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**CONSTANT BEAM PATTERN ARRAY METHOD**

**STATEMENT OF GOVERNMENT INTEREST**

[0001] The invention may be manufactured and used by or for the Government of the United States of America for governmental purposes without the payment of any royalties thereon or therefor.

**CROSS REFERENCE TO OTHER PATENT APPLICATIONS**

[0002] None.

**BACKGROUND OF THE INVENTION**

**(1) Field of the Invention**

[0003] The present invention is directed to a method for creating an array having a constant beam pattern across a broad frequency range.

**(2) Description of the Prior Art**

[0004] Most directional acoustic transducers and arrays have beam patterns which are frequency dependent. The beam width becomes wide when frequency goes low for a plane piston or a line array. As a result, the spectral content of the transmitted or received signals varies with position in the beam, and thus the fidelity of an acoustic system will depend on the relative orientation of the transmitter and receiver. Constant beam width transducers have been studied and tested, where only the beam width, a limited portion of the beam pattern is maintained as

constant. This is up to the half power point of -3 dB beam width control within the main lobe over a certain frequency band.

**[0005]** Prior art constant beam width transducers typically sacrifice management of side lobes and nulls to the goal of providing a constant beam width. Often it is desirable to have a specific beam pattern. In a typical lobed beam pattern having a main lobe and side lobes, it is desirable to steer the main lobe toward a target of interest while placing noise sources at beam pattern nulls. The side lobes can also be controlled for other environmental sounds.

**[0006]** Thus, it is desirable to have an array that is capable of transmitting and receiving acoustic signals with a constant beam pattern across a broad range of frequencies.

#### **SUMMARY OF THE INVENTION**

**[0007]** It is a first object of the present invention to provide a constant beam pattern for an acoustic array over a broad range of frequencies.

**[0008]** Another object is to allow such array to have a far field beam pattern specified by a user.

**[0009]** Accordingly, a broadband constant beam pattern acoustic array method is provided that includes utilizing an array of transducers in a known three dimensional axisymmetric

configuration with each transducer element having an associated signal. A user can specify a far field beam pattern for the array. Weightings are calculated for each transducer in the array as being proportional to the voltage that gives the beam pattern power level associated with the bearing for each transducer. Signal power levels for each transducer are modified in accordance with the weightings. The array can be operated for receiving and transmitting signals with a constant beam pattern over a broad range of frequencies.

#### **BRIEF DESCRIPTION OF THE DRAWINGS**

**[0010]** Reference is made to the accompanying drawings in which are shown an illustrative embodiment of the invention, wherein corresponding reference characters indicate corresponding parts, and wherein:

**[0011]** FIG. 1 is a graph of a Dolph-Chebyshev shading function utilized in array design.

**[0012]** FIGS. 2A-2C are polar graphs of beam patterns of -50dB to 0dB scale, modeled in accordance with the shading function specified in FIG. 1.

**[0013]** FIG. 3 is a graph of a cubed cosine shading function utilized in array design.

[0014] FIGS. 4A-4C are -50dB to 0dB scale polar graphs of beam patterns modeled in accordance with the shading function specified in FIG. 3.

[0015] FIG. 5 is a graph of a narrow beam width shading function utilizing a Legendre polynomial  $P_{62}$ .

[0016] FIGS. 6A-6C are -50dB to 0dB scale polar graphs of beam patterns modeled in accordance with the shading function specified in FIG. 5.

[0017] FIG. 7 is a block diagram of an apparatus for practicing the method herein.

[0018] FIG. 8 is a polar graph illustrating user specification of a beam pattern.

#### **DETAILED DESCRIPTION OF THE INVENTION**

[0019] Disclosed herein is a design concept for a broadband constant beam pattern (CBP) transducer. With only one shading function for all frequencies in a broadband, the CBP beam patterns are maintained as "constant" not only within -3dB beam width to the main lobe, but also for all levels and angular positions to the side lobes. The locations of nulls can also be defined before the transducer array is operated, because of the predefined Legendre polynomials applied.

[0020] If the radial velocity on the surface of a radius  $a$  sphere is equal to  $U_0V(\theta)\exp(-i\omega t)$ , where  $U_0$  is a constant value to

the peak of particle velocity, and  $V(\theta)$  is the axially symmetric dimensionless angular radial particle velocity distribution on the surface of the sphere,  $\omega$  is the angular frequency, then the general corresponding acoustic pressure outside of the sphere will be:

$$p(R, \theta, t) = e^{-i\omega t} \sum_{\nu}^{\infty} B_{\nu} P_{\nu}(\cos(\theta)) h_{\nu}(kR) \quad (1)$$

after solving the Helmholtz equation for the axially symmetrical case, here  $R$  and  $\theta$  are for spherical coordinates,  $B_{\nu}$  is a coefficient,  $h_{\nu}$  is first kind spherical Hankel function of  $\nu$  degree,  $c$  is the sound speed of the surrounding fluid, and  $k = \omega / c$  is the wave number. The boundary condition on the sphere surface is

$$\left. \frac{\partial p(R, \theta)}{\partial R} \right|_{R=a} = i\rho\omega U_0 V(\theta); \quad (2)$$

where  $\rho$  is the medium density,  $a$  is the radius of the sphere. The angular radial particle velocity distribution  $V(\theta)$  can be expanded by the following Legendre polynomials series  $P_{\nu}(\cos(\theta))$

$$V(\theta) = \sum_{\nu=0}^{\infty} A_{\nu} P_{\nu}(\cos\theta) \quad (3)$$

and the quantities  $A_v$  are the coefficients in the Legendre series expansion of  $V(\theta)$ , and be found in Eq. (4)

$$A_v = \frac{(2v+1)}{2} \int_0^\pi V(\theta) P_v(\cos(\theta)) \sin(\theta) d\theta \quad (4)$$

By utilizing Eq. (2) and Eq. (3) as boundary conditions, Eq. (1) becomes

$$p(R, \theta, t) = i\rho c U_0 e^{-i\omega t} \sum_{v=0}^{\infty} A_v P_v(\cos(\theta)) \frac{h_v(kR)}{h'_v(ka)} \quad (5)$$

here  $h'_v(x)$  is the derivative of  $h_v(x)$ , with respect to the argument of  $x$ . The acoustic far field beam pattern in logarithmic format is defined as

$$BP(\theta) = 20 \log_{10} \left| \frac{p(R, \theta)}{[p(R, \theta)]_{max}} \right|_{R \rightarrow \infty} \quad (6)$$

Utilizing Eq. (5), the above equation becomes

$$BP(\theta) = 20 \log_{10} \left| \frac{\sum_{v=0}^{\infty} A_v P_v(\cos(\theta)) \frac{h_v(kR)}{h'_v(ka)}}{[\sum_{v=0}^{\infty} A_v P_v(\cos(\theta)) \frac{h_v(kR)}{h'_v(ka)}]_{max}} \right|_{R \rightarrow \infty} \quad (7)$$

[0021] The spherical Hankel function asymptotic forms become,

$$\left\{ \begin{array}{l} \mathbf{h}_v(\mathbf{x})|_{x \rightarrow \infty} \approx \frac{e^{i(x-a_v\pi)}}{x} \Big|_{x=kR} \\ \mathbf{h}'_v(\mathbf{x})|_{x \rightarrow \infty} \approx \frac{(ix-1)e^{i(x-a_v\pi)}}{x^2} \Big|_{x=ka} \end{array} \right. \quad (8)$$

and under the far field conditions

$$\mathbf{kR} \rightarrow \infty, \quad (9)$$

for  $\mathbf{h}_v(\mathbf{x})|_{x=kR}$  and

$$\mathbf{ka} \rightarrow \infty, \quad (10)$$

for  $\mathbf{h}'_v(\mathbf{x})|_{x=ka}$ . To all degrees of  $\mathbf{v}$ , Legendre polynomials have

$$\mathbf{P}_v(\mathbf{cos}\theta)|_{\text{Maximum}} = \mathbf{1} \quad (11)$$

when  $\theta = 0$  is the maximum view direction at main lobe. Applying Eq. (8) and Eq. (11), Eq. (7), become

$$\mathbf{BP}(\theta) \approx 20\log_{10} \left| \frac{\sum_{v=0}^{\infty} \mathbf{A}_v \mathbf{P}_v(\mathbf{cos}\theta)}{\sum_{v=0}^{\infty} \mathbf{A}_v} \right|_{\substack{\mathbf{kR} \rightarrow \infty \\ \mathbf{ka} \rightarrow \infty}} = 20\log_{10} \left| \frac{\mathbf{V}(\theta)}{\sum_{v=0}^{\infty} \mathbf{A}_v} \right|. \quad (12)$$

**[0022]** The physics behind Eq. (12) is that the far field acoustic beam pattern is the same as the normalized particle velocity (or shading) angular distribution on the surface of the spherical transducer or array, and the beam pattern becomes frequency independent under Hankel asymptotic conditions of Eq. (9) and Eq. (10). To achieve a certain shapes of angular



distribution far field beam patterns, the same types of angular excitation (or shading for receivers) on the normal surface of the spherical transducer or array need to be engineered under the asymptotic conditions. The asymptotic condition of Eq. (9) can always be satisfied, because of the definition of far field beam pattern Eq. (6). However, the second asymptotic condition to Hankel function in Eq. (10) may be restrained by physical dimension of the sphere and the operating frequency that  $ka$ , or  $a/\lambda$ , becomes an important design parameter for constant beam pattern (CBP) transducer engineering. Here,  $a$  is the radius of the spherical transducer or array, and  $\lambda$  is the wavelength of the frequency interested.

**[0023]** The concept of acoustic constant beam pattern (CBP) transducer is introduced where its beam patterns are independent of frequency in a wide band. The theory and numerical simulations for the constant beam pattern transducer design are studied and described. The far-field beam pattern shows the same as the normal directional radial particle velocity distribution, or shading function on the spherical transducer or array surface, under the spherical Hankel function asymptotic approximation conditions. Any arbitrary shading functions are expandable by Legendre series, per Sturm-Liouville theory. Classic Dolph-Chebyshev shading of equal side lobes can be achieved by Legendre polynomial expansion for spherical

transducers. The narrower the beam width, the higher degree Legendre polynomials that must be involved such that a larger control parameter of  $a/\lambda$  may be required, in order to control small ripples to the side lobes for achieving ideal constant beam pattern (CBP) transducers.

**[0024]** Several numerical examples are simulated by the Rayleigh integral method to verify the wave equation beam pattern solution in Eq. (12) under various  $a/\lambda$  conditions

(13)

$$p(x, y, z) = -\frac{i\rho\omega}{2\pi} \int_{S_0} \frac{e^{ikR}}{R} (U_0 V(\theta)) \hat{n} \cdot d\vec{S}_0,$$

Where  $V(\theta)$  is defined in Eq. (2) as the radial particle velocity distribution in its normal direction with  $\hat{n}$  as a unit vector on the surface  $S_0(x_0, y_0, z_0)$  for the sphere of radius  $a$ , and

(14)

$$R = \sqrt{(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2},$$

is the distance between the active surface element and the far field point.

**[0025]** Simulations have shown that this method and apparatus is effective for a variety of shadings including shading by a single Legendre polynomial,  $P_5 = (\cos(\theta))$ ; shading by a Chebyshev polynomial of the first kind,  $T_5 = (\cos(\theta))$ ; shading by the classic Dolph-Chebyshev technique for -26dB sidelobe control,  $T_9(z_0 \cos(\theta))$ ;

shading by a cosine function of cubic power,  $(\cos(\theta))^3$ ; shading by a Gaussian shaped spherical surface particle velocity function; and shading by a Legendre polynomial of high degree for narrow bandwidth. These shadings have been modeled utilizing a spherical transducer or array design with a known radius, resulting in an axisymmetric beam pattern. Thus, this method and apparatus can be used to create an arbitrary, axisymmetric beam pattern utilizing the described system.

**[0026]** FIG. 1 shows the normalized shading function  $V(\theta)$  by Chebyshev polynomial  $T_9(z_0 \cos(\theta))$  for a -26 dB sidelobe suppression

$$V(\theta) = \begin{cases} T_9(z_0 \cos(\theta)) & 0 \leq \theta \leq \pi/2 \\ 0 & \text{Other} \end{cases} \quad (15)$$

where  $z_0$  equals 1.09. The  $V(\theta)$  in equation (15) can be expanded by a summation of Legendre polynomials per equation (3) and equation (4). By Rayleigh integral numerical method of equation (13), the associated polar graph beam patterns at various frequencies are displayed in FIG. 2A for 80 kHz, FIG. 2B for 400 kHz, and FIG. 2C for 2000 kHz, where the beam patterns are kept as constant shapes with all side-lobes around -26 dB for frequencies greater than 140 kHz, or dimensionless parameter  $a/\lambda$  greater than 32.6. Here, the spherical transducer has a radius of 0.349 meters, and all of the polar graphs of logarithmic scale from -50 dB to the main lobe peak of 0 dB.

**[0027]** Another modeled beam pattern is a cubed cosine function. The simulation example here is for a high power order cosine form on the surface of a rigid hemisphere of radius 0.349 meters:

$$V(\theta) = \begin{cases} (\cos(\theta))^3 & 0 \leq \theta \leq \pi/2 \\ 0 & \text{Other} \end{cases} \quad (16)$$

This can also be expressed by Legendre polynomials as:

$$(\cos(\theta))^3 = 0.6P_1(\cos(\theta)) + 0.4P_3(\cos(\theta)). \quad (17)$$

The radial shading function of  $(\cos(\theta))^3$  is shown in FIG. 3. Associated constant beam patterns for the  $(\cos(\theta))^3$  shading function are shown in polar graph FIG. 4A for 40 kHz, FIG. 4B for 160 kHz, and FIG. 4C for 400 kHz. The dimensionless ratio of the radius to the wavelength,  $a/\lambda$ , is about 9.3 at 40 kHz and 92.8 at 400 kHz. This model again shows that a constant beam pattern spherical transducer can be realized over a decade frequency band.

**[0028]** A narrow beam width simulation for a 0.94 meter radius hemispherical transducer or array example was formulated utilizing a single high degree Legendre polynomial  $P_{62}$ , as follows:

$$V(\theta) = \begin{cases} P_{62}(z_0 \cos(\theta)) & 0 \leq \theta \leq \pi/2 \\ 0 & \text{Other} \end{cases} \quad (18)$$

where  $z_0$  is 1.0025. FIG. 5 shows the hemispherical shading distribution. Far field beam pattern representations are shown in FIG. 6A for 200 kHz, FIG. 6B for 600 kHz, and FIG. 6C for

2000 kHz. (Representations are used because the detail of the side-lobes in original data is too fine to show accurately.) Utilizing this method, a shading function can be calculated for axisymmetric beam patterns that are constant over a wide frequency range.

**[0029]** FIG. 7 provides a diagram showing one embodiment of an apparatus for this method. A user can provide a chosen symmetric beam pattern at terminal 10. The beam pattern can be selected from known shading functions or the user can specify maximum lobe directions and nulls for the beam pattern. Other parameters such as a maximum or average power level can be selected by the user. A processor 12 calculates shading weights in accordance with the provided beam pattern, provided power level, the equations provided above, and the known transducer array 14. Transducer array 14 is made from a plurality of transducers 16, each oriented towards a specific angle. Preferably transducers 16 are positioned in a regular three dimensional spherical array in order to simplify computation of the shading weights. Processor 12 can divide shading weights among several transducers 16 in order to create a fit with the beam pattern. Array 14 is joined to a bank of amplifiers 18. For maximum versatility, each amplifier 20 in the bank of amplifiers 18 is joined to one transducer 16; however, amplifiers 20 can also be joined to groups of transducers such

as those at a specific bearing from a given bearing. Processor 12 is joined to bank of amplifiers 18 to provide shading weights associated with each transducer 16. These shading weights become are translated into voltage gains or attenuations by the bank of amplifiers 18. A signal source 22 is further joined to bank of amplifiers 18. Signal source 22 is provided to each amplifier 20 without a time delay. Each amplifier 20 amplifies or attenuates the source 22 in accordance with the calculated shading weight.

**[0030]** FIG. 8 provides a beam pattern 24 having transducer array 14 superimposed thereon. This illustrates how a user or other routine can specify an arbitrary beam pattern for calculation by the current method. User can place a point 26 indicating main lobe bearing angle, which is then aligned with z-axial direction for the spherical coordinates used for equations (1) and equation (13). If the main lobe bearing angle does not correspond directly with an array transducer, the amplification or shading can be split among the next nearest array transducers. Auxiliary points 28 and 30 can be placed to indicate main lobe shape. Side lobes 32 can be specified using side lobe maximum points 34 and shapes 36. Nulls 38 can also be specified. After generally specifying a beam pattern, the user can identify an axis of rotation 40 for rotation of the two

dimensionally specified beam pattern into the three dimensional, axisymmetric beam pattern required by this method.

**[0031]** Other methods, such as drawing a beam pattern can also be utilized. After specifying the preferred beam pattern, processor 12 can utilize a variety of methods to fit known shading functions to the points identified by the user.

Transducers in the array can be associated with the user defined rotated beam pattern. Shading or amplification values for each transducer can be calculated by processor 12. A required operating band width and maximum array radius can also be used as design constraints. The specified beam pattern can be realized according to the method described above.

**[0032]** Other data entry methods can be utilized to specify the beam pattern. For example, a user can specify an envelope indicating the maximum beam or a null for a given bearing and a rotation axis. The processor can then develop a shading function that will fit the specified three dimensional envelope.

**[0033]** It will be understood that many additional changes in the details, materials, steps and arrangement of parts, which have been herein described and illustrated in order to explain the nature of the invention, may be made by those skilled in the art within the principle and scope of the invention as expressed in the appended claims.

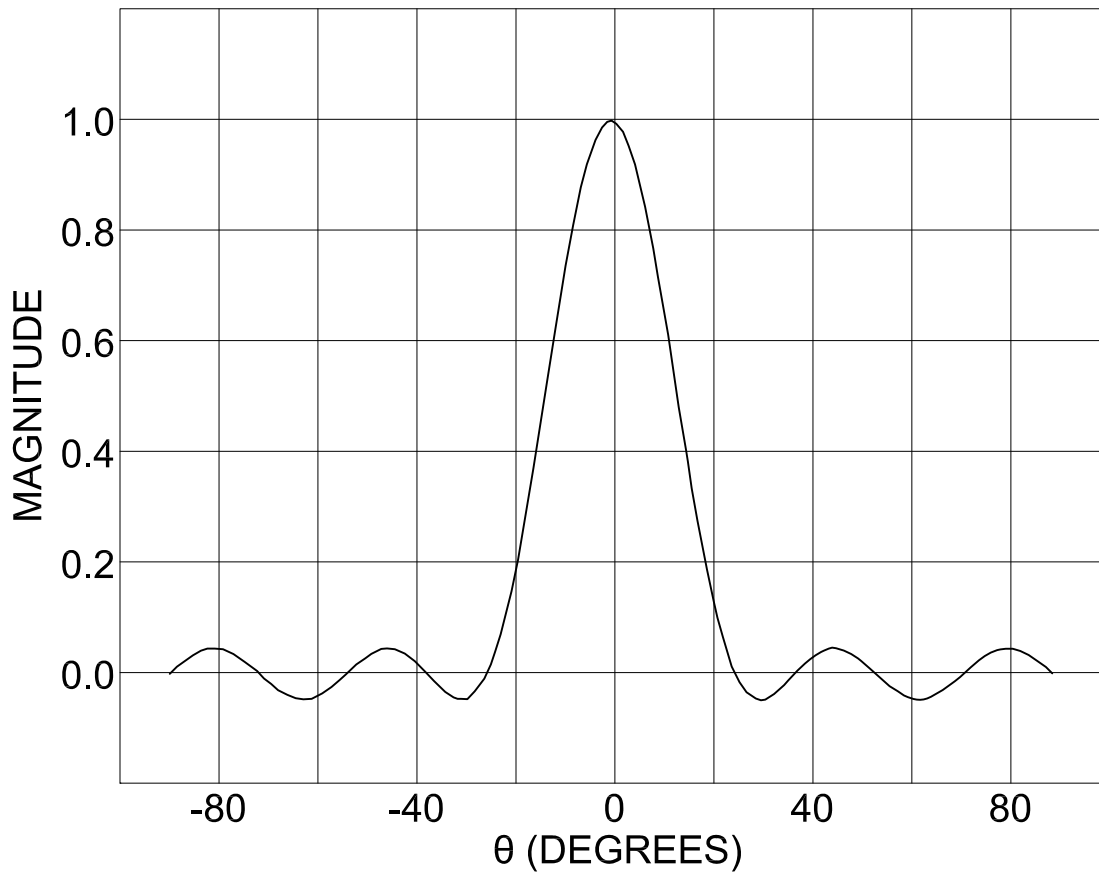
**[0034]** The foregoing description of the preferred embodiments of the invention has been presented for purposes of illustration and description only. It is not intended to be exhaustive, nor to limit the invention to the precise form disclosed; and obviously, many modification and variations are possible in light of the above teaching. Such modifications and variations that may be apparent to a person skilled in the art are intended to be included within the scope of this invention as defined by the accompanying claims.



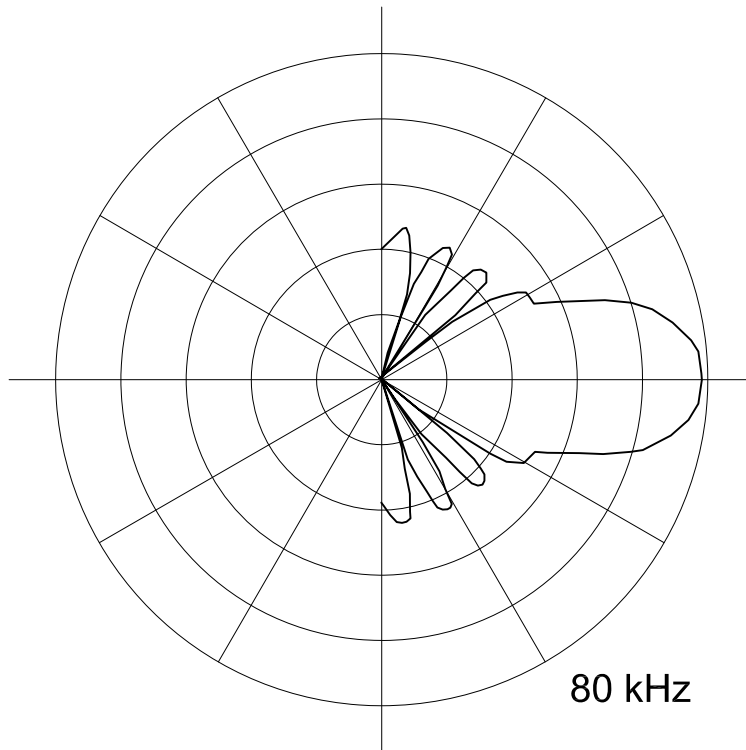
**CONSTANT BEAM PATTERN ARRAY METHOD**

**ABSTRACT OF THE DISCLOSURE**

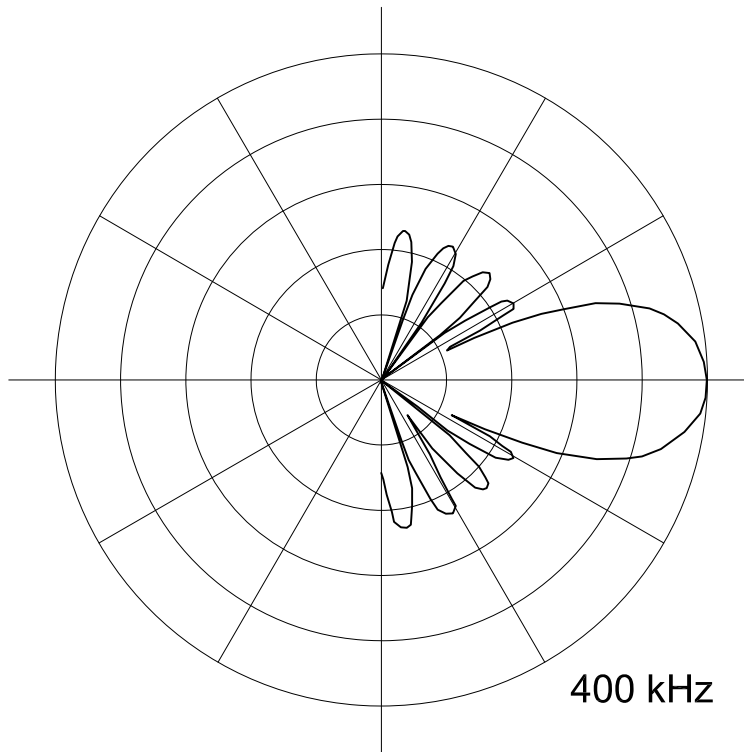
A method for providing a broadband constant beam pattern acoustic array includes providing an array of transducers in a known three dimensional axisymmetric spherical configuration with each transducer element having an associated signal. A user can specify a far field beam pattern for the array. Weightings are calculated for each transducer in the array as being proportional to the voltage that gives the beam pattern power level associated with the bearing for each transducer. Signal power levels for each transducer are modified in accordance with the weightings. The array can be operated for receiving and transmitting signals with a constant beam pattern over a broad range of frequencies.



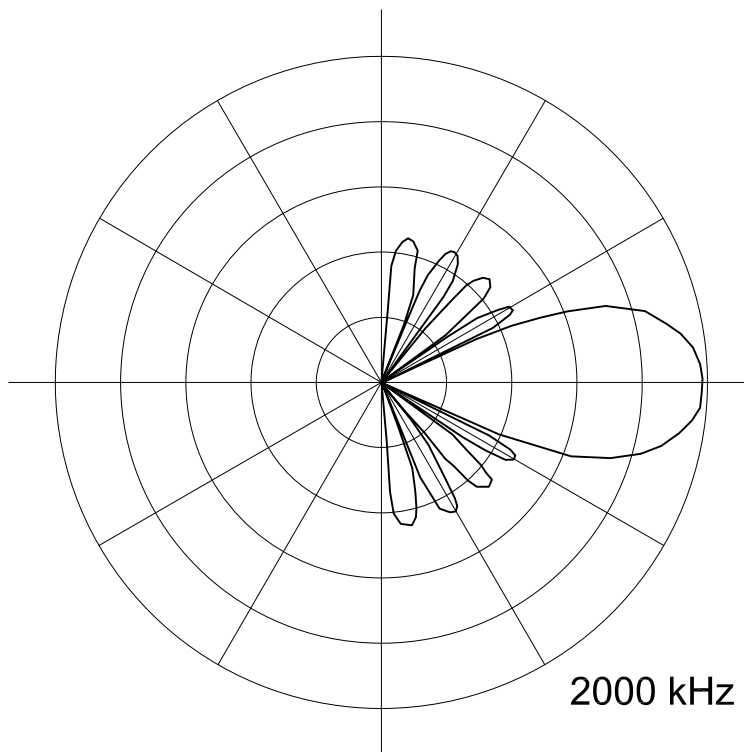
**FIG. 1**



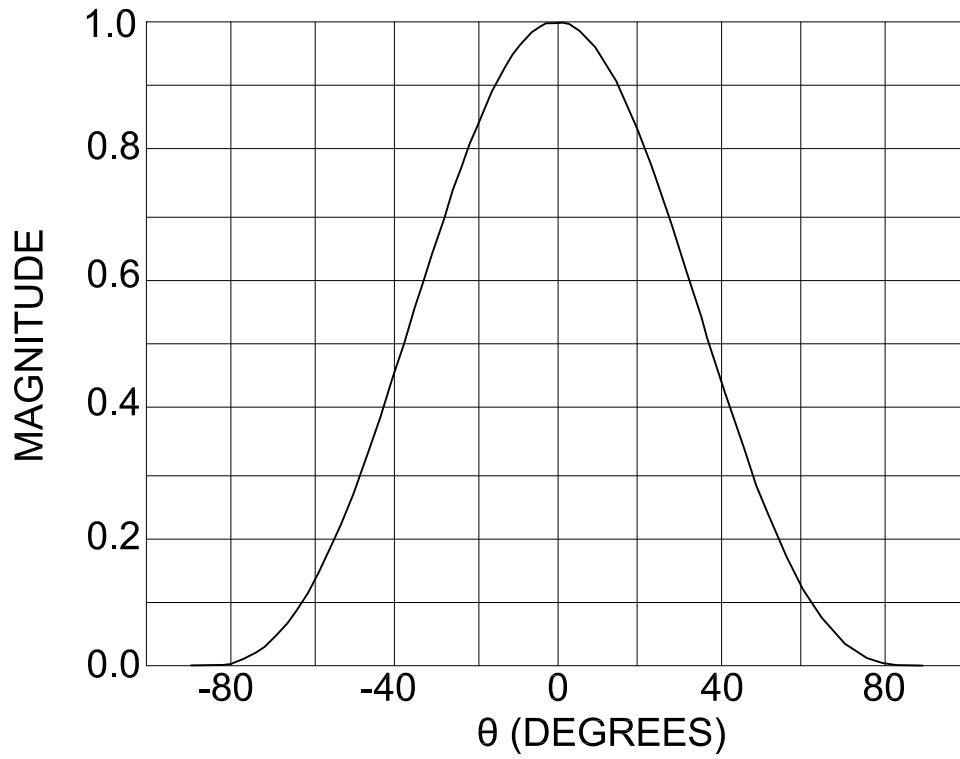
**FIG. 2A**



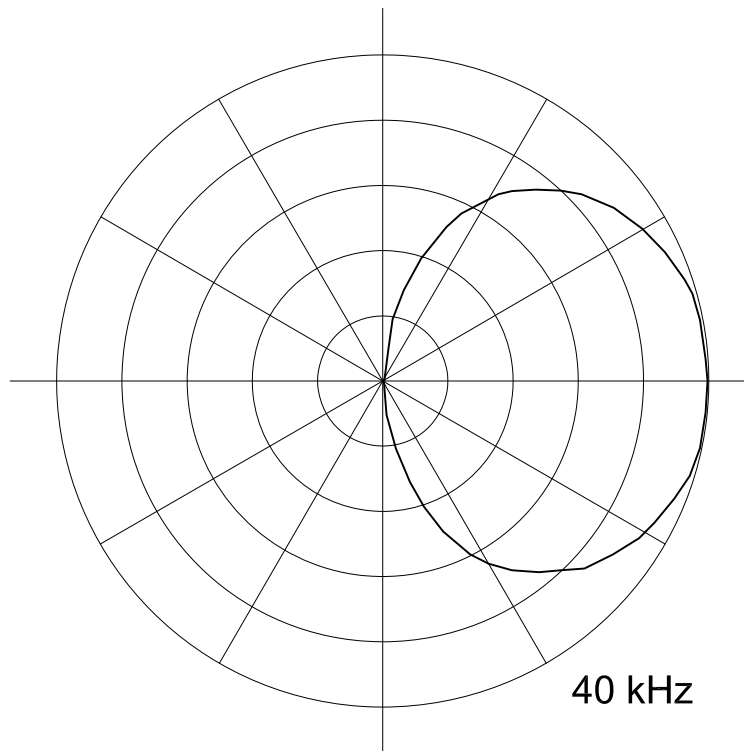
**FIG. 2B**



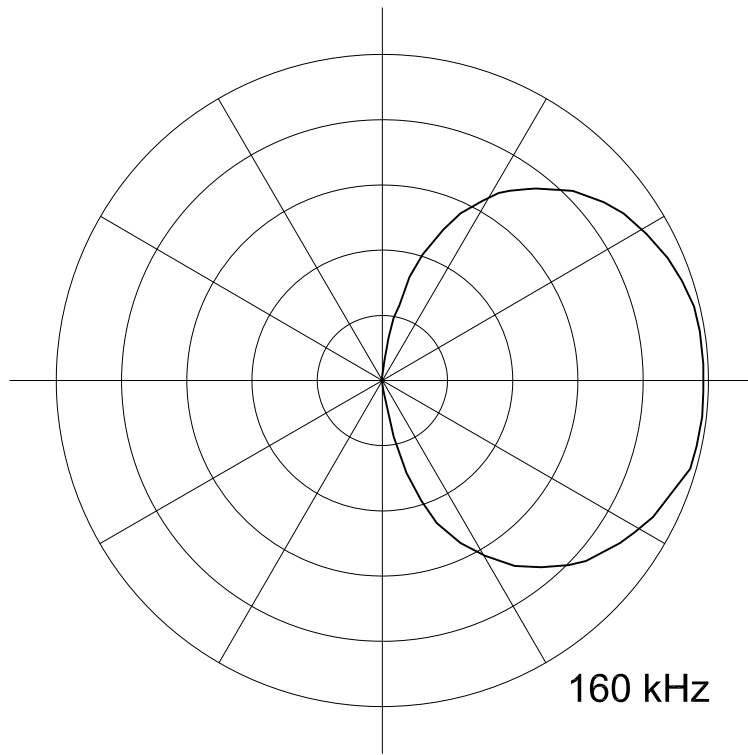
**FIG. 2C**



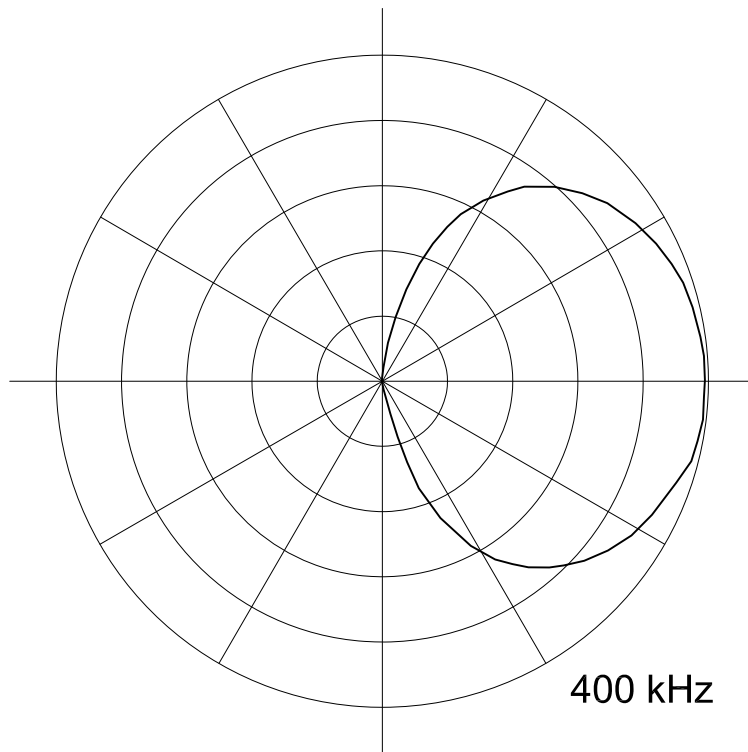
**FIG. 3**



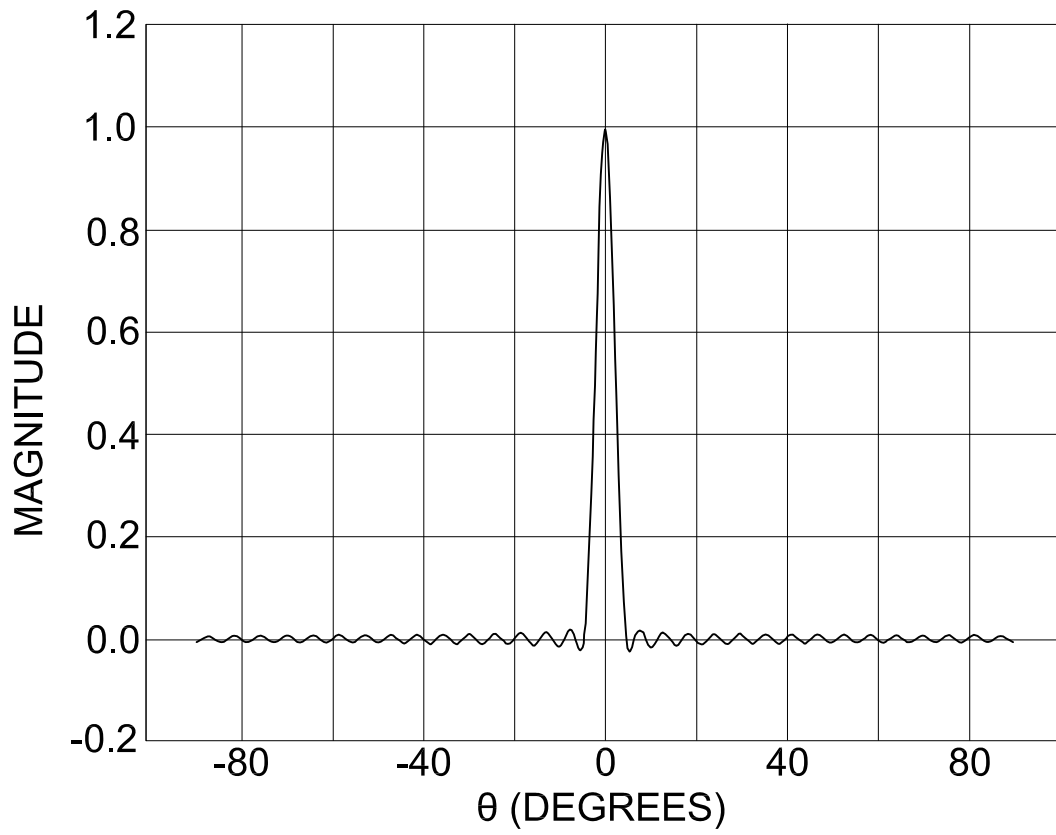
**FIG. 4A**



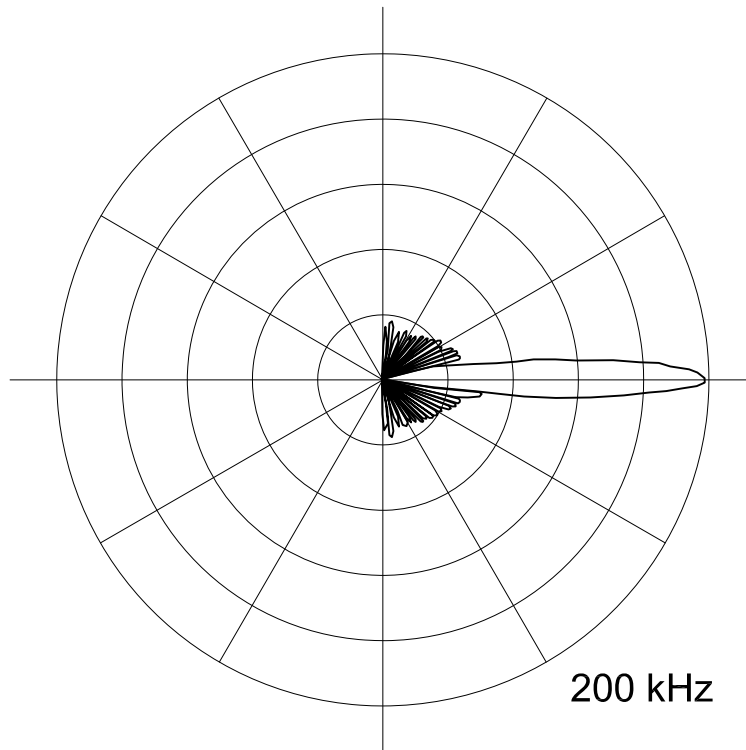
**FIG. 4B**



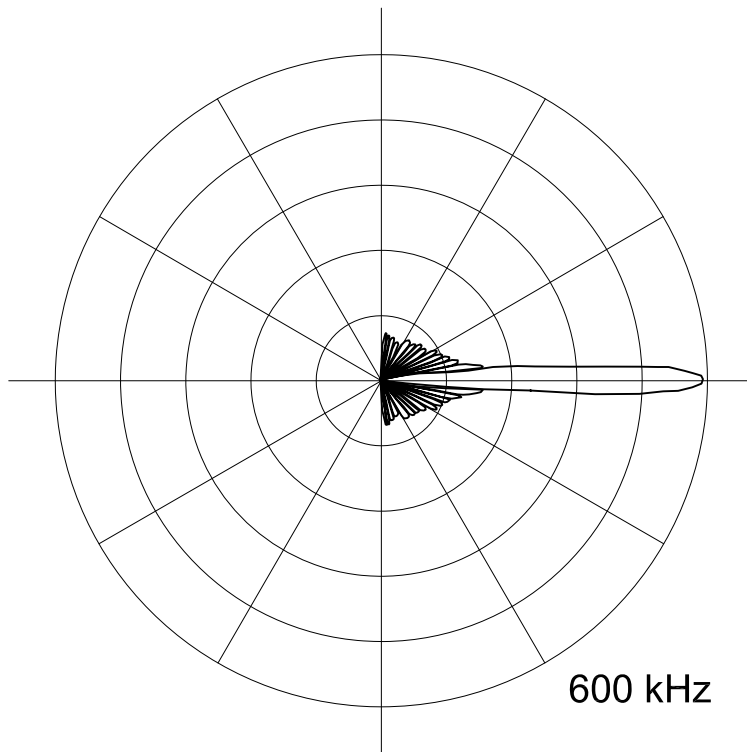
**FIG. 4C**



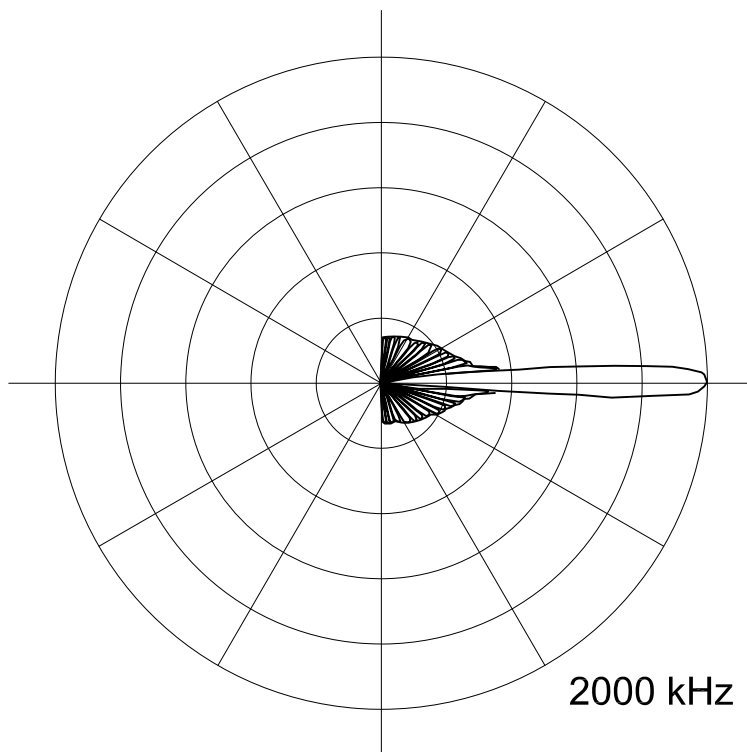
**FIG. 5**



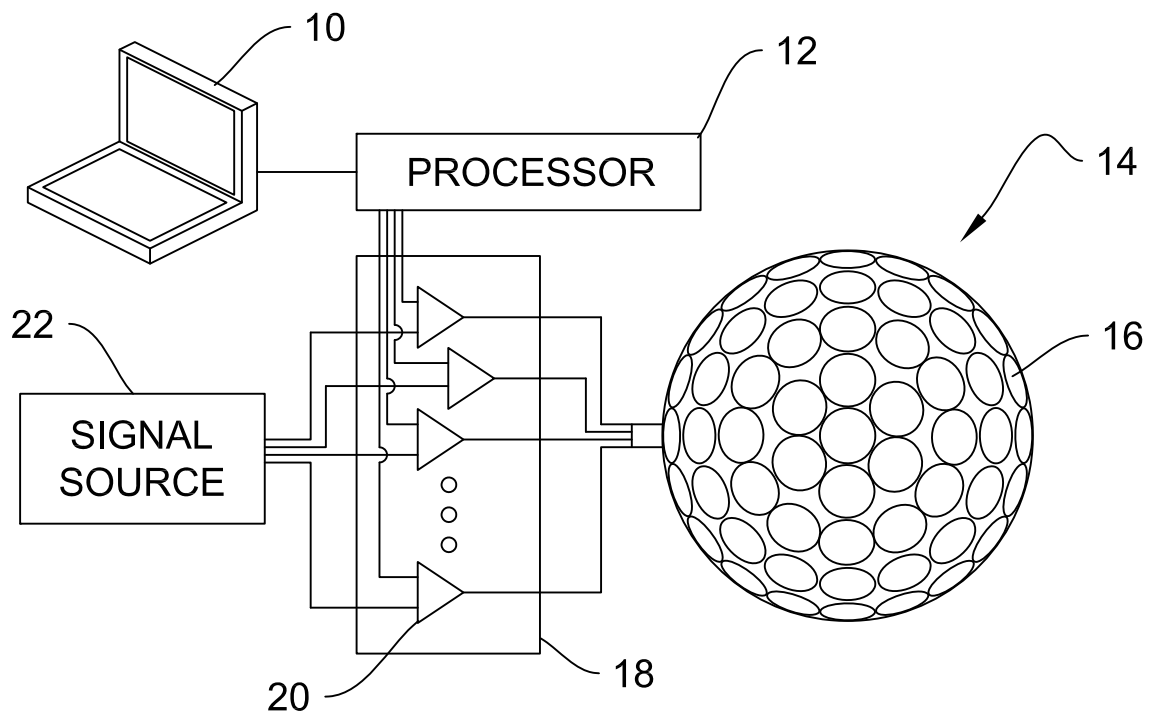
**FIG. 6A**



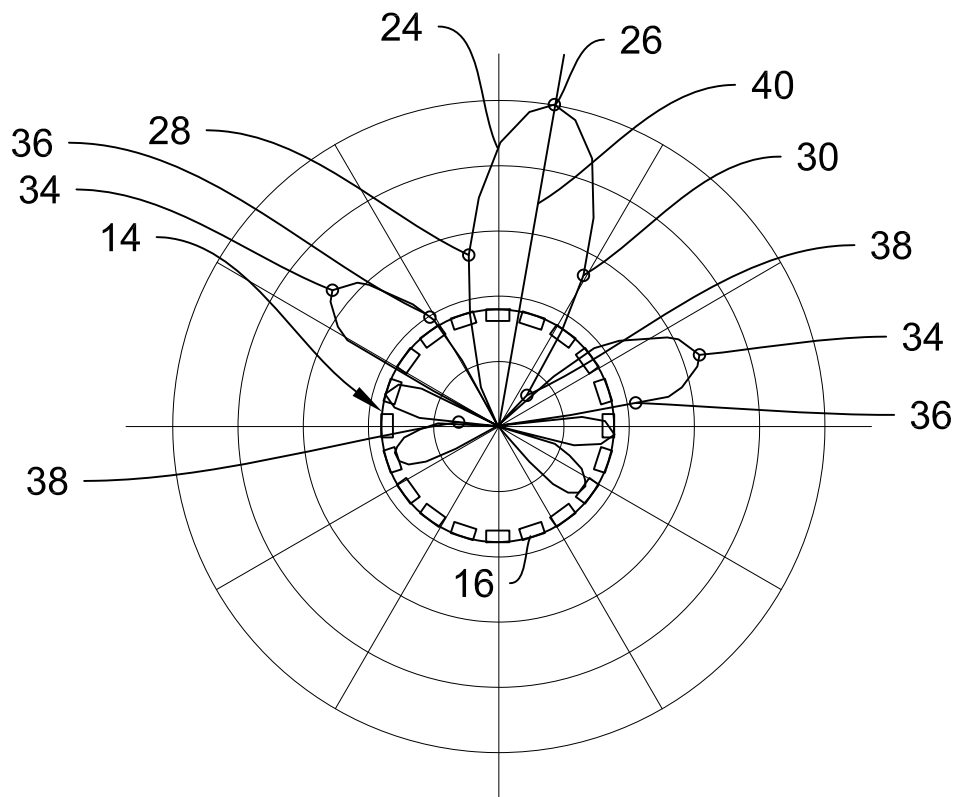
**FIG. 6B**



**FIG. 6C**



**FIG. 7**



**FIG. 8**