clearly has a supply and demand problem, and this researcher started thinking about the problem with respect to how future commercial air refueling tankers could be used to increase the overall supply faster than the USAF could through additional KC-46A procurement. However, as more information was gathered from previous reports, articles, and academic papers, it became clear that the demand side of the equation required analysis before any supply proposals could be explored.

Increased supply will certainly help in the short-term, and commercial air refueling has the potential to solve only certain portions of the overall demand, namely peacetime training purposes for a host of reasons. It is unlikely that any commercial enterprise will invest the time, partner with industry for aircraft acquisition, breakthrough long-standing barriers to market-entry, or even spend what would probably amount to a significant research and development cost without the DoD's support and a better understanding of the potential market. Therefore, gaining a better understanding of the unsupported air refueling requests (market demand) should be a first step in actually making headway in this process, which began over 20 years ago.

In order to fully understand the market demand of the USAF tanker enterprise, the researcher first focused on literature detailing two decades of recapitalization efforts along with multiple commercial air refueling studies as background to the problem. With that background in mind, the second part of this literature review will detail why demand forecasting is important, in particular, to commercial industries with a focus on the aviation industry. Lastly, the literature review will focus on the background and theory behind the primary forecasting technique used in the following analysis which is the BoxJenkins methodology for Autoregressive, Integrated, Moving Average (ARIMA) forecast models.

## Air Refueling Tanker Recapitalization

Unfortunately, the multiple plans to replace the over 400 "Eisenhower-era KC-135's" have faced significant challenges, controversies, corruption, and ultimately several failed acquisition programs dating back to the early 2000s (Grismer, 2011, p. 63). These recapitalization plans range from Boeing's 767 lease proposal that failed in 2001 to Northrop Grumman/European Aeronautic Defense and Space Company's (EADS) 2008 contract award that was later cancelled under protest from Boeing (Grismer, 2011, p. 63). Then, in February 2011, the Boeing company was finally awarded the contract to build the KC-46A, a contract valued at approximately \$35 billion (Gertler, 2013, p. 1). Again, after more delays, the USAF only recently took delivery of the first KC-46A on 10 Jan 2019, while still working to reconcile "major technical problems" (Insinna, 2019). This brief background of the KC-46A's acquisition is important because, in the over 19 years that it has taken for the USAF to take the delivery of a new tanker, the overall fleet size

has decreased by 151 boom tanker aircraft. Moreover, as the USAF acquires the projected 179 KC-46A aircraft, 59 KC-10A and approximately 96 KC-135R/T aircraft are projected to be retired, further reducing the overall tanker fleet (Pawlyk, 2018). Therefore, although the USAF will increase the number of booms available by 24, from 455 to 479 with the current plan, the capacity of fuel available actually decreases by over 2.2M pounds of fuel, from 100,204,000 pounds at the end of fiscal year 2018 to a projected 98,001521 pounds of fuel at the end of fiscal year 2029 (Gertler, 2013, p. 6). As stated above, work on this problem started decades ago with the full knowledge of how important it was to replace this high-demand, low-density resource quickly. It is truly unfortunate that years' worth of studies and recapitalization efforts have languished for so long, resulting in the retirement of more aircraft and only recently resulting in a new tanker acquisition.

The USAF, DoD, and other research agencies have conducted multiple studies on increasing the tanker enterprise over the years. In an effort to highlight some of the most applicable reports presented in chronological order, the researcher started in 1996. That year the GAO conducted a study entitled, "U.S. Combat Air Power: Aging Refueling Aircraft are Costly to Maintain and Operate." The basic conclusions of this report to Congress included an observation that, "although the services' air refueling tanker aircraft meet current needs, satisfying future requirements may be difficult" as the aircraft age and require increasingly more money to operate (Meredith, Stone, Dey, Newell, & Ragsdale, 1996, p. 34). The report also recommended that Congress consider dual use airlift and tanker aircraft in future acquisitions programs (Meredith et al., 1996, p. 35). Reports to Congress such as these highlighted the potential for a deficit of tankers in the

future and opened up studies of alternate methods to meet future tanker requirements, including contracted or commercial tankers.

The United States Transportation Command's "Concept Development Report on Contracted Aerial Refueling" was one of the first DoD studies and was completed on 1 Oct 1997, revised on 1 March 1998, and revised again on 21 June 1999. At the time of the original report, the USAF still maintained approximately 610 boom air refueling tankers and had only started talks with industry about a replacement for the KC-135, then roughly 40 years old. Moreover, the 1997 report represents the first time that a commercial organization, Omega Air, Inc., was included in USTRANSCOM working groups. This USTRASCOM report provided in-depth operational, policy, legal, contractual, and cost considerations for what a contract air refueling provider might provide to USTRASCOM, along with the feasibility of such a contract in a "CRAF-like, indefinite delivery, indefinite quantity (IDIQ) contract" (USTRANSCOM TCJ5, 1999). This report focused on using contract tankers to complete only probe and drogue air refueling which represent a small portion of the annual requirement and the only type of air refueling that Omega Air was at the time (and remains) capable of performing. Due to the focus remaining on a small subset of the air refueling requirement (probe and drogue), legal concerns about how commercial tankers would be integrated into combat operations, and concerns over reduced training opportunities for USAF crews the idea was rejected by USTRANSCOM, AMC, and the Joint Staff. In a November 1997 "Report to Congress on Private Sourcing of Airlift of Military Personnel and Cargo," required by the 1996 National Defense Authorization Act, the Office of the Secretary of Defense (OSD) made their position on commercial air refueling clear. OSD highlighted

that the combat integration, training, and full range of mission sets make "private-sector sources" for air refueling not suitable (USTRANSCOM TCJ5, 1999, p. 23).

Then, in 2004, the Office of the Under Secretary of Defense for Acquisition, Technology, and Logistics received their requested report from the "Defense Science Board Task Force on Aerial Refueling Requirements." The report focused on evaluating the current state and future requirements "of the USAF tanker fleet, the corrosion and maintenance issues associated with it, the studies pertaining to the KC-135, and several near-term options that the DoD has with regard to recapitalizing the fleet" (Defense Science Board, 2004, p. 4). Although the report found that the corrosion which resulted from the age and exposure to naturally-occurring elements was being controlled, the cost of that process along with the increasing costs of maintenance, lead to the task force to recommend multiple options for replacing the KC-135 fleet within a "reasonable timeframe" (Defense Science Board, 2004, p. 42). The proposed options included purchasing and converting used aircraft for air refueling, such as available DC-10-30's, contracting commercial organizations for specific mission sets, and working with large airframe manufactures for procurement of the next generation of air refueling tanker in the near-term (Defense Science Board, 2004, p. 36). Significantly, this report continued to highlight the need for multiple courses of action while considering alternate methods of recapitalization from a traditional acquisition approach. Furthermore, the 2004 Defense Science Board Report directly lead to a follow-on RAND study for an analysis of alternatives for the future of the KC-135.

In 2006 RAND Project Air Force published their report "Analysis of Alternatives (AOA) for KC-135 Recapitalization." The Under Secretary of Defense for Acquisition,

Technology, and Logistics at the time had requested the AOA based on the findings of the 2004 DSB study. Furthermore, the AOA was to include considerations of the tanker requirements from the forthcoming 2005 Mobility Capabilities Requirements Study. The RAND AOA study looked specifically at two research questions: "What is the most costeffective alternative for recapitalizing the KC-135 fleet?" and "When should the recapitalization assets be acquired?" (Kennedy et al., 2006, p. 16) After reviewing multiple options from new and used commercial-derivatives to new designs to unmanned and even commercial sources, the study recommended new commercial-derivatives in the 300,000 - 1,000,000 pound max gross takeoff weight categories as the most costeffective (Kennedy et al., 2006, p. 21). Smaller tankers, unmanned, and stealthy tankers were all considered to be not cost-effective in the report. The idea of commercial tankers was rejected primarily on the assumption that all tankers "must be capable of carrying out wartime missions" and, therefore, commercial tankers were considered cost prohibitive due to required defensive equipment (Kennedy et al., 2006, p. 22). Furthermore, the report found that the timing of the recapitalization of the tanker fleet is more dependent on factors such as the risk of catastrophic technical problems or critical maintenance issues rather than overall life-cycle costs of operating the legacy fleet (Kennedy et al., 2006, p. 25). However, the AOA conceded that the purchasing of a new tanker fleet quickly after waiting too long for recapitalization would be much more costly than purchasing it before the legacy tanker fleet wears out (Kennedy et al., 2006, p. 25). The AOA represents the most recent government report highlighting the need to recapitalize the KC-135 tanker fleet and, while it considered alternatives to a normal acquisition

process, it failed to look for broad and innovative solutions to a supply and demand problem that was only getting worse by the year.

Following the 2006 AOA report, the tanker recapitalization focus was on the 2008 contract that was initially awarded to EADS, challenged by Boeing, and then rescinded by the USAF. Again, timing is important because the leadup to the contract award coincided with the USAF's decision to retire a majority of the KC-135E model aircraft due to corrosion issues with the engine bolts. Although a small number were upgraded to KC-135R aircraft, approximately 114 were grounded and ultimately sent to long-term storage in Arizona, further reducing the overall fleet (Knight & Bolkcom, 2008, p. 31). The next eleven years were marked by continued calls for recapitalization, followed by yet another contract bid for a new tanker replacement and countless delays that only now have resulted in the USAF taking the possession of the first few KC-46As. This history of multiple working groups, analysis, reports, and recapitalization efforts at multiple levels within multiple organizations is important for the reader to understand that just because an innovative or non-standard solution such as outsourcing was deemed not cost effective or rejected in the past, that conclusion does not necessarily apply in today's environment which is radically different than ten or twenty years ago.

## **Aviation Demand Forecasting**

With a clearer understanding of the environment of dwindling tanker fleet supply and increased demand that the Joint Force faces today, the focus must be on using the tanker fleet as efficiently as possible, and this includes forecasting. The air transportation industry is heavily reliant on forecasting in order to use its assets as efficiently as

possible. "Every day, at all levels of management, within all segments of the air transportation industry, decisions are made about what is likely to happen in the future" (Wensveen, 2015, p. 268). Forecasting expected demand can look at multiple different time periods and include an evaluation using several different methods. "The choice of forecasting methods should be based on several factors, including availability of data, accuracy of available data, management sophistication, intended forecast use, and availability of electronic processing" (Wensveen, 2015, p. 270).

For the reasons above, the decision as to the specific forecasting method is almost as important as the data available. There are countless studies of forecasting methods for the aviation industry to include Richard Wickham's 1995 MIT thesis, "Evaluation of Forecasting Techniques for Short-Term Demand of Air Transportation." Wickham utilized eleven different models -3 time series models, 2 regression models, and 6 different pickup models - in his analysis of an eighteen week, short-term air transportation demand forecast for up to an eight week future time period (Wickham, 1995, p. 55). Wickham's research was focused on determining the best forecasting method while varying the forecasting period and the size of the historical data set to see how these changes affected the accuracy of the different methods; he limited his methods to a simple mean and various exponential smoothing techniques. His research findings showed that each model's accuracy decreased as the forecast horizon was increased, yet for short periods of less than four weeks, the results were very similar (Wickham, 1995, p. 110). This is to be expected in that longer forecasts are more challenging, in particular when the data is highly variable as was the case in Wickham's study. However, Wickham's third finding was much more interesting. Some of the models that decreased

in accuracy did so as the sample size was increased. He states that "if the focus is shifted to the local booking activity, *micro-trends* can be observed where the data immediately before the point of observation gives some indications of the preceding booking behavior" (Wickham, 1995, p. 111). The point is that, with his short-term demand data and its volatile behavior, the more recent observations were more important to the shortterm forecast than the older observations. This is also why his exponential smoothing models resulted in less forecast error; those models weight the more recent observations greater than the older observations. It should be noted that Wickham chose not to include corrections for seasonal fluctuations in his analysis based on the narrowed focus of only eighteen weeks of data, although a larger data set might require such corrections and have different results as "seasonal variation occurs quite naturally in the demand for air travel" (Wickham, 1995, p. 23). Wickham's analysis provides a comparison of multiple forecasting methods and their application to the air transportation industry, while the next article speaks to the applicability of one method of analysis over another to the aviation industry.

In "Predicting Air-Transport Demand," Pitfield focused on a comparison of ARIMA models and regression models for the purpose of forecasting air-transport passengers by the route. Pitfield pointed out numerous cases where regression analysis is used for demand forecasting and identifying explanatory variables in the air transportation industry such as UK domestic passengers who use business services and passenger traffic between airports (Pitfield, 1993, p. 459). The main challenge Pitfield identified with extremely complicated systems like air transportation was that defining all relevant variables accurately for regression models is difficult if not impossible. His

paper examines domestic air travel time series data from the UK by applying both ARIMA modeling techniques and regression modeling to the same data set for comparison (Pitfield, 1993, p. 460). The final comparison of the error statistics showed significantly lower error in every category for the ARIMA model versus the regressionbased models, leading to Pitfield to conclude that the ARIMA model is a superior forecasting tool over the regression model that struggled with defining the explanatory variables outside of the airlines' control (Pitfield, 1993, pp. 465–466). This article highlights the difficulty in defining and quantifying the multiple variables associated with forecasting in the airline industry. It also highlights the potential accuracy and utilization of ARIMA forecasting models, despite multiple ill-defined variables in such an industry.

### **Historical Perspective of Time Series Analysis**

The concept of time series forecasting is not new. In fact, there are documented cases of merchants utilizing rudimentary forecasting and quantitative reasoning in order to determine expected values for their profits and losses as the markets changed going back to at least mid-17<sup>th</sup> century and probably earlier (Klein, 1997, p. 1). Though many statisticians have contributed to the early work in time series analysis, George Udny Yule is credited as one of the first statisticians to utilize applied correlation and regression for what he called the "time-correlation problem" (Klein, 1997, p. 222). Yule's work in the 1920s laid the foundation for applied statistics and time series in multiple fields of study by providing an initial understanding of non-stationary data. "Udny Yule's specification of an autoregressive stochastic process was a by-product of his attempts to explain why statisticians, and in particular those who worked with economic and social data, often got

strange correlations from time series data" (Klein, 1997, pp. 264–265). In doing so, "Yule's specification of the autoregressive process...was to become one of the most common tools of forecasting with univariate time series models" (Klein, 1997, p. 270). This work enabled others, including Herman Wold who, in 1938, defined the discrete stationary process, which is a stochastic or random process. "Wold spelled out the autoregressive, moving-average, modeling approach that would be used in the coming decades to investigate stochastic processes, his work spurred further development in spectral analysis, he gave justification for the use of sum of squares in the analysis of stationary times series, and he put correlograms front and center as the key means of model specification" (Klein, 1997, p. 289). These statisticians are highlighted for their foundational work that was continued by the statisticians George Box and Gwilym Jenkins, who popularized the use of the ARIMA process for economic and business process in their seminal work "Time Series Analysis: Forecasting and Control" (1970).

## AutoRegressive, Integrated, Moving Average (ARIMA)

"ARIMA processes are mathematical models used for forecasting" (Hyndman, 2001, p. 1). Due to George Box and Gwilym Jenkins' extensive study of these models and their applicability to time series forecasting, the ARIMA processes are often called Box-Jenkins Methods. "The ARIMA approach to forecasting is based on the following ideas: 1) The forecasts are based on linear functions of the sample observations; 2) The aim is to find the simplest modes that provide an adequate description of the observed data" (Hyndman, 2001, p. 1). This concept of the fewest parameters possible is also known as parsimony. "Parsimony may often be achieved by representation of the linear
process in terms of a small number of autoregressive and moving average terms" (Box & Jenkins, 1970, p. 46). The intent is to reduce the model's complexity and, therefore, to avoid the potential of over-fitting the model with excessive parameters. The ARIMA process is based in three parameters: autoregressive (AR), integrated (I), and moving average (MA); each are explained in detail with their applicable equations in Chapter 3 (Commandeur & Koopman, 2007, p. 122).

By putting all three parameters together, the ARIMA process forms a powerful forecasting tool with a defined mathematical structure allowing for seasonal and multivariate time series forecasting (Hyndman, 2001, p. 2). The research presented in this paper did not focus on multivariate or forecasting with multiple variables; however, it does focus on multiple seasonal time series independently for each of the five ARST defined statuses. Seasonal analysis uses the same basic structure as ARIMA but adds an extra set of AR, I, and MA parameters to model the seasonal elements of the time series, each characterized by a capital letter. Therefore, the shorthand for a seasonal ARIMA is (p,d,q)(P,D,Q). In order to determine the value of those parameters, Box and Jenkins developed a methodology to approach the process.

### The Box-Jenkins Methodology

As discussed above, the Box-Jenkins modeling methodology was developed by two statisticians, George Box and Gwilym Jenkins, as a way to apply an organized and iterative approach for determining the correct ARIMA values given a forecasting problem. "Box-Jenkins modeling involves identifying an appropriate ARIMA process, fitting it to the data, and then using the fitted model for forecasting" (Hyndman, 2000, p.

1). Box and Jenkins applied the process through an iterative three-step process that they called model selection, parameter estimation, and model checking; in recent years the first step of data preparation and last stage of the model application or forecasting has been added (Hyndman, 2000, p. 1). The full process, as described by Makridakis, Wheelwright, and Hyndman, is presented in Table 2 below.



Table 2: The Box Jenkins Methodology for ARIMA models

Source: (Makridakis, Wheelwright, & Hyndman, 1998, p. 314)

# **Data Preparation**

Data transformations such as taking the logarithm, square root, or other

mathematical transformations of the data in order to stabilize variance are often required

for business or economic data as the first step in data preparation (Hyndman, 2000, p. 1). Data may then be differenced which involves subtracting successive observations in order to remove patterns such as trend or seasonality thus increasing ease of modeling the data without unpredictable trends or fluctuations (Hyndman, 2000, p. 1). The data transformation and differencing techniques that are included in the data preparation steps are not required with every data set; however, a thorough analysis of the data set is required before a model is selected to determine what type of preparation, if any, is required.

# Model Selection

After the data is prepared, ARIMA models are determined based on the use of autocorrelation functions (ACF) and partial autocorrelation functions (PACF) of differenced data series to determine the best data fit (Hyndman, 2000, p. 1). "If the future values can be described only in terms of a probability distribution, the time series is said to be non-deterministic or simply a statistical time series" (Box & Jenkins, 1970, p. 24).

### **Parameter Estimation**

This involves determining the ARIMA coefficients of (p,d,q) or (p,d,q)(P,D,Q) for seasonal models which best fit the series data (Hyndman, 2000, p. 1). There are multiple methods for determining the goodness-of-fit of a model, including the Akaike's Information Criterion and Schwarz Bayesian Information Criterion, both of which are utilized in the research below.

# Model Checking

Checking the model is an iterative process of identifying where or how the model is not as good of a fit to the data series and re-accomplishing the model selection,

parameter estimation, and model checking steps until the best fit is obtained (Hyndman, 2000, p. 1).

### Forecasting

Utilizing the best model to accomplish a forecast is the end result of the process and is often accomplished via a computer program for large data sets (Hyndman, 2000, p. 1). Once the data is forecast, it can be compared to reserved data observations or future observations in order to validate the model via a multitude of error statistics. Validation is important because all the goodness-of-fit measures used in the iterative model checking process are only designed to determine how well the model fits the data used to forecast. Whereas, the validation process is used to determine how well the end result accurately predicts future events, the end goal of forecasting.

### Summary

This literature review presented a background on over twenty years of USAF tanker recapitalization efforts from failed contracts to numerous reports analyzing tanker recapitalization strategies. All of these efforts were focused on producing a new tanker or tanker alternative before the tanker fleet was reduced by aging aircraft and the increasing cost of maintenance. Unfortunately, multiple delays have reduced the DoD's options and strained the air refueling system, only recently producing a new tanker in the KC-46A. Efficient use of the remaining assets to include demand forecasting strategies that have been used in the air transportation industry for decades may alleviate some of the strain. Several articles on different forecasting strategies were presented along with a background on time series analysis and ARIMA models. This background helps to explain the power and utilization of ARIMA models specifically for the air transportation industry. Finally, a discussion of the Box-Jenkin's methodology was presented in preparation for the mathematical methodology in Chapter 3.

### **III.** Methodology

"Prediction is very difficult, especially if it is about the future" - Niels Bohr

# **Chapter Overview**

This research employed time series forecasting techniques, including autoregressive integrated moving average (ARIMA), models developed by statisticians George Box and Gwilym Jenkins for business and economic data application in the 1970s (Nau, 2014b). Time series forecasting develops a model based on past observations of the same variable over equally spaced time periods in order to describe an underlying relationship (Zhang, 2003). Such a model can then be used to predict future outcomes of that variable over future time periods of the data series. This chapter will explain how the researcher analyzed the available tanker training ARST data which is comprised primarily of priority 3 and 4 air refueling training missions versus operational missions and is focused on the air refueling request status portion of the data set. Next, this chapter will include a brief discussion on data transformations to provide stationary time series including differencing and logarithmic transformations for application in multiple forecasting methods. Lastly, the chapter will focus on the description and understanding of the various time series forecasting methods that were utilized in increasing complexity to analyze the available data.

# **Data Scope**

As previously stated, this researcher's data source focused on tanker training data received from the 618 Air Operations Center's (AOC) Air Refueling Scheduling Tool. 136,466 rows of data from 30 Jun 2010 to 10 Mar 2018 were initially provided. The data

provided also contained information about the air refueling requests including receiver organization, aircraft type, fuel required, priority, duration, etc. for a total of 22 database columns. As stated in Chapter 1, the request status is most effective in determining whether the air refueling request was supported. Due to differences in how the request status was originally recorded prior to April 2012, the data set was scoped to start on 08 Apr 2012 and end on 10 March 2018. These dates were chosen to correspond to the Sunday through Saturday weekly schedule, which will be more thoroughly explained in Chapter 4. This subset of the data left 117,446 data points for analysis - 95,906 for forecasting and 21,540 for validation. Data after 10 Mar 2018 remained unavailable to the researcher due to a system software change; however, it is still being collected and should ultimately be used for further validation and updates to the presented forecast models.

# **Data Transformations**

Many time series forecasting techniques depend on a stationary time series, meaning that the series is not dependent on the specific observed time (Hyndman & Athanasopoulos, 2018). Therefore, a time series with a significant trend or seasonal component would be non-stationary as observed at different times of the year. A series with no predictable pattern like "white noise" would be stationary and should have "constant statistical properties such as mean, variance, and autocorrelation over time" (Nau, 2018). This is important because it can make the series easier to forecast if the statistical properties remain constant.

One technique to achieve a stationary series is to take the difference of the time series. This is accomplished by subtracting the value of the time Y at time period t from Y at time period t – 1; if this operation produces a stationary series that is random and not autocorrelated, then it is also called the random walk model described below (Nau, 2018). It is also possible to take the second difference of a time series by taking the first difference again of the first difference just taken; however, this can easily cause an unintended error if not properly analyzed. Other data transformations include mathematical operations to stabilize the variance in a time series, thereby increasing the ability to predict future events as the subsequent observances would have a smaller range of highs and lows from the mean. For example, taking the logarithm (Log 10) or natural log (LN) of the data set can reduce variance as differencing can stabilize the mean thus reducing data trend and seasonality (Hyndman & Athanasopoulos, 2018).

# **Time Series Forecasting Methods**

There are several different methods for forecasting data over time. Those in this section are some of the most commonly used and are presented from the simplest to the more complex; several concepts are reliant on one-another.

### The Naïve Method

Naïve forecasts simply predict future outcomes based on the last observation recorded (Hyndman & Athanasopoulos, 2018). Simple forecasts such as these can serve as an important benchmark for comparison against more complex forecasting methods that may be prone to more error. The naïve model should represent the worst case of an error a researcher is willing to accept in any given forecast. These models are quick and

require little to no computing power to produce as they assume that the next data point is equal to the last observed point (Singh, 2018). The naïve method can be surprisingly adequate when applied to a stable or limited data series. The equation for the naïve method is:

Equation 1: The Naïve Method
$$\widehat{Y}_t = Y_{t-1} \tag{1}$$

Where

 $\hat{Y}_t$  = the forecast value at time t

### The Random Walking Method

The random walking method is the first difference of the naïve method, which outputs the difference between successive data points from the original observations, as shown below (Hyndman & Athanasopoulos, 2018).

### **Equation 2: First Difference**

$$Y_t = Y_t - Y_{t-1} \tag{2}$$

Assuming this output is stationary, the random walking model can be re-written as:

### **Equation 3: The Random Walking Method**

$$\dot{Y}_t = Y_{t-1} + \varepsilon_t \tag{3}$$

Where

 $\varepsilon_t$  = accounts for random error or white noise

This random walking model has the same benefit presented for the naïve method -a benchmark for the worst case of error - yet it is designed for non-stationary data.

# The Simple Average Method

Instead of using the last value to predict the next value, the simple average method uses the average of all previous values observed in order to predict the expected value of a future value (Singh, 2018). This method is best suited for series that maintain a nearly constant mean and will not account well for data with an upward or downward trend. The equation for the simple average method is below and encompasses a simple arithmetic average of all observed values that number a total of p.

**Equation 4: The Simple Average Method** 

$$\hat{Y}_{t+1} = \frac{1}{p} \sum_{t=1}^{p} Y_t \tag{4}$$

Where

p = previous values

# The Simple Moving Average Method

A potentially more useful application building upon this method is the simple moving average method, which allows for the use of a specified subset of the data often but not always the more recent values to account for variation in earlier data. Another way to think about this method is that it considers a "sliding window" of data that is specified by k for a stationary data series in the below equation (Singh, 2018).

Equation 5: The Simple Moving Average Method

$$\hat{Y}_{t+1} = \frac{Y_t + Y_{t-1} + Y_{t-2} + \dots + Y_{t-k+1}}{k}$$
(5)

Where

 $\hat{Y}_{t+1}$  = the forecast value at time t + 1

It is important to note that if k = 1, the output is the same as the naïve method above; the other extreme is that if k = p or all previous observations, the output would be the same as the simple average or mean method above. Furthermore, it is possible to weigh past observations differently, resulting in the weighted moving average method (Singh, 2018).

# The Simple Exponential Smoothing (SES) Method

In an effort to bridge the gap between the simple average and weighted moving average methods, simple exponential smoothing takes into account all data by using weighted averages that decrease exponentially to account more for recent, potentially relevant data, and less for older, potentially irrelevant, data (Singh, 2018). This process weighs recent data the most, a benefit of a simple moving average, yet considers all data in the set, unlike moving average. In order to define the equation for the simple exponential smoothing method several new terms must be introduced. First,  $\alpha$  is a "smoothing constant" that is between 0 and 1 (Nau, 2014a, p. 8). It is important to note that, once again, if  $\alpha$ =1 then the SES model will equal the naïve model (or random walking if differenced) and if  $\alpha$ =0 then the SES will equal the simple average or mean model. The second issue is to define the series L that represents a level or local mean of the series "computed recursively" from the previous data as shown in equation 6 (Nau, 2014a, p. 7).

# Equation 6: Level Mean of the Series $L_t = \alpha Y_t + (1 - \alpha)L_{t-1}$ (6)

Where

```
0 \le \alpha \le 1
```

The estimated local mean at time t is calculated by "interpolating between the just-observed value and the previous estimated level, with weights of  $\alpha$  and 1- $\alpha$ , respectively" (Nau, 2014a, p. 8). Larger values of  $\alpha$  will increase the weight of more

recent observations while, conversely, lower values will increase the weight of older observations. A significant benefit of the SES model is that  $\alpha$  can be varied with new data input to minimize forecast error. Assuming the forecast series has no trend, the forecast for time t+1 can be estimated by the local mean at time t in equation 7.1 (Nau, 2014a, p. 8). This leads to the simple exponential smoothing equation by substituting equation 6 in for  $L_t$ :

Equation 7: The Simple Exponential Smoothing (SES) Method

$$\hat{Y}_{t+1} = L_t \tag{7.1}$$

$$\hat{Y}_{t+1} = \alpha Y_t + (1 - \alpha) \hat{Y}_t$$
(7.2)

Where

 $0 \leq \, \alpha \, \leq \, 1$ 

### Double (Brown's) Linear Exponential Smoothing Method

While simple moving average and simple exponential smoothing methods rely on data without a trend, Brown's linear exponential smoothing model computes both a level and trend denoted as  $L_t$  and  $T_t$ , respectively (Nau, 2014a, p. 16). To accomplish this, the series S at time t is exponentially smoothed using SES and the same  $\alpha$ , once for S<sup>'</sup> and a second time for S<sup>''</sup>.

Equation 8: Double (Brown's) Linear Exponential Smoothing Method

$$S'_{t} = \alpha Y_{t} + (1 - \alpha) S'_{t-1}$$
(8.1)

$$S''_{t} = \alpha S'_{t} + (1 - \alpha) S''_{t-1}$$
(8.2)

$$L_t = 2S'_t - S''_{t-1} \tag{8.3}$$

$$T_t = (\alpha / (1 - \alpha))(S'_t - S''_{t-1})$$
(8.4)

$$\hat{Y}_{t+k} = L_t + kT_t \tag{8.5}$$

Where

 $S'_t$  = Single smoothed series at time t  $S''_t$  = Double smoothed series at time t  $L_t$  = Estimated level at time t  $T_t$  = Estimated trend at time t  $\alpha$  = Smoothing constant  $0 \le \alpha \le 1$ 

# Linear (Holt's) Exponential Smoothing Method

Holt's linear exponential smoothing method is similar to Brown's in that it also estimates level and trend. However, it employs two different smoothing parameters, one for level ( $\alpha$ ) and one for trend ( $\beta$ ), allowing those estimates to vary at independent rates thus fitting more data patterns (Nau, 2014a, p. 16).

Equation 9: Linear (Holt's) Exponential Smoothing Method

$$L_t = \alpha Y_t + (1 - \alpha)(L_{t-1} + T_{t-1})$$
(9.1)

$$T_t = \beta (L_t - L_{t-1}) + (1 - \beta) T_{t-1}$$
(9.2)

$$\hat{Y}_{t+k} = L_t + kT_t \tag{9.3}$$

Where

 $\begin{array}{ll} L_t = & \text{Estimated level at time t} \\ T_t = & \text{Estimated trend at time t} \\ \alpha = & \text{Smoothing constant 0} \leq \, \alpha \, \leq \, 1 \end{array}$ 

 $\beta$  = Trend smoothing constant  $0 \le \beta \le 1$ 

# Damped Trend Linear Exponential Smoothing Method

A downside of Holt's linear exponential smoothing method is that it forecasts a constant trend infinitely into the future resulting in over or under forecasting over longer

periods of time (Hyndman & Athanasopoulos, 2018). In order to combat this forecasting problem, the damped trend method includes a parameter ( $\phi$ ) to adjust the trend line toward 0 slope as time periods increase. If  $\phi = 1$  the damped trend method will equal the Holt's linear method, while possible values for  $\phi$  range from  $0 < \phi < 1$ .

### **Equation 10: Damped Trend Linear Exponential Smoothing Method**

$$L_t = \alpha Y_t + (1 - \alpha)(L_{t-1} + \phi T_{t-1})$$
(10.1)

$$T_t = \beta (L_t - L_{t-1}) + (1 - \beta) \phi T_{t-1}$$
(10.2)

$$\hat{Y}_{(k)} = L_t + \sum_{i=1}^{k} \phi^i T_t$$
(10.3)

Where

 $\begin{array}{ll} L_t = & \text{Estimated level at time t} \\ & T_t = & \text{Estimated trend at time t} \\ & \alpha = & \text{Smoothing constant } 0 \leq \alpha \leq 1 \\ & \beta = & \text{Trend smoothing constant } 0 \leq \beta \leq 1 \\ & \phi = & \text{Dampening parameter } 0 < \phi < 1 \end{array}$ 

### Seasonal Exponential Smoothing Method

Up to this point, methods have built on one another to address mean, moving average, and exponential smoothing. In particular, the exponential smoothing models have increased in complexity to accommodate series with different trends. The seasonal exponential smoothing addresses those series with a seasonal or cyclical component but no overall trend increasing or decreasing by utilizing a level and seasonal term (Hyndman & Athanasopoulos, 2018). **Equation 11: Seasonal Exponential Smoothing Method** 

$$L_t = \alpha (Y_t - S_{t-p}) + (1 - \alpha) L_{t-1}$$
(11.1)

$$S_t = \delta(Y_t - L_t) + (1 - \delta)S_{t-p}$$
(11.2)

$$\hat{Y}_{(k)} = L_t + S_{t-p+k}$$
(11.3)

Where

 $L_t = Estimated level at time t$ 

 $S_t = Estimated length of seasonality at time t$ 

 $\alpha$  = Smoothing constant  $0 \le \alpha \le 1$ 

 $\delta$  = Seasonality smoothing constant  $0 \le \delta \le 1$ 

# The Holt-Winters Seasonal Method

In an effort to put all of these components together into one method, Holt and Winters developed a triple exponential smoothing method with a term for level  $(L_t)$ , trend  $(T_t)$ , and seasonal  $(S_t)$  components, along with respective independent smoothing parameters  $(\alpha, \beta, \delta)$  (Hyndman & Athanasopoulos, 2018). There are two different representations depending on the series behavior of the seasonal component. "The additive method is preferred when the seasonal variations are roughly consistent through the series, while the multiplicative method is preferred when the seasonal variations are changing proportional to the level of the series" (Hyndman & Athanasopoulos, 2018). Data samples with little trend or seasonality will result in similar outputs via either method. The difference as presented in the equations below is that the additive form is expressed in absolute terms while the multiplicative is expressed in relative terms (Hyndman & Athanasopoulos, 2018).

### Equation 12: The Holt-Winters Seasonal Method - Additive

Additive Seasonality - Constant Amplitude

$$L_t = \alpha(Y_t - S_{t-p}) + (1 - \alpha)(L_{t-1} + T_{t-1})$$
(12.1)

$$T_t = \beta (L_t - L_{t-1}) + (1 - \beta) T_{t-1}$$
(12.2)

$$S_t = \delta(Y_t - L_t) + (1 - \delta)S_{t-p}$$
(12.3)

$$\hat{Y}_{(k)} = L_t + kT_t + S_{t-p+k}$$
(12.4)

Where

 $L_t = Estimated level at time t$ 

 $T_t = Estimated trend at time t$ 

 $S_t = Estimated length of seasonality at time t$ 

 $\alpha$  = Smoothing constant  $0 \le \alpha \le 1$ 

- $\beta$  = Trend smoothing constant  $0 \le \beta \le 1$
- $\delta$  = Seasonality smoothing constant  $0 \le \delta \le 1$

p = Number of seasons per year

# Equation 13: The Holt-Winters Seasonal Method - Multiplicative

Multiplicative Seasonality - Constant Cycles (in overall percentage teams)

$$L_{t} = \alpha \frac{Y_{t}}{S_{t-p}} + (1-\alpha)(L_{t-1} + T_{t-1})$$
(13.1)

$$T_t = \beta (L_t - L_{t-1}) + (1 - \beta) T_{t-1}$$
(13.2)

$$S_{t} = \delta \frac{Y_{t}}{L_{t}} + (1 - \delta)S_{t-p}$$
(13.3)

$$\hat{Y}_{(k)} = L_t S_{t-p+k}$$
(13.4)

Where

 $\begin{array}{l} L_t = \mbox{ Estimated level at time t} \\ T_t = \mbox{ Estimated trend at time t} \\ \\ S_t = \mbox{ Estimated length of seasonality at time t} \\ \\ \alpha = \mbox{ Smoothing constant } 0 \leq \alpha \leq 1 \\ \\ \beta = \mbox{ Trend smoothing constant } 0 \leq \beta \leq 1 \\ \\ \delta = \mbox{ Seasonality smoothing constant } 0 \leq \delta \leq 1 \\ \\ p = \mbox{ Number of seasons per year} \end{array}$ 

# ARIMA (The Box-Jenkins Methodology)

Auto-Regressive Integrated Moving Average (ARIMA) models are a variation of discrete-time filtering methods created by electrical engineers in the 1930s and 40s. Statisticians George Box and Gwilym Jenkins popularized these models starting in the 1970s for application to business and economic data (Nau, 2014b). Almost all of the previously covered methods can be described by an ARIMA model, as shown in Table 3 below. The application of the ARIMA model is broken down into three parts – auto-regressive (AR), integrated (I), and moving average (MA), often with the standard notation p, d, q to explain the order and degree of each segment of the model (Nau, 2014b). An autoregressive (AR) term is a forecast of the indicated variable utilizing a linear regression of a number of past values of the variable (Hyndman & Athanasopoulos, 2018). Therefore, the AR term is focused on fitting the forecast to previous observations in the series. The integrated (I) term is a factor when the series is not stationary and must be differenced to stabilize the mean or reduce trend and seasonality. ARIMA models rarely need to be differenced more than twice, and over-differencing can output strong

negative autocorrelation along with a strong moving average signature (Nau, 2014b).

Lastly, the moving average (MA) term utilizes past forecast errors in the series and

regression for forecasting future values (Hyndman & Athanasopoulos, 2018).

Application of a specific ARIMA model is an iterative process to develop the best model,

involving trial and error in order to minimize forecast error displayed in the

autocorrelation (ACF) and partial autocorrelation (PACF) residual plots (Nau, 2014b).

Forecasting Method	Equivalent ARIMA
Mean Model / White Noise	ARIMA $(0,0,0) + c$
Random Walk Model	ARIMA (0,1,0)
Simple Exponential Smoothing	ARIMA (0,1,1)
Double (Brown) Linear Exponential Smoothing	ARIMA (0,2,2)
Linear (Holt) Exponential Smoothing	ARIMA (0,2,2)
Damped Trend Linear Exponential Smoothing	ARIMA (1,1,2)
Seasonal Exponential Smoothing	ARIMA (1,1,p+1)(0,1,0)p
Holt-Winters Additive Seasonal Method	ARIMA (1,1,p+1)(0,1,0)p

**Table 3: Forecasting Method and ARIMA Equivalence** 

Source: (Jones & Arnold, 2019)

# Autoregressive (AR)

The AR part of the model is often written as *p* and "describes how each

observation is a function of the previous p observations" (Hyndman, 2001, p. 1). The

equation for p > 1 is:

#### **Equation 14: Autoregressive Standard Equation**

$$Y_t = c + \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \dots + \phi_p Y_{t-p} + \varepsilon_t$$
(14)

Where

 $Y_t = Observed$  value at time t

 $Y_{t-1}$  = The previous value at time t

 $\varepsilon_t$  = Random error

c = Constant $\phi_p = Constant$ 

### Integrated (I)

The I part of the model is often written as *d* and "determines whether the observed values are modelled directly, or whether the differences between consecutive observations are modelled instead" (Hyndman, 2001, p. 1). This term is used to correct for non-stationary series, that is those with trend or seasonality, which is quite common in business and economic data. "Forecasting has been of particular importance to industry, business, and economics where many times series are often represented as non-stationary and, in particular, as having no natural mean" (Box & Jenkins, 1970, p. 7). Series are rarely differenced more than twice to obtain stationarity.

# Moving Average (MA)

The MA part of the model is often written as q and "describes how each observation is a function of the previous q errors" (Hyndman, 2001, p. 1). The equation for q > 1 is:

Equation 15: Moving Average Standard Equation  

$$Y_t = c + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2} + \ldots + \theta_q \varepsilon_{t-q} + \varepsilon_t$$
(15)

Where

 $Y_t = Observed$  value at time t

 $\varepsilon_{t-q}$  = Random error at a previous time t – q

 $\varepsilon_t$  = Random error at time t

c = Constant

 $\Theta_q = \text{Constant}$ 

#### Seasonal ARIMA

For data series that exhibit seasonal fluctuations, ARIMA can be used to model the data in a way similar to the seasonal exponential smoothing or the Holt-Winters methods, which include a seasonal component. In order to apply seasonal ARIMA, or SARIMA, three more terms are added - seasonal auto-regressive (P), seasonal integrated (D), and seasonal moving average (Q). The seasonal function thus is expressed as ARIMA(p,d,q)(P,D,Q) where the uppercase and lowercase terms are independent of one another and have recommended values based on residual ACF/PACF outputs (Nau, 2014b).

### Time Series Forecasting Model Approach

The researcher utilized JMP 12.0 statistical software developed by SAS to analyze the 2012 – 2018 ARST statuses of Confirmed, Bought, Filled, Published, and Cancelled grouped by seven-day weeks (Sunday thru Saturday). Weekly analysis was chosen for seasonal reasons apparent in the data set. Some days resulted in zero requests (major holidays, etc.) and daily forecasting with a zero value for the variable of interest results in software calculation errors amongst the various formulas. An initial exploration into monthly forecasting resulted in a less than optimal sample size with only 72 months of data, 12 of which were to be withheld for validation. Furthermore, there are some monthly cycles and some seasonality, but the true cycles appeared to be weekly, as requests on the weekends were much less than mid-week (and often non-existent thus skewing data and making an analysis of a smaller time scale not useful with the techniques evaluated). Therefore, weekly aggregation of data allowed for 309 observations, 52 of which were withheld for validation.

Each of the five series were plotted in JMP, log (ln) transformed and differenced to achieve stationary series data, before applying the above exponential smoothing and ARIMA methods in order to determine the best models to compare to the withheld 52weeks of data and to determine the best forecast model moving forward for each request status. An explanation of the output results and how they were used to determine the best possible models for validation is included in the next chapter. Furthermore, analysis and comparison of error for each status is included with a graph depicting the forecast, validation data, and confidence interval for each request status.

### **Measures of Error and Model Comparison**

The output from the JMP program produces multiple measures to compare against one another by defining error in multiple ways. For example, the JMP model comparison output includes degrees of freedom, variance, Akaike's 'A" Information Criterion (AIC), Schwartz's Bayesian Information Criterion (SBC), RSquare, -2LogLikelihood (-2LogLH), Mean Absolute Percentage Error (MAPE), and Mean Absolute Error (MAE). Whereas the AIC and SBC are used primarily for model comparison, the MAPE and MAE are measures of error to determine how well the model actually represents the historical observations. Each of these measures will be discussed below in more detail; for now, it is important to note that none of these measures can be certain in predicting future values. For that, the model predictions must be compared to data reserved for model validation.

# Degrees of Freedom (DF)

Degrees of Freedom are measured from n (the number of observations in the series) – k (the "number of fitted parameters in the model") (JMP Support, 2018). The model degrees of freedom are normally reduced primarily by the number of forecast periods which are included in the value of k. However, other data alterations like differencing, AR, and MA terms also increase model parameters and reduce the degrees of freedom.

### Variance

An estimate of variance in the model is calculated by dividing the sum squared error (SSE), which is calculated by summing the squared residuals by the degrees of freedom, SSE / (n-k). This results in a sample estimate of the variance or the random changes in the model (JMP Support, 2018).

### *RSquare*

Also known as the coefficient of determination ( $R^2$ ) is calculated by 1-SSE/SST where SSE is the "sum of the squares of the residuals" and SST is the total sum of the squares (JMP Support, 2018).

#### **Equation 16: Sum Squared Error**

n

SSE = 
$$\sum_{i=1}^{n} (y_i - \hat{y}_i)^2$$
 (16)

### **Equation 17: Total Sum of Squares**

SST = 
$$\sum_{i=1}^{n} (y_i - \bar{y}_i)^2$$
 (17)

RSquare is presented as a correlation between the explained variance and total variance of Y values in a squared form to provide a proportion of the variation that can be explained (Makridakis et al., 1998, pp. 199–200). This makes RSquare an especially useful tool when using regression and understanding. For example, it shows how linear variance in y is explained by x. However, it can be a more difficult tool with time series forecasting; which can have an extremely low RSquare's that must be taken into context with other forecasting methods as one piece of the puzzle. One aspect to watch out for is a negative Rsquare resulting from a SSE that is larger than the SST; this result may indicate a poorly fitting model (JMP Support, 2018).

#### -2LogLikelihood

The above measures focus primarily on variance while the next three focus on model comparison. -2LogLikelihood, or -2LogLH as it is depicted in JMP, is an iterative optimization method to find the maximum likelihood of the probability density function (PDF) by observing the actual sample observations given a defined set of parameters (Makridakis et al., 1998). The likelihood function is applied to time series forecasting which also uses historical observational data modeled after a known probability distribution function for a given set of parameters or variables (Myung, 2003, p. 92). The process uses a method called maximum likelihood estimation (MLE) which derives a probability distribution that best fits the data observation in order to determine other selection criteria such as AIC and SBC (Myung, 2003, p. 93). The actual calculation varies depending on the projected PDF and potential for local and global maximums. In practice, -2LogLikelihood is actually taking negative two times the natural log of the likelihood function which is then evaluated "at the best fit parameter estimates with the

smallest value representing a better fit model" (JMP Support, 2018). The major drawback of this parameter estimation technique is that -2LogLikelihood fails to penalize models with excessive parameters. This can result in 'overfitting' a model with additional parameters in order to result in a low -2LogLikelihood value, yet there is no corresponding value in how well the model fits future data; this is where AIC and SBC have value.

# Akaike's "A" Information Criterion (AIC)

The Akaike's "A" Information Criterion (AIC) is a measure of a model's goodness-of-fit which can be used to decide between competing forecast models (Makridakis et al., 1998, p. 589). This is because the AIC estimates the quality of the model by including twice the number of model parameters in the equation, thereby reducing the chance of 'overfitting' or 'underfitting' the model with the lowest AIC representing the best fit model.

Equation 18: Akaike's "A" Information Criterion (AIC)  

$$AIC = -2LogLikelihood + 2k$$
 (18)

Where

### Schwartz's Bayesian Information Criterion (SBC)

The Schwartz's Bayesian Information Criterion (SBC), like AIC, is a measure of a model's goodness-of-fit which can be used to decide between competing forecast models; it is also called an order selection criteria (Makridakis et al., 1998, p. 592). Like AIC, SBC estimates model quality while reducing the chance of 'overfitting' or 'underfitting' with a more complex model. SBC includes both the number of parameters and number of observations in the equation, thereby producing a slightly different result than AIC and often an overall less complex model which should be prone to less error.

Equation 19: Schwartz's Bayesian Information Criterion (SBC)  

$$SBC = -2LogLikelihood + k * ln(n)$$
 (19)

Where

k = Number of estimated parameters in the modeln = Number of observations in the model

With a complete understanding of the above measures of model goodness-of-fit, the researcher utilized JMP software for the model comparisons below. The software output is rank ordered by AIC followed by SBC, both computed using an MLE approach for calculating best fit model parameters using -2LogLikelihood. This approach was used for both forecast model selection and order selection criteria when deciding between multiple ARIMA models.

# **Model Validation**

The above measures of goodness-of-fit are focused on how well each model compares to the historical data it is based upon and not how well the model might fit future data. 52-weeks of data were reserved from each of the five statuses being forecasted for the purpose of forecast validation. The error statistics MAE and MAPE can be used to see which forecasting technique resulted in the least error and should be considered for modeling future requests for that type of status. Each of the five demand statuses was modeled independently and, therefore, have slightly different models for best fit of the data. It also should be noted that all five statuses were transformed by

taking the natural log of the data in order to reduce variance for each of the forecasting methods.

### Mean Absolute Error (MAE)

Although there are several ways of communicating forecast error, this research will focus on two of the most common, mean absolute error (MAE) and mean absolute percentage error (MAPE). Each of these measures of error represents the difference between the actual observation and the prediction or forecast for that observation. The first, mean absolute error, is a simpler representation of error as a measure of the difference between the observation and prediction without regard to the sign of the error (Makridakis et al., 1998, p. 605). The absolute value of the value's difference results in the positive and negative errors not being cancelled out. However, MAE is not particularly good at comparing different sized data sets or unit values; for that MAPE is a better option.

#### Equation 20: Mean Absolute Error (MAE)

$$MAE = \frac{1}{n} \sum_{i=1}^{n} |y_i - \hat{y}_i|$$
(20)

Where

### n = Number of observations in the model

### Mean Absolute Percentage Error (MAPE)

Mean absolute percentage error is the average of all the percentage errors of a data set without regard to the sign of the errors (Makridakis et al., 1998, p. 605). MAPE includes an absolute value so that the positive and negative errors are not cancelled out. Moreover, the percentage of errors allows for better comparison across multiple forecasts of different magnitudes or even units as the percentage error can be compared equally. The drawback is that observations of zero result in an undefined MAPE. Both MAE and MAPE are common measures for forecasting error and are utilized throughout this analysis for their simplicity and commonality.

Equation 21: Mean Absolute Percentage Error (MAPE)

$$MAPE = \frac{100}{n} \sum_{i=1}^{n} \left| \frac{y_i - \hat{y}_i}{y_i} \right|$$
(21)

Where

n = Number of observations in the model

#### Summary

After evaluating the 618 AOC's ARST data provided from Apr 2012 to Mar 2018, the researcher saw patterns and seasonality in the data when separated by request status. The five statuses of Confirmed (the receiver and tanker have agreed to the details of the request which is supported), Bought (the tanker unit has agreed to support the request by clicking "buy" in ARST and, therefore, is supporting the request), Filled (the request is entered with an identified tanker unit in ARST assuming some initial coordination; however, the request may or may not be supported), Published (the request was not entered with an identified tanker unit and was not supported in the system, an unsupported air refueling request), and Cancelled (the request that was cancelled by either the tanker or receiver unit and may or may not have been able to be supported).

These five statuses were then aggregated by weeks from 8 Apr 2012 to 10 Mar 2018 in order to develop 309 weeks of data for the 5 statuses. The researcher then

explored the major time series forecasting methods, including exponential smoothing and ARIMA, to understand how they applied to one another and to determine which might be the most valuable in forecasting future demand by each of the five statuses designated in the data set. The next chapter will apply this methodology along with measures of the error to determine the best potential models to compare to the withheld validation data.

#### **IV. Analysis and Results**

"I have seen the future and it is very much like the present, only longer." - Kehlog Albran

# **Chapter Overview**

This chapter presents the results of the researcher's time series forecasting for each of the five identified statuses represented in the ARST data set described in Chapter 1. The time series forecasting was accomplished in JMP to create models that would best represent the data while forecasting the next 52-weeks of air refueling requests. This analysis was grouped by seven-day weeks starting on the Sunday of each week. Upon initial review, the researcher determined that neither monthly groupings of data nor daily groupings of data were feasible for the analysis method due to small sample size for monthly groupings (72 total) and zero observations on certain days for particular daily groupings. Therefore, the weekly method was chosen for an adequate sample size of 309 observations with no weekly status groupings resulting in zero observations over the seven-day week. Once the data was grouped by week, multiple forecasting methods were applied to the data sets. Each forecasting method results in some error. Therefore, this chapter will begin with a discussion of measures of error and a discussion as to how different methods best fit a forecast for the source data. The best models were then used to forecast 52-weeks of data that was reserved from the initial forecast; this allowed for model validation and analysis of how error as a measure of best fit applied to the forecast data over the validation period.

# **Data Preparation**

In order to utilize Box-Jenkins ARIMA Methodology for the time series forecasting models, the data had to be first analyzed for stationarity and seasonality. The researcher expected seasonal fluctuations over monthly and weekly time periods, which were visible in the data with peaks of requests surrounding April and October and a lack of requests overall in December and January.



# **Figure 3: Monthly Distribution of Air Refueling Requests**

The most apparent cyclical or seasonal fluctuations are seen within the weekly period. Graphing the data by the day of the week clearly shows that most requests are for Tuesday through Thursday, with the fewest for Saturday and Sunday, as shown in Figure 4. This figure represents the daily distribution of all 309 weeks of request data, each represented by a different color line in order to show highs and lows of the requests by the day of the week. Therefore, although there is some monthly seasonal fluctuation in the data, the real cyclical or seasonal activity happens at the weekly level. This observation was confirmed with forecasting models with less error over a 52-week period versus a 12-month period. This drives the 52-period calculations in the forecasting techniques with a seasonal component seen below.





The next aspect to look for in the data was stationarity, which means that the data set's mean and variance are constant throughout the observed time periods in order to apply the ARIMA forecasting techniques. This is important because the larger the variance, the more difficult it will be to predict future occurrences. Therefore, the smaller the variance, the more accurate the forecast should be. One of the easiest ways to check for stationarity is to look at the autocorrelation function (ACF); if the ACF quickly decreases toward zero, the data set is stationary. Alternatively, if the values decrease slowly, as shown below, the data set is non-stationary (Ngo, 2013, p. 1). This example is

from the Confirmed status and depicts the slow decrease in ACF values in the bottom left column, only decreasing within an acceptable margin of error at the 10<sup>th</sup> lag point.



**Figure 5: Non-Stationary Data Set** 

In order to correct for non-stationary data, the first or second difference can be taken to reduce the variance and make the data more constant around the mean, thereby transforming the data into a stationary time series (Ngo, 2013, p. 1). Below is the first difference of the same data set shown above. It now decreases quickly after the first lag and is centered around the mean of -0.53 with a smaller standard deviation.



**Figure 6: Stationary Data Set** 

Differencing is a more complex process when applied to seasonal data. The five data sets being analyzed are non-stationary and seasonal data. Therefore, a data transformation must be applied prior to differencing and application of a forecasting technique in that order to reduce the data variance such as log, exp, square root, etc. (Ngo, 2013, p. 3). The researcher applied all 17 potential data transformation techniques in JMP to each of the five data sets and determined that natural log of the data reduced the variance the most for each of the five seasonal data sets. As an example, below is the first difference of the same Confirmed data set showing the data centered around a mean of effectively zero and a standard deviation of 0.4.



**Figure 7: Seasonal Stationary Data Set** 

# Model Selection, Estimation, and Diagnostics

After the above analysis, the researcher determined that the five air refueling request statuses were seasonal data on a weekly basis to be approximated by 52 equal time periods per year. The air refueling requests were disaggregated into the five statuses of Confirmed, Bought, Filled, Published, and Cancelled because each type of request has its own mean, variance, and specific input variables. A single forecast model could be fit for overall request into the ARST system; however, it would have additional error embedded into the model. Furthermore, the focus of this research was to highlight unsupported air refueling requests. The researcher is assuming that Confirmed and Bought requests are supported, that Filled and Cancelled include both supported and unsupported requests at unknown percentages until the system is updated with that fidelity, and that the Publish status includes all known unsupported requests. Therefore, disaggregation was required to get at the known unsupported requests.

Furthermore, all data sets required seasonal data transformation via taking the natural log of the original data for analysis and raising that number to the number e in order to transform the result back to the original form after the forecasting was complete. Also, all data sets with the exception of the Cancelled time series required differencing to develop stationary data sets. A full breakdown and validation of each data set are set out below.

### **The Confirmed Status**

As discussed in Chapter 1, air refueling requests that have been supported by both the receiver unit and the tanker unit have the Confirmed status and were likely completed

air refueling events. Prior to early April 2012, the only statuses shown in the ARST data were Confirmed and Published which is why the number of requests is very high in the early weeks of data; correspondingly, it is also why the other three statuses are low and take some time to catch up.



Figure 8: Time Series of Weekly Confirmed Status, 08 Apr 12 – 11 Mar 17

The above JMP graph of the Confirmed status over weekly intervals from Sunday through Saturday shows high request numbers in the first few months of 2012 moderating into a more cyclical fashion around the mean as time progresses. Note that this representation has not been transformed in any way and yet still shows a high standard deviation along with seasonality. In order to run multiple forecasting techniques, the data set was transformed with the natural log of the original observation, thus decreasing the variance of the data about the mean.
### **Model Selection**



**Figure 9: Confirmed Status Model Comparison** 

The model comparison chart above allows for the assessment of different forecasting techniques in JMP. Each of the forecasting techniques described in Chapter 3 was applied to the Confirmed data. Moreover, a minimum of 1250 different iterations of seasonal ARIMA permutations was applied in accordance with the Box-Jenkins Methodology (available in Appendix C) in order to arrive at the lowest AIC/SBC values while verifying significant P values for each parameter. In this case, the seasonal ARIMA(2,1,2)(1,1,0)52 with no intercept resulted in the lowest AIC/SBC values along with the lowest MAPE and MAE error values for the data used to develop the forecast from 08 Apr 12 – 11 Mar 17. This indicates that the seasonal ARIMA model is the best fit for the forecast data with the least number of parameters and the least overall error.



Figure 10: Confirmed Status Seasonal ARIMA(2,1,2)(1,1,0)52 No Intercept

Looking closer at the selected seasonal ARIMA model, the basic information about the model is available, and each of the parameters is significant. The AR1,2 parameter is the least of all at just above 3%. However, it is not possible to exclude one AR or MA parameter and keep the other parameters with this type of ARIMA forecasting. Nevertheless, all meet a 95% or greater significance. Additionally, the depicted residuals appear to be evenly distributed about zero and show no clear patterns or skew.

#### Model Validation

Based on the JMP output and forecast measures such as AIC/SBC, the seasonal ARIMA model appears to be the best fit for the data used during the forecast period. However, the only true test of a forecast model is to compare the forecast, for 52-weeks in this case, to actual data not used in the development of the model. The researcher utilized the MAPE and MAE error statistics in order to show which model represents the least error and, therefore, is the most accurate forecast for the subsequent 52-weeks, from 12 March 17 to 10 March 18.

Model	MAPE	MAE
Mean Model / White Noise (0,0,0) + c	4.687855	0.187497
Random Walk Model (0,1,0)	4.605422	0.178326
Simple Exponential Smoothing ( $0 \le \alpha \le 1$ )	4.946746	0.199302
Double (Brown) Linear Exponential Smoothing	5.722216	0.234371
Linear (Holt) Exponential Smoothing	5.359548	0.218370
Damped Trend Linear Exponential Smoothing	4.946746	0.199302
Seasonal Exponential Smoothing (52 weeks, $(0 \le \delta \le 1))$	6.622882	0.268338
Winters Method (Additive)	5.120940	0.205673
Seasonal ARIMA (2,1,2)(1,1,0)52 No Intercept	4.310800	0.173742

 Table 4: Confirmed Status – 52-Week Validation Set Error Statistics

As shown above, the seasonal ARIMA model both provided the best model of the forecast data and resulted in the least error for the 52-week validation data set, leading the researcher to conclude that it was the best model analyzed to forecast future Confirmed air refueling requests.



Figure 11: Confirmed Status Forecast with 95% Confidence Interval

The above graph depicts the entire data set over the 309 weeks, including a 95% confidence interval for the forecast versus the actual request values in the last 52-weeks. In reality, there were 3058 Confirmed requests with a forecast for 2863. Therefore, the forecast was a conservative estimate by 195 requests or 6% less than actual.

# **The Bought Status**

Air refueling requests with the Bought status have been supported by the tanker unit, but not yet Confirmed by the receiver unit. For the purposes of understanding the tanker capacity and ability to support receiver demand, these requests were considered as likely completed air refueling events.



Figure 12: Time Series of Weekly Bought Status, 08 Apr 12 – 11 Mar 17

The above JMP graph of the Bought status over weekly intervals from Sunday through Saturday shows low request numbers through late 2013 as this new status was only included in the ARST since the spring of 2012. Also apparent is some cyclical seasonality. This graph of the Bought data has not been transformed in any way and yet still shows a high standard deviation, along with seasonality, to illustrate the difference from the figure below. In order to run multiple forecasting techniques, this data set was transformed with the natural log of the original observation, thereby decreasing the variance of the data about the mean.



#### Model Selection

Figure 13: Bought Status Model Comparison

The model comparison chart above allows for the assessment of different forecasting techniques in JMP. Each of the forecasting techniques described in Chapter 3 was also applied to the Bought data. Moreover, a minimum of 1250 different iterations of seasonal ARIMA permutations was applied in accordance with the Box-Jenkins Methodology (available in Appendix D) in order to arrive at the lowest AIC/SBC values while verifying significant P values for each parameter. In this case, the seasonal ARIMA(0,1,1)(0,1,1)52 with no intercept resulted in the lowest AIC/SBC values along with the lowest MAPE, although three models resulted in a slightly lower MAE for the data used to develop the forecast from 08 Apr 12 – 11 Mar 17. The lower MAEs are all remarkably close to the seasonal ARIMA's value; this represents something to consider during model validation. Moreover, the model comparison indicates that the seasonal ARIMA model is the best fit for the forecast data with the least number of parameters and the least overall error.



Figure 14: Bought Status Seasonal ARIMA(0,1,1)(0,1,1)52 No Intercept

Looking closer at the selected seasonal ARIMA model, the basic information about the model is available. Each of the parameters is significant and well above a 95% significance level. Additionally, the depicted residuals appear to be evenly distributed about zero and show no clear patterns or skew.

### Model Validation

Based on the JMP output and forecast measures such as AIC/SBC, the seasonal ARIMA model appears to be the best fit for the data used during the forecast period. Again, the 52-weeks of validation data will be used to calculate the lowest MAPE and MAE error statistics. The resulting model represents the least error and, therefore, is the most accurate forecast for the subsequent 52-weeks (12 March 17 to 10 March 18).

Model	MAPE	MAE
Mean Model / White Noise (0,0,0) + c	12.347367	0.535522
Random Walk Model (0,1,0)	7.735098	0.316859
Simple Exponential Smoothing ( $0 \le \alpha \le 1$ )	4.487647	0.180085
Double (Brown) Linear Exponential Smoothing	4.002118	0.160925
Linear (Holt) Exponential Smoothing	9.651085	0.398166
Damped Trend Linear Exponential Smoothing	4.487640	0.180085
Seasonal Exponential Smoothing (52 weeks, $(0 \le \delta \le 1))$	6.040294	0.256403
Winters Method (Additive)	4.033481	0.170903
Seasonal ARIMA (0,1,1)(0,1,1)52 No Intercept	3.918795	0.164768

 Table 5: Bought Status – 52-Week Validation Set Error Statistics

As shown above, the seasonal ARIMA model both provided the best model of the forecast data and resulted in the least MAPE and second lowest MAE error for the 52-week validation data set, leading the researcher to conclude that it was the best model analyzed to forecast future Bought air refueling requests.



Figure 15: Bought Status Forecast with 95% Confidence Interval

The above graph depicts the entire data set over the 309 weeks, including a 95% confidence interval for the forecast versus the actual request values in the last 52-weeks. In reality, there were 3882 Bought requests with a forecast for 4392. Therefore, the forecast overestimated by 510 requests or 13%.

# **The Filled Status**

Air refueling requests submitted with tanker details and the presumption of precoordination between the receiver and tanker unit have the Filled status. The challenge with this category is that receivers can enter tanker unit information from which they intend to receive air refueling service without any knowledge about whether the tanker unit can actually support the AR event. Therefore, some of the requests in the Filled status were probably supported by the tanker units and others were not. It is just not possible with the current database information to determine how much demand was unsupported.



Figure 16: Time Series of Weekly Filled Status, 08 Apr 12 - 11 Mar 17

The above JMP graph of the Filled status over weekly intervals from Sunday through Saturday shows low request numbers through late 2013 as this new status was only included in the ARST since the spring of 2012. Some cyclical seasonality is apparent, along with very high numbers from 2014 to 2015 as users gained an understanding of the status. This graph of the Filled data has not been transformed in any way and yet still shows a high standard deviation along with seasonality to illustrate the difference from the figure below. In order to run multiple forecasting techniques, this data set was transformed with the natural log of the original observation decreasing the variance of the data about the mean.

### **Model Selection**

	<b>1</b>		Mean Std N Zero Mean Single Mea Trend ADF	4.93 0.58 ADF -0.2 n ADF -5.4 -6.	84988 65371 257 16586 77023 18176					
01Jan2013 01Jan2014 01Jan2015 01Jar Week Of	n2016	01Jan2017								
Time Series Basic Diagnostics Model Comparison										
Time Series Basic Diagnostics Model Comparison Model	DF	Variance	AIC	SBC	RSquare	-2LogLH	AIC Rank	SBC Rank	MAPE	MAE
Time Series Basic Diagnostics Model Comparison Model Seasonal ARIMA(0, 1, 1)(1, 0, 1)52 No Intercept	<b>DF</b> 253	<b>Variance</b> 0.0546544	AIC 12.670775	<b>SBC</b> 23.306307	RSquare 0.797	-2LogLH 6.6707746	AIC Rank	SBC Rank	<b>MAPE</b> 3.920432	<b>MAE</b> 0.179548
Time Series Basic Diagnostics Model Comparison Model Seasonal ARIMA(0, 1, 1)(1, 0, 1)52 No Intercept Seasonal Exponential Smoothing(52, Zero to One )	DF 253 202	Variance 0.0546544 0.0626176	AIC 12.670775 30.070623	<b>SBC</b> 23.306307 36.706863	<b>RSquare</b> 0.797 0.455	-2LogLH 6.6707746 26.070623	AIC Rank	SBC Rank	<b>MAPE</b> 3.920432 3.715940	<b>MAE</b> 0.179548 0.187774
Time Series Basic Diagnostics Model Comparison Model Seasonal ARIMA(0, 1, 1)(1, 0, 1)52 No Intercept Seasonal Exponential Smoothing( 52, Zero to One ) Winters Method (Additive)	DF 253 202 201	Variance 0.0546544 0.0626176 0.0629291	AIC 12.670775 30.070623 32.070624	<b>SBC</b> 23.306307 36.706863 42.024984	<b>RSquare</b> 0.797 0.455 0.455	-2LogLH 6.6707746 26.070623 26.070624	AIC Rank 1 2 3	<b>SBC Rank</b> 1 2 3	<b>MAPE</b> 3.920432 3.715940 3.715940	<b>MAE</b> 0.179548 0.187774 0.187774
Time Series Basic Diagnostics Model Comparison Seasonal ARIMA(0, 1, 1)(1, 0, 1)52 No Intercept Seasonal Exponential Smoothing( 52, Zero to One ) Winters Method (Additive) Simple Exponential Smoothing( Zero to One )	DF 253 202 201 255	Variance 0.0546544 0.0626176 0.0629291 0.0871012	AIC 12.670775 30.070623 32.070624 103.37278	SBC 23.306307 36.706863 42.024984 106.91796	<b>RSquare</b> 0.797 0.455 0.455 0.736	-2LogLH 6.6707746 26.070623 26.070624 101.37278	AIC Rank 1 2 3 4	<b>SBC Rank</b> 1 2 3 4	<b>MAPE</b> 3.920432 3.715940 3.715940 4.415334	MAE 0.179548 0.187774 0.187774 0.202960
Time Series Basic Diagnostics Model Comparison Model Seasonal ARIMA(0, 1, 1)(1, 0, 1)52 No Intercept Seasonal Exponential Smoothing (52, Zero to One ) Winters Method (Additive) Simple Exponential Smoothing (Zero to One ) Damped-Trend Linear Exponential Smoothing	DF 253 202 201 255 253	Variance 0.0546544 0.0626176 0.0629291 0.0871012 0.0877897	AIC 12.670775 30.070623 32.070624 103.37278 107.37278	SBC 23.306307 36.706863 42.024984 106.91796 118.00832	<b>RSquare</b> 0.797 0.455 0.455 0.736 0.736	-2LogLH 6.6707746 26.070623 26.070624 101.37278 101.37278	AIC Rank 1 2 3 4 5	<b>SBC Rank</b> 1 2 3 4 5	MAPE 3.920432 3.715940 3.715940 4.415334 4.415334	MAE 0.179548 0.187774 0.187774 0.202960 0.202960
Time Series Basic Diagnostics Model Comparison Seasonal ARIMA(0, 1, 1)(1, 0, 1)52 No Intercept Seasonal Exponential Smoothing( 52, Zero to One ) Winters Method (Additive) Simple Exponential Smoothing (Zero to One ) Damped-Trend Linear Exponential Smoothing Linear (Holt) Exponential Smoothing	DF 253 202 201 255 253 253	Variance 0.0546544 0.0626176 0.0629291 0.0871012 0.0877897 0.0877213	AIC 12.670775 30.070623 32.070624 103.37278 107.37278 111.93534	SBC 23.306307 36.706863 42.024984 106.91796 118.00832 119.01787	<b>RSquare</b> 0.797 0.455 0.455 0.736 0.736 0.713	-2LogLH 6.6707746 26.070624 101.37278 101.37278 107.93534	AIC Rank 1 2 3 4 5 6	<b>SBC Rank</b> 1 2 3 4 5 6	MAPE 3.920432 3.715940 3.715940 4.415334 4.415334 4.535114	MAE 0.179548 0.187774 0.202960 0.202960 0.202960 0.205382
Time Series Basic Diagnostics Model Comparison Model Seasonal ARIMA(0, 1, 1)(1, 0, 1)52 No Intercept Seasonal Exponential Smoothing (52, Zero to One ) Winters Method (Additive) Simple Exponential Smoothing (Zero to One ) Damped-Trend Linear Exponential Smoothing Linear (Holt) Exponential Smoothing Double (Brown) Exponential Smoothing	DF 253 202 201 255 253 253 253 254	Variance 0.0546544 0.0626176 0.0629291 0.0877012 0.0877897 0.0877213 0.0897262	AIC 12.670775 30.070623 32.070624 103.37278 107.37278 111.93534 115.90153	SBC 23.306307 36.706863 42.024984 106.91796 118.00832 119.01787 119.44279	<b>RSquare</b> 0.797 0.455 0.455 0.736 0.736 0.713 0.707	-2LogLH 6.6707746 26.070623 26.070624 101.37278 101.37278 107.93534 113.90153	AIC Rank 1 2 3 4 5 6 7	5BC Rank 1 2 3 4 5 6 7	MAPE 3.920432 3.715940 3.715940 4.415334 4.415334 4.535114 4.535114	MAE 0.179548 0.187774 0.202960 0.202960 0.205382 0.209191
Time Series Basic Diagnostics Model Comparison Seasonal ARIMA(0, 1, 1)(1, 0, 1)52 No Intercept Seasonal Exponential Smoothing (52, Zero to One ) Winters Method (Additive) Simple Exponential Smoothing (Zero to One ) Damped-Trend Linear Exponential Smoothing Linear (Holt) Exponential Smoothing Double (Brown) Exponential Smoothing (1) No Intercept	DF 253 202 201 255 253 253 253 254 256	Variance 0.0546544 0.0626176 0.0629291 0.0871012 0.0877817 0.0877213 0.0897262 0.1148503	AIC 12.670775 30.070623 32.070624 103.37278 107.37278 111.93534 115.90153 172.48033	SBC 23.306307 36.706863 42.024984 106.91796 118.00832 119.01787 119.44279 172.48033	<b>RSquare</b> 0.797 0.455 0.455 0.736 0.736 0.733 0.707 0.653	-2LogLH 6.6707746 26.070623 26.070624 101.37278 107.93534 113.90153 172.48033	AIC Rank 1 2 3 4 5 6 7 8	SBC Rank 1 2 3 4 5 6 7 8	MAPE 3.920432 3.715940 3.715940 4.415334 4.415334 4.535114 4.598607 5.115907	MAE 0.179548 0.187774 0.202960 0.202960 0.205382 0.209191 0.234225

Figure 17: Filled Status Model Comparison

The model comparison chart above allows for the assessment of different forecasting techniques in JMP. Each of the forecasting techniques described in Chapter 3 was also applied to the Filled data. Moreover, a minimum of 1250 different iterations of seasonal ARIMA permutations was applied in accordance with the Box-Jenkins Methodology (available in Appendix E) in order to arrive at the lowest AIC/SBC values while verifying significant P values for each parameter. In this case, the seasonal ARIMA(0,1,1)(1,0,1)52 with no intercept resulted in the lowest AIC/SBC values along with the lowest MAE, although two other models resulted in a slightly lower MAPE for the data used to develop the forecast from 08 Apr 12 to 11 Mar 17. The lower MAPE values remained relatively close to the seasonal ARIMA's MAPE error value. This indicates that the seasonal ARIMA model is the best fit for the forecast data with the least number of parameters and the least overall error.



Figure 18: Filled Status Seasonal ARIMA(0,1,1)(1,0,1)52 No Intercept

Again, looking closer at the selected Filled seasonal ARIMA model, the basic information about the model is available. Each of the parameters is significant and well above a 95% significance level; the lowest is the MA2,52 parameter, which is only 1.3%, still significant and required to maintain the MA1,1 parameter. Additionally, the depicted residuals appear to be evenly distributed about zero and show no clear patterns or skew.

# Model Validation

Based on the JMP output and forecast measures such as AIC/SBC, the Filled seasonal ARIMA model appears to be the best fit for the data used during the forecast period. Again, the 52-weeks of validation data will be used to calculate the lowest MAPE and MAE error statistics. The resulting model represents the least error and, therefore, is the most accurate forecast for the subsequent 52-weeks (12 March 17 to 10 March 18).

Model	MAPE	MAE
Mean Model / White Noise (0,0,0) + c	3.936391	0.195276
Random Walk Model (0,1,0)	3.355093	0.159267
Simple Exponential Smoothing ( $0 \le \alpha \le 1$ )	3.102410	0.147881
Double (Brown) Linear Exponential Smoothing	4.418292	0.211397
Linear (Holt) Exponential Smoothing	3.319933	0.157722
Damped Trend Linear Exponential Smoothing	3.102410	0.147881
Seasonal Exponential Smoothing (52 weeks, $(0 \le \delta \le 1)$	3.734782	0.185105
Winters Method (Additive)	3.734782	0.185105
Seasonal ARIMA (0,1,1)(1,0,1)52 No Intercept	2.222276	0.108962

Table 6: Filled Status – 52-Week Validation Set Error Statistics

As shown above, the seasonal ARIMA model both provided the best model of the forecast data and resulted in the least error for the 52-week validation data set, leading the

researcher to conclude that it was the best model analyzed to forecast future Filled air refueling requests.



Figure 19: Filled Status Forecast with 95% Confidence Interval

The above graph depicts the entire data set over the 309 weeks, including a 95% confidence interval for the forecast versus the actual request values in the last 52-weeks. Of note, there were 8223 Filled requests with a forecast for 7970. Therefore, the forecast was a conservative estimate by 253 requests or 3% less than actual.

# The Published Status

Air refueling requests that remain in the Published status are those requests with no tanker details and that are understood to have been unsupported in the system. This category is in short term focus for this research because, unlike the Filled status and the Cancelled status yet to come, where it was unclear how many requests went unsupported, all of those requests that remained in the Published status were unsupported by tanker units.

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Figure 20: Time Series of Weekly Published Status, 08 Apr 12 - 11 Mar 17

The above JMP graph of the Published status over weekly intervals from Sunday through Saturday shows some higher request numbers in 2012 as the system moved away from Confirmed and Published being the only possible categories. The data is fairly consistent about the mean, with some cyclical seasonality. The above graph of the Published data has not been transformed in any way and yet still shows a high standard deviation along with seasonality to illustrate the difference from the figure below. In order to run multiple forecasting techniques, this data set was transformed with the natural log of the original observation decreasing the variance of the data about the mean.

#### Model Selection

45 4 35 3 2 25 2 15 5	N.	تعجنل	Mean Std N Zero Mean Single Mea Trend ADF	3.62 0.45 ADF -1.1 n ADF -9.5 -9.7	15343 11277 257 04706 24027 64479					
01Jan2013 01Jan2014 01Jan2015 01Jar Week Of ime Series Basic Diagnostics Aodel Comparison	12016	01Jan2017								
Madal	DE	Variance	AIC	CP/C	DCausara	21	ALC Bank	CBC Dank	MADE	MAE
Model	DF	Variance	AIC	SBC	RSquare	-2LogLH	AIC Rank	SBC Rank	MAPE	MAE
Model Seasonal ARIMA(1, 1, 2)(1, 1, 0)52	DF 199	Variance	AIC	SBC 224.68159	<b>RSquare</b> 0.093	-2LogLH	AIC Rank	SBC Rank	<b>MAPE</b> 9.246431	MAE 0.316599
Model Seasonal ARIMA(1, 1, 2)(1, 1, 0)52 Seasonal Exponential Smoothing( 52, Zero to One )	DF 199 202	Variance 0.1435461 0.1405198	AIC 208.09099 220.52234	SBC 224.68159 227.15858 222.47670	<b>RSquare</b> 0.093 0.002	-2LogLH 198.09099 216.52234	AIC Rank	SBC Rank	MAPE 9.246431 9.351976	MA 0.316599 0.31995
Model Seasonal ARIMA(1, 1, 2)(1, 1, 0)52 Seasonal Exponential Smoothing( 52, Zero to One ) Winters Method (Additive) Finando Exponential Careo thing( 2ero to One )	DF 199 202 201	Variance 0.1435461 0.1405198 0.1412189	AIC 208.09099 220.52234 222.52234 258.66317	<b>SBC</b> 224.68159 227.15858 232.47670 262.20834	<b>RSquare</b> 0.093 0.002 0.002	-2LogLH 198.09099 216.52234 216.52234	AIC Rank 1 2 3	SBC Rank	MAPE 9.246431 9.351976 9.351976 8.904626	MAI 0.316599 0.31995 0.31995
Model Seasonal ARIMA(1, 1, 2)(1, 1, 0)52 Seasonal Exponential Smoothing( 52, Zero to One ) Winters Method (Additive) Simple Exponential Smoothing( Zero to One )	DF 199 202 201 255 253	Variance 0.1435461 0.1405198 0.1412189 0.1596962	AIC 208.09099 220.52234 222.52234 258.66317	SBC 224.68159 227.15858 232.47670 262.20834 273.20870	<b>RSquare</b> 0.093 0.002 0.002 0.218	-2LogLH 198.09099 216.52234 216.52234 256.66317	AIC Rank 1 2 3 4	<b>SBC Rank</b> 1 2 3 4	MAPE 9.246431 9.351976 9.351976 8.994626 8.994626	MA 0.31659 0.31995 0.31995 0.28830
Model Seasonal ARIMA(1, 1, 2)(1, 1, 0)52 Seasonal Exponential Smoothing( 52, Zero to One ) Winters Method (Additive) Simple Exponential Smoothing( Zero to One ) Damped Trend Linear Exponential Smoothing	DF 199 202 201 255 253	Variance 0.1435461 0.1405198 0.1412189 0.1596962 0.1609586	AIC 208.09099 220.52234 222.52234 258.66317 262.66317	SBC 224.68159 227.15858 232.47670 262.20834 273.29870	<b>RSquare</b> 0.093 0.002 0.002 0.218 0.218	-2LogLH 198.09099 216.52234 216.52234 256.66317 256.66317	AIC Rank 1 2 3 4 5	<b>SBC Rank</b> 1 2 3 4 5 5	MAPE 9.246431 9.351976 9.351976 8.994626 8.994626 9.4304626	MA 0.31659 0.31995 0.31995 0.288300 0.288300
Model Seasonal ARIMA(1, 1, 2)(1, 1, 0)52 Seasonal Exponential Smoothing( 52, Zero to One ) Winters Method (Additive) Simple Exponential Smoothing( Zero to One ) Damped-Trend Linear Exponential Smoothing Linear (Holt) Exponential Smoothing	DF 199 202 201 255 253 253 253	Variance 0.1435461 0.1405198 0.1412189 0.1596962 0.1609586 0.1609765	AIC 208.09099 220.52234 225.52234 258.66317 262.66317 268.75299	<b>SBC</b> 224.68159 227.15858 232.47670 262.20834 273.29870 275.83552	<b>RSquare</b> 0.093 0.002 0.218 0.218 0.193 0.193	-2LogLH 198.09099 216.52234 216.52234 256.66317 256.66317 264.75299	AIC Rank 1 2 3 4 5 6	<b>SBC Rank</b> 1 2 3 4 5 6 7	MAPE 9.246431 9.351976 9.351976 8.994626 8.994626 9.130169	MAI 0.316599 0.31995 0.31995 0.288300 0.288300 0.288300 0.295633
Model Seasonal ARIMA(1, 1, 2)(1, 1, 0)52 Seasonal Exponential Smoothing(52, Zero to One ) Winters Method (Additive) Simple Exponential Smoothing(Zero to One ) Damped-Trend Linear Exponential Smoothing Linear (Holt) Exponential Smoothing Double (Brown) Exponential Smoothing	DF 199 202 201 255 253 253 253 254	Variance 0.1435461 0.1405198 0.1412189 0.1596962 0.1609586 0.1609765 0.1694224	AIC 208.09099 220.52234 222.52234 258.66317 262.66317 268.75299 278.14542	<b>SBC</b> 224.68159 227.15858 232.47670 262.20834 273.29870 275.83552 281.68669	RSquare 0.093 0.002 0.218 0.218 0.193 0.159	-2LogLH 198.09099 216.52234 216.52234 256.66317 256.66317 264.75299 276.14542	AIC Rank 1 2 3 4 5 6 7 7	<b>SBC Rank</b> 1 2 3 4 5 6 7 7 7	MAPE 9.246431 9.351976 9.351976 8.994626 8.994626 9.130169 9.263584	MAI 0.316599 0.319951 0.288300 0.288300 0.295633 0.298679
Model Seasonal ARIMA(1, 1, 2)(1, 1, 0)52 Seasonal Exponential Smoothing( 52, Zero to One ) Winters Method (Additive) Simple Exponential Smoothing( Zero to One ) Damped-Trend Linear Exponential Smoothing Linear (Holt) Exponential Smoothing Double (Brown) Exponential Smoothing ARIMA(0, 0, 0)	DF 199 202 201 255 253 253 253 254 256	Variance 0.1435461 0.1405198 0.1412189 0.1596962 0.1609586 0.1609765 0.1694224 0.2043111	AIC 208.09099 220.52234 222.52234 258.66317 262.66317 268.75299 278.14542 322.18787	<b>SBC</b> 224.68159 227.15858 232.47670 262.20834 273.29870 275.83552 281.68669 325.73695	<b>RSquare</b> 0.093 0.002 0.218 0.218 0.193 0.159 -0.00	-2LogLH 198.09099 216.52234 216.52234 256.66317 256.66317 264.75299 276.14542 320.18787	AIC Rank 1 2 3 4 5 6 7 8	SBC Rank 1 2 3 4 5 6 7 8	MAPE 9.246431 9.351976 9.351976 8.994626 9.130169 9.263584 10.329647	MA 0.316599 0.31995 0.288300 0.288300 0.288300 0.295633 0.298679 0.335868

#### Figure 21: Published Status Model Comparison

The model comparison chart above allows for the assessment of different forecasting techniques in JMP. Each of the forecasting techniques described in Chapter 3 was also applied to the Filled data. Moreover, a minimum of 1250 different iterations of seasonal ARIMA permutations was applied in accordance with the Box-Jenkins Methodology (available in Appendix F) in order to arrive at the lowest AIC/SBC values while verifying significant P values for each parameter. In this case, the seasonal ARIMA(1,1,2)(1,1,0)52 resulted in the lowest AIC/SBC values but not the lowest MAPE and MAE error statistics. In fact, the seasonal ARIMA model was 4<sup>th</sup> and 5<sup>th</sup> lowest for the data used to develop the forecast from 08 Apr 12 to 11 Mar 17. This indicates that the seasonal ARIMA model is the best fit for the forecast data with the least number of parameters but with slightly higher error values than represented by other models. The true test of the better model will be a comparison of the error values that result from the validation set presented below.



Figure 22: Published Status Seasonal ARIMA(1,1,2)(1,1,0)52

Again, looking closer at the selected Filled seasonal ARIMA model, the basic information about the model is available. Each of the parameters is significant and well above a 95% significance level. Additionally, the depicted residuals appear to be evenly distributed about zero and show no clear patterns or skew.

# Model Validation

Based on the JMP output and forecast measures such as AIC/SBC, the Published seasonal ARIMA model again appears to be the best fit for the data used during the forecast period. Again, the 52-weeks of validation data will be used to calculate the lowest MAPE and MAE error statistics. The resulting model represents the least error and, therefore, is the most accurate forecast for the subsequent 52-weeks from 12 March 17 to 10 March 18.

Model	MAPE	MAE
Mean Model / White Noise (0,0,0) + c	11.397139	0.473538
Random Walk Model (0,1,0)	16.486559	0.682855
Simple Exponential Smoothing ( $0 \le \alpha \le 1$ )	9.216920	0.383172
Double (Brown) Linear Exponential Smoothing	7.459565	0.308530
Linear (Holt) Exponential Smoothing	10.420547	0.433338
Damped Trend Linear Exponential Smoothing	9.216920	0.383172
Seasonal Exponential Smoothing (52 weeks, $(0 \le \delta \le 1))$	9.062291	0.373491
Winters Method (Additive)	9.062291	0.373491
Seasonal ARIMA (1,1,2)(1,1,0)52	6.762654	0.276404

Table 7: Published Status – 52-Week Validation Set Error Statistics

As shown above, the seasonal ARIMA model both provided the best model of the forecast data and resulted in the least error for the 52-week validation data set, leading the researcher to conclude that it was the best model analyzed to forecast future Published air refueling requests.



Figure 23: Published Status Forecast with 95% Confidence Interval The above graph depicts the entire data set over the 309 weeks, including a 95% confidence interval for the forecast versus the actual request values in the last 52-weeks. In reality, there were 3150 Filled requests with a forecast for 2897. Therefore, the forecast was a conservative estimate by 253 requests or 8% less than actual.

# The Cancelled Status

Air refueling requests that remain in the Cancelled status are those requests that were cancelled by either the tanker or receiver unit. Unfortunately, the system fails to account for why an AR event was cancelled, which could be for a number of reasons to include the inability of the tanker unit to support the air refueling event. Therefore, there are certainly unsupported requests in the Cancelled status that cannot be accounted for at this time.



Figure 24: Time Series of Weekly Cancelled Status, 08 Apr 12 – 11 Mar 17

The above JMP graph of the Cancelled status over weekly intervals from Sunday through Saturday shows a fairly consistent distribution of data about the mean with some cyclical seasonality. The above graph of the Cancelled data has not been transformed in any way and yet still shows a high standard deviation along with seasonality to illustrate the difference from the figure below. In order to run multiple forecasting techniques, this data set was transformed with the natural log of the original observation decreasing the variance of the data about the mean.

# Model Selection

r										
4.5 4.5 4.5 3.5 2.5 01Jan2013 01Jan2014 01Jan2015 01Jar	2016	01Jan2017	Mean Std N Zero Mean Single Mea Trend ADF	4.02 0.35 ADF -0.4 n ADF -9. -9.7	08277 33596 257 84238 .67892 704816					
ime Series Basic Diagnostics										
Iodel Comparison	DF	Variance	AIC	SBC	RSquare	-2LogLH	AIC Rank	SBC Rank	MAPE	MAE
Nodel Comparison Model	<b>DF</b>	Variance	AIC	SBC	RSquare	-2LogLH	AIC Rank	SBC Rank	<b>MAPE</b>	MAE
Model Comparison Model Seasonal ARIMA(2, 0, 0)(2, 0, 0)52 Seasonal Exponential Smoothing(52, Zero to One.)	DF 252 202	Variance 0.0819538 0.0757818	AIC 108.75940 138.22695	SBC 126.50478 144.86319	<b>RSquare</b> 0.284	-2LogLH 98.759397 134.22695	AIC Rank	SBC Rank	<b>MAPE</b> 5.799628 6.666302	MAI 0.22258 0.262050
Aodel Comparison Model Seasonal ARIMA(2, 0, 0)(2, 0, 0)52 Seasonal Exponential Smoothing( 52, Zero to One ) Winters Method (Additive)	DF 252 202 201	Variance 0.0819538 0.0757818 0.0761588	AIC 108.75940 138.22695 140.22695	SBC 126.50478 144.86319 150.18131	<b>RSquare</b> 0.284 -0.14	-2LogLH 98.759397 134.22695 134.22695	AIC Rank	SBC Rank	MAPE 5.799628 6.666302 6.666302	MA 0.22258 0.262050 0.262050
Nodel Comparison Model Seasonal ARIMA(2, 0, 0)(2, 0, 0)52 Seasonal Exponential Smoothing( 52, Zero to One ) Winters Method (Additive) Simple Exonential Smoothing( Zero to One )	DF 252 202 201 255	Variance 0.0819538 0.0757818 0.0761588 0.1106493	AIC 108.75940 138.22695 140.22695 164.39601	SBC 126.50478 144.86319 150.18131 167.94119	<b>RSquare</b> 0.284 -0.14 -0.14	-2LogLH 98.759397 134.22695 134.22695 162.39601	AIC Rank	<b>SBC Rank</b> 1 2 3 4	MAPE 5.799628 6.666302 6.666302 6.729038	MA 0.22258 0.26205 0.26205 0.26205
Model           Seasonal ARIMA(2, 0, 0)(2, 0, 0)52           Seasonal Exponential Smoothing(52, Zero to One )           Winters Method (Additive)           Simple Exponential Smoothing(Zero to One )           Damped: Trand Linear Exponential Smoothing	DF 252 202 201 255 253	Variance 0.0819538 0.0757818 0.0761588 0.1106493 0.1115246	AIC 108.75940 138.22695 140.22695 164.39601 168.39608	SBC 126.50478 144.86319 150.18131 167.94119 179.03161	<b>RSquare</b> 0.284 -0.14 -0.14 0.095 0.095	-2LogLH 98.759397 134.22695 134.22695 162.39601 162.39608	AIC Rank 1 2 3 4 5	SBC Rank 1 2 3 4	MAPE 5.799628 6.666302 6.666302 6.729038 6.729088	MA 0.22258 0.26205 0.26205 0.25870 0.25870
Model Comparison Model Seasonal ARIMA(2, 0, 0)(2, 0, 0)52 Seasonal Exponential Smoothing( 52, Zero to One ) Winters Method (Additive) Simple Exponential Smoothing( Zero to One ) Dauble (Rown) Exponential Smoothing	DF 252 202 201 255 253 254	Variance 0.0819538 0.0757818 0.0761588 0.1106493 0.1115246 0.120478	AIC 108.75940 138.22695 140.22695 164.39601 168.39608 189.64748	<b>SBC</b> 126.50478 144.86319 150.18131 167.94119 179.03161 193.18874	<b>RSquare</b> 0.284 -0.14 -0.14 0.095 0.095 -0.03	-2LogLH 98.759397 134.22695 134.22695 162.39601 162.39608 187.64748	AIC Rank 1 2 3 4 5 6	<b>SBC Rank</b> 1 2 3 4 5 6	MAPE 5.799628 6.666302 6.729038 6.729038 7.192615	MA 0.22258 0.26205 0.26205 0.25870 0.25870 0.25870
Model Comparison           Seasonal ARIMA(2, 0, 0)(2, 0, 0)52           Seasonal Exponential Smoothing( 52, Zero to One )           Winters Method (Additive)           Simple Exponential Smoothing (Zero to One )           Damped-Trend Linear Exponential Smoothing           Double (Brown) Exponential Smoothing           ABIMA(0, 0, 0)	DF 252 202 201 255 253 254 256	Variance 0.0819538 0.0757818 0.0761588 0.1106493 0.1115246 0.120478 0.1253508	AIC 108.75940 138.22695 140.22695 164.39601 168.39608 189.64748 196.63618	SBC 126.50478 144.86319 150.18131 167.94119 179.03161 193.18874 20018525	<b>RSquare</b> 0.284 -0.14 -0.14 0.095 0.095 -0.03 -0.03	-2LogLH 98.759397 134.22695 134.22695 162.39601 162.39608 182.39608 182.64748 194.63618	AIC Rank 1 2 3 4 5 6 7	SBC Rank 1 2 3 4 5 6 7	MAPE 5.799628 6.666302 6.729038 6.729088 7.192615 7.034649	MA 0.22258 0.26205 0.26205 0.25870 0.25870 0.25870 0.27628 0.26810
Model           Seasonal ARIMA(2, 0, 0)(2, 0, 0)52           Seasonal Exponential Smoothing (52, Zero to One )           Winters Method (Additive)           Simple Exponential Smoothing (Zero to One )           Damped-Trend Linear Exponential Smoothing           Double (Brown) Exponential Smoothing           IDuble (Brown) Exponential Smoothing           V(1) No Intercent	DF 252 202 201 255 253 254 256 256	Variance 0.0819538 0.0757818 0.0761588 0.1106493 0.1115246 0.120478 0.1225508 0.1313543	AIC 108.75940 138.22695 140.22695 164.39601 168.39608 189.64748 196.63618 206.85320	SBC 126.50478 144.86319 150.18131 167.94119 179.03161 193.18874 200.18525 206.85320	<b>RSquare</b> 0.284 -0.14 -0.14 0.095 0.095 -0.03 -0.00 -0.00	-2LogLH 98,759397 134.22695 134.22695 162.39601 162.39608 187.64748 194.63618 206.8532	AIC Rank 1 2 3 4 5 6 7 8	SBC Rank 1 2 3 4 5 6 7 8	MAPE 5.799628 6.666302 6.666302 6.729038 6.729088 7.192615 7.034649 7.133811	MAI 0.22258 0.262050 0.262050 0.258708 0.258708 0.258708 0.268104 0.276278

#### **Figure 25: Cancelled Status Model Comparison**

The model comparison chart above allows for the assessment of different forecasting techniques in JMP. Each of the forecasting techniques described in Chapter 3 was also applied to the Filled data. Moreover, a minimum of 1250 different iterations of seasonal ARIMA permutations was applied in accordance with the Box-Jenkins Methodology (available in Appendix G) in order to arrive at the lowest AIC/SBC values while verifying significant P values for each parameter. In this case, the seasonal ARIMA(2,0,0)(2,0,0)52 resulted in the lowest AIC/SBC values and the lowest MAPE and MAE error statistics used to develop the forecast from 08 Apr 12 to 11 Mar 17. This indicates that the seasonal ARIMA model is the best fit for the forecast data with the least number of parameters and the least overall error. Of note, the Cancelled status was the only one of the five that did not require differencing in order to reduce the variance for the forecasting techniques.



Figure 26: Cancelled Status Seasonal ARIMA(2,0,0)(2,0,0)52

Again, looking closer at the selected Filled seasonal ARIMA model, the basic information about the model is available. Each of the parameters is significant and well above a 95% significance level. Of note, the AR,1,2 and AR2,52 are the highest at 3.1% and 1.2%, respectively, although both still significant at the 95% level. Additionally, the depicted residuals appear to be evenly distributed about zero with a notable low outlier around Christmas 2012.

# Model Validation

Based on the JMP output and forecast measures such as AIC/SBC, the Cancelled seasonal ARIMA model again appears to be the best fit for the data used during the forecast period. Again, the 52-weeks of validation data will be used to calculate the lowest MAPE and MAE error statistics. The resulting model represents the least error and, therefore, is the most accurate forecast for the subsequent 52-weeks from 12 March 17 to 10 March 18.

Model	MAPE	MAE
Mean Model / White Noise (0,0,0) + c	5.992025	0.236018
Random Walk Model (0,1,0)	13.984956	0.539294
Simple Exponential Smoothing ( $0 \le \alpha \le 1$ )	8.129361	0.307461
Double (Brown) Linear Exponential Smoothing	30.313196	1.197508
Linear (Holt) Exponential Smoothing	6.412654	0.255705
Damped Trend Linear Exponential Smoothing	8.138256	0.307804
Seasonal Exponential Smoothing (52 weeks, $(0 \le \delta \le 1)$	7.216771	0.283724
Winters Method (Additive)	7.216771	0.283724
Seasonal ARIMA (2,0,0)(2,0,0)52	5.727990	0.228003

Table 8: Cancelled Status – 52-Week Validation Set Error Statistics

As shown above, the seasonal ARIMA model both provided the best model of the forecast data and resulted in the least error for the 52-week validation data set, leading the researcher to conclude that it was the best model analyzed to forecast future Cancelled air refueling requests.





The above graph depicts the entire data set overall 309 weeks, including a 95% confidence interval for the forecast versus the actual request values in the last 52-weeks. Of note, there were 3227 Filled requests with a forecast for 2931. Therefore, the forecast was a conservative estimate by 296 requests or 9% less than actual. Significantly, the Cancelled status was much more volatile than the other statuses due to increased unpredictability resulting in more forecasting error.

# Forecasting

As stated above, each of the five ARST request statuses was best modeled by seasonal ARIMA models as verified by the lowest overall MAPE and MAE error when applied to the validation data set. There were differences in the parameters of the seasonal model between each of the statuses but, overall, the seasonal ARIMA proved to be the best model for forecasting future air refueling requests in the ARST as shown in the consolidated Table 9 below. The number of air refueling requests on a weekly or annual basis is helpful for long-term planners and answers the researcher's first research question, about which forecasting technique is best to predict future demand, seasonal ARIMA in this case. Since air refueling is accounted for financially by the number of airframe hours utilized in the expenditure of the service, flight hours executed is a useful metric for communicating future demand in financial terms. Therefore, the researcher's second question applies to methods to convert the number of forecast requests into the number of requested hours for air refueling with a reasonable degree of certainty.

Confirm	ed Status	Bought Status		Filled Status		Published Status		Cancelle	ed Status
Actual	Predicted	Actual	Predicted	Actual	Actual Predicted Actual Predicted		Actual	Predicted	
3058	2863	3882	4392	8223	7970	3150	2897	3227	2931
1	95	-5	10	2	53	2	53	2	96
6	%	-1	3%	3	8%	8	8%		%
Total	Actual Req	uests	215	540	Total P	redicted R	equests	21	053
Difference (Actual - Predicted) 487									
	Percent Change 2%								

Table 9: Air Refueling Requests by ARST Status, Difference, and Percent Change

To attempt to answer this second research question, the researcher analyzed multiple statistical measures of the hours requested in the ARST. The use of statistical measures as a method was chosen because a number of factors go into a specific hourly request for AR, to include the size or number of aircraft to be refueled, length of the track, operational or training requirements, etc. Furthermore, these varied requirements do not necessarily have anything to do with when a request is made in the time series or what status it is assigned, unlike the requests by the statuses that were forecast above. Therefore, multiple mean and median statistical figures were analyzed to determine the best hourly predictor based on the number of forecast requests by status. The researcher

analyzed the statistical measures of one year, two years, three years, four years, and all data prior to the forecast period beginning 12 Mar 2017. These distributions and summary statistics are available in their entirety in Appendixes H - L. Furthermore, the researcher also provided hourly duration requests for the 52-week validation period from 12 Mar 17 - 10 Mar 18 in Appendix M for comparison. By plotting the hourly duration requests and reviewing the summary statistics for each category, it is apparent that there is a wide range of values from zero to several thousand. This results from input error or potentially from users not understanding or not seriously considering the actual duration of air refueling they are requesting. Whatever the case, although each range has significant outliers, the data is tightly grouped around the mean and median in each case. Interestingly, the median value is 60 minutes in every case except the one-year confirmed and validation confirmed ranges where the median value is 66 and 70 minutes, respectively. This is most likely a result of the request for one hour of air refueling service being a common practice for multiple aircraft and a fairly standard air refueling track length. In order to determine the best statistical measure for predicting hourly duration based on the number of forecasted air refueling requests, the researcher computed six test cases utilizing the median value of 60 minutes for all five statuses, followed by the average number of minutes requested by each requested status for all of the test data (08 Apr - 11 Mar 17), four years of test data (17 Mar 13 - 11 Mar 17), three years of test data (16 Mar 14 - 11 Mar 17), two years of test data (15 Mar 15 - 11 Mar 17), and one year of test data (13 Mar 16 - 11 Mar 17). Lastly, the average of the 52weeks of validation data was computed as a mark of comparison with the six computed estimates. The comparison is shown below in Table 10. Interestingly, although the 60minute median in each category resulted in the closest hourly estimation for the Published category with an underestimate of 231 hours, the median technique also resulted in the worst overall estimation of hourly duration with an underestimation of 2,897 over the 52-week validation period. Even if the 66-minute Confirmed status median is used, as was seen in the 1-year distribution, the total hourly duration is still underestimated by 2,610 hours. Therefore, it is apparent that utilizing the mean over some range of values will result in a closer estimate of future hourly duration given a forecast of future requests.

	Conf	irmed	Bou	ught	Fil	led	Publ	ished	Cano	celled	Total	Hours
Data	Actual	Predicted	Actual	Predicted	Actual	Predicted	Actual	Predicted	Actual	Predicted	Actual	Predicted
Mean (60)	3,540	2,863	4,827	4,392	9,044	7,970	3,128	2,897	3,411	2,931	23,950	21,053
Difference	6	77	4	35	10	)74	2	31	4	80		2,897
All Test	3,540	2,675	4,827	4,633	9,044	9,426	3,128	2,658	3,411	2,677	23,950	22,068
Difference	erence 86		1	94	-3	82	4	70	7	33		1,882
4 Years	3,540	3,060	4,827	4,707	9,044	9,594	3,128	2,777	3,411	2,787	23,950	22,925
Difference	4	80	1	20	-5	50	3	51	6	23		1,025
3 Years	3,540	3,166	4,827	4,806	9,044	9,869	3,128	2,768	3,411	2,889	23,950	23,498
Difference	3	74	2	21	-8	25	3	60	5	21		451
2 Years	3,540	3,249	4,827	4,980	9,044	10,588	3,128	2,791	3,411	3,138	23,950	24,745
Difference	2	91	-1	52	-15	544	3	38	2	73		(795)
1 Year	3,540	3,282	4,827	5,439	9,044	9,534	3,128	2,874	3,411	3,074	23,950	24,203
Difference	2	58	-6	12	-4	.90	2	55	337		(253)	
Validation	3,540	3,314	4,827	5,461	9,044	8,766	3,128	2,877	3,411	3,098	23,950	23,515
Difference	2	26	-6	34	2	78	2	52	3	13		434

**Table 10: Air Refueling Hourly Duration Estimates** 

Reviewing the five distributions of the mean hourly requested duration of air refueling service showed that the averages over a shorter time span closer to the reserved data resulted in a better overall predictor of the total number of hours that would be requested. With the exception of the two years of data distribution that drastically overestimated the Filled status, the difference between the actual hours requested and predicted number of hours requested based on previous averages decreased as the data range decreased toward the preceding one year prior. It is worth noting that the one year prior only overestimated the number of hours requested by 253 or about 1%, which is much less than the control of the average validation data that underestimated by 434 hours. Additional testing in future years will be required to see if the previous year's distribution data remains the best estimator for hourly air refueling duration requests, although this technique is certainly a starting point in estimating air refueling hours given a forecast number of requests. The full table for one year of air refueling hourly duration request distribution is shown below in Table 11.

	Confirmed (68.8)		Bough	ought (74.3) Filled (71.8) Published (59.5		ed (59.5)	Cancelle	ed (62.9)		
Data	Actual	Predicted	Actual	Predicted	Actual	Predicted	Actual	Predicted	Actual	Predicted
Requests	3058	2863	3882	4392	8223	7970	3150	2897	3227	2931
Hours	3,540	3,282	4,827	5,439	9,044	9,534	3,128	2,874	3,411	3,074
Difference	2	58	-6	512	-4	90	2	55	3	37
% Change	7	'%	-1	3%	Ľ,	5%	8	8%	10	0%
Tot	Total Actual Hour Reque			239	950	Total Pred	icted Hou	r Requests	24	203
	Difference	e (Actual - P	redicted)	-	-253					
	Pe	rcent Chan	ge		-1%					

Table 11: Air Refueling Estimates (Averages Calculated in Minutes)

#### **Research Question Analysis**

This research focused on specific time series forecasting techniques addressed in Chapter 3 in order to determine which technique would best model the five identified air refueling statuses presented in the air refueling scheduling tool (ARST). The researcher utilized the MAPE and MAE error statistics in order to validate the best fit forecast model for each of the five statues by minimizing the error between the forecasting model and the 52-weeks of validation data from 12 March 17 to 10 March 18. In each of the five cases, the seasonal ARIMA model was the best overall fit of a forecasting model, resulting in the lowest MAPE and MAE error overall for the nine models evaluated.

The second research question focused on how the forecast number of requests could then be used as a predictor for the number of hours of air refueling service

requested. Multiple methods were experimented with including numbers distributions and summary statistics. The mean of the previous one-year of each defined request status multiplied by the number of requests represented the best model evaluated. This method resulted in 90% accuracy or better for each status with the exception of the Bought status that was 87% accurate and a total hour overestimate of 253 hours or 1% more than actual. No other method evaluated resulted in a closer estimation or with all five categories greater than 90% accuracy.

#### Summary

This chapter presented multiple measures of forecasting comparison to include -2Log Likelihood, AIC, and SBC, along with the statistical error measures of MAPE and MAE. The researcher presented the initial data analysis and determination of weekly seasonality along with the concepts of differencing and data transformation. Next, each of the five ARST request statuses were presented, including their model selection in JMP and model validation including error analysis, and then graphed to fully understand how close the prediction was to the actual reserved data. In each case, the seasonal ARIMA model was the best fit for the forecast data and was validated with the lowest overall error utilizing the MAPE and MAE error statistics. Lastly, in accordance with the second research question, the total number of hours requested was estimated utilizing the previous one year's mean hourly duration requests by ARST status multiplied by the forecast number of requests in each status. This method resulted in the best estimation overall with regard to the actual number of hours requested and represented a baseline for future analysis along with a mathematically supported hourly estimation of forecast air refueling demand that could then be used for forthcoming decision making. A discussion of the financial implications, conclusions, and implications of this research will be included in the next chapter.

### V. Conclusions and Recommendations

"All models are wrong, some are useful" – George E. P. Box

# **Chapter Overview**

This research into tanker training demand forecasting not only fills a void in the current tools available to USTRANSCOM, AMC, and the 618<sup>th</sup> AOC, but also allows for forecasts that can be updated to obtain more precise results as more data is gathered in future years. Additionally, the forecast models can be adapted for both training and operational requirements in order to build a more holistic view of the tanker force supply and demand. This research sought to define the most accurate of nine forecasting techniques evaluated for predicting future air refueling requests in five different statuses as defined by the 618<sup>th</sup> AOC's air refueling scheduling tool. Furthermore, based on the most precise forecasting model, the research pursued a calculation by which the number of forecast air refueling flight hours could be predicted with up to 90% accuracy annually.

This chapter will focus on the specific conclusions of the above research, concentrating on the significant applications of the forecast models and specifically what they could indicate for future demand. Specific recommendations for action are presented to include an analysis and application of DoD fixed wing hourly reimbursement rates and their application for future air refueling service cost comparisons. Lastly, various avenues for future research into the tanker supply and demand problem will be explored, to include 1) a review of request prioritization, 2) the multitude of barriers to air refueling service market entry, and 3) a lack of a DoD tanker pricing strategy. This last issue not only results in the DoD/USAF being unable to participate in other air refueling markets but also has the potential to cause the DoD to be ill-prepared to work with commercial air refueling organizations if/when the latter bring a service to market.

#### **Conclusions of Research**

The first research question focused on identifying the best forecasting model for the various air refueling training requests for each of the five designated statuses defined by the air refueling scheduling tool. After evaluating nine different forecasting models, multiple goodness-of-fit measures demonstrated that the seasonal ARIMA forecasting models were the best fit for the modeled data. Furthermore, the MAPE and MAE error measures used to validate the forecast models resulted in the lowest error in every case for the seasonal ARIMA models with one exception. The MAE for the Bought status was the second lowest by four one-thousandths of a point with a much lower MAPE for comparison. Overall, these low error statistics confirm that the seasonal ARIMA models are the best models to represent future air refueling demand.

Secondly, the research evaluated six different statistical measures of hourly duration data in order to predict future requested flying hour duration based on the forecast number of requests by ARST status. The results showed that in four of the five categories the future requested hours could be predicted within 90 percent accuracy. The fifth category (Bought) was predicted with 87 percent accuracy. Overall, using the previous year's data resulted in an overestimation of 253 flight hours or 1% off from the actual 23,950 hours requested, as shown in Table 11.

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# Significance of Research

Not only was the seasonal ARIMA model the most accurate model evaluated, but it also highlighted a significant trend with regard to the number of air refueling requests. The researcher noticed that in four of the five status categories, the seasonal ARIMA model under forecasted the future number of requests; only the Bought status was over forecast as noted in Table 9. This could be the result of an increasing trend in the number of air refueling requests. Therefore, the researcher re-examined the request data over the period used for the forecast. Five sets of 52-weeks of request data are represented below in Figure 28 with the last set of data 3/12/17 - 3/10/18 being the validation data set.



Figure 28: Total Number of ARST Requests in 52-Week Periods

This data represents a historical low in the 2013 timeframe due in part to sequestration and also potentially to the ARST program being new to the tanker and receiver community as of 2010, which could have resulted in fewer requests being captured. In 2014 - 2015 the number of requests spiked, again potentially due to

increased training that was delayed or suspended due to sequestration in 2013. March 2015 – 2017 showed very stable total requests with a slight jump in the following year. This increase in the number of total air refueling requests from Mar 2017 to Mar 2018 is also the period of time used to validate the forecast models and could explain why most of the categories were under forecasted. It also appears that there is an upward trend in the number of air refueling requests represented by the green trendline. Additional future years of data will be required to determine if this upward trend will continue; however, it seems logical as more air refueling requests are being captured in the ARST system along with greater overall training requirements for the Joint Force. The total number of air refueling requests by ARST category over the same range are displayed in Figure 29.



Figure 29: Total Number of Requests by ARST Status in 52-Week Periods

This representation starts to explain some of the overestimation in the Bought status as it was on an upward trend until March of 2017, leading the model to predict a higher than actual value for the Bought Status. Significantly, this representation reveals that the Confirmed and Cancelled requests are fairly consistent across the data rage and the Filled requests have leveled out in the last year. The main upward trend is the Published requests that represent unsupported air refueling requests. Again, additional data for future years will be required to see if this upward trend continues. Moreover, years of study could refine the above forecasting and continue to build upon the time series data set. Lastly, it represents an opportunity to execute more sorties with a potentially growing unsupported market, thus increasing readiness of the Joint Force or for outside support from a future commercial contractor.

#### **Recommendations for Action**

Planners at USTRANSCOM, AMC, and the 618<sup>th</sup> AOC have an opportunity to utilize these forecasting models not only for tanker training but also for operational use outside of training and other larger data sets that were not available to this researcher. Moreover, in order to refine the forecasts and, thereby, increase the accuracy of the output, several more years of data need to be gathered and analyzed.

The second part of this research focused on being able to communicate, in financial terms, what the air refueling requests and, in particular, the unsupported requests mean or could mean in the future. The objective, given an ability to forecast the number of requests by status, was to convert those requests into a predicted number of flight hours of service. That was accomplished with a reasonable degree of certainty with the results depicted in Table 11. The researcher chose flight hours as the units of analysis because the DoD publishes fixed wing reimbursement rates for all aircraft each fiscal year, which equates to the hourly rate of operating the aircraft for one additional hour. To put that another way, the DoD already has a quantitative financial measure for aircraft flight hours. Several Air Mobility Command (AMC) aircraft have alternate rates that they can charge other uniformed services or agencies for operations such as airlift under the Defense Working Capital Fund or Transportation Working Capital Fund (TWCF), but USAF tankers completing a refueling mission do not fall under a separate working capital fund at this time. Therefore, the researcher chose the DoD component of the fixed wing reimbursable rate published each fiscal year by the Office of the Secretary Defense (OSD) for the KC-135 and KC-10 as a baseline for this discussion. The rate for the fiscal year 2019 equates to \$13,419 for the KC-135R, \$13,463 for the KC-135T, and \$16,078 for the KC-10A (McAndrew, 2018, p. 4). Moreover, it is important to understand that this rate does not include many sustainment, logistics, operational, ownership, or other equipment capitalization costs; it is only a function of Aviation Fuel, Other Operational Material, Consumable Materials and Repair Parts, Depot Level Repairs (DLR), Intermediate Level Maintenance, Depot Maintenance, and Contract Maintenance Services as depicted in Figure 30 below (AFCAA/FMCY, 2018, p. 10).


### Figure 30: DoD Cost Metric Categories

### Source: (AFCAA/FMCY, 2018, p. 12)

As is apparent, there are a number of other costs that are not included in the fixed wing reimbursable rate that would have to be considered if the DoD choose to compare costs directly with a commercial provider at some later date, yet this comparison cannot be accomplished until a commercial organization develops or acquires a boom tanker and, moreover, is outside of the scope of this research. What can be addressed at this time is what it would cost the DoD financially to execute the unsupported flight hours under the Published status in the ARST utilizing existing aircraft, sustainment, and support facilities and personnel. It is important to note in any such analysis that the flight hours predicted as part of the research and analysis in Chapter 4 only equates to the flying hours used during the process of aerial refueling with a receiver and does not include the flight time required for takeoff, setup en route to an air refueling track, recovery back to an airfield, and approach and landing. All of these events add additional flight time to the tanker aircraft. Therefore, again as a baseline for discussion, the researcher assumed a conservative forty-five minutes for takeoff and flight to the air refueling track and another forty-five minutes to return to an appropriate airfield, approach, and landing. The process of adding an additional one hour and thirty minutes to each request will more fairly account for the total aircraft flight time required for each air refueling request and therefore a more appropriate cost. An example of this calculation is depicted in Table 12.

F	Published ARS	Γ Requests for	Air Refueling	(Unsupported	1)				
Actual	31	50							
Forecast	28	97							
	Published Air	<b>Refueling Ho</b>	ur Estimates (L	Jnsupported)					
Actual	Actual 3128 Actual + 1.5 7853								
Forecast	28	74	Foreca	st + 1.5	721	.9.5			
Estimated ar	nd Actual Fixed	l Wing Reimbu	ursement Cost	for Unsuppor	ted Requests				
Airframe	КС-1	.35R	KC-1	.35T	KC-10A				
FY19 Rate	\$13,	,419	\$13,	,463	\$16,078				
	Actual	Estimated	Actual	Estimated	Actual	Estimated			
	\$105,379,407	\$96,878,471	\$105,724,939	\$97,196,129	\$126,260,534	\$116,075,121			
Difference	\$8,50	0,937	\$8,52	8,811	\$10,18	85,413			
Percent Change 8% 8% 8%						%			

Table 12: Actual and Forecast Reimbursement Cost for Unsupported Requests

The above baseline for calculating reimbursement costs would need to be modified based on a more appropriate mix of tankers available, but the concept is the same. For example, tanker support would not come solely from KC-10As or KC-135Ts, as both of these aircraft represent a smaller portion of the fulfillment of any unsupported requests as these airframes represent a smaller portion of the total tanker force. However, even accounting for the KC-10A and KC-135T as approximately 13% and 12% of the tanker force, respectively, it is still approximately \$100M per year using this cost metric to execute known unsupported requests with the above assumptions. Furthermore, again because it is important, this all assumes that the USAF has the aircraft, crews, and other operations and support capacity to execute with this cost metric. At the end of the day, it is more likely that additional tanker capacity will be required in the future. It is also likely that, when a commercial tanker is brought to the market, calculations such as those in Table 12 will be helpful in not only forecasting the air refueling hours required by the DoD but also helpful in making a fair cost comparison between USAF air refueling costs and any proposed commercial service.

Lastly, after reviewing several years of DoD fixed wing reimbursable rate for the tanker force, the researcher noted that the costs appear to be increasing. This trend is likely due to the increased cost of aviation fuel and the steadily rising maintenance costs as depicted in Figure 31.



Figure 31: DoD Fixed Wing Reimbursement Rates (FY 2007 – 2019)

Source: (McAndrew, 2018)

In order to truly understand the rate trend, the researcher corrected the values for inflation via the Bureau of Labor and Statistics Consumer Price Index (CPI) as seen in Figure 32 below.



Figure 32: CPI Inflation Corrected Reimbursement Rates

### Source: (McAndrew, 2018)

Even with the reimbursement rates corrected for inflation, there are similar positive linear trendlines for each of the three airframes studied; and the annual costs by specific tanker airframe are increasing over time. Moreover, the actual number of tankers available is not projected to increase in the near-term and may even decrease as requirements have the potential to increase; requests for service are clearly showing an overall increase. All of these trends point to an opportunity for the DoD to leverage commercial air refueling service to fill near-term gaps and moderate spikes in demand for specific peacetime training opportunities.

# **Recommendations for Future Research**

Future research in the USAF air refueling tanker supply and demand problem should continue to evaluate the feasibility of innovative and non-traditional ideas not only to increase supply but also to understand demand signals from receiver units better. This research primarily focused on better understanding demand from a forecasting perspective; however, there are multiple other elements that could help planners better understand the underlying demand signals. Starting with the data this researcher analyzed in the ARST, the database should be expanded to include prompts for users to explain why a sortie is cancelled (lack of availability of aircraft, lack of aircrew, priority, etc.) or if the sortie is not "Bought" from the Filled status. Multiple feedback mechanisms built into the database could allow the 618<sup>th</sup> AOC a great deal more specific information on which sorties were unsupported and why, for future research and analysis.

Secondly, the DoD Air Refueling Support Priority System is governed by CJCSI 4120.02D and is the system through which USTRANSCOM validates and manages air refueling requirements with the support of AMC and the 618<sup>th</sup> AOC. This system is required because there needs to be a fair process to allocate limited resources to the highest priority users. The process needs to be more transparent, and users of the system, particularly non-USAF users, need to be better trained in the benefits of the system. This lack of knowledge has probably led to some receiver units not receiving needed service and ultimately no longer requesting service because of false perceptions that they will not receive service even if requested. There is an opportunity for USTRANSCOM, AMC, and the 618<sup>th</sup> AOC to build transparency in the system, some of which already is happening through the ARST system to advertise air refueling opportunities, build

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knowledge about the request system, and result in a more successful tanker and receiver matches. Moreover, aspects of this research could be continued to include more research into the weekly request rate. For example, more capacity appears available during the weekends, Mondays, and Fridays than midweek, according to Figure 4. Simple concepts such as this, packaged together for our Joint Force, could result in less frustration about the process and more efficient use of our limited resources.

While the above future research strategies are designed to understand more clearly and to moderate the demand, the supply side of the equation could be supplemented via commercial contracted air refueling service. The likelihood of this concept succeeding is directly related to the support received from the DoD, USTRANSCOM, and AMC. This support is required not only because the DoD owns 100% of the current market but also because of enormous barriers to market entry for a commercial air refueling service. These barriers include everything from a lack of a Federal Aviation Regulation (FAR) for commercial air refueling to likely interference from large aviation industry contractors like the Boeing Corporation, as well as numerous government policies that limit the sale or lease of military equipment. Many of these issues are policy related and could be solved if the commercial industry had DoD support; to date, such support has failed to materialize despite commercial interest since the mid-1990s. Each of these issues could be researched in detail as interest continues to grow throughout the DoD.

As previously stated, USAF air refueling service is not currently governed by TWCF and only charges receiver units for the fuel transferred and not the flight hours expended as they are considered training for the aircrew. The costs of the tanker flight

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hours are covered by operational and maintenance (O&M) funds. Future research in the benefits and consequences of a TWCF-like program for the USAF tanker fleet could be very beneficial on several fronts. First of all, it would modify demand signals as some fighter units may reduce requests to what they actually need versus what is convenient due to actually having to use command funds for the transit. For example, an overseas fighter deployment from Luke AFB to Aviano AB Italy may only be requested from the East coast to Lajes Field, Azores, (only the long distance over water portion where landing fields for refueling are not available) versus the entire distance. This has the potential to reduce overall demand and use the tanker aircraft more efficiently, where they are needed. Moreover, assigning an appropriate value for air refueling service would allow the USAF to fully participate in air refueling exchanges such as the Movement Coordination Centre Europe's (MCCE) Air Transport, Air to Air Refueling, and other Exchanges of Services (ATARES) program. These programs could be an important first step into laying the groundwork for the DoD to be prepared to incorporate commercial air refueling service once it is brought to the market.

# **Summary**

This research established the best forecasting method as the seasonal ARIMA model out of the nine evaluated for predicting future air refueling requests by ARST status. Six methods then were evaluated for estimating the number of hours of air refueling duration expected annually for each ARST status with a focus on those requests that were not supported and, therefore, presented an opportunity to increase readiness training and potentially be accomplished by a commercial contractor if and when that

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capability is finally available to the market. The researcher then presented the significance of an increasing trend of unsupported requests along with a consistently increasing trend of the fixed wing reimbursement rate corrected for inflation. While more analysis of future years of data will be required to continue to validate the forecasting models, an example of how to perform a cost comparison of multiple tanker options was presented with the intent for an easier comparison against commercial air refueling services when they come available. Lastly, the researcher presented multiple directions for future research aimed both at better understanding the demand signals and also at supporting the supply of air refueling tankers through research into both policy efforts and funding changes. Ultimately, the current USAF air refueling fleet is facing likely increased requirements, more requests for service, and similar or fewer tankers to accomplish the task. Continued research into the supply and demand problems can and will result in solutions to these problems and potentially create alternatives, rather than just failing to support requests, or worse yet, making our aircrews work longer and harder. Having the means to consider new opportunities to address this supply and demand problem will hopefully allow the USAF to fulfill its air refueling mission to the Joint Force better.



# Appendix A: Quad Chart

# **Appendix B: ARIMA Model Comparison**

# Daily, Monthly, and Weekly Forecasting Comparison

The following daily and monthly forecasts were the researcher's original attempt at defining the interval upon which to forecast. Note the daily forecasts compute extremely high AIC/SBC and no MAPR due to zero observations on particular dates. The monthly forecasts have a limited number of observations and produce negative AIC/SBC values.

### Daily

4	Model Comparis	on	
	model company		

	Report	Graph	Model	DF	Variance	AIC	SBC	RSquare	-2LogLH	Weights	.2 .4 .6 .8	MAPE	MAE
			<ul> <li>— Seasonal ARIMA(0, 1, 18)(0, 1, 18)52</li> </ul>	1371	9.3570054	7556.4601	7750.7074	0.687	7482.4601	1.000000			2.731424
	▼ 🗸		<ul> <li>Seasonal ARIMA(0, 1, 7)(0, 1, 7)52</li> </ul>	1393	14.396294	8047.0126	8125.7615	0.553	8017.0126	0.000000			3.361691
	▼ ✓		<ul> <li>— Seasonal ARIMA(0, 1, 5)(0, 1, 5)52</li> </ul>	1397	17.508744	8202.2015	8259.9506	0.498	8180.2015	0.000000			3.559857
	▼ 🗸		<ul> <li>Seasonal ARIMA(0, 0, 0)(0, 1, 7)12</li> </ul>	1441	23.327236	8765.0099	8807.2389	0.370	8749.0099	0.000000			4.084376
	▼ ✓		<ul> <li>Seasonal Exponential Smoothing( 52, Zero to One )</li> </ul>	1406	37.207405	9263.1875	9273.6873	-0.07	9259.1875	0.000000			5.173543
	▼ 🗸		<ul> <li>Winters Method (Additive)</li> </ul>	1405	37.233887	9265.1875	9280.9372	-0.07	9259.1875	0.000000			5.173543
	▼ ✓		<ul> <li>— Seasonal Exponential Smoothing( 12, Zero to One )</li> </ul>	1446	36.802269	9389.7101	9400.2660	0.035	9385.7101	0.000000			4.821486
	▼ 🗸		<ul> <li>Winters Method (Additive)</li> </ul>	1445	40.010519	9518.6060	9534.4398	-0.06	9512.606	0.000000			5.554903
	▼ ✓		<ul> <li>— Simple Exponential Smoothing( Zero to One )</li> </ul>	1459	39.687893	9522.3887	9527.6749	0.002	9520.3887	0.000000			5.434729
	▼ 🗸		<ul> <li>Damped-Trend Linear Exponential Smoothing</li> </ul>	1457	39.742372	9526.3887	9542.2472	0.002	9520.3887	0.000000			5.434729
	▼ ✓		<ul> <li>Double (Brown) Exponential Smoothing</li> </ul>	1458	39.824815	9534.4110	9539.6965	-0.01	9532.411	0.000000			5.450732
	▼ 🗸		<ul> <li>Linear (Holt) Exponential Smoothing</li> </ul>	1457	39.818562	9535.2028	9545.7738	-0.01	9531.2028	0.000000			5.450894
1	a												

Monthly

mouched	mpa	ison									
Report	Graph	Model	DF	Variance	AIC	SBC	RSquare	-2LogLH	Weights 2.4.6.8	MAPE	MAF
Teport		inouti					maquane		intergine interiorie		
▼ ✓		<ul> <li>IMA(1, 1) No Intercept</li> </ul>	46	0.0307241	-28.81429	-26.96414	0.088	-30.81429	0.396306	2.562784	0.139840
▼ ✓		<ul> <li>Simple Exponential Smoothing( Zero to One )</li> </ul>	46	0.0307241	-28.81429	-26.96414	0.088	-30.81429	0.396306	2.562784	0.139840
		- IMA(1, 1)	45	0.0314033	-26.81602	-23.11572	0.088	-30.81602	0.145919	2.562362	0.139798
▼ 🗸		<ul> <li>Damped-Trend Linear Exponential Smoothing</li> </ul>	44	0.0324741	-24.80350	-19.25305	0.083	-30.8035	0.053346	2.574044	0.140408
▼ 🗸		<ul> <li>Linear (Holt) Exponential Smoothing</li> </ul>	44	0.0322999	-19.56648	-15.90920	-0.03	-23.56648	0.003890	2.753312	0.150514
		<ul> <li>Double (Brown) Exponential Smoothing</li> </ul>	45	0.0342284	-19.31509	-17.48645	-0.08	-21.31509	0.003430	2.760636	0.150893
		<ul> <li>Seasonal Exponential Smoothing( 12, Zero to One )</li> </ul>	33	0.032016	-15.78551	-12.67481	-0.37	-19.78551	0.000587	2.772228	0.152875
		<ul> <li>Winters Method (Additive)</li> </ul>	32	0.0330165	-13.78551	-9.119462	-0.37	-19.78551	0.000216	2.772228	0.152875
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The following outputs are for the same data; the first was without a logarithmic data transformation, while the second was transformed via the natural log. Note the difference in variance along with much lower AIC/SBC values.

Report	Graph	Model	DF	Variance	AIC	SBC	RSquare	-2LogLH	Weights .2 .4 .6 .8	MAPE	MAE	
-		- Seasonal ARIMA(1, 1, 1)(1, 1, 0)52 No Intercept	201	234.91852	1713.9702	1723.9246	-0.31	1707.9702	0.119921	37.999911	12.537200	
~		<ul> <li>Seasonal ARIMA(1, 1, 1)(1, 1, 1)52 No Intercept</li> </ul>	200	231.9521	1714.6207	1727.8932	-0.30	1706.6207	0.086625	37.642359	12.476097	
~		<ul> <li>Seasonal ARIMA(2, 1, 1)(1, 1, 0)52 No Intercept</li> </ul>	200	234.47737	1714.8390	1728.1114	-0.30	1706.839	0.077670	37.137134	12.443354	
		<ul> <li>Seasonal ARIMA(1, 1, 1)(2, 1, 0)52 No Intercept</li> </ul>	200	233.33847	1714.9705	1728.2430	-0.30	1706.9705	0.072725	37.752986	12.503639	
			199	232.07498	1715.6703	1732.2609	-0.30	1705.6703	0.051254	36.918045	12.395629	
		- Seasonal ARIMA(1, 1, 1)(3, 1, 0)52 No Intercept	199	230.09033	1715.9683	1732.5589	-0.30	1705.9683	0.044158	37.429446	12.378324	
		<ul> <li>Seasonal ARIMA(2, 1, 1)(2, 1, 0)52 No Intercept</li> </ul>	199	233.44256	1716.0167	1732.6073	-0.30	1706.0167	0.043102	37.028825	12.425991	
		- Seasonal ARIMA(1, 1, 1)(2, 1, 1)52 No Intercept	199	209.66977	1716.0419	1732.6325	-0.30	1706.0419	0.042564	37.489186	12.408913	
		- Seasonal ARIMA(3, 1, 1)(1, 1, 0)52 No Intercept	199	234.9173	1716.1800	1732.7706	-0.30	1706.18	0.039723	37.215997	12.450609	
		<ul> <li>Seasonal ARIMA(1, 1, 1)(0, 1, 1)52 No Intercept</li> </ul>	201	228.84567	1716.5523	1726.5066	-0.32	1710.5523	0.032977	37.285776	12.367590	
		- Seasonal ARIMA(2, 1, 1)(3, 1, 0)52 No Intercept	198	228.92917	1716.7439	1736.6526	-0.29	1704.7439	0.029965	36.579324	12.274908	
			198	209.72647	1716.9386	1736.8473	-0.30	1704.9386	0.027185	36.727706	12.329608	
		- Seasonal ARIMA(3, 1, 1)(1, 1, 1)52 No Intercept	198	232.66911	1717.0892	1736.9979	-0.29	1705.0892	0.025213	37.049808	12.430996	
		- Seasonal ARIMA(3, 1, 1)(2, 1, 0)52 No Intercept	198	234.12836	1717.4517	1737.3604	-0.29	1705.4517	0.021033	37.131878	12.449769	
			200	228.84599	1717.6236	1730.8961	-0.32	1709.6236	0.019301	36.584798	12.287037	
		<ul> <li>Seasonal ARIMA(3, 1, 1)(3, 1, 0)52 No Intercept</li> </ul>	197	228.26506	1717.8966	1741.1235	-0.29	1703.8966	0.016838	36.727480	12.305810	
		- Seasonal ARIMA(1, 1, 1)(4, 1, 0)52 No Intercept	198	1.3126201	1717.9583	1737.8671	-0.30	1705.9583	0.016326	37.427960	12.377655	
		Seasonal ARIMA(1, 1, 1)(3, 1, 1)52 No Intercept	198	205.89825	1717.9672	1737.8759	-0.30	1705.9672	0.016254	37.429126	12.378188	
			198	236.11665	1718.1356	1738.0443	-0.30	1706.1356	0.014942	37.144556	12.421381	
		- Seasonal ARIMA(3, 1, 1)(2, 1, 1)52 No Intercept	197	209.91629	1718.2295	1741.4564	-0.29	1704.2295	0.014256	36.870185	12.364426	
Ē.		- Seasonal ARIMA(2, 1, 1)(4, 1, 0)52 No Intercept	197	0.3203738	1718,7326	1741,9595	-0.29	1704.7326	0.011085	36.585435	12.275387	

eport	Graph	Model	DF	Variance	AIC	SBC	RSquare	-2LogLH	Weights .2 .4 .6 .8	MAPE	MAE	
] [		<ul> <li>Seasonal ARIMA(1, 1, 2)(1, 1, 0)52</li> </ul>	199	0.1435461	208.09099	224.68159	0.093	198.09099	0.040876	9.246431	0.316599	
[		<ul> <li>— Seasonal ARIMA(1, 1, 2)(1, 1, 2)52</li> </ul>	197	0.1275094	209.38064	232.60748	0.101	195.38064	0.021450	9.148346	0.312746	
[		<ul> <li>Seasonal ARIMA(2, 1, 1)(1, 1, 0)52</li> </ul>	199	0.144668	209.38066	225.97126	0.087	199.38066	0.021450	9.244203	0.316767	
[		<ul> <li>— Seasonal ARIMA(1, 1, 2)(1, 1, 1)52</li> </ul>	198	0.1434427	209.64837	229.55709	0.094	197.64837	0.018762	9.244105	0.316268	
[		<ul> <li>— Seasonal ARIMA(1, 1, 1)(1, 1, 0)52 No Intercept</li> </ul>	201	0.1488412	209.69939	219.65375	0.073	203.69939	0.018290	9.285754	0.318511	
[		<ul> <li>Seasonal ARIMA(2, 1, 2)(1, 1, 0)52</li> </ul>	198	0.1441446	209.71891	229.62763	0.096	197.71891	0.018112	9.219734	0.315748	
[		<ul> <li>Seasonal ARIMA(1, 1, 2)(2, 1, 0)52</li> </ul>	198	0.1438471	209.81466	229.72338	0.094	197.81466	0.017265	9.248561	0.316521	
[		<ul> <li>Seasonal ARIMA(1, 0, 1)(1, 1, 0)52</li> </ul>	201	0.1477288	210.04406	223.33610	0.084	202.04406	0.015394	9.262517	0.314895	
[		<ul> <li>Seasonal ARIMA(1, 1, 2)(1, 1, 0)52 No Intercept</li> </ul>	200	0.1482357	210.12460	223.39708	0.078	202.1246	0.014787	9.218173	0.316846	
Ī		<ul> <li>Seasonal ARIMA(2, 1, 1)(1, 1, 0)52 No Intercept</li> </ul>	200	0.1486082	210.34407	223.61655	0.077	202.34407	0.013250	9.226350	0.316780	
[		<ul> <li>Seasonal ARIMA(2, 0, 1)(1, 1, 0)52</li> </ul>	200	0.1475591	210.43806	227.05311	0.094	200.43806	0.012642	9.143014	0.311310	
[		<ul> <li>Seasonal ARIMA(1, 1, 2)(0, 1, 1)52</li> </ul>	199	0.1410218	210.60250	227.19310	0.078	200.6025	0.011644	9.259274	0.316468	
[		<ul> <li>— Seasonal ARIMA(1, 1, 2)(2, 1, 1)52</li> </ul>	197	0.1303883	210.62257	233.84941	0.097	196.62257	0.011528	9.200971	0.314583	
[		<ul> <li>Seasonal ARIMA(2, 1, 1)(1, 1, 2)52</li> </ul>	197	0.1274447	210.65521	233.88205	0.095	196.65521	0.011341	9.149128	0.313007	
[		<ul> <li>Seasonal ARIMA(1, 1, 2)(0, 1, 2)52</li> </ul>	198	0.1439475	210.71358	230.62230	0.089	198.71358	0.011015	9.263495	0.316855	
[		<ul> <li>Seasonal ARIMA(1, 0, 1)(1, 1, 2)52</li> </ul>	199	0.1283309	210.76423	230.70229	0.094	198.76423	0.010739	9.230328	0.313292	
[		<ul> <li>— Seasonal ARIMA(0, 1, 3)(1, 1, 0)52 No Intercept</li> </ul>	200	0.1489018	210.85022	224.12270	0.075	202.85022	0.010287	9.213806	0.316004	
[		<ul> <li>Seasonal ARIMA(1, 0, 2)(1, 1, 0)52</li> </ul>	200	0.1477274	210.87729	227.49234	0.092	200.87729	0.010149	9.160656	0.311710	
[		<ul> <li>Seasonal ARIMA(1, 1, 2)(2, 1, 2)52</li> </ul>	196	0.1206916	210.90249	237.44745	0.099	194.90249	0.010022	9.130501	0.312135	
[		<ul> <li>Seasonal ARIMA(1, 1, 1)(1, 1, 0)52</li> </ul>	200	0.148947	210.97806	224.25054	0.076	202.97806	0.009650	9.380163	0.320728	
Ĩ		<ul> <li>Seasonal ARIMA(2, 1, 1)(1, 1, 1)52</li> </ul>	198	0.1446743	210.98634	230.89506	0.089	198.98634	0.009611	9.244413	0.316549	

# Appendix C: Confirmed Time Series: Best Seasonal ARIMA Values from JMP

The following JMP output includes the best seasonal ARIMA values computed for the Confirmed status sorted by lowest AIC followed by lowest SBC. The researcher explored all options in accordance with the Box-Jenkins Methodology in order to produce a model with significant parameters that was the least complex possible. This required at least 625 permutations for an equation with an intercept and another 625 without an intercept. Additional permutations were calculated to verify the best result.

mie oe	eries Log	[Confirmed]									
5.5 5 4.5 4 3.5 3 2.5 2.5 2	The second se	and the standard of the stand	Mo Sto Ze Sir Tre	ean ro Mean AD ngle Mean A end ADF	4.00376 0.4150 2 F -1.0166 DF -9.4385 -9.4524	92 42 57 887 26 77					
1.5	01Jan201	3 01Jan2014 01Jan2015 01Jan2016 01Ja Week Of	12017								
me Sei	ries Basi	Diagnostics									
odel C	omparis	on									
Report	Graph	Model	DF	Variance	AIC	SBC	RSquare	-2LogLH	Weights .2 .4 .6	.8 MAPE	N
		- Seasonal ARIMA(2, 1, 2)(1, 1, 0)52 No Intercep	t 199	0.0854796	89.275002	105.86560	0.008	79.275002	0.022616	5.596895	0.215
	-	- Seasonal ARIMA(1, 1, 3)(1, 1, 0)52 No Intercep	t 199	0.0856568	89.581056	106.17166	0.008	79.581056	0.019407	5.580605	0.214
	-	<ul> <li>Seasonal ARIMA(2, 1, 2)(0, 1, 2)52 No Intercep</li> </ul>	t 198	0.0842005	89.842351	109.75107	0.012	77.842351	0.017030	5.561535	0.214
	-	<ul> <li>Seasonal ARIMA(1, 1, 3)(0, 1, 2)52 No Intercep</li> </ul>	t 198	0.0844253	90.197673	110.10639	0.011	78.197673	0.014258	5.546106	0.213
	-	<ul> <li>Seasonal ARIMA(2, 1, 2)(1, 1, 0)52</li> </ul>	198	0.0854809	90.395477	110.30420	0.013	78.395477	0.012915	5.590353	0.215
	-	<ul> <li>Seasonal ARIMA(2, 1, 4)(1, 1, 0)52 No Intercep</li> </ul>	t 197	0.0854712	90.753717	113.98056	0.023	76.753717	0.010797	5.545271	0.213
	-	<ul> <li>Seasonal ARIMA(1, 1, 3)(1, 1, 0)52</li> </ul>	198	0.0856924	90.785796	110.69452	0.011	78.785796	0.010625	5.575611	0.214
	-	<ul> <li>Seasonal ARIMA(3, 1, 4)(0, 1, 2)52 No Intercep</li> </ul>	t 195	0.0828478	90.829368	120.69245	0.035	72.829368	0.010397	5.459281	0.210
_	L -	<ul> <li>Seasonal ARIMA(2, 1, 4)(0, 1, 2)52 No Intercep</li> </ul>	t 196	0.0838033	90.929210	117.47417	0.028	74.92921	0.009890	5.518595	0.212
	<u> </u>	<ul> <li>Seasonal ARIMA(2, 1, 2)(2, 1, 0)52 No Intercep</li> </ul>	t 198	0.0857294	91.009632	110.91835	0.010	79.009632	0.009500	5.592037	0.215
	-	<ul> <li>Seasonal ARIMA(1, 1, 4)(1, 1, 0)52 No Intercep</li> </ul>	t 198	0.0858719	91.078729	110.98745	0.010	79.078729	0.009178	5.598588	0.215
	E	C LADIALIZA A DUA A ALEDALI I.				111.01651	0.009	/9.10//94	0.009045	5.590280	0.215
		<ul> <li>Seasonal ARIMA(2, 1, 2)(1, 1, 1)52 No Intercep</li> </ul>	t 198	0.0858143	91.107794	11101001	0.040	77 440000	0.000000 1 1 1		
		<ul> <li>Seasonal ARIMA(2, 1, 2)(1, 1, 1)52 No Intercep</li> <li>Seasonal ARIMA(2, 1, 2)(0, 1, 2)52</li> <li>Seasonal ARIMA(2, 1, 4)(1, 1, 2)52</li> </ul>	t 198 197	0.0858143	91.107794 91.119989	114.34683	0.016	77.119989	0.008990	5.556096	0.213
		<ul> <li>Seasonal ARIMA(2, 1, 2)(1, 1, 1)52 No Intercep</li> <li>Seasonal ARIMA(2, 1, 2)(0, 1, 2)52</li> <li>Seasonal ARIMA(3, 1, 4)(1, 1, 0)52 No Intercep</li> <li>Seasonal ARIMA(1, 1, 4)(1, 1, 0)52 No Intercep</li> </ul>	t 198 197 t 196	0.0858143 0.0843725 0.085157	91.119989 91.175293 91.213604	114.34683 117.72025	0.016	77.119989 75.175293	0.008990 0.008745	5.556096	0.213
		<ul> <li>Seasonal ARIMA(2, 1, 2)(1, 1, 1)52 No Intercep</li> <li>Seasonal ARIMA(2, 1, 2)(0, 1, 2)52</li> <li>Seasonal ARIMA(3, 1, 4)(1, 1, 0)52 No Intercep</li> <li>Seasonal ARIMA(0, 1, 4)(0, 1, 2)52 No Intercep</li> <li>Seasonal ARIMA(0, 1, 4)(1, 1, 2)52 No Intercep</li> </ul>	t 198 197 t 196 t 198	0.0858143 0.0843725 0.085157 0.0844716	91.107794 91.119989 91.175293 91.213604 91.267266	114.34683 117.72025 111.12232	0.016 0.029 0.006	77.119989 75.175293 79.213604	0.008990 0.008745 0.008579	5.556096 5.485315 5.562268 5.504005	0.213
		<ul> <li>Seasonal ARIMA(2, 1, 2)(1, 1, 1)52 No Intercep Seasonal ARIMA(2, 1, 2)(0, 1, 2)52</li> <li>Seasonal ARIMA(3, 1, 4)(1, 1, 0)52 No Intercep</li> <li>Seasonal ARIMA(3, 1, 2)(1, 1, 0)52 No Intercep</li> <li>Seasonal ARIMA(3, 1, 2)(1, 1, 0)52 No Intercep</li> <li>Seasonal ARIMA(3, 1, 2)(1, 1, 0)52 No Intercep</li> </ul>	t 198 197 t 196 t 198 t 198 t 198	0.0858143 0.0843725 0.085157 0.0844716 0.0859266	91.107794 91.119989 91.175293 91.213604 91.267266 91.346950	114.34683 117.72025 111.12232 111.17599	0.016 0.029 0.006 0.009	77.119989 75.175293 79.213604 79.267266 75.24695	0.008990 0.008745 0.008579 0.008352	5.556096 5.485315 5.562268 5.594095 5.525042	0.213
		<ul> <li>Seasonal ARIMA(2, 1, 2)(1, 1, 1)52 No Intercep Seasonal ARIMA(2, 1, 2)(0, 1, 2)52</li> <li>Seasonal ARIMA(3, 1, 4)(1, 1, 0)52 No Intercep Seasonal ARIMA(3, 1, 4)(1, 1, 2)52 No Intercep Seasonal ARIMA(3, 1, 2)(1, 1, 0)52 No Intercep Seasonal ARIMA(3, 1, 2)(1, 1, 0)52 No Intercep Seasonal ARIMA(3, 1, 2)(1, 1, 0)52 No Intercep</li> </ul>	t 198 197 t 196 t 198 t 198 196 t 198	0.0858143 0.0843725 0.085157 0.0844716 0.0859266 0.0852114	91.107794 91.119989 91.175293 91.213604 91.267266 91.346950 91.351882	114.34683 117.72025 111.12232 111.17599 117.89191 111.26060	0.016 0.029 0.006 0.009 0.030	77.119989 75.175293 79.213604 79.267266 75.34695 79.351882	0.008990 0.008745 0.008579 0.008352 0.008026	5.556096 5.485315 5.562268 5.594095 5.525043 5.577020	0.213 0.211 0.213 0.215 0.215
		<ul> <li>Seasonal ARIMA(2, 1, 2)(1, 1, 1)52 No Intercep Seasonal ARIMA(2, 1, 2)(0, 1, 2)52</li> <li>Seasonal ARIMA(3, 1, 4)(1, 1, 0)52 No Intercep Seasonal ARIMA(3, 1, 4)(1, 1, 0)52 No Intercep Seasonal ARIMA(3, 1, 2)(1, 1, 0)52 No Intercep Seasonal ARIMA(2, 1, 4)(1, 1, 0)52</li> <li>Seasonal ARIMA(1, 1, 3)(2, 1, 0)52 No Intercep Seasonal ARIMA(1, 1, 3)(1, 1, 1)52 No Intercep Seasonal ARIMA(1, 1, 3)(1, 1, 1)52 No Intercep</li> </ul>	t 198 197 t 196 t 198 t 198 t 198 t 198 t 198 t 198	0.0858143 0.0843725 0.085157 0.0844716 0.0859266 0.0852114 0.0859344 0.0859344	91.107794 91.119989 91.175293 91.213604 91.267266 91.346950 91.351882 91.437334	114.34683 117.72025 111.12232 111.17599 117.89191 111.26060 111.34605	0.016 0.029 0.006 0.009 0.030 0.009	77.119989 75.175293 79.213604 79.267266 75.34695 79.351882 79.437334	0.008990 0.008745 0.008579 0.008352 0.008026 0.008006 0.007671	5.556096 5.485315 5.562268 5.594095 5.525043 5.577020 5.577020	0.213 0.211 0.213 0.215 0.215 0.212 0.214
		<ul> <li>Seasonal ARIMA(2, 1, 2)(1, 1, 1)52 No Intercep</li> <li>Seasonal ARIMA(2, 1, 2)(0, 1, 2)52</li> <li>Seasonal ARIMA(3, 1, 4)(1, 1, 0)52 No Intercep</li> <li>Seasonal ARIMA(3, 1, 4)(1, 1, 0)52 No Intercep</li> <li>Seasonal ARIMA(3, 1, 2)(1, 1, 0)52 No Intercep</li> <li>Seasonal ARIMA(1, 1, 2)(1, 1, 0)52</li> <li>Seasonal ARIMA(1, 1, 3)(2, 1, 0)52 No Intercep</li> <li>Seasonal ARIMA(1, 1, 3)(1, 1, 1)52 No Intercep</li> </ul>	t 198 197 t 196 t 198 t 198 t 198 t 198 t 198 t 198 t 200	0.0858143 0.0843725 0.085157 0.0844716 0.0859266 0.0852114 0.0859344 0.0860076 0.0866752	91.107794 91.119989 91.175293 91.213604 91.267266 91.346950 91.351882 91.437334 91.445537	114.34683 117.72025 111.12232 111.17599 117.89191 111.26060 111.34605 104.71802	0.016 0.029 0.006 0.009 0.030 0.009 0.008	77.119989 75.175293 79.213604 79.267266 75.34695 79.351882 79.437334 83.445537	0.008990 0.008745 0.008579 0.008352 0.008026 0.008006 0.007671 0.007640	5.556096 5.485315 5.562268 5.594095 5.525043 5.577092 5.574792 5.594718	0.213 0.211 0.213 0.215 0.212 0.214 0.214 0.214

### Appendix D: Bought Time Series: Best Seasonal ARIMA Values from JMP

The following JMP output includes the best seasonal ARIMA values computed for the Bought status sorted by lowest AIC followed by lowest SBC. The researcher explored all options in accordance with the Box-Jenkins Methodology in order to produce a model with significant parameters that was the least complex possible. This required at least 625 permutations for an equation with an intercept and another 625 without an intercept. Additional permutations were calculated to verify the best result.



### Appendix E: Filled Time Series: Best Seasonal ARIMA Values from JMP

The following JMP output includes the best seasonal ARIMA values computed for the Filled status sorted by lowest AIC followed by lowest SBC. The researcher explored all options in accordance with the Box-Jenkins Methodology in order to produce a model with significant parameters that was the least complex possible. This required at least 625 permutations for an equation with an intercept and another 625 without an intercept. Additional permutations were calculated to verify the best result.



# Appendix F: Published Time Series: Best Seasonal ARIMA Values from JMP

The following JMP output includes the best seasonal ARIMA values computed for the Published status sorted by lowest AIC followed by lowest SBC. The researcher explored all options in accordance with the Box-Jenkins Methodology in order to produce a model with significant parameters that was the least complex possible. This required at least 625 permutations for an equation with an intercept and another 625 without an intercept. Additional permutations were calculated to verify the best result.

ime S	eries Lo	g[Published]										
4.5 4 3.5 3 2.5 2 1.5	et p	and the second	Mean Std N Zero I Single Trend	Mean ADF Mean ADF ADF	3.6215343 0.4511277 257 -1.104706 -9.524027 -9.764479							
1-+	01Jan20	013 01Jan2014 01Jan2015 01Jan2016 01Jan20 Week Of	17									
me Se	ries Ba	sic Diagnostics										
odel (	Compar	ison										
Repor	t Graph	Model	DF	Variance	AIC	SBC	RSquare	-2LogLH	Weights	.2 .4 .6 .8	MAPE	MAE
	~		199	0.1435461	208.09099	224.68159	0.093	198.09099	0.026484	1111	9.246431	0.316599
		Seasonal ARIMA(1, 1, 2)(3, 1, 0)52	197	0.1330542	208.90652	232.13336	0.099	194.90652	0.017615		9.130876	0.312147
		<ul> <li>— Seasonal ARIMA(1, 1, 2)(1, 1, 2)52</li> </ul>	197	0.1275094	209.38064	232.60748	0.101	195.38064	0.013898		9.148346	0.312746
		— Seasonal ARIMA(2, 1, 1)(1, 1, 0)52	199	0.144668	209.38066	225.97126	0.087	199.38066	0.013897		9.244203	0.316767
		<ul> <li>— Seasonal ARIMA(1, 1, 2)(0, 1, 3)52</li> </ul>	197	0.1151825	209.56592	232.79276	0.101	195.56592	0.012668		9.156361	0.313073
		<ul> <li>— Seasonal ARIMA(1, 1, 2)(1, 1, 1)52</li> </ul>	198	0.1434427	209.64837	229.55709	0.094	197.64837	0.012156		9.244105	0.316268
$\checkmark$		<ul> <li>— Seasonal ARIMA(1, 1, 1)(1, 1, 0)52 No Intercept</li> </ul>	201	0.1488412	209.69939	219.65375	0.073	203.69939	0.011850		9.285754	0.318511
		Seasonal ARIMA(2, 1, 2)(1, 1, 0)52	198	0.1441446	209.71891	229.62763	0.096	197.71891	0.011735		9.219734	0.315748
-	H		198	0.1438471	209.81466	229.72338	0.094	197.81466	0.011186		9.248561	0.316521
-	H		190	0.1441342	209.03022	229.73094	0.093	197.03022	0.010989		9.230443	0.310000
H	H	Seasonal ARIMA(2, 1, 1)(3, 1, 0)52 No Intercent	200	0.1330029	210.12460	223 39708	0.033	202 1246	0.009580		9,130010	0.316846
1	H	— Seasonal ARIMA(2, 1, 2)(1, 1, 0)52 No Intercept	200	0.1486082	210 34407	223 61655	0.077	202 34407	0.008585		9 226350	0.316780
ř.	H	<ul> <li>Seasonal ARIMA(1, 1, 2)(0, 1, 1)52</li> </ul>	199	0.1410218	210.60250	227,19310	0.078	200.6025	0.007544		9.259274	0.316468
m i	H I	- Seasonal ARIMA(1, 1, 2)(2, 1, 1)52	197	0.1303883	210.62257	233.84941	0.097	196.62257	0.007469		9.200971	0.314583
		- Seasonal ARIMA(2, 1, 1)(1, 1, 2)52	197	0.1274447	210.65521	233.88205	0.095	196.65521	0.007348		9.149128	0.313007
		- Seasonal ARIMA(1, 1, 2)(4, 1, 0)52	196	0.0014278	210.70304	237.24800	0.100	194.70304	0.007174		9.120955	0.311852
		- Seasonal ARIMA(1, 1, 2)(0, 1, 2)52	198	0.1439475	210.71358	230.62230	0.089	198.71358	0.007137		9.263495	0.316855
			197	0.1158291	210.72269	233.94953	0.096	196.72269	0.007104		9.153552	0.313180
		Jeasonal ARIMA(2, 1, 1)(0, 1, 5)52							the second se			
		- Seasonal ARIMA(3, 1, 1)(1, 1, 0)52	198	0.1448093	210.75165	230.66037	0.090	198.75165	0.007002		9.278803	0.317817

### Appendix G: Cancelled Time Series: Best Seasonal ARIMA Values from JMP

The following JMP output includes the best seasonal ARIMA values computed for the Cancelled status sorted by lowest AIC followed by lowest SBC. The researcher explored all options in accordance with the Box-Jenkins Methodology in order to produce a model with significant parameters that was the least complex possible. This required at least 625 permutations for an equation with an intercept and another 625 without an intercept. Additional permutations were calculated to verify the best result.



Confirmed				
	Quantiles	3	Summary Statist	ics
-20 0 20 40 60 80 110 140 170 200 230 260	100.0% maximum 99.5% 97.5% 90.0% 75.0% quartile 50.0% median 25.0% quartile 10.0% 2.5% 0.5% minimum	260 1 210 3 98.925 3 99.925 1 76 1 66 7 60 45 0 0 0 0	Mean 6 Std Dev 2: Std Err Mean 0: Joper 95% Mean 6 Jower 95% Mean 6 N	3,782372 1,013394 4120058 3,590202 1,974542 3120
Paught				
Bought	Quantilar		Cummany Sta	tistics

**Appendix H: 1 Year of Air Refueling Hourly Duration Request Distribution** 



### Published



Quan	tiles		Summary Stat	istics
100.0% 99.5% 97.5% 90.0% 75.0% 50.0% 25.0% 10.0% 2.5% 0.5%	quartile quartile median quartile	260 240 110 75 60 60 45 45 45 15 0	Mean Std Dev Std Err Mean Upper 95% Mean Lower 95% Mean N	59.515975 24.828205 0.5421833 60.579249 58.452702 2097
0.076	minimum	0		

-20 0 20 40 60 80 110 140 170 200 230 260

#### Cancelled



	Summary Stat	istics
290	Mean	62,92604
210	Std Dev	23,482973
120	Std Err Mean	0.4335339
90	Upper 95% Mean	63.776101
60	Lower 95% Mean	62.075978
60	N	2934
60		
45		
30		

0

8037

Quantiles

99.5% 97.5%

90.0%

75.0%

50.0%

25.0%

2.5% 0.5%

0.0%

100.0% maximum

quartile

median

quartile

minimum

Distributions				
Confirmed				
	Quantiles		Summary Stat	istics
	100.0% maximum 99.5% 97.5% 90.0% 75.0% quartile 50.0% median 25.0% quartile 10.0% 2.5% 0.5%	270 210 105 90 75 60 45 30 0	Mean Std Dev Std Err Mean Upper 95% Mean Lower 95% Mean N	68.085583 22.487502 0.2876637 68.649506 67.521661 6111
0 30 60 90 120 150 180 210 240 270	0.0% minimum	0		

# Appendix I: 2 Years of Air Refueling Hourly Duration Request Distribution



	Quantiles		Summary Stat	istics
15/00 25000 35000 45000 55000	100.0% maximum 99.5% 97.5% 90.0% 75.0% quartile \$0.0% median 25.0% quartile 10.0% 2.5% 0.5% minimum	60000 169.04 120 90 60 60 45 40 0 0 0	Mean Std Dev Std Err Mean Upper 95% Mean Lower 95% Mean N	68.028166 667.80276 7.4223218 82.577825 53.478506 8095



-5000 5000



Quan	tiles		Summary Stat	istics
100.0% 99.5% 97.5% 90.0% 75.0% 50.0% 25.0% 10.0% 2.5% 0.5% 0.0%	maximum quartile median quartile minimum	60000 300 120 90 75 60 60 40 30 0 0	Mean Std Dev Std Err Mean Upper 95% Mean Lower 95% Mean N	79.708179 493.89281 3.7016752 86.963822 72.452535 17802

### Published



Quant	tiles		Summary Stat	istics
100.0% 99.5% 97.5% 90.0% 75.0% 50.0% 25.0% 10.0% 2.5% 0.5% 0.0%	maximum quartle median quartile minimum	300 240 105 75 60 60 45 30 0 0 0	Mean Sid Dev Sid Err Mean Upper 95% Mean Lower 95% Mean N	57,800468 24,38635 0,3930727 58,571118 57,029817 3849





Quantiles		Summary Stat	istics	
100.0% 99.5% 97.5% 90.0% 75.0% 50.0% 25.0% 10.0% 2.5% 0.5% 0.0%	maximum quartile median quartile minimum	300 276 120 90 60 60 45 20 0 0	Mean Std Dev Std Err Mean Upper 95% Mean Lower 95% Mean N	64.232061 32.067838 0.4176647 65.050837 63.413285 5895

Confirmed				
100 001	Quantiles		Summary Stat	istics
	100.0% maximum 99.5% 97.5% 90.0% quartile 50.0% median 25.0% quartile 10.0% 2.5%	6090 194,25 90,375 90 75 60 60 45 30 0	Mean Std Dev Std Err Mean Upper 95% Mean Lower 95% Mean N	66.357126 68.971825 0.7479285 67.823248 64.891004 8504

# Appendix J: 3 Years of Air Refueling Hourly Duration Request Distribution



Jough	Quantiles		Summary Stat	istics
	100.0% maximum 99.5% 90.0% 75.0% quartile 50.0% median 25.0% quartile 10.0% 2.5% 0.5%	60000 150 120 90 64 60 45 35 0 0	Mean Std Dev Std Err Mean Upper 95% Mean Lower 95% Mean N	65,651857 581,26058 5,6216229 76,671283 54,632431 10691

### Filled



Quantiles		Summary Stat	istics	
100.0% 99.5% 97.5% 90.0% 75.0% 50.0% 25.0% 10.0% 2.5% 0.5% 0.0%	quartile median quartile minimum	60000 240 120 90 75 60 60 40 30 0 0	Mean Std Dev Std Err Mean Upper 95% Mean Lower 95% Mean N	74,296916 382,77961 2,2138938 78,636244 69,957588 29894

-5000 5000 15000 25000 35000 45000 55000

### Published



Quantiles		Summary Stat	istics	
100.0% 99.5% 97.5% 90.0% 75.0%	quartile	300 210 110 75 60	Summary Stan Mean Std Dev Std Err Mean Upper 95% Mean Lower 95% Mean	57.327986 24.580385 0.3245802 57.964286 56.691686
50.0% 25.0% 10.0% 2.5% 0.5% 0.0%	median quartile minimum	60 45 30 0 0	N	5735

#### Cancelled



Quantiles			Summary Stat	istics
100.0% 99.5% 97.5% 90.0% 50.0% 25.0% 10.0% 2.5% 0.5% 0.0%	maximum quarble median quarble minimum	300 270 120 90 60 45 30 0 0	Mean Std Dev Std Eir Mean Upper 95% Mean Lower 95% Mean N	59,144738 31,550115 0,3318859 59,79531 58,494167 9037

onfirmed				
	Quantiles		Summary Stat	istics
	100.0% maximum 99.5% 97.5% 90.0% quartile 50.0% median 25.0% quartile 10.0% 2.5%	6090 137.52 90 90 75 60 60 45 30 0	Mean Std Dev Std Err Mean Upper 95% Mean Lower 95% Mean N	64.12296 61.03469 0.575927 65.25188 62.99404 1123

# Appendix K: 4 Years of Air Refueling Hourly Duration Request Distribution

#### Bought

	Quantiles		Summary Statistics	
	100.0% maximum 99.5% 97.5% 90.0% 75.0% quartile 50.0% median 25.0% quartile 10.0% 2.5% 0.5%	60000 150 120 90 65 60 45 35 0 0	Mean Std Dev Std Err Mean Upper 95% Mean Lower 95% Mean N	64.307934 533.08563 4.7272072 73.573972 55.041896 12717
5000 15000 25000 35000 45000 55000	0.0% minimum	0		

### Filled

-5000



-500 0 500 1500 2500 3500 4500 5500 6500

100.0%         maximum         60000         Mean         72.223           99.5%         240         Std Dev         338.06           97.5%         120         Std Err Mean         1.7236           90.0%         90         Upper 95% Mean         75.602           75.0%         quartile         75         Lower 95% Mean         68.845           50.0%         median         60         N         38           25.0%         quartile         60         N         38           25.0%         quartile         60         0         38           25.0%         quartile         60         0         38           25.0%         guartile         60         0         38           25.5%         30         0         55%         0	Quantiles		Summary Statistics		
0.0% minimum 0	100.0% 99.5% 97.5% 90.0% 75.0% 50.0% 25.0% 10.0% 2.5% 0.5% 0.0%	quartile median quartile minimum	60000 240 120 90 75 60 60 40 30 0 0	Mean Std Dev Std Err Mean Upper 95% Mean Lower 95% Mean N	72.223733 338.06327 1.7236016 75.602036 68.845429 38470

-5000 5000 15000 25000 35000 45000 55000

### Published



Quan	tiles	
100.0%	maximum	3
99.5%		2
97.5%		15
90.0%		8
75.0%	quartile	9
50.0%	median	- 3
25.0%	quartile	
10.0%		
2.5%		
0.5%		
0.0%	minimum	

Ownedler

cs
512616 201623 062943 113034 912198 7887
5120

-20 20 60 100 140 180 220 260 300 340 380

#### Cancelled



Quant	tiles		Summary Stat	istics
100.0% 99.5% 97.5% 90.0% 75.0% 50.0% 25.0% 10.0% 2.5% 0.5% 0.0%	maximum quartile median quartile minimum	300 270 120 90 60 45 30 0 0	Mean Std Dev Std Err Mean Upper 95% Mean Lower 95% Mean N	57.053636 30.649764 0.2756769 57.594006 56.513267 12361

Confirmed			
-500 500 1500 2500 3500 4500 5500 6500	Quantiles 100.0% maximum 99.5% 97.5% 90.0% quartile 50.0% median 25.0% quartile 10.0% 2.5% 0.5% minimum	6090 N 125 S 90 U 75 L 60 N 45 1 0 0	ummary Statistics lean 56.052238 td Dev 57.531116 td Err Mean 0.4657665 pper 95% Mean 55.3928 15257
Bought			
-5000 5000 15000 25000 35000 45000 55000	Quantiles 100.0% maximum 99.5% 97.5% 90.0% 75.0% quartile 50.0% median 25.0% 0.5% 0.5% 0.0% minimum	60000 150 120 90 60 45 30 0 0	Summary Statistics           Mean         63.2879           Std Dev         515.154           Std Err Mean         44.1384           Upper 95% Mean         54.6361           N         136
Filled	No		
-5000 5000 15000 25000 35000 45000 55000	Quartities           100.0% maximum           99,5%           97.5%           90.0%           75.0%           90.0%           75.0%           quartile           50.0%           wedian           25.0%           Quartile           10.0%           2.5%           0.5%           0.0%           minimum	60000 240 120 90 75 60 50 36 18 18 0 9	Summary statistics           Mean         70(9593)           Stid Dev         326,246           Stid Err Mean         1.6044           Upper 95% Mean         74,1040           Lower 95% Mean         67,8146           N         4133
Published			
-20 20 60 100 140 180 220 260 300 340 390	Quantiles           100.0% maximum           99.5%           97.5%           90.0%           75.0% quartile           50.0% median           25.0% quartile           10.0%           2.5%           0.5%           0.0% minimum	360 210 120 90 60 45 5.6 0 0 0	Summary Statistics           Mean         \$5.0462           Std Dev         30.0963           Std Err Mean         0.29335           Upper 95% Mean         55.6212           Lower 95% Mean         54.4712           N         105.2012
Cancelled	Quantilar		Summany Statistics
	100.0% maximum 99.5% 97.5% 90.0% 75.0% quartile 50.0% median 25.0% quartile 10.0% 2.5% 0.5%	300 260 120 90 60 45 1 0 0	Mean         54.80597           Std Dev         30.93922           Std Err Mean         0.251336           Uoper 95% Mean         55.29863           Lower 95% Mean         54.31332           N         1515

# Appendix L: All Test Data of Air Refueling Hourly Duration Request Distribution

Confirmed				
ACCOUNT OF A CONTRACT OF A CONTRACT.	Quantiles		Summary Stat	istics
	100.0% maximum 99.5% 97.5% 90.0% 75.0% quartite 50.0% median 25.0% quartite 10.0% 2.5%	260 126 110 90 78 70 60 45 17.375 0	Mean Stol Dev Stol Err Mean Upper 95% Mean Lower 95% Mean N	69.456835 20.814105 0.3763908 70.194839 68.71883 3058

# **Appendix M: Validation Data of Air Refueling Hourly Duration Distribution**

100.0% maximum 99.5% 97.5%	55555 180	Mean	74,608449
90.0% 75.0% quartile 50.0% median 25.0% quartile 10.0% 2.5% 0.5%	120 90 60 48 30 15 0	Std Eer Mean Lipper 95% Mean Lower 95% Mean N	891.01066 14.300629 102.64591 46.570988 3883
	90.0% 75.0% quartile 50.0% median 25.0% quartile 10.0% 2.5% 0.5% 0.0% minimum	90.0%         90           75.0%         quartile         60           50.0%         median         60           25.0%         quartile         48           10.0%         30         30           2.5%         15         0.5%           0.0%         0         0	90.0%         90         Upper 95% Mean           75.0%         quartile         60         Lower 95% Mean           50.0%         median         60         N           25.0%         quartile         48           10.0%         30         30           2.5%         15         0.5%           0.0%         0         0

### Filled

[	Quantiles		Summary Stat	istics
+	100.0% maximum 99.3% 97.5% 90.0% 90.0% evantile 50.0% median 25.0% evantile 10.0% 2.5% 0.5%	3600 210 120 90 75 60 60 36 24	Mean Std Dev Std Err Mean Upper 95% Mean Lower 95% Mean N	65.989177 63.524771 0.700532 67.362396 64.615957 8223
-200 200 600 1000 1600 2200 2800 3400	0.0% minimum	0		

### Published



Quantiles			Summary Statistics			
100.0% 99.5% 97.5% 90.0% 75.0% 50.0% 25.0% 10.0% 2.5% 0.5%	maximum quartile median quartile	300 180 120 90 60 60 60 30 0 0	Maan Std Dev Std Err Mean Upper 95% Mean Lower 95% Mean N	59.57746 25.771553 0.4591825 60.477188 58.677133 3150		
0.0%	muminim	0				

#### Cancelled



<sup>-20 0 20 40 60 80 110 140 170 200 230 260</sup> 

Summary Stat	istics
Mean Std Dev Std Dev Std Err Mean Upper 95% Mean Lower 95% Mean N	63.411838 22.178904 0.3904277 64.177349 62.646326 3227
Std Err Mean Upper 95% Mean Lower 95% Mean N	0.390427 64.17734 62.64632 322
	Summary Stat Mean Std Dev Std Err Nean Upper 95% Mean Lower 95% Mean N

Quantiles

75.0%

\$0.0%

25.0% 10.0% 2.5% 0.5% 0.0%

100.0% maximum 99.5% 97.5% 90.0%

quartile

median

quartile

minimum

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Since 1998 th	ne USAF	tanker f	fleet has de	ecreased	by 155	aircraft, with the additional
anticipated a	retireme	nt of th	ne equivaler	nt of 201	KC-135	5 tankers by 2029. Meanwhile,
the new KC-46	6A is sc	heduled	to replace	only 179	boom-e	equipped tankers by 2029,
resulting in	a total	decreas	se of 131 ai	ircraft w	hich co	prresponds to a reduction of
24.5% in tota	al air r	efuelind	fuel capad	city in j	ust ove	er three decades. This
research exam	nines hi	storical	. tanker tra	aining re	auests,	drawn from the 618th AOC's
Air Refuelind	n Schedu	ling Too	ol (ARST), a	and uses	multipl	le forecasting techniques.
including aut	oregres	sive int	egrated mov	zinα aver	age (AF	RIMA) models, in order to
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