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RPPR Final Report
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Major Goals: The erosion of earth materials by shear flows is common in nature, and has numerous applications including geomorphology, agriculture, and climate science [1-3]. Large sums of money are spent every year to control or mitigate erosion, with varying degrees of success. In some cases, such as controlling silting in dams and reservoirs, erosion may be desirable to remove deposited sediment. On the other hand, the prevention of erosion is crucial in places where bodies of water, landmasses, and human development meet.

Despite its importance, modeling erosion remains a difficult problem due to its tremendous complexity [4,5]. Strongly erosive flows are typically turbulent, leading to highly stochastic shear stress imparted to the earth material. The grains that make up the bed typically have highly nonspherical shapes and variable sizes or material properties. Commonly used riverbed erosion models typically do not include these complexities and instead employ dimensional analysis [6]. Previous studies have suggested that the onset of erosion (i.e., the conditions under which a static bed will begin to erode) is controlled by two nondimensional parameters: ? and Re_*. The Shields number ? is the ratio of the shear stress ? imparted to the surface layer of the bed by the flow to the (reduced) weight of the bed particles ((?_g-?_f)gD), where ?_g is the density of the bed grains, ?_f is the density of the fluid, g is the acceleration due to gravity, and D is the diameter of the grains. The shear Reynolds number ?Re?_* is the ratio of the inertial forces in the flow (where u_*^2=?/?_f is the known as the friction velocity) to viscous damping, where ? is the kinematic viscosity of the fluid. Based on a compilation of empirical data from field and laboratory measurements, the onset of erosion is given by a curve, known as the Shields curve, in the parameter space spanned by these two nondimensional parameters [7]. The Shields curve, however, is a highly oversimplified characterization of the actual dynamics of incipient bed motion, and eroding systems are routinely observed to violate the Shields curve. In addition, there is no agreed-upon theoretical explanation for the shape of the Shields curve.

Previous modeling approaches to erosion of granular beds have typically either thrown away all the details of the problem using a simple mass-balance approach or have attempted to mimic the full complexity of the natural system [8-10]. However, the former approaches tend to make predictions that fail because they miss key elements of the erosion process, while using the latter approaches makes it difficult to identify the dominant mechanisms that control erosion, which limits their generalizability.

The primary goal of our research was to simplify the problem by identifying the key elements necessary to describe hydrodynamic erosion via complementary numerical simulations and laboratory experiments. We aimed to clarify which details of this enormously complex problem are necessary to capture its dynamics and behavior, and which play only secondary roles and may thus be neglected. To sharpen our questions, we focused on the restricted problem of incipient motion, i.e., what are the conditions under which bed grains just begin to move?

Accomplishments: Our research accomplishments can be organized into four main thrusts: A. Physical justification for the Shields curve, B. System-size dependence near the critical Shields number, C. Incipient grain motion in turbulent flow, and D. Segregation and armoring of riverbeds.

A. Physical justification for the Shields curve

In our article “Onset and cessation of motion in hydrodynamically sheared granular beds” [11], we showed that by considering only a simple model flow, a linear fluid drag law, purely repulsive and elastic intergrain contact forces, and gravity, one can qualitatively reproduce the Shields curve for the onset of grain motion. We demonstrated this result by varying the strength of the hydrodynamic driving and the relative degree of fluid damping experienced by the grains.

We then extended this model to include both more general flows and grain interactions to identify the physical mechanisms that control different regimes of the Shields curve. In our article “Role of grain dynamics in determining the onset of sediment transport” [12], we included more complex grain interactions such as friction, collisional dissipation, and irregular grain shapes. A more complete picture of the fluid drag was also adopted in the model, including both Stokes drag (linear in flow velocity) and inertial drag (quadratic in flow velocity), to appropriately account for the relevant drag laws at low and high Reynolds numbers, respectively.

In Fig. 1, we show the Shields curve along with the results of our numerical model. We note that the shear Reynolds number Re_* can be written as the ratio of two timescales ($\mathrm{Re}_*=\tau_v/\tau_\Theta$) related to the grain motion: the viscous time τ_v for a grain to equilibrate to the fluid flow and the time τ_Θ that it takes for the fluid force to accelerate the grain over the typical bed roughness. At low Re_*, mobilized grains quickly equilibrate to the fluid, and so feel little acceleration between collisions and have small slip velocities. But at high Re_*, mobilized grains are less affected by hydrodynamic drag and are accelerated between collisions, making it more difficult for them to stop. This argument explains the higher critical Shields number for lower Re_*. Equivalently, one can frame this argument by saying that size-dependent grain dynamics account for the lower yield stress of granular beds composed of larger particles. With the model fluid flow, drag, and particle interactions, we find qualitative agreement between the numerical results and the Shields curve. To find quantitative agreement between the Shields curve and our numerical simulations, we found that we needed to include lift forces as well.

One of the most significant results from our computational studies is that the yield strength of granular beds is better characterized as the point at which the grains find a mechanically stable state when driven by a steady shear flow. This conclusion is distinct from the prevailing view that the initial movement of the surface grains determines the onset of sediment transport, since this motion can be transient.

B. System-size dependence near the critical Shields number
Our computational models of riverbeds are periodic in the stream-wise direction, and thus grains that leave the domain on one side reappear on the other side. By varying the length in the stream-direction near the critical Shields number, we can investigate the spatiotemporal dynamics in the large system limit, as well as uncover grain-scale mechanisms that determine the value of the critical Shields number.

For example, the stream-wise length plays an important role in how bed failure occurs. Figure 2, from “Onset and cessation of motion in hydrodynamically sheared granular beds” (Ref. [11]) shows distributions of the excess stress that beds of varying size with $\mathrm{Re}_*\geq 1$ (moderate to large grain inertia) can support above the critical Shields number, which we define as the point at which mobile grains can no longer stop. These data were obtained by preparing an initially static granular bed, slowly increasing the Shields number, and then measuring the Shields number at which permanent grain motion occurs. As the system size increases, these distributions narrow and with mean values that converge to the critical Shields number, meaning that the onset and cessation of motion in large beds always occurs at the same value of the Shields number. The distributions are well described by Weibullian “weakest-link” statistics, which implies that global bed motion is initiated by a single weak spot in the bed.

For $\mathrm{Re}_*\ll 1$, the Shields curve plateaus to a value that corresponds to the strongest possible configuration that grains can find. In this case, as system size is varied, we observe critical scaling that is common in a broad range of phase transitions. Figure 3 is adapted from a manuscript submitted to Physical Review Letters, entitled “Critical scaling near the yielding transition in granular media” [13]. In this work, we performed simulations of packings of granular materials that were prepared isotropically and then subjected to shear in a riverbed-like geometry as well as driven via simple planar shear, as depicted in Fig. 3(a). As shown in Fig. 3, we find that the yield stress of the bed is equivalent to the maximum shear stress that a granular material can withstand in the large-system limit. We demonstrated that (1) such a strongest state exists and that (2) a structural correlation length diverges as the system approaches the yield stress. These results suggest that the yielding of slowly sheared granular media is analogous to a second-order critical phase transition.

C. Incipient motion in turbulent flow

As discussed above, when a hydrodynamic stress is applied to a granular bed, the bed may move transiently as it searches for a new mechanically stable configuration of grains. However, as long as the Shields number is below its critical value, we expect that the granular bed will eventually find such a mechanically stable configuration and cease to move. This argument implies that some grains may be in marginally stable configurations that are just strong enough to support the applied stress. Thus, it is interesting to ask how this picture is modified when the driving flow is turbulent, such that the stress applied to the bed becomes stochastic. In this case, we might expect that we would see much longer transients and increased grain motion in the bed in general, as contacts between grains in marginally stable configurations are stochastically broken, before bed erosion truly begins.

To investigate this aspect of erosion, we turned to laboratory experiments in a recirculating racetrack-shaped flume where the fluid flow can be made turbulent. As expected, the transition to net sediment transport was blurred in this case, although the range in which the “critical” Shields number occurs was in agreement with expectations from the literature.

To describe the experimental results with fluctuations from turbulence, we turned to a statistical approach. Specifically, we measured the probability distribution function (PDF) of surface grain velocity, as shown in Fig. 4. If the observed grain motion were purely due to turbulent fluctuations and not affected by net sediment transport, one would expect a Gaussian velocity PDF (with possibly heavy tails due to turbulent intermittency). In contrast, for net sediment transport near onset, one would expect an exponential velocity PDF, assuming that grain velocities are uncorrelated. As shown in Fig. 4, we observed PDFs that were clearly a blend of these two expectations, where the weight in the exponential tail increases monotonically with increasing Shields number. To characterize the onset of sediment transport, we fit the velocity PDFs with a mixture model and extracted the weight of the exponential tail.

Training Opportunities: This award has providing training and research opportunities for one postdoctoral associate (PDA), one Ph.D. student, and several undergraduate and high school students.

PDA: Dr. Abram Clark was supported until July 31, 2017. He is now an Assistant Professor in the Department of Physics at the Naval Postgraduate School in Monterey, CA.

Ph.D. student: Philip Wang is a 3rd year Ph.D. student in the Department Mechanical Engineering & Materials Science at Yale University.

PI O’Hern has received supplements from the ARO URAP/HSAP programs in 2017 and 2018. In 2017, HSAP supported Ayush Dalmia from Hamden High School. In addition, high school junior Jonah Berman from Choate Rosemary Hall performed ARO research in the O’Hern research group. In the summer 2017, URAP supported Mitchell Butler from the University of Southern California. In the summer 2018, HSAP will support one high school student and REAP will support 2 high school students. These students will be selected in March 2018. In the summer 2018, URAP will support Victoria Palmer, who is a 2nd year Mechanical Engineering major at Yale University.

In 2018, PI O’Hern received ARO conference support as the co-organizer of the minisymposium, ``Physics of dense granular media” at the 10th EUROMECH Solid Mechanics Conference in Bologna, Italy, July 2-6, 2018. PI O’Hern also co-organized the invited session, `` Jamming of Frictional and Non-spherical Particles'' at the American Physical Society’s March Meeting, New Orleans, LA, March 2017.

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Results Dissemination: In 2017, the PIs published two articles, both in Physical Review Fluids. Another manuscript is under review at Physical Review Letters.

1. J. C. Salevan, A. H. Clark, M. D. Shattuck, C. S. O'Hern, and N. T. Ouellette, "Determining the onset of hydrodynamic erosion in turbulent flow," Phys. Rev. Fluids 2 (2017) 114302.
2. A H. Clark, N. T. Ouellette, M. D. Shattuck, C. S. O'Hern, ``The role of grain dynamics in determining the onset of sediment transport,'' Phys. Rev. Fluids 2 (2017) 034305; xxx.lanl.gov/abs/1602.07571
3. A. H. Clark, M. D. Shattuck, N. T. Ouellette, and C. S. O'Hern, "Critical scaling of the yielding transition in sheared granular media," submitted (2017); xxx.lanl.gov/abs/1706.09465

In addition, the PIs and PDA disseminated this research by giving presentations at domestic and international conferences, lecture series, and departmental seminars and colloquia.

Conferences and Lecture Series:

O'Hern

- Keynote speaker in session on "Mechanical response of complex and disordered materials" at 13th World Congress on Computational Mechanics, New York City, NY (July 22-28, 2018).
- Minisymposium, ``Physics of dense granular media" at the 10th EUROMECH Solid Mechanics Conference in Bologna, Italy, July 2-6, 2018.
- Lecture Series on ``Simulation techniques for dense particulate matter,'' Department of Energy and Power Engineering, Tsinghua University, Beijing, China (May 14-17, 2018).
- Keynote speaker in session on "Fundamentals of deformation and yielding in amorphous materials" at 2018 MACH Conference, Annapolis, MD (April 4-6, 2018).
- American Physical Society March Meeting, Invited Session on "Athermal and Statistical Mechanics of Granular Media", Los Angeles, CA (March 8, 2018).
- Program on "Physics of dense suspensions," Kavli Institute for Theoretical Physics, Santa Barbara (February 12-March 9, 2018).
- Workshop on "Rheology near the jamming transition," Yukawa Institute for Theoretical Physics, Kyoto University, Kyoto, Japan (August 19, 2017).
- Program on "From supercooled liquids to glasses: Current challenges for amorphous materials," Kavli Institute for Theoretical Sciences, Beijing, China (August 7-18, 2017).
- Lecture Series on ``Simulation techniques for dense particulate matter,'' Department of Thermal Engineering, Tsinghua University, Beijing, China (June 19-22, 2017).

Clark

- Contributed talk on "Critical scaling near yield in granular materials" at the American Physical Society's March Meeting, Los Angeles, CA (March 6, 2018)
- Minisymposium, ``Physics of dense granular media" at the 10th EUROMECH Solid Mechanics Conference in Bologna, Italy, July 2-6, 2018.
- Invited talk at the 15th Annual Northeastern Granular Materials Workshop at Northeastern University, June 2, 2017.
-

Colloquia and Seminars

O'Hern

- Institute of Physics, Chinese Academy of Sciences, Beijing, China (July 4, 2017).
- School of Materials Science and Engineering, Shanghai Jiao Tong University, Shanghai, China, (June 12, 2017).
- Department of Physics and Astronomy, Shanghai Jiao Tong University, Shanghai, China, (June 9, 2017).

Clark

- Department of Civil and Environmental Engineering, Stanford University (January 8, 2018).
- Center for Nonlinear and Complex Systems, Duke University (December 5, 2017)

- Department of Physics, US Naval Postgraduate School (April, 2017)

Honors and Awards: In 2017, PI O'Hern was elected a Fellow of the American Physical Society.

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Technology Transfer: Nothing to Report

PARTICIPANTS:

Participant Type: Co PD/PI
Participant: Corey Shane OHern
Person Months Worked: 12.00 **Funding Support:**
Project Contribution:
International Collaboration:
International Travel:
National Academy Member: N
Other Collaborators:

Participant Type: Co PD/PI
Participant: Nicholas Ouellette
Person Months Worked: 12.00 **Funding Support:**
Project Contribution:
International Collaboration:
International Travel:
National Academy Member: N
Other Collaborators:

Participant Type: Postdoctoral (scholar, fellow or other postdoctoral position)
Participant: Abram Clark
Person Months Worked: 12.00 **Funding Support:**
Project Contribution:
International Collaboration:
International Travel:
National Academy Member: N
Other Collaborators:

Participant Type: Graduate Student (research assistant)
Participant: Philip Wang
Person Months Worked: 12.00 **Funding Support:**
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National Academy Member: N
Other Collaborators:

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Article Title: Onset and cessation of motion in hydrodynamically sheared granular beds
Authors: Abram H. Clark, Mark D. Shattuck, Nicholas T. Ouellette, and Corey S. O'Hern
Keywords: erosion, sediment transport, bed load
Abstract: We performed molecular dynamics simulations of granular beds driven by a model hydrodynamic shear flow to elucidate general grain-scale mechanisms that determine the onset and cessation of sediment transport. By varying the Shields number (the nondimensional shear stress at the top of the bed) and particle Reynolds number (the ratio of particle inertia to viscous damping), we explore how variations of the fluid flow rate, particle inertia, and fluid viscosity affect the onset and cessation of bed motion. For low to moderate particle Reynolds numbers, a critical boundary separates mobile and static states. Transition times between these states diverge as this boundary is approached both from above and below. At high particle Reynolds number, inertial effects become dominant and particle motion can be sustained well below flow rates at which mobilization of a static bed occurs. We also find that the onset of bed motion (for both low and high particle Reynolds numbers) is described by Wei
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Date Submitted: 3/2/18 12:00AM Date Published: 1/11/16 11:05AM
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Article Title: Protocol dependence of the jamming transition
Authors: Thibault Bertrand, Robert P. Behringer, Bulbul Chakraborty, Corey S. O'Hern, and Mark D. Shattuck
Keywords: jamming transition, granular media
Abstract: We propose a theoretical framework for predicting the protocol dependence of the jamming transition for frictionless spherical particles that interact via purely repulsive contact forces. We study isostatic jammed packings obtained via two protocols: isotropic compression and simple shear. We show that for frictionless systems, all jammed packings can be obtained via either protocol. However the probability to obtain a particular jammed packing depends on the packing-generation protocol. We predict the average shear strain required to induce jamming in initially unjammed packings from the measured probability to jam at packing fraction phi from isotropic compression. We compare our predictions to results from numerical simulations of jamming and find quantitative agreement. We also show that the packing fraction range, over which strain-induced jamming occurs, tends to zero for frictionless packings with overdamped dynamics.
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Authors: J. C. Salevan, A. H. Clark, M. D. Shattuck, C. S. O'Hern, and N. T. Ouellette
Keywords: sediment transport, turbulence, granular materials
Abstract: We revisit the longstanding question of the onset of sediment transport driven by a turbulent fluid flow via laboratory measurements. We use particle-tracking velocimetry to quantify the fluid flow as well as the motion of individual grains. As we increase the flow speed above the threshold for sediment transport, we observe that an increasing fraction of grains is transported downstream, although the average downstream velocity of the transported grains remains roughly constant. However, we find that the fraction of mobilized grains does not vanish sharply at a critical flow rate. Additionally, the distribution of the fluctuating velocities of nontransported grains becomes broader with heavier tails, meaning that unambiguously separating mobile and static grains is not possible. As an alternative approach, we quantify the statistics of grain velocities by using a mixture model consisting of two forms for the grain velocities: a decaying-exponential tail, which represents grains transp
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Institution: Yale University
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Authors: J. C. Salevan, A. H. Clark, M. D. Shattuck, C. S. O"Hern, and N. T. Ouellette
Acknowledged Federal Support: **Y**

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Final Progress Report

(1) Major Goals

The erosion of earth materials by shear flows is common in nature, and has numerous applications including geomorphology, agriculture, and climate science [1-3]. Large sums of money are spent every year to control or mitigate erosion, with varying degrees of success. In some cases, such as controlling silting in dams and reservoirs, erosion may be desirable to remove deposited sediment. On the other hand, the prevention of erosion is crucial in places where bodies of water, landmasses, and human development meet.

Despite its importance, modeling erosion remains a difficult problem due to its tremendous complexity [4,5]. Strongly erosive flows are typically turbulent, leading to highly stochastic shear stress imparted to the earth material. The grains that make up the bed typically have highly nonspherical shapes and variable sizes or material properties. Commonly used riverbed erosion models typically do not include these complexities and instead employ dimensional analysis [6]. Previous studies have suggested that the onset of erosion (i.e., the conditions under which a static bed will begin to erode) is controlled by two nondimensional parameters: $\Theta = \frac{\tau}{\Delta\rho g D}$ and $Re_* = \frac{u_* D}{\nu}$. The Shields number Θ is the ratio of the shear stress τ imparted to the surface layer of the bed by the flow to the (reduced) weight of the bed particles ($(\rho_g - \rho_f)gD$), where ρ_g is the density of the bed grains, ρ_f is the density of the fluid, g is the acceleration due to gravity, and D is the diameter of the grains. The shear Reynolds number Re_* is the ratio of the inertial forces in the flow (where $u_*^2 = \tau/\rho_f$ is the known as the friction velocity) to viscous damping, where ν is the kinematic viscosity of the fluid. Based on a compilation of empirical data from field and laboratory measurements, the onset of erosion is given by a curve, known as the Shields curve, in the parameter space spanned by these two nondimensional parameters [7]. The Shields curve, however, is a highly oversimplified characterization of the actual dynamics of incipient bed motion, and eroding systems are routinely observed to violate the Shields curve. In addition, there is no agreed-upon theoretical explanation for the shape of the Shields curve.

Previous modeling approaches to erosion of granular beds have typically either thrown away all the details of the problem using a simple mass-balance approach or have attempted to mimic the full complexity of the natural system [8-10]. However, the former approaches tend to make predictions that fail because they miss key elements of the erosion process, while using the latter approaches makes it difficult to identify the dominant mechanisms that control erosion, which limits their generalizability.

The primary goal of our research was to simplify the problem by identifying the key elements necessary to describe hydrodynamic erosion via complementary numerical simulations and laboratory experiments. We aimed to clarify which details of this enormously complex problem are necessary to capture its dynamics and behavior, and which play only secondary roles and may thus be neglected. To sharpen our questions, we focused on the restricted problem of incipient motion, i.e., what are the conditions under which bed grains just begin to move?

(2) Accomplished Under Goals

Our research accomplishments can be organized into four main thrusts: A. Physical justification for the Shields curve, B. System-size dependence near the critical Shields number, C. Incipient grain motion in turbulent flow, and D. Segregation and armoring of riverbeds.

A. Physical justification for the Shields curve

In our article "Onset and cessation of motion in hydrodynamically sheared granular beds" [11], we showed that by considering only a simple model flow, a linear fluid drag law, purely repulsive and elastic intergrain contact forces, and gravity, one can qualitatively reproduce the Shields curve for the onset of grain motion. We demonstrated this result by varying the strength of the hydrodynamic driving and the relative degree of fluid damping experienced by the grains.

We then extended this model to include both more general flows and grain interactions to identify the physical mechanisms that control different regimes of the Shields curve. In our article "Role of grain dynamics in determining the onset of sediment transport" [12], we included more complex grain interactions such as friction, collisional dissipation, and irregular grain shapes. A more complete picture of the fluid drag was also adopted in the model, including both Stokes drag (linear in flow velocity) and inertial drag (quadratic in flow velocity), to appropriately account for the relevant drag laws at low and high Reynolds numbers, respectively.

In Fig. 1, we show the Shields curve along with the results of our numerical model. We note that the shear Reynolds number Re_* can be written as the ratio of two timescales ($Re_* = \frac{\tau_\nu}{\tau_\Theta}$) related to the grain motion: the viscous time τ_ν for a grain to equilibrate to the fluid flow and the time τ_Θ that it takes for the fluid force to accelerate the grain over the typical bed roughness. At low Re_*, mobilized grains quickly equilibrate to the fluid, and so feel little acceleration between collisions and have small slip velocities. But at high Re_*, mobilized grains are less affected by hydrodynamic drag and are accelerated between collisions, making it more difficult for them to stop. This argument explains the higher critical Shields number for lower Re_*. Equivalently, one can frame this argument by saying that size-dependent grain dynamics account for the lower yield stress of granular beds composed of larger particles. With the model fluid flow, drag, and particle interactions, we find qualitative agreement between the numerical results and the Shields curve. To find quantitative agreement between the Shields curve and our numerical simulations, we found that we needed to include lift forces as well.

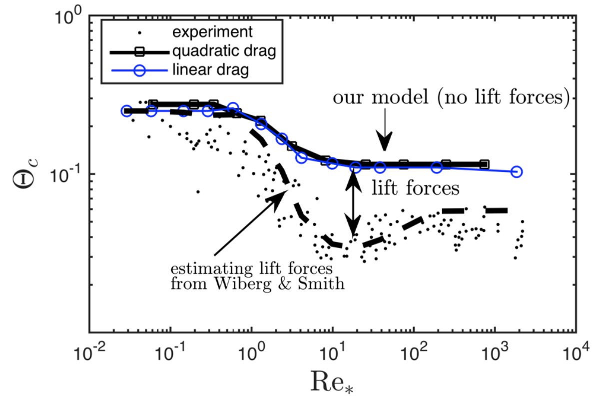

Figure 1: A collection of experimental and field data (filled dots) from Dey, *Fluvial hydrodynamics*: *Hydrodynamic and Sediment Transport Phenomena* (Springer, Berlin 2014) showing the variation of the critical Shields number for grain motion Θ_c with the shear Reynolds number Re_*. The solid curves show the boundaries between systems with and without sustained grain motion from our discrete element simulations in 3D, excluding lift forces. The black curve with square markers shows results for quadratic drag, and the blue curve with open circles shows results for linear drag in the limit of small restitution coefficient. The dashed line represents a theoretical curve for θ_c that

includes lift forces for monodisperse sediment derived by Wiberg and Smith in *Water Resour. Res.* 23 (1987) 1471.

One of the most significant results from our computational studies is that the yield strength of granular beds is better characterized as the point at which the grains find a mechanically stable state when driven by a steady shear flow. This conclusion is distinct from the prevailing view that the initial movement of the surface grains determines the onset of sediment transport, since this motion can be transient.

B. System-size dependence near the critical Shields number

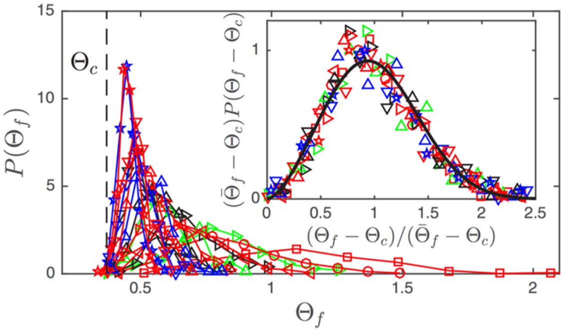

Figure 2: The probability distribution of the Shields number at bed failure (static to mobile transition) Θ_f for many different system sizes. The dashed vertical line represents the mobile to static transition at Θ_c. The inset shows that $P(\Theta_f)$ collapses when rescaled by $\Theta_f - \Theta_c$, where Θ_f is the mean of each distribution. The solid line is a Weibull distribution with shape parameter $\alpha \approx 2.6$.

Our computational models of riverbeds are periodic in the stream-wise direction, and thus grains that leave the domain on one side reappear on the other side. By varying the length in the stream-direction near the critical Shields number, we can investigate the spatiotemporal dynamics in the large system limit, as well as uncover grain-scale mechanisms that determine the value of the critical Shields number.

For example, the stream-wise length plays an important role in how bed failure occurs. Figure 2, from “Onset and cessation of motion in hydrodynamically sheared granular beds” (Ref. [11]) shows distributions of the excess stress that beds of varying size with $\mathrm{Re}_* \geq 1$ (moderate to large grain inertia) can support above the critical Shields number, which we define as the point at which mobile grains can no longer stop. These data were obtained by preparing an initially static granular bed, slowly increasing the Shields number, and then measuring the Shields number at which permanent grain motion occurs. As the system size increases, these distributions narrow and with mean values that converge to the critical Shields number, meaning that the onset and cessation of motion in large beds always occurs at the same value of the Shields number. The distributions are well described by Weibullian “weakest-link” statistics, which implies that global bed motion is initiated by a single weak spot in the bed.

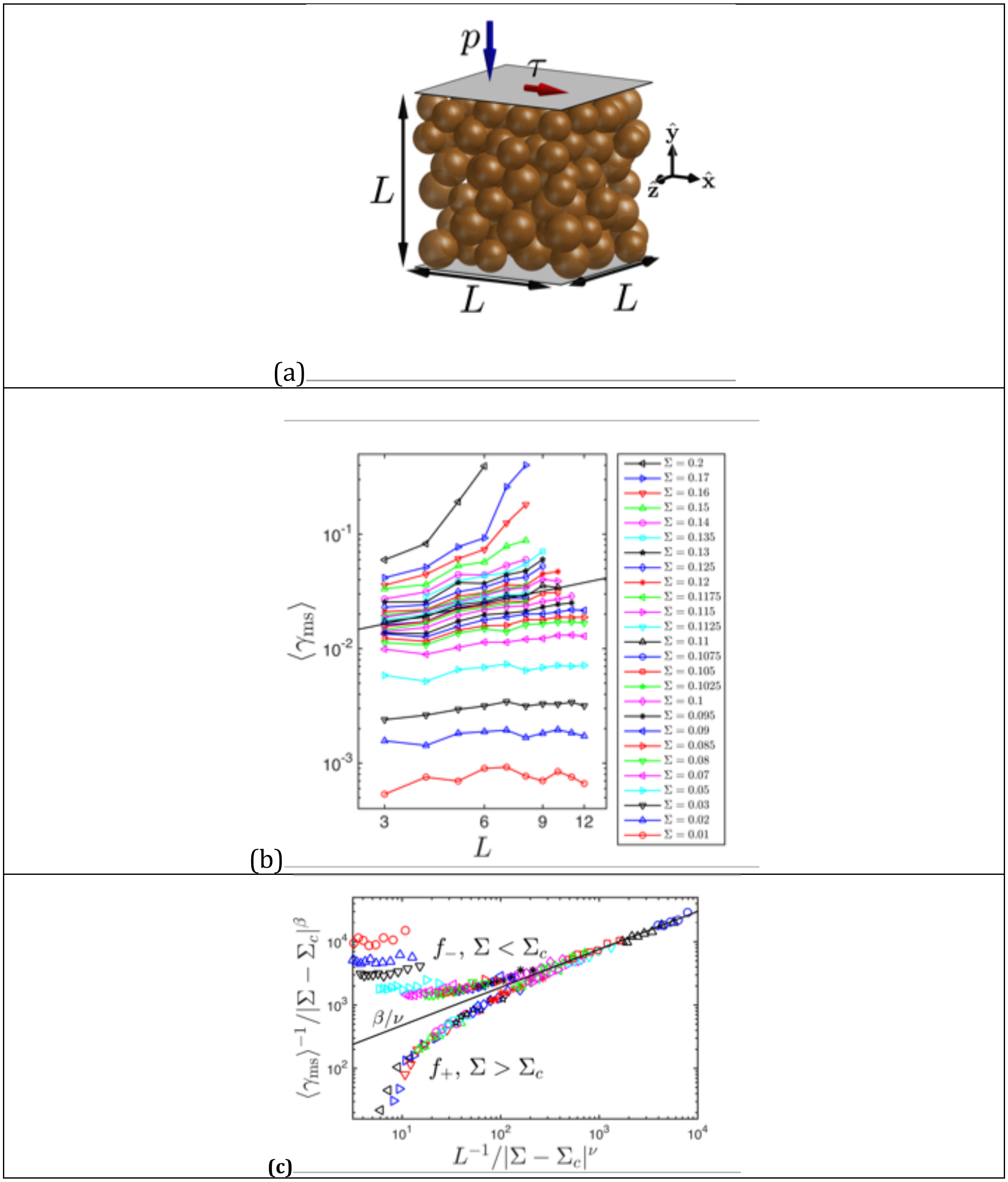

Figure 3: (a) A schematic of the simulation procedure to study fluid sheared granular beds with the x-, y-, and z-directions indicated. We first create mechanically stable (MS) packings under only a fixed normal force per area p. We then apply a shear force per area τ and search for an MS packing at a given $\Sigma = \tau/p$ and nondimensional length $L = l/D$, where l is the box edge length and D is the grain diameter. (b) The average strain $\langle \gamma_{\rm ms} \rangle$ between the initial and final MS packings plotted as a function of L over a range of Σ, where $\Sigma_c \approx 0.1085$. The solid black line shows $\langle \gamma_{\rm ms} \rangle \propto L^{0.6}$. (c) The same data shown in (b) (with the same symbol and color conventions) collapses for $0.07 < \Sigma < 0.2$ when plotted using the scaled variables $\langle \gamma_{\rm ms} \rangle^{-1}/|\Sigma - \Sigma_c|^{\beta}$ and $L^{-1}/|\Sigma - \Sigma_c|^{\nu}$, where $\beta \approx 0.9$ and $\nu \approx 1.5$ are scaling exponents. This scaling behavior demonstrates the existence of a diverging length scale $\xi \propto |\Sigma - \Sigma_c|^{-\nu}$.

For Re$_*$ $\ll$ 1, the Shields curve plateaus to a value that corresponds to the strongest possible configuration that grains can find. In this case, as system size is varied, we observe critical scaling that is common in a broad range of phase transitions. Figure 3 is adapted from a manuscript submitted to *Physical Review Letters*, entitled “Critical scaling near the yielding transition in granular media” [13]. In this work, we performed simulations of packings of granular materials that were prepared isotropically and then subjected to shear in a riverbed-like geometry as well as driven via simple planar shear, as depicted in Fig. 3(a). As shown in Fig. 3, we find that the yield stress of the bed is equivalent to the maximum shear stress that a granular material can withstand in the large-system limit. We demonstrated that (1) such a strongest state exists and that (2) a structural correlation length diverges as the system approaches the yield stress. These results suggest that the yielding of slowly sheared granular media is analogous to a second-order critical phase transition.

C. Incipient motion in turbulent flow

As discussed above, when a hydrodynamic stress is applied to a granular bed, the bed may move transiently as it searches for a new mechanically stable configuration of grains. However, as long as the Shields number is below its critical value, we expect that the granular bed will eventually find such a mechanically stable configuration and cease to move. This argument implies that some grains may be in marginally stable configurations that are *just* strong enough to support the applied stress. Thus, it is interesting to ask how this picture is modified when the driving flow is turbulent, such that the stress applied to the bed becomes stochastic. In this case, we might expect that we would see much longer transients and increased grain motion in the bed in general, as contacts between grains in marginally stable configurations are stochastically broken, before bed erosion truly begins.

To investigate this aspect of erosion, we turned to laboratory experiments in a recirculating racetrack-shaped flume where the fluid flow can be made turbulent. As expected, the transition to net sediment transport was blurred in this case, although the range in which the “critical” Shields number occurs was in agreement with expectations from the literature.

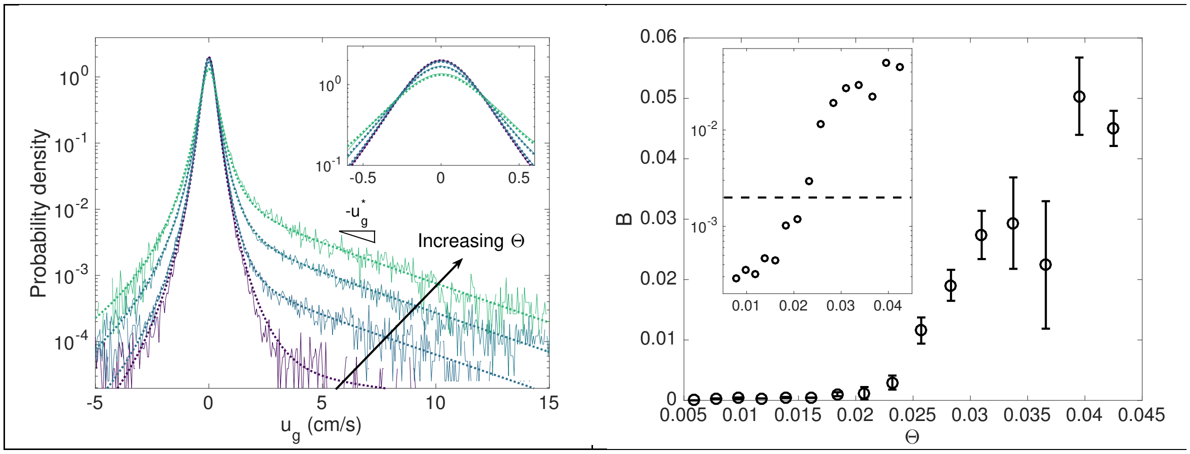

Figure 4: (left) Probability distribution functions $P(u_g)$ of experimentally determined grain velocities u_g (solid curves), for Shields numbers $\Theta = 0.0059$, 0.0232, 0.0283, and 0.0425 increasing from bottom to top. Dashed curves show fits of the data to the equation: $P(u_g) = Af(u_g, \zeta, \sigma) + Bg(u_g^*, u_g)$, where A and B are mixture fractions that sum to unity and give the relative fraction of particles in

the static core and the mobile tail of the distributions. The function $f(u_g,\zeta,\sigma)$ is the Students's t-distribution that describes the core of P(u_g), where ζ represents the shape parameter and σ represents the characteristic width. The function $g(u_g^*,u_g)$ controls the shape of the tail, which decays exponentially. The inset shows a close-up of the same data near the core of the probability distribution function (PDF). (right) Fit parameter B from the PDFs, which governs the relative fraction of particles in the tail of the velocity distribution. The inset shows the same data plotted on semi-logarithmic axes, with a horizontal line at $B = 2\times10^{-3}$, which marks the initiation of the constant region for u_g^*, the average velocity of the mobile grains. The non-vanishing behavior of B between $\Theta = 0.018$ and 0.023 shows that the transition to net transport is blurred, likely due to turbulence in the experiments.

To describe the experimental results with fluctuations from turbulence, we turned to a statistical approach. Specifically, we measured the probability distribution function (PDF) of surface grain velocity, as shown in Fig. 4. If the observed grain motion were purely due to turbulent fluctuations and not affected by net sediment transport, one would expect a Gaussian velocity PDF (with possibly heavy tails due to turbulent intermittency). In contrast, for net sediment transport near onset, one would expect an exponential velocity PDF, assuming that grain velocities are uncorrelated. As shown in Fig. 4, we observed PDFs that were clearly a blend of these two expectations, where the weight in the exponential tail increases monotonically with increasing Shields number. To characterize the onset of sediment transport, we fit the velocity PDFs with a mixture model and extracted the weight of the exponential tail. These results were published in the article "Determining the onset of hydrodynamic erosion in turbulent flow" [14].

4. Segregation and armoring of riverbeds

Riverbed armoring, i.e. the tendency of the surface of a riverbed to be populated by larger grains that shield smaller grains underneath [15], occurs frequently in rivers. We are interested in determining the mechanisms that control armoring and size segregation in fluid-driven granular beds. One process involves sieving, where small particles fall through gaps between larger particles. A related wedging process also occurs where larger particles are pushed to the surface by smaller particles [16].

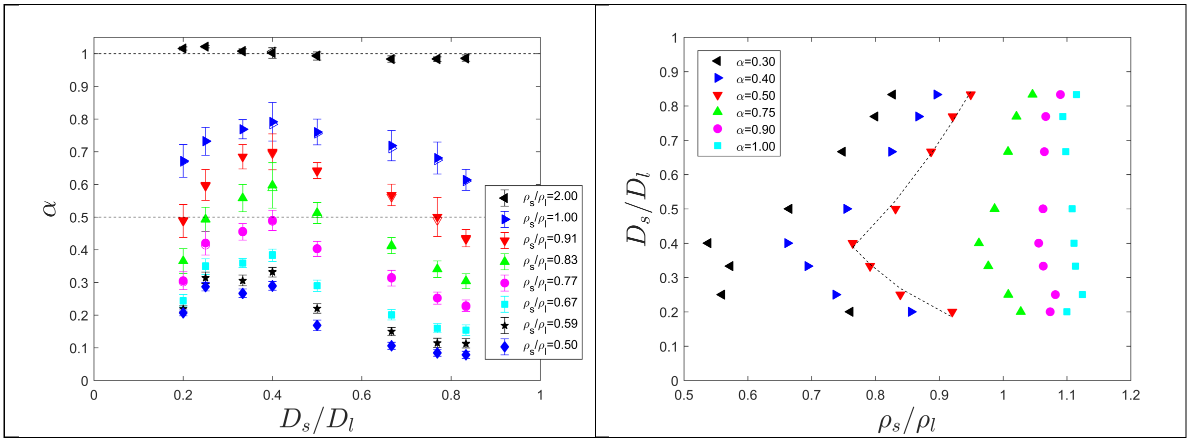

Figure 5: (left) Degree of segregation α for a bidisperse mixture of small (s) and large (l) grains as a function of both the grain size ratio D_s/D_l and density ratio ρ_s/ρ_l with the global volume fraction $\Phi_l = V_l/(V_l+V_s)$=0.5. α>0.5 to corresponds to more large grains near the top of the system, whereas α<0.5 corresponds to more small grains near the top of the system. As the density ratio decreases (i.e. the large grains become heavier), the system can no longer sustain the large grains near the top surface, and they begin to sink to the lower layers. (right) Same data as in the left panel,

but showing the values of D_s/D_l and ρ_s/ρ_l that yield particular values of α. The dotted line connects points that give the ideally mixed state, α=0.5.

We first study segregation in fluid-driven binary mixtures of frictionless grains using discrete element simulations. We prepare a static packing by seeding grains randomly in a container and then allow them to settle under gravity. We subject the packing to a model fluid flow with a linear velocity profile and Shields number $\Theta > \Theta_c$, and shear the packing until the system reaches steady state. We quantify the degree of segregation [17] using the parameter $\alpha = (1 - \frac{CV_s - CV_l}{(CV_s - CV_l)_0})/2$, where the "0" subscript indicates the α=0 state with all small grains near the top and all large grains near the bottom. (See the left panel of Fig. 5.) In contrast, $\alpha = 1$ corresponds to having all large particles near the top and all small particles near the bottom. As shown in the right panel of Fig. 5, we find that to maintain the mixed state ($\alpha = 0.5$) in the upper branch ($0.4 < D_S/D_l < 0.9$), the large grains must become heavier as they increase in size. In the lower branch ($0.2 < D_S/D_l < 0.4$), more intuitive behavior is observed. The mixed state is maintained by making the large particles lighter as their size is increased. A related observation is that the system can change from α>0.5 to α <0.5 at fixed ρ_s/ρ_l by varying the grain size ratio.

(3) Training Opportunities

This award has providing training and research opportunities for one postdoctoral associate (PDA), one Ph.D. student, and several undergraduate and high school students.

PDA: Dr. Abram Clark was supported until July 31, 2017. He is now an Assistant Professor in the Department of Physics at the Naval Postgraduate School in Monterey, CA.

Ph.D. student: Philip Wang is a 3rd year Ph.D. student in the Department Mechanical Engineering & Materials Science at Yale University.

PI O'Hern has received supplements from the ARO URAP/HSAP programs in 2017 and 2018. In 2017, HSAP supported Ayush Dalmia from Hamden High School. In addition, high school junior Jonah Berman from Choate Rosemary Hall performed ARO research in the O'Hern research group. In the summer 2017, URAP supported Mitchell Butler from the University of Southern California. In the summer 2018, HSAP will support one high school student and REAP will support 2 high school students. These students will be selected in March 2018. In the summer 2018, URAP will support Victoria Palmer, who is a 2nd year Mechanical Engineering major at Yale University.

In 2018, PI O'Hern received ARO conference support as the co-organizer of the minisymposium, ``Physics of dense granular media" at the 10th EUROMECH Solid Mechanics Conference in Bologna, Italy, July 2-6, 2018. PI O'Hern also co-organized the invited session, `` Jamming of Frictional and Non-spherical Particles" at the American Physical Society's March Meeting, New Orleans, LA, March 2017.

(4) Dissemination

In 2017, the PIs published two articles, both in *Physical Review Fluids*. Another manuscript is under review at *Physical Review Letters*.

1. J. C. Salevan, A. H. Clark, M. D. Shattuck, C. S. O'Hern, and N. T. Ouellette, "Determining the onset of hydrodynamic erosion in turbulent flow," *Phys. Rev. Fluids* 2 (2017) 114302.
2. A H. Clark, N. T. Ouellette, M. D. Shattuck, C. S. O'Hern, ``The role of grain dynamics in determining the onset of sediment transport," *Phys. Rev. Fluids* 2 (2017) 034305; xxx.lanl.gov/abs/1602.07571
3. A. H. Clark, M. D. Shattuck, N. T. Ouellette, and C. S. O'Hern, "Critical scaling of the yielding transition in sheared granular media," submitted (2017); xxx.lanl.gov/abs/1706.09465

In addition, the PIs and PDA disseminated this research by giving presentations at domestic and international conferences, lecture series, and departmental seminars and colloquia.

Conferences and Lecture Series:

O'Hern

- Keynote speaker in session on "Mechanical response of complex and disordered materials" at 13th World Congress on Computational Mechanics, New York City, NY (July 22-28, 2018).
- Minisymposium, ``Physics of dense granular media" at the 10th EUROMECH Solid Mechanics Conference in Bologna, Italy, July 2-6, 2018.
- Lecture Series on ``Simulation techniques for dense particulate matter," Department of Energy and Power Engineering, Tsinghua University, Beijing, China (May 14-17, 2018).
- Keynote speaker in session on "Fundamentals of deformation and yielding in amorphous materials" at 2018 MACH Conference, Annapolis, MD (April 4-6, 2018).
- American Physical Society March Meeting, Invited Session on "Athermal and Statistical Mechanics of Granular Media", Los Angeles, CA (March 8, 2018).
- Program on "Physics of dense suspensions," Kavli Institute for Theoretical Physics, Santa Barbara (February 12-March 9, 2018).
- Workshop on "Rheology near the jamming transition," Yukawa Institute for Theoretical Physics, Kyoto University, Kyoto, Japan (August 19, 2017).
- Program on "From supercooled liquids to glasses: Current challenges for amorphous materials," Kavli Institute for Theoretical Sciences, Beijing, China (August 7-18, 2017).
- Lecture Series on ``Simulation techniques for dense particulate matter," Department of Thermal Engineering, Tsinghua University, Beijing, China (June 19-22, 2017).

Clark

- Contributed talk on "Critical scaling near yield in granular materials" at the American Physical Society's March Meeting, Los Angeles, CA (March 6, 2018)
- Minisymposium, ``Physics of dense granular media" at the 10th EUROMECH Solid Mechanics Conference in Bologna, Italy, July 2-6, 2018.
- Invited talk at the 15th Annual Northeastern Granular Materials Workshop at Northeastern University, June 2, 2017.

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Colloquia and Seminars

O'Hern

- Institute of Physics, Chinese Academy of Sciences, Beijing, China (July 4, 2017).
- School of Materials Science and Engineering, Shanghai Jiao Tong University, Shanghai, China, (June 12, 2017).
- Department of Physics and Astronomy, Shanghai Jiao Tong University, Shanghai, China, (June 9, 2017).

Clark

- Department of Civil and Environmental Engineering, Stanford University (January 8, 2018).
- Center for Nonlinear and Complex Systems, Duke University (December 5, 2017)
- Department of Physics, US Naval Postgraduate School (April, 2017)

(5) Honors and Awards

In 2017, PI O'Hern was elected a Fellow of the American Physical Society.

(6) Technology Transfer (patent applications, inventions, licenses, interaction with DoD laboratories): N/A

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REPORT OF INVENTIONS AND SUBCONTRACTS

(Pursuant to "Patent Rights" Contract Clause) (See Instructions on back)

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P.O. Box 208327
New Haven CT 06520-8327

d. AWARD DATE *(YYYYMMDD)*: 20131114

2.a. NAME OF GOVERNMENT PRIME CONTRACTOR: Yale University

c. CONTRACT NUMBER: W911NF-14-1-0005

b. ADDRESS *(Include ZIP Code)*
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☐ a. INTERIM ☒ b. FINAL

4. REPORTING PERIOD *(YYYYMMDD)*
a. FROM 20131114
b. TO 20171113

SECTION I - SUBJECT INVENTIONS

5. "SUBJECT INVENTIONS" REQUIRED TO BE REPORTED BY CONTRACTOR/SUBCONTRACTOR *(If "None," so state)*

NAME(S) OF INVENTOR(S) *(Last, First, Middle Initial)* a.	TITLE OF INVENTION(S) b.	DISCLOSURE NUMBER, PATENT APPLICATION SERIAL NUMBER OR PATENT NUMBER c.	ELECTION TO FILE PATENT APPLICATIONS *(X)* d. (1) UNITED STATES (a) YES	(1) UNITED STATES (b) NO	(2) FOREIGN (a) YES	(2) FOREIGN (b) NO	CONFIRMATORY INSTRUMENT OR ASSIGNMENT FORWARDED TO CONTRACTING OFFICER *(X)* e. (a) YES	e. (b) NO
None								

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(b) NAME OF EMPLOYER
(c) ADDRESS OF EMPLOYER *(Include ZIP Code)*

g. ELECTED FOREIGN COUNTRIES IN WHICH A PATENT APPLICATION WILL BE FILED

(1) TITLE OF INVENTION
(2) FOREIGN COUNTRIES OF PATENT APPLICATION

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6. SUBCONTRACTS AWARDED BY CONTRACTOR/SUBCONTRACTOR *(If "None," so state)*

NAME OF SUBCONTRACTOR(S) a.	ADDRESS *(Include ZIP Code)* b.	SUBCONTRACT NUMBER(S) c.	FAR "PATENT RIGHTS" d. (1) CLAUSE NUMBER	d. (2) DATE *(YYYYMM)*	DESCRIPTION OF WORK TO BE PERFORMED UNDER SUBCONTRACT(S) e.	SUBCONTRACT DATES *(YYYYMMDD)* f. (1) AWARD	f. (2) ESTIMATED COMPLETION

SECTION III - CERTIFICATION

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b. TITLE: OSP Award Closeout Manager

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