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The Kelvin’s formula of resultant force acting on polarizable solid is one of the key instruments in modeling various electromagnetic phenomena. One of the conceptual difficulties rendering the Kelvin’s formula was formulated as the Kelvin’s formula paradox. Recently, one more attempt of justifying the K-paradox has been published that creates the impression that the K-paradox is wrong. This technical note presents evidence that this impression is incorrect and is based on a misunderstanding of the Kelvin’s formula paradox.

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1. Introduction

The Kelvin's formula (K-formula) of resulting force acting on polarizable solid is one of the key instruments in modeling various electromagnetic phenomena. One of the conceptual difficulties rendering the Kelvin's formula was formulated as the Kelvin's formula paradox (the K-paradox).[1] Paradoxes are the focused and highly concentrated indications of serious misunderstanding of the problems under study. Serious researchers should not and do not tolerate the presence of paradoxes in their theories, and do all that is possible to resolve them.

The first attempt to resolve the K-paradox was not particularly successful, which was addressed in a previous report.[2] Recently, one more attempt of justifying the K-paradox has been published.[3] The idea of the publication[3] is based on replacement of the Kelvin's K-formula with the Landau and Lifshitz[4] LL-formula. In this short note we discuss why this idea is misleading.

2. Why is the Kelvin's Formula Paradoxical?

The Kelvin's formula paradox,[1] also called the K-paradox or the self-force paradox, deals with the resultant force acting upon a polarizable body in electrostatic or magnetostatic fields. We remind the reader that according to the K-formula the total (resultant) force $\vec{F}_{res}$, acting on the electrically polarized body, is given by the formula

$$\vec{F}_{res} = \int_{\Omega} d\Omega \vec{P} \cdot \nabla \vec{E}, \tag{1}$$

where $\vec{E}$ is the electric field, $\vec{P}$ is the polarization density, and the integration is taken over the whole polarized body.

What is paradoxical about the K-formula? The paradoxical chain (the K-chain) is the following: the derivation of the K-formula begins with postulating of the Coulomb law, which automatically satisfies Newton's laws (the first link of the K-chain). It proceeds with the physically (not necessarily mathematically) consistent derivation of the force acting on the elementary (discrete) dipole (the second link of the K-chain). The system of elementary dipoles still obeys Newton's laws. At last, standard homogenization (i.e., replacement of finite sums with integrals) is finalized, resulting in the K-formula (the third link of the K-chain). The K-paradox claims that the K-formula gives a nonvanishing resultant self-force acting on a polarized body, even in the absence of any external electric field (the fourth, final, link of the K-chain). Thus, we have the typical paradoxical chain (the

K-chain): the first link of the K-chain satisfies the Newton law whereas the final link violates it.

So far, all prior counter-arguments regarding the K-paradox were reduced to the criticism of usage of our model of nondeformable solids.[1] This argument of resolving the K-paradox can be formulated as follows: "The K-formula is correct, and the K-paradox arises through the assumption of non-deformability. When deformability is taken into account, it will automatically eliminate the K-formula paradox." This is a misdirected attempt of resolving the K-paradox. We addressed this argument in our previous report.[2]

A more recent objection[3] against the K-paradox is formulated in quite a vague way by the mathematical standards. To our understanding, the publication[3] suggests to use the LL-formula for the resultant force:

$$\vec{F}_{res} = \int_{\Omega} d\Omega \vec{P} \cdot \nabla \vec{E}_{ext}, \tag{2}$$

instead of the K-formula. In Eq. 2, $\vec{E}_{ext}$ is the *external* electrostatic field (i.e., the field, created by all sources of the electrostatic field *except for* the dipoles of the body in question). We call it the LL-formula since it was published many decades ago in the classical textbook of Landau and Lifshitz.[4]

Obviously, the LL-formula gives a vanishing value of the resultant force when all the external forces are absent. No calculations whatsoever are necessary to make this obvious conclusion since $\vec{E}_{ext} = 0$ in this case. In other words, there is no analogy to the K-paradox when dealing with the LL-formula.

Based on the vanishing self-action for the LL-formula, it is concluded in the publication[3] that the early published[1] K-paradox is wrong. This is a shocking logical construct. The LL-formula has nothing in common with the previously mentioned K-chain proceeding from the Coulomb law to the K-formula. The K-paradox is formulated for the K-formula, not for the LL-formula.

3. Conclusion

Criticism of the K-paradox[3] is misleading. It is based on a misunderstanding of the K-paradox and the confusion of the K-formula with the LL-formula.

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