Final Report for ONR Project N00014-15-1-2268 Strengthened Benders Cuts for Stochastic Integer Programs with Continuous Recourse

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Abstract

This report summarizes the research findings obtained in the project "Strengthened Benders Cuts for Stochastic Integer Programs with Continuous Recourse." The first topic investigated was a comparison of the strength of relaxation obtained when using split cuts to improve a mixed-integer programming formulation in an extended vs. in a projected space. In general, split cuts in the extended space can lead to better relaxations. This has implications for stochastic integer programming in that it implies that one should seek to find split cuts valid for the Benders subproblems before projecting to obtain Benders cuts in the space of first-stage variables. On the other hand, using split cuts on the master problem also has a potential benefit in that doing so can combine relaxations from multiple scenarios. These findings also motivated investigation into new techniques for using extended formulations to obtain improved valid inequalities. Finally, specialized techniques were investigated for solving a particular class of stochastic integer program known as the vehicle routing problem with stochastic demands. In particular, improved valid inequalities and a relaxed pricing scheme were discovered to be effective at reducing the integrality gap of such problems.

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1 Distribution Statement

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2 Accomplishments

2.1 What were the major goals and objectives of the project?

The goal of this project was to improve the state of the art for solving stochastic integer programming (SIP) problems by gaining insights into how cutting planes can be used within the context of decomposition algorithms for solving such problems. In particular, one goal was to understand the relative merits of using cutting planes that are derived in the extended variable space, but based on data from one scenario at a time, or using cutting planes in the projected variable space that is used within a decomposition algorithm. The other goal was to investigate techniques for most effectively using cutting planes within a decomposition algorithm. The approach used to conduct this investigation was to compare the strength of the relaxation that can be obtained with a given class of cuts (split cuts) in the different variable spaces (extended vs. projected). In addition, a computational study is to be performed on two different example applications (facility location and network interdiction) to test and compare the different approaches.

Another goal of this project was to investigate the use of extended formulations for generating valid inequalities for integer programs beyond those arising in stochastic integer programming, to understand the potential for using such extended formulations to obtain stronger relaxations using existing classes of valid inequalities.

The final goal of this project was to investigate specialized techniques for solving structured stochastic integer programs, such as the stochastic vehicle routing problem.

2.2 What was accomplished towards achieving these goals?

Activities: The first major activity has been the investigation of the use of split cuts within stochastic programming decomposition algorithms. Two approaches were considered. The first, called "project-and-cut", involves first computing cuts to approximate the projected formulation, then applying split cuts to that compact formulation. The second, called "cut-and-project", involves first computing split cuts in the extended formulation, using variables from the first-stage and only a single scenario at a time, and then projecting the resulting formulation. We investigated conditions under which either approach may yield better relaxations, and compared the two approaches in extensive computational experiments.

Inspired by the results of the first activity, the co-PIs Dash and Günlük have been investigating extended formulations of general binary mixed-integer sets. In terms of optimization, LP relaxation of these extended formulations do not lead to better bounds as their projection onto the original space is precisely the LP relaxation of the original set. However, adding cutting planes in the extended space can lead to stronger bounds.

In addition, the co-PIs Dash and Günlük also analyze the strength of split cuts in a lift-and-project framework. They observe that the Lovász-Schrijver and Sherali-Adams lift-and-project operator hierarchies can be viewed as applying specific elementary split cuts to appropriate extended formulations and demonstrate how to strengthen these hierarchies using additional split cuts.

Finally, PI Luedtke has been investigating branch-and-cut and branch-and-cut-and-price algorithms for solving the chance-constrained stochastic vehicle routing problem. In this problem a set of trucks is to be routed to serve customers having random demand quantities. A chance constraint is imposed which requires that the probability that the total demand assigned to a truck should exceeds the truck capacity should be smaller than a given tolerance.

Significant Results: Cuts in Extended vs. Projected Spaces We found that when considering cuts derived from a single scenario, the relaxation that can be obtained by using split cuts in the extended space is always at least as good as that which can be obtained by using split cuts in the projected space. As an illustration of this result, Figure 1 provides an illustration of a polyhedron Q^{LP} and its projection P^{LP}. If we assume that x_1 and x_2 are integer variables, then there are no integer solutions in either polyhedron, so the integer program with Q^{LP} or P^{LP} as its constraint set is infeasible. If we add split cuts in the extended space, Q^{LP}, then the cuts $z_1 \leq 0$ and $z_1 \geq 1$ can be derived, thus demonstrating that the integer program is infeasible. On the other hand, in the projected space, there is no split cut that can cut off the point $(x_1, x_2) = (1/2, /1/2)$. Thus, this example demonstrates that adding split cuts in the extended space can yield a strictly stronger relaxation than in the projected space. In the context of stochastic integer programming, this implies that when considering a single scenario, it is best to first derive split cuts in the extended space then apply Benders cuts to project the relaxation, as opposed to the reverse.

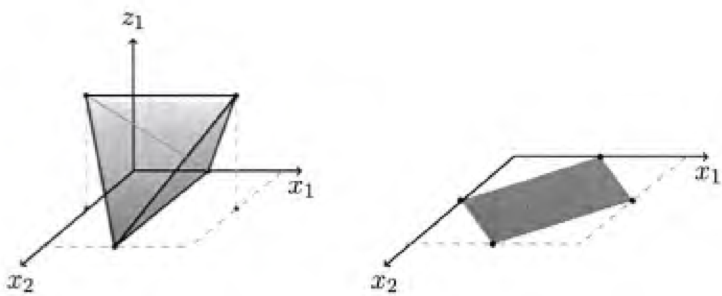

Figure 1: The set Q^{LP} and its projection P^{LP}

On the other hand, when using split cuts within a decomposition framework, they must be used within one scenario at a time when deriving them in the extended variable space, in order to preserve decomposability. This restriction is not in place for split cuts used in the projected space, and so in that context there is potential for split cuts in the projected space to be important as well. Figure 2 provides an illustration of the potential need to consider cuts that are based on multiple scenarios simultaneously. The variables z_1 and z_2 can be interpreted as the second-stage variables for scenarios 1 and 2, respectively. Assume the variable x_1 is an integer decision variable. If one considers only scenario 1, or only scenario 2, then there are no valid split cuts that can be derived to improve the relaxation. On the other hand, suppose now that one considers the two scenarios jointly, and projects each to the common space of variables x_1 and x_2. This yields the two figures in the middle row, and the figure on the bottom represents their intersection. In this case, as x_1 is an integer decision variable, the valid inequality $x_1 \leq 0$ could be derived. This example demonstrates that using valid inequalities in the projected space may be necessary to combine information from multiple scenarios in a stochastic integer program.

We demonstrate these theoretical findings with an extensive computational study on two different example applications. In the first application, a stochastic capacitated facility location problem, we found that using split cuts in the extended space leads to orders of magnitude reduction in solution time when compared to methods that do use cuts only in the projected space. This reduction is due to the significant reduction in the optimality gap of the relaxation obtained by using split cuts in the extended space. Table 1 provides the average root optimality gaps obtained by different strategies for generating cuts. The column 'BEN' refers to using just Benders cuts (so the LP relaxation); the column 'MP' refers to using split cuts derived in the projected space; the column 'SP' refers to using cuts derived in the extended space for one scenario at a time. We see that the percentage optimality gaps are significantly smaller when

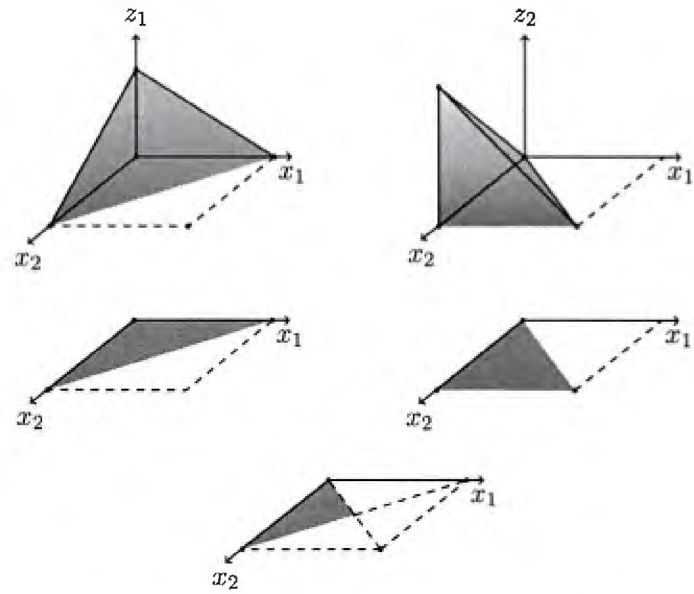

Figure 2: Sets Q_1^{LP}, Q_2^{LP} on top, their projections P_1^{LP}, P_2^{LP} in the middle and $P_1^{LP} \cap P_2^{LP}$ on the bottom.

using split cuts in the extended space, demonstrating that the theoretical possibility of this use of split cuts being stronger can indeed be very significant in realistic test instances.

Table 1: Average % root gap obtained by different methods for CAP instances with 250 scenarios. At each CAP # row, the averages of four instances are reported.

CAP #	BEN	MP	SP
101-104	22.40	20.74	0.07
111-114	8.72	8.01	0.41
121-124	18.92	18.26	0.99
131-134	25.23	24.18	0.30
Mean	18.82	17.80	0.44

In the second application, a stochastic network interdiction problem, we found that cuts in the projected space are more important for the efficient solution of the instances, although the cuts in the extended space are also helpful for that problem class. Table 2 presents the average root optimality gaps obtained by different strategies for generating cuts in instances of a stochastic network interdiction problem. In this case, we also consider the effect of adding Cplex cuts, which are cuts that are derived in the projected space. We see that for all of the settings, the use of Cplex cuts in the projected space significantly reduces the root optimality gap. Thus, for this problem class, the use of cuts that use information from multiple different scenarios is very important for improving the relaxation gaps.

Table 2: Effect of Cplex cuts on average % root gap obtained by different methods for SNIP instances. At each (snipno,Budget) row, the averages of five instances are reported.

		Without Cplex Cuts			With Cplex Cuts		
snipno	Budget	BEN	MP	SP	BEN	MP	SP
3	30	23.2	13.3	12.7	5.18	4.64	3.25
	40	27.2	17.0	15.7	7.37	6.47	5.50
	50	27.8	18.3	16.5	6.58	6.15	5.27
	60	28.4	18.5	16.8	6.12	5.46	4.98
	70	29.0	19.2	17.5	5.42	4.82	4.31
	80	31.2	20.7	18.3	6.93	6.07	5.18
	90	33.1	23.8	20.9	8.15	7.85	7.26
	Mean	28.6	18.7	16.9	6.53	5.92	5.11
4	30	25.8	14.2	13.5	4.97	4.75	3.08
	40	29.2	17.7	16.1	7.49	6.39	4.42
	50	31.4	19.3	18.0	8.13	6.71	5.53
	60	32.5	22.0	19.4	8.11	7.84	6.14
	70	32.9	22.5	19.0	6.38	6.29	4.70
	80	33.4	23.7	18.6	4.54	6.21	3.02
	90	36.8	26.7	22.4	7.24	9.19	5.67
	Mean	31.7	20.9	18.1	6.69	6.77	4.65

Other Significant Results Co-PIs Dash and Günlük introduced a new lift-and-project framework and demonstrated that for mixed binary integer programs, the number of steps required to obtain the convex hull when using the split cut strengthened operator is bounded by one half the number of binary integer variables. This bound is half the bound that is known for the standard lift-and-project operator.

They also show that for every 0-1 mixed-integer set with n integer and k continuous variables, there is an extended LP formulation with $2n + k - 1$ variables whose elementary split closure is integral. Their proof is constructive but it requires an inner description of the LP relaxation. They also extend this idea to general mixed-integer sets and characterize how to construct the best extended LP formulation for such sets with respect to lattice-free cuts.

For the chance-constrained vehicle routing problem, new bounds on the number of trucks required to serve a subset of customer have been derived, which yield significantly stronger capacity inequalities in the formulation. We numerically demonstrated that using these bounds leads to significant improvement in solution times when compared to using a simple alternative. When considering a branch-and-cut-and-price framework, the question of pricing chance-constrained feasible q-routes has been shown to be strongly NP-hard, eliminating the possibility of a pseudo polynomial time separation routine which has been successful for deterministic vehicle routing. However, a method of obtaining a relaxed version of the chance constraint has been proposed, so that this relaxed constraint can be used in the pricing routine, maintaining tractability at the expense of decreasing the quality of the relaxation. Numerical tests of the proposed branch-and-cut-and-price algorithm demonstrate that it can lead to significantly smaller relaxation gaps than the pure branch-and-cut approach within a two-hour time limit on relatively larger instances.

2.3 What opportunities for training and professional development did the project provide?

Merve Bodur, a postdoctoral researcher at Georgia Tech, and former PhD student of PI Luedtke, has received training on conducting research and writing papers in integer and stochastic programming from the PI and co-PIs as part of the collaboration on this project.

2.4 How were the results disseminated to communities of interest?

Research results were published in journals and conference proceedings (see the product sections). In addition, the PI's and other project participants presented results of the research at several conferences, as detailed below.

- O. Günlük (co-PI), "Cutting planes from extended LP formulations," Aussois Combinatorial Optimization Workshop, January 2015.
- M. Bodur, "Cutting Planes from extended LP formulations" (poster), Mixed Integer Programming Workshop 2015, Chicago, Illinois, July 2015.
- J. Luedtke (PI), "Branch-and-cut-and-price for the chance-constrained vehicle routing problem", 2015 Mixed Integer Programming Workshop, Chicago, USA, July 2015.
- S. Dash (co-PI), "Cutting planes from extended LP formulations," 2015 International Symposium on Mathematical Programming, Pittsburgh, PA, July 2015.
- J. Luedtke (PI), "Branch-and-cut-and-price for the chance-constrained vehicle routing problem", Optima Congreso 2015, Antofagasta Chile, October 2015.
- M. Bodur, "Cutting Planes from extended LP formulations" (research performed in collaboration with co-PIs Dash and Günlük), INFORMS 2015 Annual Meeting, Philadelphia, Pennsylvania, November 2015.
- S. Dash (co-PI), "A new lift-and-project operator," 2016 Informs Optimization Society meeting, Princeton, NJ, March 2016.
- O. Günlük (co-PI), "Cutting planes from extended LP formulations," University of Minnesota, April 2016.
- J. Luedtke (PI), "Exact algorithms for the chance-constrained vehicle routing problem", 2016 Integer Programming and Combinatorial Optimization Conference, Liege, Belgium, June 2016.
- S. Dash (co-PI), "Cutting planes from extended LP formulations," Summer School, Integer Programming and Combinatorial Optimization Conference, Waterloo, June 2017.
- O. Günlük (co-PI), "Cutting planes in the extended space," International Symposium on Mathematical Programming, July 2018.

Other conference presentations where support from this grant was acknowledged are listed below.

- S. Dash (co-PI), "Optimization over structured subsets of positive semidefinite matrices," ICCOPT, Tokyo, Aug 2016.
- S. Dash (co-PI), "Cutting planes from binary extended formulations," Informs Annual Meeting, Houston, Oct 2017.
- S. Dash (co-PI), "A new lift-and-project operator," Workshop on Designing and Implementing Algorithms for Mixed-Integer Nonlinear Optimization, Dagstuhl, Feb 2018.
- S. Dash (co-PI), "Cutting planes from binary extended formulations," 8th Cargese-Porquorolles Combinatorial Optimization workshop, Porquorelle, France, Aug 2018.

2.5 Honors: What honors or awards were received under this project in this reporting period?

PI Luedtke was a Fulbright Scholar during this award.

3 Technology Transfer

Nothing to report.

4 Participants

Principal Investigators:

- James Luedtke. Person-months worked: 3. National Academy member? No.
- Oktay Günlük. Person-months worked: 1. National Academy member? No.
- Sanjeeb Dash. Person-months worked: 1. National Academy member? No.

5 Students

Nothing to report.

6 Products

Publications

1. M. Bodur, S. Dash, O. Günlük, and J. Luedtke. "Strengthened Benders cuts for stochastic integer programs with continuous recourse", *INFORMS Journal on Computing*, 29:77-91, 2017. DOI: 10.1287/ijoc.2016.0717. Funding acknowledged: Yes. Published: Yes.
2. M. Bodur and J. Luedtke, "Integer programming formulations for minimum deficiency interval coloring," *Networks*, 2018. DOI: 10.1002/net.21826. Funding acknowledged: Yes. Published: Yes.
3. T. Dinh, R. Fukasawa, and J. Luedtke, "Exact algorithms for the chance-constrained vehicle routing problem," *Mathematical Programming*, 2017. DOI: 10.1007/s10107-017-1151-6. Funding acknowledged: Yes. Published: Yes.
4. M. Bodur, S. Dash, and O. Günlük "A new lift-and-project operator", *European Journal of Operations research*, 257(2), 420-428 (2017). Funding acknowledged: Yes. Published: Yes.
5. M. Bodur, S. Dash, and O. Günlük "Cutting planes derived from extended LP formulations", *Mathematical Programming*, 161(1-2), 159-192 (2017). Funding acknowledged: No. Published: Yes.

Conference Papers:

1. T. Dinh, R. Fukasawa, and J. Luedtke, "Exact algorithms for the chance-constrained vehicle routing problem," Q. Louveaux and M. Skutella (eds), *IPCO2016: The 18th Conference on Integer Programming and Combinatorial Optimization, Lecture Notes in Computer Science*, Vol. 9682, 89-101, 2016. DOI: 10.1007/978-3-319-33461-5_8. Funding acknowledged: Yes. Published: Yes.

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14. ABSTRACT

This report summarizes the research findings obtained in the project "Strengthened Benders Cuts for Stochastic Integer Programs with Continuous Recourse." The first topic investigated was a comparison of the strength of relaxation obtained when using split cuts to improve a mixed-integer programming formulation in an extended vs. in a projected space. In general, split cuts in the extended space can lead to better relaxations. This has implications for stochastic integer programming in that it implies that one should seek to find split cuts valid for the Benders subproblems before projecting to obtain Benders cuts in the space of first-stage variables. On the other hand, using split cuts on the master problem also has a potential benefit in that doing so can combine relaxations from multiple scenarios. These findings also motivated investigation into new techniques for using extended formulations to obtain improved valid inequalities. Finally, specialized techniques were investigated for solving a particular class of stochastic integer program known as the vehicle routing problem with stochastic demands. In particular, improved valid inequalities and a relaxed pricing scheme were discovered to be effective at reducing the integrality gap of such problems.

15. SUBJECT TERMS

Mixed-integer programming; Stochastic programming; Valid inequalities; Extended formulations; Vehicle routing

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