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ATOMIZATION OF LIQUID JETS AND DROPLETS

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Abstract

The object of this report is to make a critical review and, if possible, an extension of the literature concerning the atomization of liquid jets by high velocity gas streams. The subject is considered under three headings: (1) The instability of jets; (2) the mechanism of atomization of liquid jets; (3) atomization of drops.

The first part is based essentially on Rayleigh's analysis. Dimensional analysis is used to develop an equation for the breakup-length of a jet. In the second part Castleman's analysis is closely followed. In the third part Littaye's work is reviewed and an alternative analysis concerning the mechanism of drop atomization is presented. It is shown that this is compatible with the conclusions of Lenard.

The Instability of Jets

The instability of liquid cylinders has considerable bearing on atomization phenomena. It is apparent that the continuous part of a liquid jet may be treated as a liquid cylinder.

Lord Rayleigh (1) in his classical paper on "The Instability of Jets" recognizes two causes for the unstable condition of liquid jets. The first of these is due to the operation of surface tension. It is to be expected that a liquid cylinder will be unstable with respect to any deformation which results in a decrease in surface, i.e., surface energy. Such instability is expected to develop, especially when the jet and its environment differ significantly in their physical properties. It should be noted that this type of instability is statical in character, and is independent of the general translatory motion of the jet.

The second type of instability has its cause in the translatory motion, and is therefore dynamical in character. Sir W. Thompson (2) has shown the conditions under which a frictionless wind tends to render the level surface of water unstable. Thus, an air stream may cause deformations on a liquid jet in a process similar to the formation of waves on a liquid surface.

More recently, additional causes for instability have been suggested. Weber (3) examined the effect of air friction, and found that friction decreases the wave length of the oscillations and also shortens the breakup distance. Weber also extended Rayleigh's theory to viscous liquids and calculated the time of disintegration for rationally symmetrical oscillations. Bird (4), Thiemann (5), Coldthwaite (6), and Schweitzer (7) suggest that atomization of the jet may be due to turbulence in the liquid as it issues from the nozzle.

It is apparent that the causes of instability are rather complex. Quantitative description has so far been attempted only on highly idealized models. Considering the fact that even the most involved hydrodynamical considerations are necessarily based on oversimplifications such as the assumptions of potential flow and rotational symmetry, it should be realized that the quantitative results of such theoretical considerations have no immediate engineering value. Such considerations, nevertheless, contribute greatly to the qualitative understanding of atomization, and may form the basis of attack by dimensional analysis, which, coupled with suitable experiments, seems to be the best approach to an engineering solution of atomization problems. In view of the above, it was decided to present the essential features of the most fundamental theories followed by an illustration of their use in dimensional analysis.

Consider instability caused by surface tension (1). Taking the axis z along the axis of the liquid cylinder, suppose that at a time θ the cylinder undergoes an initial, "accidental"disturbance of the form

$$r = a + \alpha \cos kz \qquad (1)$$

where α is a small quantity variable with time and k is defined by

$$k = \frac{2\pi}{\lambda} \qquad (2)$$

where λ is the wave length of the disturbance. Postulating

$$\alpha = \alpha_1 e^{q\theta} \qquad (3)$$

for the growth of the disturbance (1), the problem is to investigate under what conditions such growth is possible and what conditions result in the most rapid deformation.

Instability due to surface tension occurs when the deformation results in a decrease of surface. Denoting the surface corresponding on the average to a unit length along the z axis by A, Rayleigh finds

$$A = 2\pi a + \tfrac{1}{2}\pi a \kappa^2 x^2 \tag{4}$$

Here a is subject to the condition that the volume, S, corresponding on the average to a unit length, is constant:

$$S = \pi a^2 + \tfrac{1}{2}\pi x^2 \tag{5}$$

whence

$$a = \sqrt{\frac{S}{\pi}\left(1 - \frac{1}{2}\frac{\pi x^2}{S}\right)} \tag{6}$$

From 4 and 6 one obtains with "sufficient approximation"

$$A = 2\sqrt{\pi S} + \frac{\pi x^2}{2a}\left(\kappa^2 a^2 - 1\right) \tag{7}$$

Or if A_0 is the surface for the undisturbed condition

$$A - A_0 = \frac{\pi x^2}{2a}\left(\kappa^2 a^2 - 1\right) \tag{8}$$

Hence, the system is stable with respect to deformations for which

$$\kappa a = \frac{2\pi}{\lambda} a > 1 \tag{9}$$

On the other hand, instability results if

$$\lambda > 2\pi a \tag{10}$$

Thus, a deformation whose wave length is larger than the average circumference of the jet causes instability.

The second part of the problem is to find an expression for q as a function of the surface tension σ, the density ρ, the average radius a, and the dimensionless quantity κa. Rayleigh obtains the solution by Lagrange's method. It might be pointed out that this method as used assumes the absence of all frictional effects. Therefore, the effects of viscosity must of necessity be neglected.

Assigning arbitrarily to the potential energy, V, the value zero in the undeformed state one, obtains

$$V = \sigma (S - S_0) \tag{11}$$

and, from 8

$$V = -\sigma \frac{\pi \alpha^2}{2a} (1 - \kappa^2 a^2) \tag{12}$$

Denoting the velocity potential by ϕ one obtains for the kinetic energy, T, (either by partial integration of the classical expression of kinetic energy, or more directly from Green's theorem)

$$T = \frac{1}{2} \rho \int 2\pi a \, \phi \left(\frac{\partial \phi}{\partial r} \right)_a dz \tag{13}$$

Substituting into 13 the expression for ϕ obtained from the continuity equation ($\phi = k J_0(i\kappa r) \cos \kappa z$) it is seen that

$$T = \frac{1}{2} \rho \pi a^2 \frac{J_0(i\kappa a)\dot{\alpha}^2}{i\kappa a \, J_0'(i\kappa a)} \tag{14}$$

Putting 12, 14 and 3 into Lagrange's equation of motion, Rayleigh obtains

$$q^2 = \frac{\sigma}{\rho a^3} \frac{(1 - \kappa^2 a^2) i\kappa a \, J_0'(i\kappa a)}{J_0(i\kappa a)} \tag{15}$$

It is worth noting that dimensional analysis alone would have yielded

$$q^2 = \frac{\sigma}{\rho a^3} F(\kappa a) \tag{16}$$

From 15, Rayleigh shows that q is a maximum when

$$\lambda = 4.508 (2a) \tag{17}$$

Equation 1 is, of course, the expression for a rather special type of deformation. However, all rotationally symmetrical deformations may be expressed as a sum of deformations by a suitable expansion in a Fourier series. Thus, the above conclusions are valid for any rotationally symmetrical deformation. This, however, is a serious limitation itself. Haenlein's (9) experiments show, for instance, that in many cases the deformation is of an entirely different type. In a rotationally symmetrical deformation, a line passing through the center of gravity of the jet will remain a straight line throughout the deformation, and the disturbance manifests itself as a change in cross-section. In the other type of disturbance, which is referred to by Weber (3) as "Zerwellen", the cross-section remains unchanged, and the deformation manifests itself in the wave-like appearance of the center line. While Weber attempted to treat this case, the solution is admittedly unsatisfactory.

To summarize: Rayleigh (1) has shown that, given an initial disturbance of the type

$$r = a + \alpha \cos \frac{2\pi}{\lambda} z$$

the jet becomes unstable if

$$\lambda > 2\pi a$$

the growth rate of disturbance being a maximum when

$$\lambda = 4.508 \times 2a$$

This type of instability is due to surface tension and is independent of the translatory motion of the jet.

The second type of instability described by Rayleigh (1) depends upon this very translatory motion and is dynamical in character. A brief outline of Rayleigh's analysis is given below. The emphasis is placed on a qualitative understanding of the nature of the instability rather than on any quantitative results, which, due to the improbable assumptions made, are of little value.

Consider two fluids moving in the x direction with a plane surface of separation as represented by

$$z = 0 \tag{18}$$

This implies that the velocities V and V' of the two fluids are parallel to the x axis. Rayleigh assumes that at any time after an initial disturbance the equation of the surface of separation is given by

$$h = H' e^{int} e^{ikx} \tag{19}$$

It is naturally understood that only the real parts of 19 have physical significance. For the velocity potential of the fluid on the positive side, Rayleigh writes

$$\phi = K e^{int} e^{ikx} e^{-kz} + Vx \tag{20}$$

K may be obtained from the fact that for the normal component of the velocity (the positive direction of z being downwards)

$$\left(\frac{\partial\phi}{\partial z}\right)_{z=0} = \frac{dh}{d\theta} = \frac{\partial h}{\partial\theta} + V\frac{\partial h}{\partial x} \qquad (21)$$

Putting 19 and 20 into 21, solving for K and substituting into 20

$$\phi = i\kappa^{-1}(n+\kappa V)He^{in\theta}e^{i\kappa x}e^{-\kappa z} + Vx \qquad (22)$$

Similarly for the other fluid

$$\phi' = -i\kappa^{-1}(n+\kappa V')He^{in\theta}e^{i\kappa x}e^{\kappa z} + V'x \qquad (23)$$

Note that, so far, only geometrical considerations have been used in conjunction with mass balances. An additional relation is obtained by satisfying the condition of the equality of pressures. Denoting the density by ρ, the hydrodynamical equation of pressure for the first fluid is

$$p = C' - \rho\frac{d\phi}{d\theta} - \frac{1}{2}\rho U^2 \qquad (24)$$

and approximately, when z = 0,

$$\frac{1}{2}U^2 = \frac{1}{2}\left(\frac{\partial\phi}{\partial x}\right)^2 + \frac{1}{2}\left(\frac{\partial\phi}{\partial z}\right)^2 = \frac{1}{2}V^2 - V(n+\kappa V)He^{in\theta}e^{i\kappa x} \qquad (25)$$

Similarly the pressure of the other fluid is

$$p' = C' - \rho'\frac{d\phi'}{d\theta} - \frac{1}{2}\rho U'^2 \qquad (26)$$

where

$$\frac{1}{2}U'^2 = \frac{1}{2}V'^2 + V'(n+\kappa V')H'e^{in\theta}e^{i\kappa x} \qquad (27)$$

From the condition of equality of pressures, Rayleigh obtains

$$\rho(n+\kappa V)^2 + \rho'(n+\kappa V')^2 = 0 \qquad (28)$$

which is the equation connecting n and $\kappa\left(=\frac{2\pi}{\lambda}\right)$. For any value of the wave length λ, there is a value n which determines the exponential growth of the disturbance. The results of the above derivation may be qualitatively summarized as follows:

A disturbed surface of separation of two fluids must change in such a manner that the disturbance increases with a rate which is a function of the velocities and densities of the two fluids, and the wave length of the disturbance. This is a necessary consequence of of the principle of conservation of mass, and the condition of equality of pressures at the surface of separation.

Rayleigh shows a similar treatment for a cylindrical surface of separation. It is to be noted that friction played no part in the above derivation. The above considerations show that reasonable explanations of the instability of jets are possible, provided that an initial disturbance may be assumed. Bidone's (1) investigations throw light on a possible cause of initial deformation. Bidone investigated the behavior of jets of water issuing horizontally from orifices in thin plates. It was found that, if the orifice is circular, the jet, though diminished in cross-sectional area, retains the circular form. However, the experiments showed that for non-circular orifices the jet undergoes peculiar transformations. In the case of an elliptical apperture, with the major axis horizontal, the sections of the jet taken at increasing distances gradually loose their ellipticity until at a certain distance the section is circular. Farther out the section again becomes elliptical, but this time the major axis is in a vertical position. Thus, it is seen that orifices that are not perfectly round may give an initial deformation to the jet. Initial deformations may also be caused by turbulent liquid or gas eddies.

The break-up distance, l, of the jet should be an important design variable. It should influence the choice of the position of the liquid nozzle in the gas stream, as well as the length of the throat producing the high velocity. It will now be shown how theory and dimensional analysis may be combined to give important quantitative information concerning the magnitude of the break-up distance. Let surface tension be accepted as the main cause for the instability of the jet. The break-up distance then clearly depends upon the rate growth of the disturbance as measured by q in Equation 15. This suggests σ, ρ, and r_0 as variables that might affect the break-up distance, l. Furthermore, the theory tells us that q is independent of the velocity of the jet. This suggests that, since for large velocities one may expect large initial deformations, the break-up distance will also be a function of the velocity. Or, since the fluid head, H, determines the velocity, one may use H rather than V. Accepting turbulence as the cause for initial deformation the viscosity, μ, should also be considered as a factor in determining break-up distance. Thus, a relation of the type

$$F(\ell, r_0, \sigma, \rho, H, \mu) = 0 \qquad (29)$$

is desired. The variables with their dimensions (mass, length, time system) are shown in Table I.

TABLE I

l	r_0	σ	ρ	H	μ
L	L	$\frac{M}{\theta^2}$	$\frac{M}{L^3}$	$\frac{L^2}{\theta^2}$	$\frac{M}{\theta L}$

Since there are 6 variables and 3 fundamental dimensions one can conclude that 29 will take the form

$$F'(\pi_1, \pi_2, \pi_3) = 0 \tag{30}$$

or the variables will occur in three dimensionless groups (π - s). Constructing dimensionless groups around l, σ, and μ one obtains

$$F'\left[\left(\frac{l}{r_0}\right), \left(\frac{\rho r_0 H}{\sigma}\right), \left(\frac{r_0 H^{1/2} \rho}{\mu}\right)\right] = 0 \tag{31}$$

Alternatively

$$\frac{l}{r_0} = F''\left\{\left(\frac{\rho r_0 H}{\sigma}\right), \left(\frac{r_0 H^{1/2} \rho}{\mu}\right)\right\} \tag{32}$$

This is as far as dimensional analysis will take us. However, it is now possible to make an assumption which is reasonable and results in transforming 32 into a more useful relation.

Let it be assumed that the initial deformation is proportional to the size of the liquid eddies that cause it. Then, taking the size of the eddy to be proportional to the initial velocity, that is to $H^{1/2}$, it becomes reasonable to expect that the break-up distance should also be proportional to $H^{1/2}$. Then 32 becomes

$$\frac{l}{r_0} = \left(\frac{\rho r_0 H}{\sigma}\right)^{1/2} \cdot F'''\left(\frac{r_0 H^{1/2} \rho}{\mu}\right) \tag{33}$$

Rearranging, and substituting d7 for a $H^{1/2}$ (which we may do, since the quantities are proportional)

$$l = \frac{r_0^{3/2} \rho^{1/2} H^{1/2}}{\sigma^{1/2}} \cdot F'''(Re) \tag{34}$$

This result is very reasonable in view of the observations of Savart (10). For a given fluid and a given orifice the length is approximately proportional to the square root of the head. When the fluid is changed the length varies as $\rho^{1/2}/\sigma^{-1/2}$. Everything else being the same, the length is proportional to the diameter of the orifice.

It is seen that 34 is consistent with Savart's observation, there being a slight discrepancy only in case of the last statement. In view of the fact that some of Rayleigh's theoretical equations also indicate that $\ell \propto r_a^{1/2}$, Equation 34 may be expected to be correct. Schweitzer, *et al.* (7), have shown that the break-up length depends upon the Reynolds number as shown by 34. Unfortunately, they did not correlate their results by means of an equation of the type 34. Such correlation would serve to determine F''' and thus solve one of the important design problems in jet atomization. It seems probable that over a considerable range of Re it is possible to write

$$\ell = k \frac{r_o^{3/2} \rho^{1/2} H^{1/2}}{\sigma^{1/2}} (Re)^n \tag{35}$$

The Mechanism of the Atomization of Liquid Jets

In carrying the arguments of Rayleigh to their logical conclusion it would seem that atomization of a jet occurs when the disturbance α becomes equal to the radius, a, of the jet. It appears that atomization by this mechanism has been actually observed (9), and some authors accept it and exclude all others (11). However, it now seems probable that the above mechanism is the exception rather than the rule, and the resulting "regular" break-up can be observed only under carefully controlled conditions.

It seems reasonable to assume that as the amplitude of the waves on a jet increase beyond a certain amount, the particles of water on the top of the waves become less firmly attached to the main body than other particles. These particles may then be torn off by friction. As a matter of fact, Castleman proposed that these particles are torn off in the form of extremely fine ligaments of a very short life, which in turn break-up into droplets. Castleman (12) presents photographs to support this thesis which clearly show the existance of such filaments. Castleman also extends the theoretical considerations of Rayleigh to the case of these ligaments and shows that their life period is extremely short. In the following, Castleman's arguments will be outlined briefly.

Consider a ligament (approximately cylindrical) of length, L, and radius, a. Assuming a single swelling on such a filament, one obtains:

$$\lambda = \frac{l}{2} \text{ and } \kappa a = \frac{2\pi a}{l/2} \tag{36}$$

Then, from Equation 15

$$q = \left(\frac{\sigma}{\rho a^3}\right)^{1/2} \cdot F\left(\frac{a}{l/2}\right) = \left(\frac{\sigma}{\rho a^3}\right)^{1/2} F(Z) \tag{37}$$

where q is defined by Equations 3 and 1.

Before proceeding further with this analysis, it might be of interest to examine the assumption on which 37 is based and which is perhaps the weakest link in Castleman's chain of argument. Rayleigh's analysis, on which Equation 15 is based, concerns an infinite liquid cylinder. Nevertheless, it seems permissible to apply the conclusions of the analysis to cylinders large enough that the disturbance is distributed over several wave lengths. But is it allowable to apply the analysis to a filament with only a single swelling, that is, a half-wave disturbance? Castleman examines the point rather superficially and concludes that his procedure is justified. His argument, however, is little more than a statement and thus he leaves the question quite open. As far as the rest of the assumptions are concerned, Castleman's analysis involves every simplification inherent to Rayleigh's analysis. It might be pointed out that the assumption concerning potential flow is to a considerable extent more drastic when applied to ligaments that break up with extreme speeds, than in the case of its application to the considerably larger jet.

Assuming that a ligament of radius, a, and length, $Z \cdot 2a$, becomes a drop of radius, r, Castleman obtains

$$2Za \cdot \pi a^2 = \frac{4}{3}\pi r^3 \tag{38}$$

whence

$$a = \left(\frac{2}{3Z}\right)^{\frac{1}{3}} r \tag{39}$$

so that, for the same final value of r,

$$\frac{q_1}{q_2} = \frac{F_1(Z)^{1/2}}{F_2(Z)^{1/2}} \tag{40}$$

Limiting the discussion to that shape which will break-up with the greatest apeed - that is, that for which q is the largest - Castleman obtains (from 15) F = 0.343 and z = 4.5. Taking for λ the value 5×10^4 cm., Castleman calculates 2.65×10^{-4} cm. for a and 6.8×10^5 sec.$^{-1}$ for q.

Equation 3 may be written

$$\theta = \frac{1}{z} \ln \frac{x}{x_o} \qquad (41)$$

Noting that the ligament breaks when x grows to 2.65×10^{-4} cm., while x_o cannot be much less than 10^{-8} cm., since the molecular diameter is around 10^{-7} cm., Castleman obtains for the time of collapse $\theta = 1.5 \times 10^{-5}$ sec.

Castleman's views on atomization may be summarized in his own words:

"The actual process of atomization in an air stream seems rather simple. A portion of the large mass is caught up (say at a point where its surface is ruffled) by the airstream and, being anchored at the other end, is drawn out into a fine ligament. This ligament is quickly cut off by the rapid growth of a dent in its surface, and the detached mass, being quite small, is swiftly drawn up into a spherical drop. (A quite similar phenomenon occurs when a large drop is detached from a tube. The chief difference is that the ligament connecting the small drop to the main mass is much finer than that connecting the large drop to the liquid in the tube, and, hence, the time of detachment is enormously less.) The higher the air speed, the finer the ligaments, the shorter their lives, and the smaller the drops formed, within the limits discussed above" (12).

Atomization of Drops

According to Siestrunck (11), the liquid jet before break-up consists of a series of constrictions and swellings. These disturbances are said to be due to "the amplification of the natural oscillations of the jet by the drag". The amplitudes of the disturbances increase until the constrictions disappear , and the continuous jet breaks up into droplets which shoot off at regular intervals from the jet.

The droplets are said to be subjected to two types of pressures that tend to explode them. The first of these, p_D , is due to the drag force. This force and the equal and opposite inertia force tend to squeeze the drop into an elongated form.

The second type of pressure, p_c, is due to centrifugal effects. Presumably while colliding with the eddies in the gas stream, the droplets acquire an angular velocity proportional to the velocity of the gas stream:

$$\omega = KV \tag{42}$$

The centrifugal forces which are due to this rotation are then responsible for the second type of pressure, p_c, which tends to explode the drop.

The pressure equivalent, p_σ, of the surface tension, on the other hand, tends to restore the drop to its original shape. According to Siestrunck the drop explodes if $p_D + p_c$ exceeds p_σ by a fraction α. From such a force balance Siestrunck obtains

$$V - C = \sqrt{\frac{4\sigma\alpha}{\rho r C_D} - \frac{2}{3}\frac{\rho'}{\rho}\frac{K^2 r^2}{C_D}V^2} \tag{43}$$

where

σ = surface tension of liquid
V = critical velocity of gas
C = critical velocity of liquid drops
C_D = drag coefficient
r = radius of the drop
ρ = density of the gas
ρ' = density of the liquid
K = constant

Let us examine at this point the reasoning that led to Equation 43. First of all it is obvious that if the only forces opposing surface tension were the centrifugal forces and forces due to drag effects, the drop would collapse in the absence of these forces, since there would be nothing to oppose the compressive effect of surface tension. That such is not the case is well known to anyone who has observed a stationary droplet. To the contrary, in a stationary droplet, the compressive effect of the surface tension is exactly balanced by the static pressure which is the result of the compression. Thus a stationary drop is already in a force balance and any additional forces will only tend to destroy the equilibrium resulting in deformation of the droplets. This deformation, however, starts immediately as the drop is given an initial acceleration. The deformation may occur in such a way as to restore the balance.

There is thus little reason to believe that Equation 43 represents the true conditions at the break-up. Furthermore, the relative velocity at break-up could be calculated only if the final shape of the drop and the corresponding drag force were known exactly. From the following it will be seen that there are good reasons to believe that for water droplets, at least, the final shape is similar to an inverted cup, and is thus very far from the spherical shape on which Equation 43 is based. There also we shall see that theoretical considerations concerning the formation of rain storms indicate that atomization of droplets is probably not due to drag forces.

Littaye (13) uses the analysis of Siestrunck to develop an equation relating drop size to the relative velocity, the surface tension of the liquid and the density of the gas. He assumes that the rotational effects are negligible and therefore

$$\frac{2}{3}\frac{\rho'}{\rho}\frac{K a^2}{C_D} V \ll \frac{4\sigma x}{\rho r C_D} \tag{44}$$

Then from 43

$$\frac{\rho (V-C)^2 D}{\sigma} = \text{const} \tag{45}$$

The assumption that rotational effects are negligible requires justification. Littaye proceeds as follows: From Newton's second law:

$$m \frac{dC}{d\theta} = C_D \frac{\rho}{2} (V-C)^2 S_0 \tag{46}$$

where S_0 is the area of a great circle of the drop, and m is its mass. Integrating 46 with the boundary condition

$$C = 0 \text{ when } \theta = 0$$

one obtains for C

$$V - C = \frac{V}{1 + k\theta V}\ ; \quad k = \frac{C_D \rho S_0}{2m} \tag{47}$$

Let V be the gas velocity at which break-up occurs, and let $\theta = \tau$ be the duration of the explosion. Then from 47

$$C = \frac{dx}{d\theta} = \frac{k\theta V^2}{1 + k\theta V} \tag{48}$$

from which

$$x = V\theta - \frac{1}{k}\ln(1 + k\theta V) = \frac{1}{2k}(k\theta V)^2 + \cdots \qquad (49)$$

hence

$$x = l = \frac{1}{2k}(kV\tau)^2 \qquad (50)$$

gives the distance, l, over which the break-up occurs. For the case of water droplets in air

$$k = \frac{C_D \rho r_0}{2m} = 0.000138 D \qquad (51)$$

Also V can be obtained from 45 experimentally:

$$V = 1.06 \times 10^3 D^{-\frac{1}{2}} \qquad (52)$$

Littaye assumes that the period of explosion equals the period of vibration of the droplet:

$$\tau = \sqrt{\frac{3\pi m}{8T}} \qquad (53)$$

Substituting 51, 52, 53 into 50 he obtains

$$l = 2.6 D \qquad (54)$$

From this he concludes that the distance, l, over which the break-up occurs is too short for the drop to acquire a sufficient angular velocity, and therefore the use of Equation 45 is considered justified.

At this point we would like to point out that there is no theoretical justification of the assumption involved in 53. The break-up might occur only after quite a large number of vibrations, and below we shall endeavor to show that this is actually the case. If it is assumed that, say, 5 vibrations take place during break-up,

$$l = 15 D \qquad (55)$$

is obtained for the break-up length, an ample distance for the drop to acquire a significant amount of rotation.

We shall now attempt to give an analysis of the actual mechanism of drop atomization free from the objections enumerated above.

Consider an initially spherical droplet placed in an air stream. Due to drag, the air will accelerate the droplets. Figure 1 shows the two external forces acting on the slightly deformed drop.

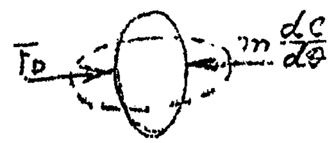

Figure 1

The two equal and opposite forces cause an initial deformation indicated in the figure. Such deformation is accompanied by an increase in surface. As a result the surface tension will tend to restore the drop to its initial spherical shape just as the spring forces tend to restore a spring mass system to its equilibrium position. Also in analogy to the spring mass system, there will be a tendency toward oscillations about the equilibrium configuration which in this case is spherical. Thus the drop, when in the position indicated by the full line in Figure 1 has a natural tendency to assume the shape indicated by the dotted line. Such oscillation, however, cannot occur exactly this way since the change in shape (from the one indicated by the full line to the one indicated by the dotted line) is opposed by the forces which caused the deformation in the first place.

In order to be able to oscillate and still not oppose the drag and inertia forces, the drop must execute a quarter turn for every half period of oscillation. This motion will result in a combination of rotation and oscillation which are so synchronized that for a stationary (non-rotating) observer the drop will always be in a position shown in Figure 1.

We are now in a position to make a quantitative estimate for the speed of rotation and also to estimate the effects of the centrifugal forces. The period of oscillation is given by

$$\tau = \sqrt{\frac{3\pi m}{8\sigma}} \qquad (56)$$

The synchronization condition requires that there be one revolution of the droplet for every two periods of vibration. Thus, if f denotes the number of revolutions per second, we have from 56:

$$f_r \doteq \frac{1}{2\tau} = \frac{1}{2}\sqrt{\frac{8\sigma}{3\pi\frac{4}{3}\pi\rho' r^3}} \qquad (57)$$

For water at 30°C. $\sigma = 72 \frac{\text{dynes}}{\text{cm.}}$ and $\rho' = 1$ gm./cm.³. Then for a 4 μ diameter drop

$$f_r = \frac{1}{2}\sqrt{\frac{2\times 72}{\pi^2\times 1\times 8\times 10^{-12}}} = 0.67\times 10^6 \frac{rev.}{sec} \qquad (58)$$

The pressure due to centrifugal effects is (11)

$$p_c = \frac{\rho'\omega_r^2 r^2}{3} \qquad (59)$$

From 57

$$\omega_r^2 = (2\pi f_r)^2 = \left(\frac{\pi}{\tau}\right)^2 = \left\{\frac{\pi}{\sqrt{\frac{3\pi\frac{4}{3}\pi\rho' r^3}{8\sigma}}}\right\}^2 = \frac{2\sigma}{\rho r^3} \qquad (60)$$

Putting 60 into 59

$$p_c = \frac{2\rho'\sigma r^2}{3\rho' r^3} = \frac{2}{3}\frac{\sigma}{r} \qquad (61)$$

which thus turns out to be independent of the density. For a 4 μ diameter water drop at 30°C.

$$p_c = \frac{2}{3}\,\frac{72}{4\times 10^{-4}} = 2.4\times 10^5 \frac{dynes}{cm^2} \qquad (62)$$

The pressure due to surface tension is

$$p_\sigma = \frac{2\sigma}{r} = \frac{2\times 72}{2\times 10^{-4}} = 7.2\times 10^5 \frac{dynes}{cm^2} \qquad (63)$$

Comparing 62 and 63, it is seen that, according to this analysis, centrifugal effects play an important role in the atomization of the drop, being able to overcome approximately 33% of the resistance of surface tension. This result is not limited to a 4 μ diameter drop since for any drop

$$\frac{P_c}{P_\sigma} = \frac{\frac{2\sigma}{3r}}{\frac{2\sigma}{r}} = \frac{1}{3} = 33.3\% \tag{64}$$

The analysis thus shows that 33.3% of the resistance to explosion is overcome by the centrifugal forces, irrespective of the size or material of the droplet. In the above considerations, the effects of viscosity have not been taken into consideration. However, it is well known (18) that as long as the viscosity (damping) is not excessive, viscosity will not affect the period of vibration materially. For extremely viscous liquids the period of vibration would be smaller than that predicted by 56 and thus the effect of centrifugal forces would be correspondingly less.

Due to the synchronism referred to above, the action of the drag and inertia forces is always in phase with the action of the surface forces. Thus we are dealing with a case of forced vibrations with viscous damping. For such cases, to the approximation that harmonic vibrations are assumed,

$$X_0 = \frac{P_0}{c\,\omega_r} \tag{65}$$

holds (18). Here X_0 is the resonant amplitude, P_0 the amplitude of the applied force, c the coefficient of viscous damping, and ω_r the circular frequency. Since ω_r is $2\pi/\tau$, we have from 56

$$X_0 = \frac{P_0}{2c}\sqrt{\frac{\rho' r^3}{2\sigma}} \tag{66}$$

where c is some function of the viscosity. Rearranging,

$$\frac{X_0}{r} = \frac{P_0}{2c}\sqrt{\frac{\rho' r}{2\sigma}} \tag{67}$$

Equation 67 states that the fractional increase in amplitude is proportional to the drag force, $C_D \frac{\rho}{2}(V-C)^2 S_0$. There is now a strong temptation to assume that when the fractional increase in the radius exceeds a certain critical value, $\left(\frac{X_0}{r}\right)_{crit}$,the process will lead to atomization. Thus from 67, when

$$\frac{C_D \frac{\rho}{2}(V-C)^2 S_0}{2c}\sqrt{\frac{\rho' r}{2\sigma}} = \left(\frac{X_0}{r}\right)_{crit} = const. \tag{68}$$

atomization will result. It remains to express c as a function of viscosity. By definition c is the damping force per unit velocity. To the approximation that the deformed shape is disc like rather than spherical, we have from Poiseuille's law

$$c \propto \mu r \tag{69}$$

Since $S_0 = \pi r^2$, we have from 69 and 68

$$\frac{C_D \rho (V-C)^2 r^{1.5} \rho'^{0.5}}{\mu\, \sigma^{0.5}} = const. \tag{70}$$

It is of course realized that 70 breaks down if $\left(\frac{X_0}{r}\right)_{crit.}$ is larger than, say, 15%, since then the assumption of harmonic motion becomes untenable.

Equation 70 gives the relative velocity necessary for the break-up of a drop of given size as a function of the densities of both the gas and the liquid, the drag coefficient, the viscosity and the surface tension of the liquid. Comparison of Equations 70 and 45 shows that the equation developed above is in many respects similar to Littaye's equation, but differs from it by the fact that it contains the density and viscosity of the liquid. Equation 70 also shows that large viscosities make large relative speeds necessary if atomization is desired.

Let us now consider what happens to the drop after the fractional increase in radius exceeds the critical value. It is to be noted that the centrifugal effects, being non-periodic, have no effect on the amplitude of the oscillation, but rather affect the shape of the equilibrium configuration. It is to be expected that the centrifugal forces will drive the liquid toward the periphery of the drop causing it to take a shape similar to that shown in Figure 2.

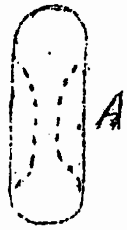

Figure 2.

This results in a gradual thinning of the liquid near the center. Eventually most of the liquid concentrates at the periphery and the thin center membrane is then blown out into a shape somewhat like that shown in Figure 3.

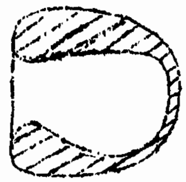

Figure 3.

The results of this analysis are identical with the conclusions reached by Lenard (14) who, however, approaches the problem from an entirely different point of view. Lenard's interest is focused on the production of static electricity in clouds. He shows (15) that the formation of static electricity in clouds cannot be explained by the collision or tearing apart of the droplets. However, a consideration of the theoretically predicted and experimentally confirmed double layer on droplets leads him to the conclusion that the production of static electricity can be explained if it is assumed that, instead of being torn apart, the droplets are actually "blown" apart, as suggested by the shape of the drop in Figure 3.

Lenard (16) shows that the double layer on a drop consists of a negative layer on the surface of the drop paralleled by a positive layer slightly farther inward. Thus, splitting the drop into, say, two parts (or several parts) would not result in the production of static electricity since the drop is electrically neutral to start with. However, if the drop atomizes by being "blown" apart, then, as suggested in Figure 2, the positive electricity is forced toward the periphery, thus leaving the center bulge negative, and the explosion results in fine negatively charged droplets formed out of the bulge and larger, positively charged, droplets formed from the liquid at the periphery. Lenard actually succeeded in observing the blowing apart of liquid droplets.

Schweitzer, *et al.*, (7) attempted to observe electrical charges on oil sprays by catching the spray on an electroscope. These experiments showed negative results. The arrangement of their apparatus indicates a possible reason for the failure. The electroscope was so arranged that it caught the total spray and thus the charges, if any, neutralized each other. We suggest that experiments be conducted in such a manner that the smaller droplets may be tested by the electroscope in the absence of the larger ones. Rather interesting experiments might consist in subjecting the spray to a strong electric field. This should result in the collection of the small droplets on the positive plate and in the collection of the large droplets on the negative plate.

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TABLE OF NOMENCLATURE

a = radius of jet, or ligament, about which oscillations occur, a function of the amplitude of oscillations.

A = surface area corresponding (on the average) to a unit length of jet.

c = coefficient of viscous damping.

C = velocity of liquid drop.

C_D = drag coefficient.

C' = a constant.

D = diameter of drop.

e = base of natural logarithm.

f_r = rotational frequency.

F = function of

F_D = drag force.

h = position of surface of separation of two fluids

H = total fluid lead

H' = amplitude of waves on surface of separation of two fluids.

J_o = zero order Bessel function of the first kind

k = constant

K = constant

l = length of the continuous part of the jet.

L = length of a filament, also denotes the dimension of length.

m = mass of liquid drop.

M = dimension of mass.

n = dynamic growth coefficient.

p = static pressure.

p_c = pressure caused by centrifugal forces.

p_σ = pressure caused by surface tension.

p_D = pressure caused by drag forces.

P_o = amplitude of periodic force causing forced vibration.

q = static growth coefficient.

r = cylindrical coordinate; radius of drop.

r_o = radius of nozzle.

Re = Reynolds number.

S = volume corresponding (on the average) to a unit length of jet.

S_o = area of a great circle of a drop.

T = kinetic energy.

V = velocity

z = cylindrical or Cartesian coordinate

$Z = \dfrac{L}{2a}$

α = amplitude of disturbance.

α_o = magnitude of initial disturbance.

$\kappa = \dfrac{2\pi}{\lambda}$

λ = wave length

ω_r = circular frequency of rotation.

ω_v = circular frequency of vibration.

ρ = density of gas.

ρ' = density of liquid.

σ = surface tension.

τ = period of vibration.

μ = coefficient of viscosity.

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ABSTRACT:

A critical review and an extension of the literature concerning the atomization of liquid jets by high velocity gas streams is made. The subject is considered under three headings, namely, the instability of jets, the mechanism of atomization of liquid jets, and the atomization of drops. The first part is based on Rayleigh's analysis. Dimensional analysis is used to develop an equation for the breakup-length of a jet. In the second part Castleman's analysis is closely followed. In the third part Littaye's work is reviewed and an alternative analysis concerning the mechanism of drop atomization is presented. It is shown that this is compatible with the conclusions of Lenard.

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