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EXPEDITIONARY LOGISTICS WAREHOUSE INVENTORY OPTIMIZATION

by

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The Personal Gear Issue (PGI) Division at each Expeditionary Support Unit is responsible for ordering, inventory management, and distribution of all personal gear issued. This work adapts an existing optimization model to determine appropriate ordering policies, specifically, the order point, *s*, and order quantity, *Q*, for each line item. The model seeks to minimize cost of inventory subject to target fill rate for each item, maximum number of orders per month, and other constraints. We consider realistic conditions such as current demand, 90 orders per month, and 99% fill rate. Under these conditions, the optimal *s* and *Q* values result in a significant reduction in the cost of inventory, from $8.2M (current PGI's inventory value) down to $1.23M. In addition, our model shows, via sensitivity analysis, the effect that varying fill-rate targets, monthly order limit, and demand have on cost. For example, we observe a nonlinear increase in cost as a function of demand increase. We recommend that PGI Division revises its ordering policy to include formal optimization analysis and to periodically track demand in order to re-evaluate the optimal *s* and *Q* values.

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EXPEDITIONARY LOGISTICS WAREHOUSE INVENTORY OPTIMIZATION

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LIST OF ACRONYMS AND ABBREVIATIONS

EOD	Explosive Ordnance Disposal
ESU	Expeditionary Support Unit
EODESU-1	Explosive Ordnance Disposal Expeditionary Support Unit One
GAMS	General Algebraic Modeling System
NAVSUP	Naval Supply Systems Command
NIIN	National Item Identification Number
PGI	Personal Gear Issue
SIOM	Site Demand-Based Level Inventory Optimization Model
SIOMsQ	Site Demand-Based Level Inventory Optimization Model with (s, Q) Pre-processing
sQUID	s and Q Unit Identification Model
WIOM	Wholesale Inventory Optimization Model

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EXECUTIVE SUMMARY

The Expeditionary Support Unit (ESU) mission is to plan, coordinate, integrate, synchronize, and provide logistics support for U.S. Navy Explosive Ordnance Disposal (EOD) units, both when located in homeport and when deployed to all areas of responsibility. In order to execute this vital mission, ESU comprises seven departments: supply, medical, administration, materiel, operations, maintenance, and training. The supply department is responsible for financial management, procurement, inventory management, shipping and receiving, contract management, and customer support for all EOD mobile units, mobile diving and salvage units, training and evaluation units and EOD shore detachments for their respective areas of operation (Commander, Explosive Ordnance Disposal Group One Public Affairs 2013).

The Personal Gear Issue (PGI) Division at each ESU is responsible for ordering, inventory management, and distribution of all personal gear issued to more than 1,200 personnel including roughly 400 EOD technicians, 200 Navy divers and various support personnel. Unlike Naval Supply Systems Command (NAVSUP) whose inventory comprises more than 430,000 items, PGI only contains roughly 600 line items (NAVSUP 2018). The scale of inventory allows PGI to economically maintain a nearly 100% fill rate, the fraction of customer demand met through immediate stock availability (Silver et al. 1998), versus a standard NAVSUP fill rate goal of 85%. The operations required by EOD technicians are dynamic and often short-notice, so inability to provide support to a mission with required gear can lead to mission delay or, at the worst, mission failure.

Obtaining the optimal order point, s, and order quantity, Q, for each item results in the best fill rate achievable within the given constraints. Site Demand-Based Level Inventory Optimization Model (SIOM) designed by Salmeron and Craparo (2016) is currently being evaluated to guide stock level decisions for NAVSUP. Ersoz (2016) developed SIOM with (s, Q) Pre-processing (SIOMsQ) to minimize the computation time required to solve SIOM. SIOMsQ limits the number of candidates evaluated per line item through pre-processing of associated fill rate, cost, penalties, and other data.

This thesis creates the *s* and *Q* Unit Identification (sQUID) model as a reformulation of SIOMsQ in order to minimize the PGI warehouse cost of inventory. sQUID maintains most of the formulation found in SIOMsQ, including maximum orders per month constraints. The key modification is that the objective function of SIOMsQ minimizes fill rate deviation penalty within a budget, while sQUID seeks to minimize inventory cost (budget) within fill rate. sQUID is implemented using the General Algebraic Modeling System (GAMS 2018a) with the CPLEX solver (GAMS 2018b) serving as the solving engine.

Our results show a pattern of diminishing return on inventory cost savings as the number of monthly orders increases. The current PGI inventory is approximately $8.2M. The percent cost reduction achieved at the various maximum monthly orders ranges from 72.6% to 90.6%. A 99% average fill rate is achievable when using the calculated *s* and *Q* values while increasing the demand up to 20%, but numerous items achieve fill rates far below the 99% fill rate required by current EOD policies. Higher levels of increased demand result in a steady decrease in the average fill rate.

As the conclusion of this research, we show that at least 47 monthly orders are required with EODESU-1's current inventory and demand, and the corresponding cost of inventory is $2.24M. We recommend implementing (*s*, *Q*) pairs that align with achieving a maximum of 90 orders per month ($1.23M cost of inventory) at 99% fill rate because this is a reasonable number of orders for one person to process. With additional assistance in processing orders, a lower cost of inventory is achievable. Re-optimizing *s* and *Q* based on 10–20% increase in demand will reduce the risk of not achieving 99% fill rate for each item during sudden increases in demand. In order to minimize the effect of fluctuating demand, we suggest to periodically track demand in order to ensure the inventory system utilizes optimal values of *s* and *Q*. Budget planners should expect a nonlinear rise in cost as a function of demand increase.

References

Commander, Explosive Ordnance Disposal Group One (EODGRU-1) Public Affairs (2013). EODESU One Keeps West Coast EOD Equipped to Fight. San Diego, CA: EODGRU-1. Retrieved from http://www.navy.mil/submit/display.asp?story_id=75290

Ersoz, T. (2016). A new approach to site demand-based level inventory optimization, Master's thesis, Naval Postgraduate School.

GAMS (2018a). General algebraic modeling system. Accessed July 22, 2018, https://www.gams.com/download/.

GAMS (2018b). CPLEX 12. Accessed July 22, 2018, https://www.gams.com/latest/docs/S_CPLEX.html.

Naval Supply Systems Command (2018). Navy supply chain management. Accessed July 22, 2018, https://www.navsup.navy.mil/public/navsup/wss/nscm/.

Salmeron J., Craparo E. (2016). Site Demand-Based Level Inventory Optimization Model, Release 1.2.3. Project Deliverable, Naval Postgraduate School.

Silver E., Pyke D., Peterson R. (1998). *Inventory Management and Production Planning and Scheduling.* (John Wiley & Sons, New York).

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I. INTRODUCTION

A. BACKGROUND

> Explosive Ordnance Disposal Expeditionary Support Unit One (EODESU-1) was commissioned in 2008 as part of the Explosive Ordnance Disposal (EOD) Total Force Vision and Commander Navy Expeditionary Combat Command's integrated strategy to transition logistics and maintenance functions from operational mobile unit commanders to a dedicated unit that is manned, trained and equipped to carry out these functions. (Commander, Explosive Ordnance Disposal Group One Public Affairs 2013)

Prior to 2008, each EOD mobile unit had its own supply department that purchased the necessary personal gear required by EOD technicians. Little standardization, no economies of scale, and varying levels of readiness resulted from the wide range of gear purchased. The establishment of EODESU-1 consolidated individual supply departments from each EOD mobile unit into a single command that is able to manage all facets of logistics needed by West Coast EOD mobile units.

The Expeditionary Support Unit (ESU) mission is to plan, coordinate, integrate, synchronize, and provide logistics support for units in homeport and forward deployed to all areas of responsibility. In order to execute this vital mission, ESU is comprised of seven departments: supply, medical, administration, materiel, operations, maintenance, and training. The supply department is responsible for financial management, procurement, inventory management, shipping and receiving, contract management, and customer support for all EOD mobile units, mobile diving and salvage units, training and evaluation units, and EOD shore detachments for their respective areas of operation (Muniz 2013). The division of interest in this analysis is the Personal Gear Issue (PGI) Division, which is responsible for the ordering, inventory, and distribution of all personal gear issued to more than 1,200 personnel, including roughly 400 EOD technicians, 200 Navy divers, and various support personnel.

Unlike traditional Navy commands that are able to procure the majority of their items from the standard Navy supply system, the specific requirements of the EOD community lead to PGI obtaining most of their items on the open market via purchase request. The EOD community is small, with very specific gear that does not apply to the majority of consumers in the Navy. The EOD community is always looking to upgrade equipment, so the expectation is that the open market will provide a majority of the gear. Therefore, the PGI inventory manager is unable to leverage the sourcing algorithms and other tools available through the standard Navy supply system to maintain the optimal quantity of items on the shelf. The decision process for determining when to order each line item depends heavily on the experience of the inventory manager. As such, the EOD community may benefit from formal mathematical optimization.

The current PGI inventory is valued in excess of $8M, and the current commanding officer is looking for ways to streamline processes and channel funds away from procuring unnecessary PGI inventory to the other departments. This thesis uses a mixed-integer linear program to determine the right level of PGI inventory, allowing the other six ESU departments to use a greater portion of the annual budget, which averages more than $27M.

B. REVIEW OF INVENTORY MODELS

Silver et al. (1998) highlight the importance of inventory management within the industry and military domains. According to Datex (2018), "implementing inventory optimization techniques in varying combinations and complexities can help supply chain operators realize significant financial and operational benefits such as reduced on-hand inventory, reduced out-of-stock instances, reduced inventory holding costs and more."

The definition of *inventory position* is the sum of on-hand and on-order quantities minus backorders. Two inventory management techniques, continuous review and periodic review, apply when modeling inventory management systems involving items with probabilistic demand. In a *periodic review system*, also called a fixed-time-period system, inventory position assessment occurs at specific time intervals. In a *continuous review system*, there is tracking of inventory position at all times.

In an (s, S) system, also called an order-up-to system, when inventory position falls below a threshold known as the order point s, the inventory system places an order to return the inventory position to a pre-designated level S. The main advantage of this system is minimal record keeping. Some drawbacks are reduced direct control, a requirement to calculate and process a different order quantity at each periodic order, and larger inventory levels because the system returns to maximum stock levels after every order. On the other hand, in an (*s*, *Q*) fixed-order-quantity system, when the inventory position of an item decreases to the order point *s*, a new order at a fixed quantity *Q* is placed. The main advantages of this system are continuous visibility of inventory position, excluding the requirement for calculating new order quantities, and reduced inventory costs. The major disadvantage is the cost associated with maintaining a continuous record of inventory position (Russell and Taylor 1999).

Figure 1 provides a comparison of an (*s*, *S*) system and an (*s*, *Q*) system. The dotted line depicts the inventory position and the solid line shows the net stock (on-hand quantity) or both the inventory position and the net stock if they are equal. The order lead time (i.e., the elapsed time from order placement to when the order is received) is represented as *L*, and is deterministic. The inventory position using the (*s*, *S*) inventory system will tend to be larger than compared to the (*s*, *Q*) inventory system because in the (*s*, *Q*) inventory system the order point is not always crossed at the specific value, often resulting in net stock less than the sum of *s* and *Q*. The value of *Q* selected will result in differing outcomes. Using a large order quantity provides a good *fill rate*, the fraction of customer demand met through immediate stock availability, (Silver et al. 1998) but it requires more funds allocated to inventory. Another concern is that larger inventories are susceptible to either shelf-life issues or obsolescence because of policy changes or technological advancements. Selecting a smaller order quantity requires frequent orders, which adds cost or may cause out-of-stock events that adversely affect fill rate. Choosing the right value for the order point is just as important. If the order point is too low, there is a larger risk for out-of-stock events. On the other hand, an order point that is too high results in unnecessary inventory that ties up limited funding. Independent of what type of inventory system is used, computer-based optimization allows for the determination of the correct pair of order point and order quantity necessary to obtain the minimal level of inventory while meeting the specified fill rate.

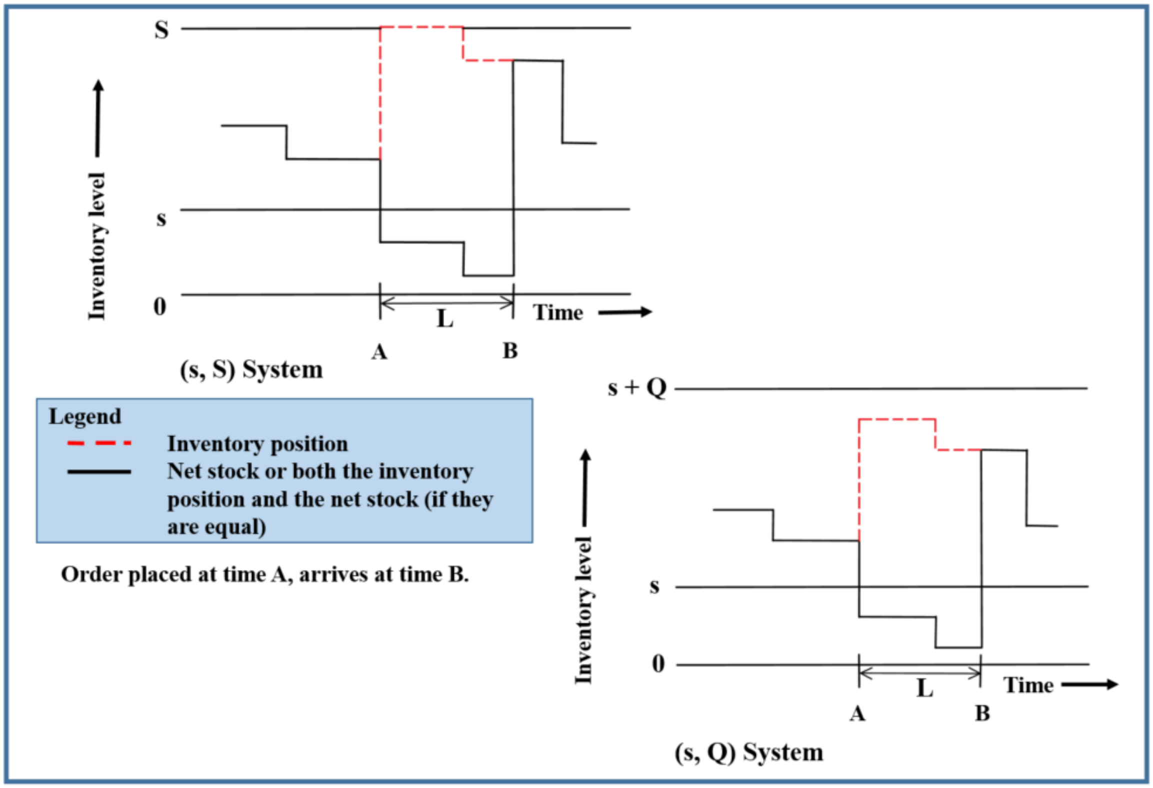

Figure 1. Example of continuous review (s, S) inventory system versus (s, Q) inventory system. Adapted from Silver et al. (1998).

All military branches solve difficult inventory management issues using optimization models and computer-based simulation. Curling (2016) creates an analytical tool that integrates optimization and discrete event simulation to better facilitate order management of repair parts within Marine Corps maintenance production plants. Bridges (1997) develops a linear program in support of the Army's Decision Support Tool that seeks a solution for the Army's inability to meet all requirements assigned to more than 200 Army installations for repair and maintenance activities. Yoon (2007) uses an optimization algorithm to aid decision makers working on the Air Force's National Defense Procurement Policy. He creates a two-stage approach to determine the optimal inventory level of concurrent spare parts. These are the initial parts purchased with the acquisition of a major item or system allowing for sustainment during the initial period. Salmeron and Craparo (2015) introduce the Wholesale Inventory Optimization Model (WIOM) composed of a series of mathematical optimization models, algorithms, and their computational implementations. WIOM calculates the optimal order points for tens of thousands of Naval Supply Systems Command (NAVSUP) line items by minimizing the expected weighted deviation from the target fill rate. Roth (2016) uses simulation to test the performance of a simple mathematical approach and two different inventory management models: simple calculation of fill rates, a legacy model called the Service Planning and Optimization model, and WIOM. The analysis concludes that WIOM outperforms the other two models by finding higher average fill rates in 22 out of 24 cases, and that average backorder lengths are up to 50% shorter.

Salmeron and Craparo (2016) also develop Site Demand-Based Level Inventory Optimization Model (SIOM), which seeks optimal values of both order quantities and order points. We intend to use the features of SIOM with (s, Q) Pre-processing (SIOMsQ) developed by Ersoz (2016). We perform a manipulation of the constraints and objective function to determine the best (s, Q) pairs to obtain the lowest cost of EOD inventory. We note the inventory size difference between the EOD and NAVSUP. Instead of the 85% fill rate NAVSUP attempts to achieve for most items, the goal for PGI is around 99%. To achieve fill rates at that level would not be financially possible for the many thousands of line items that NAVSUP maintains, but in the case of the EOD inventory, there are less than 600 line items. The expectation is that there will be minimal likelihood of an out-of-stock event for any line item. The primary goal of this thesis is to create a mixed-integer linear program to guide optimal decisions regarding setting of s and Q values for each of the PGI inventory line items in order to achieve 99% fill rate.

C. RESEARCH OBJECTIVES AND SCOPE

The objective of this research is to use formal mathematical optimization in order to recommend order points and order quantities for all the line items carried by EODESU-1 PGI in order to minimize the maximum total cost of on-hand inventory. To do this, we develop the s and Q Unit Identification (sQUID) model. sQUID is strongly based on SIOMsQ, with some modifications to capture EODESU-1's goals and constraints. We implement sQUID using General Algebraic Modeling System (GAMS 2018a) and the

CPLEX Solver (GAMS 2018b) and perform an analysis using approximately five years of historical demand and lead-time data.

II. METHODOLOGY: SQUID MODEL

A. MODEL OVERVIEW

SIOM was first developed by Salmeron and Craparo (2016) to find optimal order points and order quantities to help guide stock level decisions for NAVSUP. Because NAVSUP Weapon Systems Support maintains more than 430,000 Class IX (repair parts) National Item Identification Numbers (NIINs) (NAVSUP 2018), SIOM pre-generates candidate order-quantities for every NIIN, and then optimizes order points and order quantities (restricted to pre-generated ones). Ersoz (2016) develops SIOMsQ as a reformulation of SIOM seeking to improve computational time. SIOMsQ reduces the possible candidate (s, Q) pairs to approximate the solution of SIOM while maintaining comparable solution quality. The objective of both SIOM and SIOMsQ is to minimize the total expected penalty for missing the target fill rate, while meeting all constraints such as budget, shelf life, maximum number of orders per month, etc.

We manipulate SIOMsQ to create the sQUID model, which serves as the model used to address the problem analyzed in this research. SIOMsQ minimizes a sum of penalties assessed for missing predesignated fill rate targets, subject to a monetary budget constraint among others. In contrast, sQUID seeks to minimize the monetary cost incurred (expressed as an inventory cost assessed at the maximum stock level for all NIINs), subject to a set of fill rate constraints. Both models also place a constraint on the expected number of orders placed per unit of time.

B. GENERATING *s* AND *Q* PAIR CANDIDATES

In order to generate order quantities, SIOMsQ always selects "1" as the first candidate Q. Estimated lower and upper bounds for Q are used as the second and last candidates, and the differences between consecutive candidates of the remaining set of candidates are approximately the same.

Similarly, to generate order points, SIOMsQ considers "-1" and "zero" as the first two candidates. Note: "-1" refers to a special case, rarely used by NAVSUP, where after reaching zero inventory position, an item would need to be in a backorder status before initiating an order. Estimated lower and upper bounds for s are used as the third and last candidates, and the differences between consecutive candidates of the remaining set of candidates are approximately the same.

sQUID follows the same presented candidate rules as SIOMsQ, as well as the limits on the number of pre-generated s candidates (20) and the pre-generated Q candidates (10).

C. MATHEMATICAL MODEL

This section presents sQUID, a mathematical model adapted from the SIOMsQ formulation described by Ersoz (2016). For completeness, we also provide the SIOMsQ formulation in the appendix.

1. Indices and Index Sets

i Item (i.e., NIIN), for $i \in I$.

h Index of candidate (s, Q) pairs for a given item i, $h \in \{1, 2, ...\}$.

2. Input Data and Parameters [Units]

t_i Lead time for item i. [quarters]

$\hat{x}_i$ Expected demand for item i during lead time. [units of issue]

$\overline{h}_i$ Number of candidate pairs (order point and order quantity) for item i.

$\tilde{s}_{ih}$ s-value (order point) of the h-th candidate for item i. [units of issue]

$\tilde{Q}_{ih}$ Q-value (order quantity) of the h-th candidate for item i. [units of issue]

$\underline{s}_i$ Lower bound on order-point of item i (default is -1). [units of issue]

$\overline{s}_i$ Upper bound on order-point of item i. [units of issue]

$\underline{S}_i$ Allowance for item i. [units of issue]

$\overline{S}_i^L$ Shelf life for item i. [quarters]

$\Delta^{\underline{S}}$ 1 if allowances are activated, and 0 otherwise.

c_i Cost per unit of item i. [\$/unit]

r Maximum number of total expected orders per month for all items. [orders/month]

$\underline{f}_i$ Target fill rate for item i. [fraction, unitless]

f_{ih} Fill rate for item i if candidate h is chosen. [fraction, unitless]

3. Decision Variables

Γ_{ih} 1 if candidate pair h is selected for item i, and 0 otherwise.

Q_i Order-quantity for item i. [units of issue]

s_i Order-point for item i. [units of issue]

4. Formulation

sQUID is the following mixed-integer, linear problem:

$$\min z = \sum_i c_i \left(s_i + Q_i \right) \tag{1}$$

Subject to:

$$Q_i = \sum_{h|h \leq \overline{h}_i} \tilde{Q}_{ih} \Gamma_{ih} \qquad \forall i \tag{2}$$

$$s_i = \sum_{h|h \leq \overline{h}_i} \tilde{s}_{ih} \Gamma_{ih} \qquad \forall i \tag{3}$$

$$\sum_{h|h \leq \overline{h}_i} \Gamma_{ih} = 1 \qquad \forall i \tag{4}$$

$$\sum_{h|h \leq \overline{h}_i} f_{ih} \Gamma_{ih} \geq \underline{f_i} \qquad \forall i \tag{5}$$

$$\sum_i \sum_{h|h \leq \overline{h}_i} \frac{\hat{x}_i}{3 t_i \tilde{Q}_{ih}} \Gamma_{ih} \leq r \tag{6}$$

$$\left\lceil \underline{S}_i \right\rceil \Delta^{\underline{S}} \leq \mathrm{s}_i + \mathrm{Q}_i \leq \overline{S}_i^{L} \frac{\hat{x}_i}{3 t_i} \qquad \forall i \tag{7}$$

$$\underline{s}_i \leq s_i \leq \overline{s}_i \qquad \forall i \tag{8}$$

$$Q_i \geq 0 \qquad \forall i \tag{9}$$

$$\underline{s}_i \geq -1 \qquad \forall i \tag{10}$$

$$\Gamma_{ih} \in \{0,1\} \qquad \forall i, h \mid h \leq \overline{h}_i \tag{11}$$

D. MODEL DISCUSSION

The goal of the objective function (1) is to minimize the cost of inventory at the maximum stock level. This is a conservative (i.e., pessimistic) assumption on the dollar amount tied to inventory at any given time.

Equations (2)–(4) restrict the model in choosing only one (s, Q) pair for an item and set the corresponding values of s and Q.

Equation (5) ensures each item achieves its target fill rate $\underline{f_i}$ (99% in our baseline analysis).

Equation (6) restricts the expected number of orders placed in a month from exceeding a given limit.

Equation (7) limits the lower and upper bounds of the maximum stock level. Stock levels should be greater than the allowance and less than the minimum quantity that would cause items to perish (under expected demand assumptions).

Equations (8)–(11) set the domains of the decision variables and parameters.

Note s and Q variables are declared as continuous variables given that they will naturally take integer values, per (2) and (3).

E. MODEL ASSUMPTIONS

Regarding the data provided by EOD, the sQUID model:

- Utilizes approximately five years of historical data to create an empirical probability distribution of demand.
- Excludes any item without demand data.
- Applies a conservative six-month lead time to any item without lead-time data.

In this study, the allowance term $\underline{S_i}$ is set to zero for each item. The allowance term remains in the formulation to allow for its potential use in future studies.

F. MODEL SIZE

We implement sQUID on a Microsoft Surface Pro 3 tablet with four gigabytes of random-access memory and 1.9 gigahertz Intel Core i5-4300U processor. We use GAMS/CPLEX as the optimization engine to solve the mixed-integer linear program

(GAMS 2018b). A typical sQUID run for an instance with 555 line items and 111,000 (s, Q) pairs contains approximately 4,000 constraints, 237,000 non-zero elements, and 73,000 decision variables, of which 72,000 are integer variables. sQUID solves this in less than two seconds.

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III. COMPUTATIONAL ANALYSIS

Our analysis varies three input parameters: maximum orders per month (r), fill rate ($\underline{f_i}$), and demand data. We explore 17 values for maximum monthly orders, four values for fill rate, and five levels of demand increase. Figure 2 shows sample input data used in sQUID. For example, row 1 shows the following characteristics of NIIN LSN_0001: 0.19 quarter lead time, $873 unit cost, one average document per cycle, no shelf-life timeframe, no allowance, no parametric distribution of demand because historical demand utilized, fixed s set at "2," minimum s set at "-1," maximum s set at 10,000, fixed Q set at "1," and demand for months 1, 2, and 3.

NIIN	lead_time	unit_cost	avgDoc	shelf_life	allowance	new_pr	fixed_s	min_s	max_s	fixed_Q	DemandMon1	DemandMon2	DemandMon3
0000_00_LSN_0001	0.19	873	1	0	0	0	2	-1	10000	1	0	0	1
0000_00_LSN_0002	0.66	6.29	1	0	0	0	12	-1	10000	20	0	0	0
0000_00_LSN_0003	0.66	22.99	1	0	0	0	3	-1	10000	2	0	0	1
0000_00_LSN_0004	0.2	19.95	1	0	0	0	15	-1	10000	83	6	9	10
0000_00_LSN_0006	0.66	324.86	1	0	0	0	14	-1	10000	16	3	6	1
0000_00_LSN_0007	0.14	324.86	1	0	0	0	8	-1	10000	17	2	2	5
0000_00_LSN_0008	0.66	324.86	1	0	0	0	2	-1	10000	5	0	0	0
0000_00_LSN_0009	0.66	324.86	1	0	0	0	6	-1	10000	9	0	0	1
0000_00_LSN_0013	0.66	155.65	1	0	0	0	2	-1	10000	2	0	0	0
0000_00_LSN_0017	0.66	6	1	0	0	0	4	-1	10000	6	0	1	0
0000_00_LSN_0018	0.34	54.24	1	0	0	0	13	-1	10000	58	4	4	3
0000_00_LSN_0019	0.5	252.96	1	0	0	0	3	-1	10000	10	1	0	0

Figure 2. sQUID sample input data

A. MAXIMUM MONTHLY ORDERS

The 17 values selected for maximum monthly orders are 47, 50, 60, 70, 80, 90, 100, 110, 120, 130, 140, 150, 160, 170, 180, 190, and 200. We note the lowest feasible number of orders per month is 47, whereas 90 is a realistic number of orders that a single individual may process in a month, and 200 would require additional personnel to assist with orders. All cases assume a fill rate of at least 99% for all items. Figure 3 shows the optimal cost of inventory for various values of r. Note that the cost of inventory ranges from approximately $2.24M for 47 monthly orders to $900,000 for 200 monthly orders.

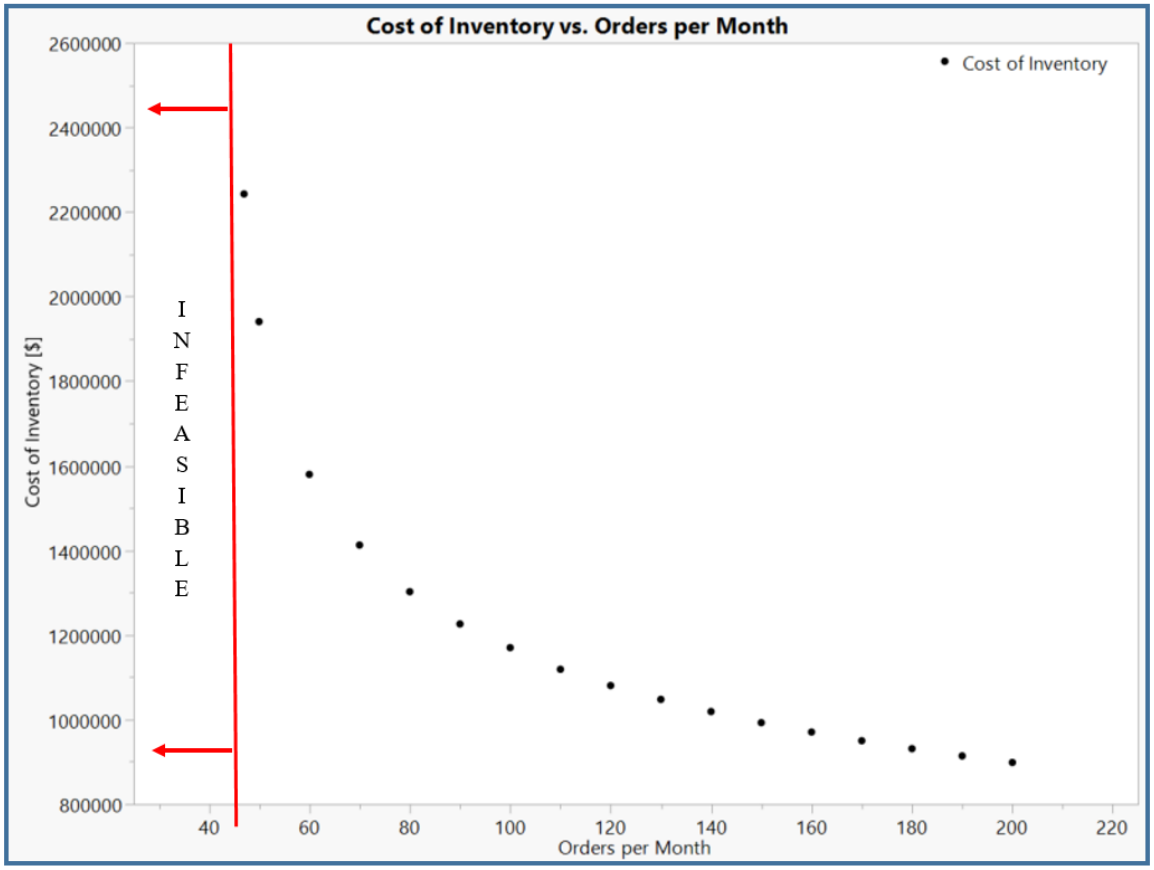

Figure 3. Cost sensitivity analysis varying maximum monthly orders at 99% fill rate

B. FILL RATE

Using the same 17 values for maximum monthly orders as in Section III.A, we run excursions with target fill rate set to 99%, 97%, 95%, 93%, and 90% for all items. Figure 4 shows that lower costs are possible as fill rate is lowered. One interesting observation is that the 97% and 95% fill rate plots are identical for maximum monthly orders between 47 and 140, but diverge afterwards. At 200 orders per month, the lowest cost of inventory achieved is $767,000.

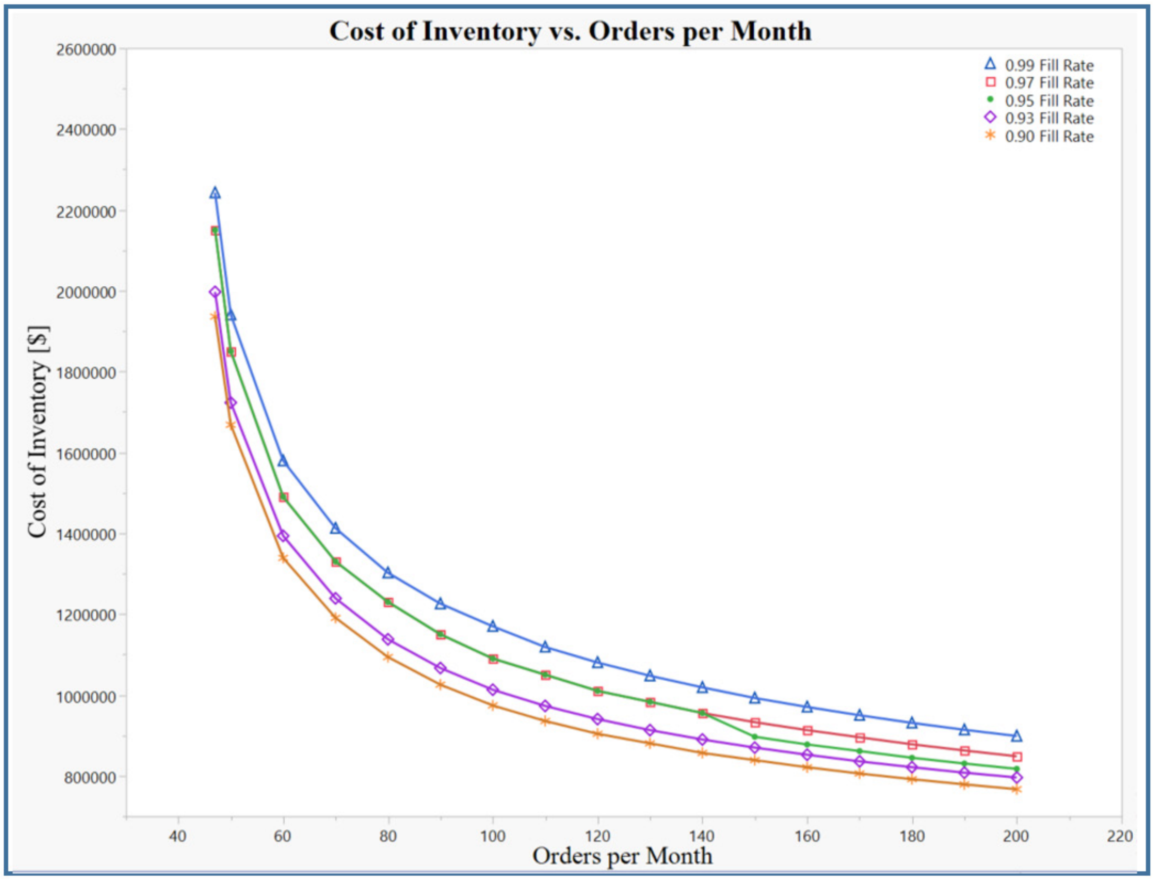

Figure 4. sQUID sensitivity analysis varying fill rate

C. INCREASED DEMAND WITH FIXED *s* AND *Q*

We next fix the *s* and *Q* values calculated from a baseline case that meets the 99% fill rate requirement for all items. We test the robustness of this solution by increasing the historical demand data by 10%, 20%, 30%, 40%, and 50%. With *s* and *Q* fixed, there is no optimization of cost. Rather, we run sQUID in order to calculate the achieved fill rate and orders per month and see how they vary as demand increases.

Figure 5 shows that as demand increases the average fill rate drops and the number of orders increases. Each dot represents an item, and triangles show the average fill rate. In all excursions, the achieved fill rate per line item varies greatly, but the average fill rates are 99%, 99%, 97%, 94%, and 91% for demand increases of 10%, 20%, 30%, 40%, and 50%, respectively. An interesting finding is that there is no change in the average fill rate and there is only a small change in the number of orders when demand increases up to

20%. For larger increases in demand, we observe a larger average decrease of 2.7% fill rate and 7.4 orders per month. This result shows that the fixed s and Q values are robust in maintaining a 99% average fill rate within demand increases up to 20%, but numerous items achieve drastically worse fill rates that are unacceptable in the current EOD policy. If demand increases, the same s and Q values no longer achieve a 99% fill rate for all items. In order to meet the goal of 99% fill rate for every item, we can re-optimize s and Q values but the result is a higher cost of inventory (see Section III.D). Therefore, we suggest to periodically track demand in order to verify that demand has not changed drastically since the s and Q values were previously set.

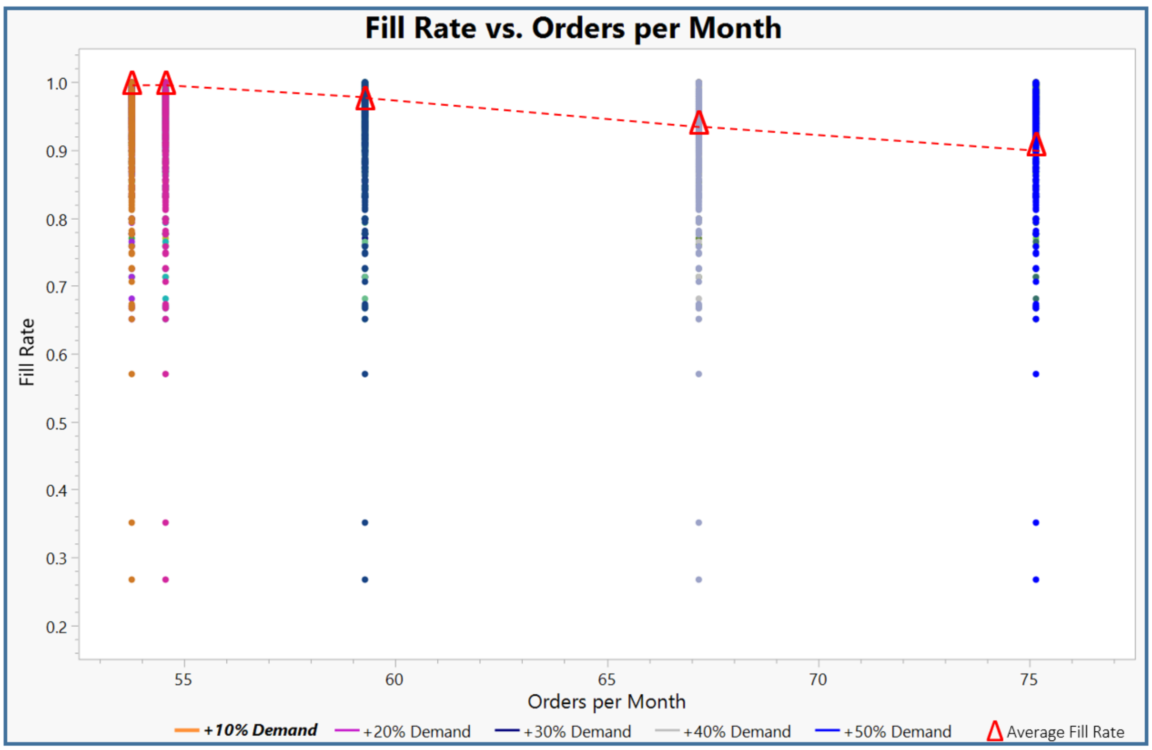

Figure 5. sQUID sensitivity analysis varying demand while fixing s and Q values for each item

D. INCREASED DEMAND WITHOUT FIXING *s* AND *Q*

Using the same 17 values selected for maximum monthly orders as in Section III.A, we allow sQUID to re-optimize *s* and *Q* (and calculate the cost of inventory) at 10%, 20%, 30%, 40%, and 50% increased demand. Figure 6 shows diminishing returns as the allowed maximum orders per month increases. An interesting finding is that the increase in inventory cost is not constant, despite a constant increase in demand. At 90 orders per month, the cost of inventory achieved is \$1.22M, \$1.39M, \$1.5M, \$1.69M, \$1.84M, and \$1.97M, respectively, as demand increases from 10% to 50%. The change in inventory cost at the five different increased demand levels is 13%, 22%, 37%, 49%, and 61%, respectively.

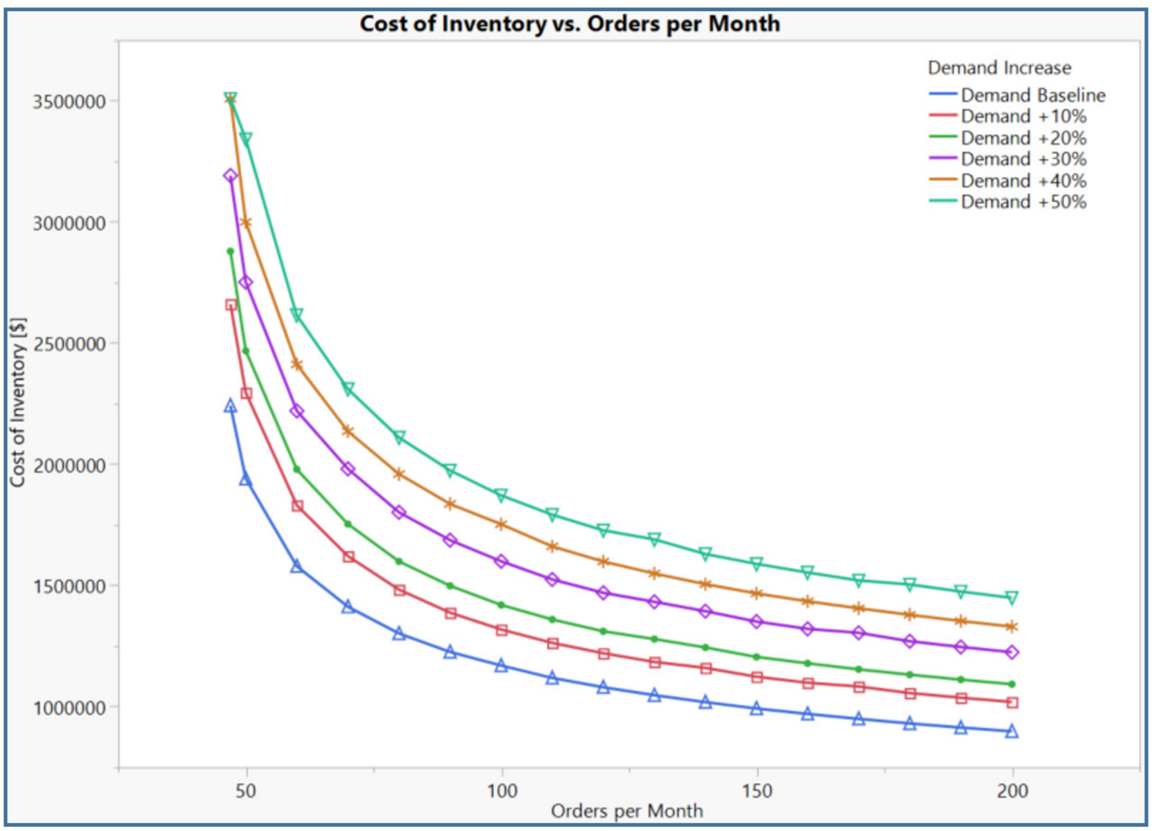

Figure 6. sQUID sensitivity analysis increasing demand

E. SUMMARY OF RESULTS

Our results show a pattern of diminishing return on inventory cost savings as the number of orders per month increases. The current PGI inventory is approximately $8.2M. The percent reduction achieved at the various maximum monthly orders ranges from 72.6% to 90.6%. A 99% average fill rate was achievable when using the calculated *s* and *Q* values while increasing the demand up to 20%, but numerous items achieved fill rates far below the 99% fill rate required by current EOD policies. Higher levels of increased demand result in a steady decrease in the average fill rate. When forecasting increasing demand, budget planners should be aware that the cost of inventory may increase in a nonlinear fashion.

IV. CONCLUSIONS AND FUTURE RESEARCH

A. CONCLUSIONS

In this thesis, we have developed sQUID, a mixed-integer linear optimization model, strongly based on SIOM (Salmeron and Craparo, 2016) and SIOMsQ (Ersoz 2016). sQUID minimizes the cost of inventory, subject to constraints on fill rate and the number of orders placed per month. Our analysis demonstrates that the cost of PGI inventory can be reduced below current levels. We show that at least 47 monthly orders are required with EODESU-1's current inventory and demand, and the corresponding cost of inventory is \$2.24M. We recommend implementing (s, Q) pairs that align with achieving a maximum of 90 orders per month (\$1.23M cost of inventory) at 99% fill rate because this is a reasonable number of orders one person can process. With additional assistance in processing orders, a lower cost of inventory is achievable. Re-optimizing s and Q based on a 10–20% increase in demand will reduce the risk of not achieving 99% fill rate for each item during sudden increases in demand. In order to minimize the effect of fluctuating demand we suggest to periodically track demand in order to ensure the inventory system utilizes optimal values of s and Q. Budget planners should expect a nonlinear rise in cost as a function of increasing demand.

B. FUTURE RESEARCH

As we have successfully implemented sQUID, future studies can focus on:

- Developing an Excel spreadsheet tool that tracks current demand periodically to adjust s and Q values, automated internal request document to procure items, provide low stock alerts, report creation, and other features minimizing administrative burden.

- Applying sQUID to EODESU-1 supply department responsible for processing all divisional purchase requests.

- Applying sQUID to other NECC commands with warehouse inventories.

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APPENDIX. SIOMSQ FORMULATION

This appendix describes the formulation of SIOMsQ as given in Ersoz (2016).

A. INDICES AND INDEX SETS

i Item (i.e., NIIN), for $i \in I$.

h Index of candidate (*s, Q*) pairs for a given item i, $h \in \{1, 2, ...\}$.

m Index for the penalty segments of fill rates (e.g., $m \in M = \{1, 2, ..., 5\}$).

l Group of items, for $l \in L$.

B. INPUT DATA AND PARAMETERS [UNITS]

t_i Lead time for item i. [quarters]

$\hat{x}_i$ Expected demand for item i during the lead time. [units of issue/lead time]

$\overline{h}_i$ Number of candidate pairs (order point and order quantity) for item i.

$\tilde{s}_{ih}$ *s*-value (order point) of the *h*-th candidate for item i. [units of issue]

$\tilde{Q}_{ih}$ *Q*-value (order quantity) of the *h*-th candidate for item i. [units of issue]

$\underline{s}_i$ Lower bound on order point of item i (default is -1). [units of issue]

$\overline{s}_i$ Upper bound on order point of item i. [units of issue]

$\underline{S}_i$ Allowance for item i. [units of issue]

$\Delta^{\underline{S}}$ 1 if allowances are activated, and 0 otherwise.

c_i Cost per unit of item i. [$/unit]

r Maximum number of total expected orders per month for all items. [orders/month]

$\bar{S}_i^L$ Shelf life for item i. [quarters]

$\bar{S}$ Maximum months of supply for any item. [months]

$\hat{s}_i^0$ Initial order point used to enforce persistence for item i. [units of issue]

δ_i^p Penalty for deviation from initial order point for item i. [unitless]

γ^p Overall persistence weight. [unitless]

$\delta_i^{\bar{S}}$ Penalty for deviation of item i above maximum months of supply. [unitless]

$\gamma^{\bar{S}}$ Overall weight for MSL deviations above maximum months of supply. [unitless]

l_i Group of item i. [unitless]

$\bar{f}_l$ Required fill rate for any item in group l. [fraction, unitless]

w_l Weight for achieving the required fill rate of any item in group l. [weight units]

C. DECISION VARIABLES

Γ_{ih} 1 if candidate pair h is selected for item i, and 0 otherwise.

Q_i Order quantity for item i. [units of issue]

s_i Order point for item i. [units of issue]

D. DERIVED DATA

f_{ih} Fill rate of item i for candidate pair h. [fraction, unitless]

$\tilde{f}_{ih}$ Penalty for selecting candidate pair h for item i. [unitless]

s_{ih}^{+} Deviation down with respect to initial order point for item i and candidate pair h. [units of issue]

s_{ih}^{-} Deviation up with respect to initial order point for item i and candidate pair h. [units of issue]

$\overline{S}_{ih}^{+}$ Deviation for MSL of item i and candidate h above the average demand during the maximum months of supply for the item. [months]

$\tilde{c}_{ih}$ Expected number of cycles per lead time period for item i if candidate pair h is selected. [unitless] Note: A cycle is defined as the time period between consecutive orders, or the lead time; whichever is shorter.

$\tilde{B}_{ih}$ Expected fraction of backorders during a cycle for item i if candidate pair h is selected. [fraction, unitless] Note: The calculation method appears in Equation (1).

X'_{ih} Random variable for the demand of item i in a cycle if candidate pair h is selected. [units of issue].

f_{ih}^{dev} Deviation from the required fill rate for item i if candidate pair h is selected. [fraction, unitless]

$\tilde{f}_{ihm}^{dev}$ Deviation penalty for item i, in fill rate penalty segment m if candidate pair h is selected. [unitless]

$\overline{f}_{im}^{-}$ Maximum deviation allowed for fill rate of item i, in penalty segment m.

(Calculated as $\overline{f}_{im}^{-} = f_{l_i} \frac{m^2}{\sum_{j \in M} j^2}$) [fraction, unitless]

$w_{l_i m}$ Penalty for deviation from the required fill rate for items of group l in penalty segment m. (calculated as $w_{l_i m} = m w_l$) [unitless]

E. SIOMSQ FORMULATION

SIOMsQ can be stated as the following mixed-integer, linear problem:

$$\text{SIOMsQ: } \min_{s,Q,\Gamma} \sum_i \sum_{h|h \le \overline{h}_i} \tilde{f}_{ih} \Gamma_{ih} + \gamma^p \sum_i \sum_{h|h \le \overline{h}_i} \frac{\delta_i^p}{\hat{s}_i^0 + 1.5} (s_{ih}^+ + s_{ih}^-) \Gamma_i^h + \gamma^{\overline{S}} \sum_i \sum_{h|h \le \overline{h}_i} \frac{\delta_i^{\overline{S}}}{\overline{S} + 1} \overline{S}_{ih}^+ \Gamma_i^h \tag{12}$$

Subject to:

$$Q_i = \sum_{h|h \le \overline{h}_i} \tilde{Q}_{ih} \Gamma_{ih} \quad \forall i \tag{13}$$

$$s_i = \sum_{h|h \le \overline{h}_i} \tilde{s}_{ih} \Gamma_{ih} \quad \forall i \tag{14}$$

$$\sum_{h|h \le \overline{h}_i} \Gamma_{ih} = 1 \quad \forall i \tag{15}$$

$$\sum_i c_i (\mathrm{s}_i + Q_i - \lceil \underline{S}_i \rceil \Delta^{\underline{S}}) \le \mathrm{b} \tag{16}$$

$$\sum_i \sum_{h|h \le \overline{h}_i} \frac{\hat{x}_i}{3 t_i \tilde{\mathrm{Q}}_{ih}} \Gamma_{ih} \le r \tag{17}$$

$$\lceil \underline{S}_i \rceil \Delta^{\underline{S}} \le \mathrm{s}_i + \mathrm{Q}_i \le \overline{S}_i^L \frac{\hat{x}_i}{3 t_i} \quad \forall i \tag{18}$$

$$\underline{s}_i \le s_i \le \overline{s}_i \tag{19}$$

$$Q_i \ge 0 \text{ and integer } \forall i \tag{20}$$

$$s_i \ge -1 \text{ and integer } \forall i \tag{21}$$

$$\Gamma_{ih} \in \{0,1\} \quad \forall i, h \mid h \le \overline{h}_i \tag{22}$$

The goals of the objective function (12) are to minimize (a) deviations from target fill rates for all items (larger penalty rates for being far away from target fill rate); (b) the penalties for deviating from current order points; and (c) the penalties for exceeding maximum months of supply. Persistence penalties and MSL deviation penalties are optional and can be removed from the objective function by setting $\gamma^p = 0$ and $\gamma^{\overline{S}} = 0$, respectively.

Equations (13)–(15) restrict the model in choosing only one (*s, Q*) pair for an item and set the values of selected *s* and *Q*.

Equation (16) establishes a budget restriction on the MSL cost of all items. Equation (17) restricts the expected number of orders placed in a month from exceeding a given limit.

Equation (18) limits the lower and upper bounds of MSL. Stock levels should be greater than the allowance and less than the minimum amount that would cause items to perish (under expected demand assumptions).

Equations (19)–(22) set the domains of the decision variables and parameters.

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LIST OF REFERENCES

Bridges, M. (1997). Optimally funding Army installation repair and maintenance activities, Master's thesis, Naval Postgraduate School.

Commander, Explosive Ordnance Disposal Group One (EODGRU-1) Public Affairs (2013). EODESU One Keeps West Coast EOD Equipped to Fight. San Diego, CA: EODGRU-1. Retrieved from http://www.navy.mil/submit/display.asp?story_id=75290

Curling, T. (2016). USMC Inventory Control Using Optimization Modeling and Discrete Event Simulation, Master's thesis, Naval Postgraduate School.

Datex (2018). Top Techniques for Inventory Optimization. Accessed June 13, 2018, https://www.datexcorp.com/top-techniques-for-inventory-optimization/

Ersoz, T. (2016). A new approach to site demand-based level inventory optimization, Master's thesis, Naval Postgraduate School.

GAMS (2018a). General algebraic modeling system. Accessed July 22, 2018, https://www.gams.com/download/.

GAMS (2018b). CPLEX 12. Accessed July 22, 2018, https://www.gams.com/latest/docs/S_CPLEX.html.

Muniz, B. (2013). Serving EOD Forces … Globally. San Diego, CA: EODESU-1. Retrieved from http://scnewsltr.dodlive.mil/2013/12/23/serving-eod-forces-globally/.

Naval Supply Systems Command (NAVSUP) (2018). Navy supply chain management. Accessed July 22, 2018, https://www.navsup.navy.mil/public/navsup/wss/nscm/.

Roth, G. (2016). A simulation of alternatives for wholesale inventory replenishment, Master's thesis, Naval Postgraduate School.

Russell, R., Taylor, B. (1999). *Operations Management.* (Prentice Hall, New Jersey).

Salmeron J., Craparo E. (2015). Wholesale Inventory Optimization Model, Release 2.7. Project Deliverable, Naval Postgraduate School.

Salmeron J., Craparo E. (2016). Site Demand-Based Level Inventory Optimization Model, Release 1.2.3. Project Deliverable, Naval Postgraduate School.

Silver E., Pyke D., Peterson R. (1998). *Inventory Management and Production Planning and Scheduling.* (John Wiley & Sons, New York).

Yoon, K. (2007). Finding the optimal CSP inventory level for multi-echelon system in Air Force using random effects regression model. *European Journal of Operational Research,* 180(3), 1076. doi:10.1016/j.ejor.2006.05.006

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