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# RPPR Final Report

as of 14-Jun-2018

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**Final Report** for Period Beginning 01-Sep-2014 and Ending 28-Feb-2018

**Title:** Statistical Energy Analysis and Asymptotic Modal Analysis of Nonlinear and Nonconservative Engineering Systems - Research Topic Area 1.3.2 Multi-Dimensional and Dissipative Dynamical Systems

**Begin Performance Period:** 01-Sep-2014

**End Performance Period:** 28-Feb-2018

**Report Term:** 0-Other

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**Distribution Statement:** 1-Approved for public release; distribution is unlimited.

**STEM Degrees:** 1

**STEM Participants:** 0

**Major Goals:** 1) Extend AMA to systems with multiple components, including dissipation in the connection

1.1) Consider a system of N parallel non-congruent plates, where adjacent plates are coupled at an arbitrary point. Derive AMA expressions for the transverse vibration of these plates where the first plate is excited by a point, band-limited random signal. Report the precision of these estimates for a varying number of excited modes in the system. Report the computational cost improvements relative to CMA in the frequency domain.

1.2) Consider two parallel non-congruent plates with N rigid connections at arbitrary points on the plate. Derive AMA expressions for the transverse vibration of these plates where the first plate is excited by a point, bandlimited random signal. Report the precision of these estimates for a varying number of excited modes in the system. Report the computational cost improvements relative to CMA in the frequency domain.

1.3) Consider two parallel non-congruent plates connected via a discrete, dissipative dynamical system (such as a damped spring, or a damped spring mass). Derive AMA expressions for the transverse vibration of these plates where the first plate is excited by a point, band-limited random signal. Report the precision of these estimates for a varying number of excited modes in the system. Report the computational cost improvements relative to CMA in the frequency domain.

2) Extend AMA to nonlinear systems

2.1) Consider an elastic plate carrying N discrete nonlinear elements (such as a damped spring-mass where either the dissipation, elasticity, or inertia is nonlinear). Derive AMA expressions for the transverse vibration of this plate and the masses where the plate is excited by a point, band-limited random signal. Report the precision of these estimates for a varying number of excited modes in the system. Report the computational cost improvements relative to CMA in the frequency domain.

2.2) Consider a nonlinear continuous component (such as a rectangular plate behaving according to the von Karman kinetic model). Derive AMA expressions for the transverse vibration of this plate. Report the precision of these estimates for a varying number of excited modes in the system. Report the computational cost improvements relative to CMA in the frequency domain.

3) Extend AMA to systems experiencing flow-induced excitation

3.1) Consider a continuous elastic structural component responding to random forces due to turbulence created in a boundary layer or an adjacent cavity that undergoes broadband response prior to and after the onset of a Hopf bifurcation of the flow or due to fluid-structure interaction. Perform time simulations of the system in order to create a benchmark. Use this as a baseline for comparison to the results of AMA/SEA expressions for estimates of the system response.

3.2) Repeat the above for a system undergoing limit cycle oscillations.

4) Gather experimental data to verify computational results of the above and other AMA investigations.

## **RPPR Final Report**

### **as of 14-Jun-2018**

Experimental configurations for testing system behavior in the asymptotic modal limit

- A single plate carrying a concentrated damped spring-mass
- Two parallel plates with one or more rigid connections
- Three or more rigidly connected plates
- a fluid-structure interaction experiment

**Accomplishments:** 1.1) AMA expressions for 3 parallel plates have been derived, and the precision/accuracy of their results relative to CMA for various numbers of excited modes in the system has been recorded. CMA expressions for N parallel plates have been derived. Moreover, limits of the applicability of AMA relative to modal density have been identified and quantified due to the influence of modal cross-coupling.

1.2) AMA expressions for 2 parallel plates with 2 rigid connections between them have been derived, and the precision/accuracy of their results relative to CMA for various numbers of excited modes in the system has been recorded. CMA expressions for N connections between the two plates have been derived.

1.3) AMA expressions for two parallel plates connected at a point by a damped spring have been derived, and the precision/accuracy of their results relative to CMA for various numbers of excited modes in the system has been recorded.

2.2) Frequency domain analysis techniques for nonlinear elastic systems have been developed that will pave the way for AMA derivations for a von Karman plate. Iterative techniques for finding solutions to nonlinear vibration of continuous systems have been developed, and these techniques have been applied to beams in a low-frequency, single harmonic limit. The resulting AMA algebraic system for the von Karman plate has been identified.

**Training Opportunities:** Nothing to Report

**Results Dissemination:** One contributor presented at the 2017 Meeting of the Acoustical Society of America (ASA) in Boston, MA on June 26, 2017. This presentation detailed the current state of the art of AMA including coupled continuous systems and nonlinear systems. We have also presented a paper to the ASME Dynamic Systems and Control Conference in Corner, VA on October 11-13, 2017, and participated in several other poster and oral presentation meetings of the ASA and the International Symposium on the Vibration of Continuous Systems.

Also five articles have been published as follows:

Culver, D and Dowell, E. "On modal cross-coupling in the asymptotic modal limit" 2018. Journal of Sound and Vibration. 416:172-191

Culver, D and Dowell, E. "Iterative techniques for analyzing nonlinear vibrating dynamical systems in the frequency domain" 2018. Nonlinear Dynamics. 91:2113-2140.

Culver, D and Dowell, E. "High frequency analysis of a plate carrying a concentrated nonlinear spring-mass system" 2016. Journal of Sound and Vibration. 379:84-105.

Culver, D and Dowell, E. "High frequency analysis of a point-coupled parallel plate system" 2017. Journal of Vibration and Acoustics. 139(5) Article 051002.

Culver, D and Dowell, E. "High frequency analysis of a dynamically-coupled parallel plate system" 2017. Proceedings of the ASME 10th Annual DSCC. Vol 3. Article V003T22A002

**Honors and Awards:** Nothing to Report

**Protocol Activity Status:**

**Technology Transfer:** Nothing to Report

**CONFERENCE PAPERS:**

**RPPR Final Report**  
as of 14-Jun-2018

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**Paper Title:** High frequency analysis of a dynamically-coupled parallel plate system

**Authors:** Dean Culver, Earl Dowell

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Nothing to report in the uploaded pdf (see accomplishments)