

REPORT DOCUMENTATION PAGE			Form Approved OMB NO. 0704-0188		
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1. REPORT DATE (DD-MM-YYYY) 15-02-2019		2. REPORT TYPE Final Report		3. DATES COVERED (From - To) 9-Nov-2015 - 8-Nov-2018	
4. TITLE AND SUBTITLE Final Report: Nonlinear and Probabilistic Analysis with Frames			5a. CONTRACT NUMBER W911NF-16-1-0008		
			5b. GRANT NUMBER		
			5c. PROGRAM ELEMENT NUMBER 611102		
6. AUTHORS			5d. PROJECT NUMBER		
			5e. TASK NUMBER		
			5f. WORK UNIT NUMBER		
7. PERFORMING ORGANIZATION NAMES AND ADDRESSES University of Maryland - College Park Office of Research Administration 3112 Lee Building 7809 Regents Drive College Park, MD 20742 -5141			8. PERFORMING ORGANIZATION REPORT NUMBER		
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS (ES) U.S. Army Research Office P.O. Box 12211 Research Triangle Park, NC 27709-2211			10. SPONSOR/MONITOR'S ACRONYM(S) ARO		
			11. SPONSOR/MONITOR'S REPORT NUMBER(S) 68099-MA.58		
12. DISTRIBUTION AVAILABILITY STATEMENT Approved for public release; distribution is unlimited.					
13. SUPPLEMENTARY NOTES The views, opinions and/or findings contained in this report are those of the author(s) and should not be construed as an official Department of the Army position, policy or decision, unless so designated by other documentation.					
14. ABSTRACT					
15. SUBJECT TERMS					
16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT UU	15. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON Radu Balan
a. REPORT UU	b. ABSTRACT UU	c. THIS PAGE UU			19b. TELEPHONE NUMBER 301-405-5492

RPPR Final Report as of 20-Feb-2019

Training Opportunities: The grant provided support for one postdoctoral associate (Joey Iverson) and three graduate students (David Bekkerman, Naveed Haghani and Chris Dock) at the University of Maryland. The grant also provided support for two graduate students (Tin Tran and Desai Cheng) both at the University of Missouri.

Joey Iverson has received and accepted a tenure-track position at the Iowa State University (Fall 2018). Desai Cheng graduated in May 2018.

The PI and co-PIs organized Research Interaction Teams (RIT) at the University of Maryland. These are weekly internal seminars/presentations by our team members:

Spring and Summer 2016: Quantum Information Systems

Fall 2016 - Fall 2018: Applied Harmonic Analysis

Close collaboration with Radhakrishnan Balu (ARL, Adelphi) during this time.

RPPR Final Report
as of 20-Feb-2019

Publication Type: Conference Paper or Presentation
Conference Name: Sampling Theory and Applications (SampTA)
Date Received: 08-Aug-2017 Conference Date: 01-Jun-2017
Conference Location: Vilnius, Estonia
Paper Title: On the structures of Grassmannian frames
Authors: Pete Casazza, John Haas,
Acknowledged Federal Support: **Y**

Publication Status: 1-Published
Date Published: 08-Aug-2017

Publication Type: Conference Paper or Presentation
Conference Name: SPIE 2017
Date Received: 08-Aug-2017 Conference Date: 07-Aug-2017
Conference Location: San Diego, CA
Paper Title: Real Phase Retrieval by orthogonal complements and hyperplanes
Authors: Sara Botelho-Andrade, Peter G. Casazza, Desai Cheng, John I. Haas, Tin T. Tran, Janet C. Tremain, Z
Acknowledged Federal Support: **Y**

Publication Status: 2-Awaiting Publicat
Date Published:

Publication Type: Conference Paper or Presentation
Conference Name: SPIE 2017
Date Received: 08-Aug-2017 Conference Date: 07-Aug-2017
Conference Location: San Diego, CA
Paper Title: Optimal projective packings from association schemes
Authors: Joseph W. Iverson, John Jasper, Dustin G. Mixon
Acknowledged Federal Support: **Y**

Publication Status: 2-Awaiting Publical
Date Published:

Nonlinear and Probabilistic Analysis with Frames

ARO Grant W911NF-16-1-0008

May 25th, 2017

Progress Report

UMD, College Park MD

Agenda

1:00pm –

- Introduction of participants
- Overview of activities & results (Radu): Papers, RIT, talks, visits, graduate students

1:15pm -

- Kasso Okoudjou:
 - Probabilistic Frames/HRT
 - Quantum Key Distributions
- Joey Iverson:
 - ETFs from Association Schemes & Zauner's Conjecture
- Rad Balu (ARL)
 - Applications of Frames to quantum key distribution and quantum graphs
- John Benedetto:
 - A generalization of Gleason's frame function in quantum mechanics
 - Super-resolution and Beurling's minimal extrapolation

3:15pm/3:30pm Break

3:30 pm

- Pete Casazza (University of Missouri):
 - Weighted Mean Square Error in Quantum Detection
 - Rank 2 projections in \mathbb{R}^{2n} from \mathbb{C}^n vectors

4:15pm –

- Radu Balan:
 - Optimal decompositions of positive semidefinite matrices ; Grothendieck inequality
 - Low Rank Matrix recovery; quantum state tomography; Lipschitz stability

4:45 pm –

- Wrap-Up

Summary of Activities

Senior Personnel

- UMD:
 - Radu Balan
 - John Benedetto
 - Wojtek Czaja
 - Kasso Okoudjou

- U. Missouri:
 - Pete Casazza

- ARL:
 - Rad Balu

Summary of Activities

Research

- Publications: 34 papers (18 accepted journal publications)
- Interactions:
 - RIT (Research Interaction Team): February 2016-present
 - Spring&Summer 2016: Quantum Information Systems
 - Fall 2016, Spring 2017: Applied Harmonic Analysis
 - Joint research projects:
 - John-Paul-Rad (Gleason's frame function)
 - Kasso – Paul – Rad (Quantum Key)
 - Joey – Rad (Quantum Markov Chain)
 - Desai-Pete-Kasso-Radu (July 2016 workshop at UMD)
- Dissemination:
 - Conference Talks: 21
 - Invited Talks: 20+

Summary of Activities

Mentoring

- Postdoctoral Associates:
 - UMD: Joey Iverson
- Selected list of graduate students:
 - UMD:
 - David Bekkerman
 - Addison Bohannon (also ARL)
 - Matt Guay (PhD 2016)
 - Naveed Haghani
 - U. Missouri:
 - Desai Cheng
- PIs have advised and graduated 6 PhD students during the past 18 months

Publications

Radu Balan (UMD)

(1) R. Balan, D. Zou, *On Lipschitz Analysis and Lipschitz Synthesis for the Phase Retrieval Problem*, available online [arXiv:1506.02092](https://arxiv.org/abs/1506.02092) [math.FA], Linear Algebra and Applications 496, 152--181 (2016).

(2) R. Balan, M. Begue, C. Clark, K. Okoudjou, *Optimization methods for frame conditioning and application to graph Laplacian scaling*, online [arXiv:1609.02233](https://arxiv.org/abs/1609.02233) [math.FA], to appear in "Novel methods in harmonic analysis with applications to numerical analysis and data processing", Lecture Notes ANHA Series, I. Pesenson and all Eds., Birkhauser 2017.

(3) R. Balan, M. Singh, D. Zou, *Lipschitz Properties for Deep Convolutional Networks*, available online [arXiv:1701.05217](https://arxiv.org/abs/1701.05217) [cs.LG], submitted (2017).

(4) R. Balan, K. Okoudjou, A. Poria, *On A Feichtinger Problem*, online arXiv:1705.06392 [math.CA] (2017).

Publications

John Benedetto (UMD)

- (1) J.J. Benedetto, A. Nava-Tudela, A. Powell, and Y. Wang, *A frame reconstruction algorithm with applications to MRI*, invited Chapter 9 in *Frames and Other, Bases in Abstract and Function Spaces: Novel Methods in Harmonic Analysis, Volume 1*, I. Pesenson, et al. editors, Springer-Birkhauser, New York, 2017, 28 pages.
- (2) J.J. Benedetto, M. Dellatorre, *Uncertainty principles and weighted norm inequalities*, invited Chapter in *Functional Analysis, Harmonic Analysis, and Image Processing: a Collection of Papers in Honor of Bjorn Jawerth, Michael Cwikel and Mario Milman*, editors, AMS Contemporary Mathematics, 2017, 24 pages
- (3) J.J. Benedetto, W. Li, *Super-resolution by means of Beurling minimal extrapolation*, submitted to *Applied and Computational Harmonic Analysis*, 2016.
- (4) J.J. Benedetto, T. Andrews and J. Donatelli , *Frame multiplication theory and vector-valued ambiguity functions*, to be submitted 2017, 45 pages
- (5) J.J. Benedetto, M. Dellomo, *Multiplicative frames and the theory and implementation of reactive sensing*, in preparation, 41 pages.
- (6) R. Balu, J.J. Benedetto, and P. Koprowski, *A generalization of Gleason's frame function in quantum mechanics*, preprint, 9 pages.

Publications

Pete Casazza (U.Missouri)

- (1) J. Cahill, P.G. Casazza, and I. Daubechies, *Phase retrieval in infinite dimensional Hilbert spaces*, Transactions of the AMS, Series B, 3 (2016) 63-76.
- (2) T. Bemrose, P.G. Casazza, D. Cheng, J. Haas, and H. Van Nguyen, *Computing the distance between frames and between subspaces of a Hilbert space*, Accepted by Contemporary Math.
- (3) T. Bemrose, P.G. Casazza, D. Cheng, J. Haas, and H. Van Nguyen, *Computing the distance between frames and between subspaces of a Hilbert space (The Complex Case)*, Submitted.
- (4) *I, Mathematician II - Further introspections on the mathematical life*, P.G. Casazza, S. Krantz, and R.D. Ruden, Eds., The Consortium for Mathematics and its Applications, Inc., Bedford, MA (2016).
- (5) Peter G. Casazza, Amineh Farzannia, John I. Haas, and Tin T. Tran, *Toward the Classification of Biangular Harmonic Frames*, Submitted ACHA. ArXiv 1610.03142. Accepted.
- (6) Sara Bothello-Andrade, Peter G. Casazza, Dorsa Ghoreishi, Shani Jose, Janet C. Tremain, *Weak phase retrieval and weak phaseless reconstruction*, arxiv: 1612.08018. Accepted.
- (7) Peter G. Casazza, Dorsa Ghoreishi, Shani Jose, Janet C. Tremain, *Norm retrieval and phase by projections*, Axioms, 6 (2017) 1-15. arxiv 1701.08014. Accepted.
- (8) P.G. Casazza and J. Haas, *On the structures of Grassmannian frames*, Proceedings of Sampta - Estonia (2017) 1-4.
- (9) P.G. Casazza and D. Cheng, *Associating vectors in C_n with rank 2 projections in \mathbb{R}^{2n} with applications*, Submitted to ACHA.
- (10) S. Bothello-Andrade, P.G. Casazza, D. Cheng, J. Haas, T. Tran, J.C. Tremain, Z. Xu, *Phase retrieval by hyperplanes*, arxiv: 1703.02657. accepted Contemporary Math.
- (11) J. Cahill, P.G. Casazza, J. Haas, and J.C. Tremain, *Constructions of biangular tight frames and their relationships with equiangular tight frames*, Submitted.

Publications

Wojciech Czaja (UMD)

- (1) J.J. Benedetto, W. Czaja, J. Dobrosotskaya, T. Doster, K. Duke, *Spatial-spectral operator theoretic methods for hyperspectral image classification*, GEM - International Journal on Geomathematics November 2016, Volume 7, Issue 2, pp 275-297.
- (2) W. Czaja, B. Manning, J.M. Murphy, K. Stubbs, *Discrete directional Gabor frames*, to appear in Applied and Computational Harmonic Analysis, 2017.
- (3) W. Czaja, T. Doster, and A. Halevy, *An Overview of Numerical Acceleration Techniques for Non-Linear Dimension Reduction*, to appear in Novel Methods in Harmonic Analysis, 2017.
- (4) W. Czaja and J. Emidih, *Heterogeneous Cancer Cell Line Data Fusion for Identifying Novel Response Determinants in Precision Medicine*, to appear in Proceedings of 13th International Symposium on Bioinformatics Research and Applications (ISBRA), 2017.
- (5) W. Czaja, W. Li, *Analysis of time-frequency scattering transforms*, submitted, 2016
- (6) W. Czaja and Y. Li, *Gabor Regression of Quantum Chemical Energies*, preprint, 2016

Publications

Joseph Iverson (UMD)

(1) J. Iverson, J. Jasper and D. Mixon, *Optimal Line Packings from Association Schemes*, submitted to Proc. London Math. Soc. - revision process.

Publications

Kasso Okoudjou (UMD)

- (1) M. Ionescu, K. A. Okoudjou, and L. G. Rogers, *Some spectral properties of pseudo-differential operators on the Sierpinski gasket*, Proc. Amer. Math. Soc., 145 (2017), no.5, 2183-2198.
- (2) R. Balan, M. Begue, C. Clark, and K. Okoudjou, *Optimization methods for frame conditioning and application to graph Laplacian scaling*, in "Novel methods in harmonic analysis with applications to numerical analysis and data processing", Lecture Notes ANHA Series, I. Pesenson and all Eds., Birkhauser 2017 (to appear).
- (3) C. Wickman and K. Okoudjou, *Duality and geodesics for probabilistic frames*, Linear Algebra Appl., (accepted), May 2017.
- (4) K. A. Okoudjou, *Extension and restriction principles for the HRT conjecture*, arXiv:1701.08129, submitted, January 2017.
- (5) M. Begue and K. A. Okoudjou, *Invertibility of graph translation and support of Laplacian Fiedler vectors*, arXiv:1703.05867, submitted, March 2017.
- (6) M. Bownik, M. S. Jakobsen, J. Lemvig, and K. A. Okoudjou, *On Wilson bases in $L^2(\mathbb{R}^d)$* , arXiv:1703.08600, submitted, March 2017.
- (7) D. Cheng and K. A. Okoudjou, *Optimal properties of the canonical tight probabilistic frame*, arXiv:1705.03437, submitted, May 2017.

Conference Talks

John Benedetto:

- (1) Invited short talk at AMS special session on Frames and Sampling at JMM Atlanta 2017-01.
- (2) Invited short talk at AMS special session on Signal Processing at JMM Atlanta 2017-01.
- (3,4) One plenary and one colloquium at CIMPA, Buenos Aires, July 2017.

Wojciech Czaja:

- (1) Invited talk at Time-Frequency Analysis and Related Topics Conference, Strobl, Austria, June 9, 2016.
- (2) Invited talk at DTRA Basic Research Technical Review, Springfield, VA, July 18, 2016.
- (3) Invited talk at SIAM Conference on Computational Science and Engineering, Atlanta GA, February 27, 2017.

Kasso Okoudjou:

- (1) QCrypt 2016 Conference (joint with Rad & Paul)
- (2) AMS Special Session on Frame Theory, College of Charleston, March 2017.
- (3) AMS Special Session on Bases in Function Spaces: Sampling, Interpolation, Expansions and Approximations, JMM, Atlanta, January 2017.

Conference Talks

Radu Balan:

- (1) Workshop on "Harmonic Analysis, Graphs, and Learning", Bonn, Germany, March 2016.
- (2) AMS Sectional Meeting on Frames, Wavelets, and Gabor Systems, North Dakota State University, Fargo, ND, April 2016.
- (3) Invited short talk at AMS special session on Frames and Sampling at JMM Atlanta 2017-01.
- (4) Invited short talk at AMS special session on Signal Processing at JMM Atlanta 2017-01.

Joseph Iverson:

- (1) AMS Spring Eastern Sectional Meeting, Special Session on Bases in Hilbert Function Spaces, May 2017.
- (2) AMS Spring Southeastern Sectional Meeting. Special Session on Frame Theory, March 2017.

Pete Casazza:

- (1) AMS Sectional Meeting on Frames, Wavelets, and Gabor Systems, North Dakota State University, Fargo, ND, April 2016.
- (2) AMS Sectional Meetings on Topological Phases of Matter and Quantum Computation, Bowdoin College, Brunswick, ME, September 2016.
- (3) Talk in special session Applied Harmonic Analysis and frame theory at the biannual IWATA, Wash.U., St. Louis, MO, July 2016.
- (4) BIRS Mexico, Oct. 2016.
- (5) Invited short talk at AMS special session on Frames and Sampling at JMM Atlanta 2017-01.