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Major Goals: Major goals of the project

Introduction: Quantum coherence is the key feature that differentiates a quantum computer from a classical one. It is the property that allows for the quantum parallelism, which, in turn, gives the speed-up of the quantum computation (e.g., the Shor's factoring algorithm). Unfortunately, the elements (qubits) of any conceivable quantum computer interact with the environment, leading to coherence loss. Thus, to enable quantum computation, coherence has to be constantly protected.

The project targets optimized coherence protection, a key enabling technology for solid-state quantum computation (QC). The idea is to design new families of quantum error-correcting codes (QECCs) that combine good parameters with a feasible near-optimal on-chip implementation, and have an additional advantage of built-in dynamical decoupling at the level of individual qubits.

Quantum error correction is the first and the main ingredient of the project. QECCs are constructed in direct analogy to classical error correction. The benefits of the latter are evidenced by a drastic improvements in quality of data and images transmitted or stored over a variety of noisy channels such as satellite communication, CDs, hard disks, and flash drives. In quantum error correction, a number of additional redundant qubits is introduced. Quantum information is encoded in certain highly-entangled states in such a way that errors which happen on one, or even on several qubits can be corrected. To this end, error syndrome needs to be measured fairly often; it allows one to decide how to correct the encoded state. While these measurements are performed on the quantum system, they are designed not to measure the actual encoded quantum state---otherwise the coherence would be destroyed. The difficulty is that measurements for a generic code are complicated, and yet they need to be performed often. Code optimization targets to build codes with good parameters (many qubits encoded without too much redundancy), and yet with simple stabilizer generators (whose measurement results constitute the error syndrome) which can be measured easier, and also in parallel.

For solid-state qubits, errors are often correlated in time. Correlated errors can be seen as a slow parameter drift resulting from various processes in the material: precession of nuclear spins, fluctuations in charge traps present in the insulator, etc. While these perturbing terms are typically small, correlation in time means that they act in the same direction over a long time. As a result, time-correlated errors can dominate the decoherence rate. While QECCs can be used to deal with such errors, fast decoherence would require a frequent measurement-correction cycle.

Dynamical decoupling (DD) is a passive error-protection technique most suitable for dealing with errors correlated in time. It is related to refocusing in nuclear magnetic resonance (NMR). The goal of DD is to prevent errors from happening in the first place. This is done by driving rapid evolution (rotation) of the quantum system to be protected in such a way that the perturbing terms in the rotating frame constantly change sign. The rotation is usually achieved with sequences of control pulses analogous to those in NMR. As a result, the perturbation no longer acts in the same direction; the associated rate of coherence loss is greately reduced. One wants to have this additional rapid rotation of qubits, and at the same time be able to perform the operations necessary for error correction and for the actual algorithm the quantum computer is designed to perform.

While DD does not require any additional qubits to work, the resource that is used is the bandwidth. Especially for solid-state qubits it is extremely important to carefully optimize the pulse shapes and sequences, which was the subject studied by the PI for some time. Incorporating the DD as the first-line coherence protection could increase the qubit coherence time by orders of magnitude, making a crucial difference for enabling the use of a particular QC architecture.

Accomplishments: Code design: The approach proposed originally was based on the code-word stabilized codes[1]. This was implemented during the first year in Ref. 2. Subsequently, the design goal was shifted to target quantum sparse-graph codes[3]. These are analogous to classical low-density-parity-check (LDPC) codes widely used in digital communication, data storage, and related areas. The advantages of quantum LDPC codes are that (i) they offer simple, low-depth structure, (ii) the syndrome measurements are simpler and can be done in parallel, and (iii) the amount of classical processing needed for error correction is expected to be smaller. Specific focus in the last two years was on designing and investigating the properties of codes related to hypergraph-product codes invented by Tillich and Zemor[4]. This resulted in publications [5, 6]. Characterization of such codes lead to publication [7] which proves a major result, the existence of a fault-tolerant error-correction threshold for quantum LDPC codes. Related work aimed at understanding this result, by studying phase transitions in non-local Ising models [9]. Additional related work targeted development of algorithms for distance verification of quantum LDPC codes[10, 11].

Dynamical decoupling: Ultimately, any error correction algorithm can be presented as a sequence of elementary gates which include a universal set of one- and two-qubit unitary operations, and single-qubit measurements. Integrating error correction with dynamical decoupling can be done by designing dynamically-protected gates. For qubits actively involved in a gate, the control pulses should provide for the desired evolution. For all of the qubits, involved in the gate or not, the pulse sequence should also average out the stray fields acting on the qubits, as well as the undesired couplings between the qubits. Some of the background research (optimized pulse shapes and sequences, and associated simulations) has been previously done by the PI [12]. For high-order cancelation in non-trivial single-qubit gates, the design techniques have been developed in Ref. 13. The main results of the supported studies are published in Refs. 14 and 15. Additional applications of dynamical decoupling have been explored in Ref. 16.

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Further details are given in the attached PDF.

Training Opportunities: Two graduate students have been trained:

Dr. Kathleen E. Hamilton (presently at Quantum Computing Institute, Oak Ridge National Laboratory, Oak Ridge, Tennessee, 37821, USA), and

Dr. David Drummond

Two postdoctoral researchers have been trained:

Dr. A.A. Kovalev, currently an Assistant Professor at the University of Nebraska--Lincoln.

Dr. Amrit De, currently a Research Scientist at the PHYSICAL OPTICS CORPORATION

Results Dissemination: Major venue for dissemination of the result was by published papers.
In addition, a number of talks have been given by the PI, co-PI, and associated graduate students and postdocs.

Honors and Awards: Nothing to Report

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Participant: Leonid P Pryadko
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Authors:
Keywords: code-word stabilized codes, quantum LDPC codes, Gilbert-Varshamov bound, quantum cyclic codes
Abstract: We consider design of the quantum stabilizer codes via a two-step, low-complexity approach based on the framework of codeword-stabilized (CWS) codes. In this framework, each quantum CWS code can be specified by a graph and a binary code. For codes that can be obtained from a given graph, we give several upper bounds on the distance of a generic (additive or nonadditive) CWS code, and the lower Gilbert-Varshamov bound for the existence of additive CWS codes. We also consider additive cyclic CWS codes and show that these codes correspond to a previously unexplored class of single-generator cyclic stabilizer codes.We present several families of simple stabilizer codes with relatively good parameters.
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Authors:
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Abstract: We study the fault tolerance of quantum low-density parity check (LDPC) codes, such as generalized toric codes with a finite rate suggested by Tillich and Zémor [in ISIT 2009: IEEE International Symposium on Information Theory (IEEE, New York, 2009)]. We show that any family of quantum LDPC codes where each syndrome measurement involves a limited number of qubits and each qubit is involved in a limited number of measurements (as well as any similarly limited family of classical LDPC codes), in which distance scales as a positive power ? of the number of physical qubits (?<1 for “bad” codes), has a finite error probability threshold. We conclude that for sufficiently large quantum computers, quantum LDPC codes can offer an advantage over the toric codes.
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Article Title: Universal Set of Scalable Dynamically Corrected Gates for Quantum Error Correction with Always-on Qubit Couplings
Authors:
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Abstract: We construct a universal set of high fidelity quantum gates to be used on a sparse bipartite lattice with always-on Ising couplings. The gates are based on dynamical decoupling sequences using shaped pulses, they protect against low-frequency phase noise, and can be run in parallel on non-neighboring qubits. This makes them suitable for implementing quantum error correction with low-density parity check codes like the surface codes and their finite-rate generalizations. We illustrate the construction by simulating the quantum Zeno effect with the [[4, 2, 2]] toric code on a spin chain.
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Authors:
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Abstract: We examine the logical qubit system of a pair of electron spins in double quantum dots. Each electron experiences a different hyperfine interaction with the local nuclei of the lattice, leading to a relative phase difference, and thus decoherence. Methods such as nuclei polarization, state narrowing, and spin-echo pulses have been proposed to delay decoherence. Instead we propose to suppress hyperfine dephasing by the adiabatic rotation of the dots in real space, leading to the same average hyperfine interaction. We show that the additional effects due to the motion in the presence of spin-orbit coupling are still smaller than the hyperfine interaction, and result in an infidelity below 0.0001 after ten decoupling cycles. We discuss a possible experimental setup and physical constraints for this proposal.
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Abstract: We introduce an ansatz for quantum codes which gives the hypergraph-product (generalized toric) codes by Tillich and Zémor and generalized bicycle codes by MacKay et al. as limiting cases. The construction allows for both the lower and the upper bounds on the minimum distance; they scale as a square root of the block length. Many thus defined codes have a finite rate and limited-weight stabilizer generators, an analog of classical low-density parity-check (LDPC) codes. Compared to the hypergraph-product codes, hyperbicycle codes generally have a wider range of parameters; in particular, they can have a higher rate while preserving the estimated error threshold.
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Technical Progress Report on the award W911NF-11-1-0027 "Lattice Codes with Built-in Dynamical Protection for Solid-State Quantum Computation".

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(Dated: May 28, 2018)

Research progressed in two directions: (a) theory of quantum error correction and (b) dynamical decoupling (DD). Research in quantum error correction concentrated on quantum low-density parity-check (LDPC) codes. Most important result is the proof of the existence of a finite fault-tolerant error-correction threshold. This makes quantum LDPC codes the only class of quantum error-correcting codes (QECCs) where such a threshold is known to exist in a finite rate code. Also, several new families of finite-rate quantum LDPC codes have been constructed. In the part of the project related to dynamical decoupling, dynamically-corrected single-qubit and two-qubit quantum gates on bipartite lattices have been constructed to enable integration with error correction cycle for any quantum LDPC code. Repeated cycles of error correction with the $[[4, 2, 2]]$ QECC have been simulated by integrating full unitary dynamics of up to five qubits driven with the constructed gate set. The corresponding error operators have also been analyzed, with a proof that this construction can be used to build fault-tolerant quantum memory using toric code.

I. MAJOR GOALS OF THE PROJECT

Introduction: Quantum coherence is the key feature that differentiates a quantum computer from a classical one. It is the property that allows for the quantum parallelism, which, in turn, gives the speed-up of the quantum computation (e.g., the Shor's factoring algorithm). Unfortunately, the elements (qubits) of any conceivable quantum computer interact with the environment, leading to coherence loss. Thus, to enable quantum computation, coherence has to be constantly protected.

The project targets optimized coherence protection, a key enabling technology for solid-state quantum computation (QC). The idea is to design new families of quantum error-correcting codes (QECCs) that combine good parameters with a feasible near-optimal on-chip implementation, and have an additional advantage of built-in dynamical decoupling at the level of individual qubits.

Quantum error correction is the first and the main ingredient of the project. QECCs are constructed in direct analogy to classical error correction. The benefits of the latter are evidenced by a drastic improvements in quality of data and images transmitted or stored over a variety of noisy channels such as satellite communication, CDs, hard disks, and flash drives. In quantum error correction, a number of additional redundant qubits is introduced. Quantum information is encoded in certain highly-entangled states in such a way that errors which happen on one, or even on several qubits can be corrected. To this end, error syndrome needs to be measured fairly often; it allows one to decide how to correct the encoded state. While these measurements are performed on the quantum system, they are designed not to measure the actual encoded quantum state---otherwise the coherence would be destroyed. The difficulty is that measurements for a generic code are complicated, and yet they need to be performed often. Code optimization targets to build codes with good parameters (many qubits encoded without too much redundancy), and yet with simple *stabilizer generators* (whose measurement results constitute the error syndrome) which can be measured easier, and also in parallel.

For solid-state qubits, errors are often correlated in time. Correlated errors can be seen as a slow parameter drift resulting from various processes in the material: precession of nuclear spins, fluctuations in charge traps present in the insulator, etc. While these perturbing terms are typically small, correlation in time means that they act in the same direction over a long time. As a result, time-correlated errors can dominate the decoherence rate. While QECCs can be used to deal with such errors, fast decoherence would require a frequent measurement-correction cycle.

Dynamical decoupling (DD) is a passive error-protection technique most suitable for dealing with errors correlated in time. It is related to refocusing in nuclear magnetic resonance (NMR). The goal of DD is to prevent errors from happening in the first place. This is done by driving rapid evolution (rotation) of the quantum system to be protected in such a way that the perturbing terms in the rotating frame constantly change sign. The rotation is usually achieved with sequences of control pulses analogous to those in NMR. As a result, the perturbation no longer acts in the same direction; the associated rate of coherence loss is greately reduced. One wants to have this additional rapid rotation of qubits, and at the same time be able to perform the operations necessary for error correction and for the actual algorithm the quantum computer is designed to perform.

While DD does not require any additional qubits to work, the resource that is used is the bandwidth. Especially for solid-state qubits it is extremely important to carefully optimize the pulse shapes and sequences, which was the subject studied by the PI for some time. Incorporating the DD as the first-line coherence protection could increase the qubit coherence time by orders of magnitude, making a crucial difference for enabling the use of a particular QC architecture.

Approach and summary of resuts: Designing a QECC is a problem of exponential complexity, and optimization in the space of QECCs is a very difficult problem.

Code design: The approach proposed originally was based on the code-word stabilized codes[1]. This was implemented during the first year in Ref. 2. Subsequently, the design goal was shifted to target quantum sparse-graph codes[3]. These are analogous to classical low-density-parity-check (LDPC) codes widely used in digital communication, data storage, and related areas. The advantages of quantum LDPC codes are that (i) they offer simple, low-depth structure, (ii) the syndrome measurements are simpler and can be done in parallel, and (iii) the amount of classical processing needed for error correction is expected to be smaller. Specific focus in the last two years was on designing and investigating the properties of codes related to hypergraph-product codes invented by Tillich and Zemor[4]. This resulted in publications [5, 6]. Characterization of such codes lead to publication [7] which proves a major result, the existence of a fault-tolerant error-correction threshold for quantum LDPC codes. Related work aimed at understanding this result, by studying phase transitions in non-local percolation [8] and Ising models[9]. Additional related work targeted development of algorithms for distance verification of quantum LDPC codes[10, 11].

Dynamical decoupling: Ultimately, any error correction algorithm can be presented as a sequence of elementary gates which include a universal set of one- and two-qubit unitary operations, and single-qubit measurements. Integrating error correction with dynamical decoupling can be done by designing *dynamically-protected* gates. For qubits actively involved in a gate, the control pulses should provide for the desired evolution. For all of the qubits, involved in the gate or not, the pulse sequence should also average out the stray fields acting on the qubits, as well as the undesired couplings between the qubits. Some of the background research (optimized pulse shapes and sequences, and associated simulations) has been previously done by the PI[12]. For high-order cancelation in non-trivial single-qubit gates, the design techniques have been developed in Ref. 13. The main results of the supported studies are published in Refs. 14 and 15. Additional applications of dynamical decoupling have been explored in Refs. 16--18.

II. DESIGN AND ANALYSIS OF QUANTUM LDPC CODES

Quantum low-density parity check (LDPC) codes[3, 4, 19],also known as quantum sparse-graph codes, are an analog of classical LDPC codes. Technically, these are just stabilizer codes, but with stabilizer generators which involve only a few qubits compared to the number of qubits used in the code. Such codes are most often degenerate: some errors have trivial effect and do not require any correction. Compared to general quantum codes, with a quantum LDPC code, each quantum measurement involves fewer qubits, measurements can be done in parallel, and also the classical processing could potentially be enormously simplified.

The most famous family of quantum sparse-graph codes is Kitaev's toric construction[20]: it has a relatively high threshold for scalable quantum computation, around 1% total error probability per quantum gate or a qubit measurement, and uses only local gates [21--24]. One disadvantage is that all toric and related surface codes encode few qubits (in technical terms, they have asymptotically zero rate[25]); thus they require many redundant physical qubits[23].

Finite-rate quantum LDPC codes are also possible[4]With these, fewer redundant qubits may be necessary to build a useful quantum computer.

Quantum sparse graph codes are commonly called quantum LDPC codes by analogy with the classical low density parity-check codes[26]. These latter codes have fast and efficient (capacity-approaching) decoders. Over the last ten years classical LDPC codes have become a significant component of industrial standards for satellite communications, Wi-Fi, and gigabit ethernet, to name a few. The success of classical LDPC codes is the reason for some of the interest in the quantum LDPC codes.

A. Fault-tolerance of quantum LDPC codes

All families of quantum LDPC codes where distance is known have asymptotically zero relative distance. The toric and related surface codes[20, 26], their finite-rate generalizations[4--6], and some unrelated constructions[27, 28] have distances scaling asymptotically as a square root of the block length n, with the record set in Ref. 29 for a single-qubit-encoding code whose distance scales as $n^{1/2}\log n$.

While the existence of a fault-tolerant error correction threshold has been in the past explicitly demonstrated for topological toric and related surface codes[20, 30, 31], the question of fault-tolerance for other codes with sublinear distance scaling is generally far from trivial. Indeed, typical random uncorrelated errors are characterized by a per-qubit probability, p; in a code of block length n a typical error involves pn qubits while a sublinear distance, $d \propto n^\alpha$ with $0 < \alpha < 1$, only guarantees that much smaller errors involving up to $d-1$ qubits can be detected, let alone corrected.

This problem has been solved[7] for LDPC codes by the PI in collaboration with Alexey Kovalev. For random uncorrelated (qu)bit errors (*e.g., quantum depolarizing channel*), we established the existence and gave a lower bound for the single (qu)bit error rate below which the decoding with probability one is possible, and analyzed the scaling of successful decoding probability with the block length. The main result is formulated as

Theorem 3 from Ref. 7: *For an infinite family of (j,ℓ)-limited LDPC codes, quantum or classical, where the distance d scales as a power law at large n, asymptotically certain recovery is possible for (qu)bit* depolarizing *probabilities $p < p_d \geq p_1$, where $4p_1(1-p_1) = p_0^2(1-p_0)^{2(z-2)} < [e(z-1)]^{-2}$, $p_1 < 1/2$, and e is the base of the natural logarithm. A threshold $p_d > 0$ also exists for code families with distance scaling logarithmically at large n.*

Here the parameter $z \equiv (\ell-1)j$ characterizes the sparsity of the code, j and ℓ respectively are the maximum weights of a column and a row of the parity-check matrix, in the case of a classical binary code. For a quantum code, j is the maximum number of independent stabilizer generators which can involve a given qubit, and ℓ is the maximum number of qubits which can be involved in a stabilizer generator. Note that this Theorem is also a new result for the much better studied classical LDPC codes.

This result has been obtained by noticing that for an LDPC code likely errors separate into small independent clusters which do not have any stabilizer generators in common. More precisely, a graph $\mathcal{G}$ is defined with the vertices labeled by qubits, and any two vertices connected iff the corresponding qubits participate in the same stabilizer generator. The maximum degree of this graph is z; the clustering property is analogous to the cluster theorem in the theory of phase transitions[32]. We also gave related bounds for fault-tolerant operation in the presence of syndrome measurement errors.

Ref. 7 established QUANTUM LDPC CODES as the first family of quantum error correction codes where a FINITE FAULT-TOLERANT ERROR CORRECTION THRESHOLD is known to COEXIST WITH THE FINITE RATE. Potentially, this could have huge implications for the resources needed for quantum computers of the future.

A similar analysis for errors when the erroneous (qu)bits are known (*erasure channel*) allowed us to establish an upper limit for the achievable rate of a quantum LDPC code with power-law scaling of the distance with block length. These results are important since, unlike for regular QEC codes, there are very few general lower (existence) or upper bounds for quantum LDPC codes[33].

The lower threshold bound in Ref. 7 is related to percolation on certain graph related to the Tanner graph of the corresponding code. In the case of quantum codes related to graphs (e.g., the hyperbolic codes[34]), the percolation threshold is more closely related to the erasure threshold. Therefore, bounds on percolation can shed some light on the decoding properties of the quantum codes. This relation was explored in Ref. 8, which established a previously unknown bound for percolation threshold on infinite graphs.

The successful maximum-likelihood decoding probability of a quantum LDPC code can be related to an average ratio of partition functions of two random-bond Ising models associated with the code. This relation was first pointed in the case of surface codes by Dennis et al.[20]. In the case of general quantum LDPC codes, this relation was explored by the PI in the preprint Ref. 7. Here, for a given single-qubit error probability p, maximum decoding probability is achieved at the temperature which corresponds to the Nishimori line[35]; temperatures away from the Nishimori line correspond to sub-optimal decoding. For a sequence of quantum LDPC codes, an asymtotically decodable region was defined as that where decoding probability converges to one. Main result [7] is that this implies a certain lower bound on the tension of defects associated with the code (such defects are related to codewords). Second, the optimality of the Nishimori line decoding was confirmed using precise inequalities.

B. Construction of quantum LDPC codes

1. *Simplified construction of stabilizer codes using the CWS formalism*

In the paper [2] we explored how the framework of CWS codes can be used to relegate the design of quantum stabilizer codes to classical binary linear codes in order to simplify the overall design. In particular, we formulated several theorems framing the parameters of an additive CWS code which can be obtained from a given graph. We also suggested a simple decomposition of the $\mathbb{F}_4$ generator matrix corresponding to the stabilizer in terms of the graph adjacency matrix and the parity check matrix of the binary code. Finally, we designed several graph families corresponding to regular lattices which result in some particularly good codes. These include graphs with circulant adjacency matrices which can be used to construct single-generator cyclic additive codes, a class of codes overlooked in previous publications. In particular, we proved the existence of single-generator cyclic additive codes with the parameters $[[km, k, m]]$, $k > 10$ and $[[t^2 + (t + 1)^2, 1, 2t + 1]]$ (the sequence of smallest toric codes). Note that these code families have distances that are not bounded, unlike any CWS code families constructed previously[1, 36].

For generic (non-cyclic) codes, the idea is that in order to construct a non-degenerate quantum stabilizer code $\mathcal{Q}$ with the parameters $[[n, k, d]]$, we first choose an order-n simple graph that obeys the following properties.

- The degree r of each vertex is at least $d - 1$.

- For any two vertices with degrees r_1 and r_2, which share r_{12} of their neighbors, the following inequality should be satisfied: $d \geq 2 + r_1 + r_2 - r_{12}$.
- The graph-state distance $d'(\mathcal{G})$ corresponding to the graph is not smaller than d.

The first two properties actually follow from the last one, but they are easier to check.

After a suitable graph is found, we construct a classical linear binary code $\mathcal{C}$ with the parameters $[n, k, d(\mathcal{C})]$, with distance $d(\mathcal{C}) \geq d$, that can correct the binary error patterns induced by the graph. The rule is simple: for a quantum error of the form $E = \prod_i X^{v_i} \prod_j Z^{u_j}$, the corresponding classical error is a vector with binary components $e_i = u_i + \sum_j R_{ij} v_j \bmod 2$, where R_{ij} is the adjacency matrix of the graph.

We justify the technique by proving the Gilbert-Varshamov (GV) bound for a minimum distance d of a quantum code that can be obtained from a specific graph[2]. The value of the bound for given n and k is exactly the same as for all stabilizer codes. While this does not guarantee that a code with record parameters can be found for every graph, it does provide a guarantee of a minimum performance. Generally, if it can be proved that a family of codes satisfies the GV bound, one expects that the family is large enough to include very good codes.

The technique is modified for cyclic codes. Here, one first selects a cyclic binary code $[n, k, d(\mathcal{C})]$, specified by the parity-check polynomial $p(x)$ which must divide $x^n - 1$, of degree $\deg p(x) = k$, and $d(\mathcal{C}) \geq d$. The second step is to choose the polynomial $r(x)$ which actually maps quantum errors to classical: the polynomial $g(x) = p(x)[\omega + r(x)]$ generates the additive $\mathbb{F}_4$ code corresponding to the stabilizer. Here ω is the generator of the field $\mathbb{F}_4$ which satisfies the equation $\omega^2 = \omega + 1$. For CWS cyclic codes, the polynomial $r(x)$ must be symmetric with respect to the free term, $r(x^{n-1}) = r(x) \bmod x^n - 1$. More general condition $p(x)p(x^{n-1})[r(x) + r(x^{n-1})] = 0 \bmod x^n - 1$ defines *single-generator additive cyclic codes*, a family of quantum codes overlooked previously (see Theorem 14 in Ref. 37).

While we were not able to prove a general GV bound for the quantum cyclic codes generated from a given binary cyclic code, we proved such a bound in two special cases corresponding to binary codes with irreducible generator polynomials, $q(x) \equiv (x^n - 1)/p(x)$. Thus, e.g., we were able to prove the existence of additive cyclic codes with the parameters of generalized repetition codes, $[[km, k, m]]$ (the actual proof works for $10 < k = m^s$, with certain m, but empirically there is no difference between these and generic values, as long as $k > 5$.) Another highlight is the discovery of a family of cyclic codes with the parameters $[[t^2 + (t+1)^2, 1, 2t + 1]]$, $t = 1, 2, \ldots$, which are actually the smallest toric codes[20, 26, 37]. This result was rather unexpected since toric (surface) codes are designed on a two-dimensional surface; in this case the surface is generated by wrapping a line around a cylinder in a highly non-trivial pattern.

2. Finite rate quantum LDPC codes

Second approach was designing codes related to quantum Hypergraph codes invented by Tillich and Zemor[4]. Two papers have been published on the subject [5, 6]. Unlike the toric and related surface codes which encode a finite number k of qubits, such codes can encode a finite *fraction* of qubits, k/n, where n is the block length, the total number of qubits directly involved in the code. At the same time these codes admit a convenient two-dimensional qubit layout (see Fig. 1, Left). While the numerical studies are still in progress, it appears that in spite of their finite rates, these codes have thresholds approaching that for the toric code.

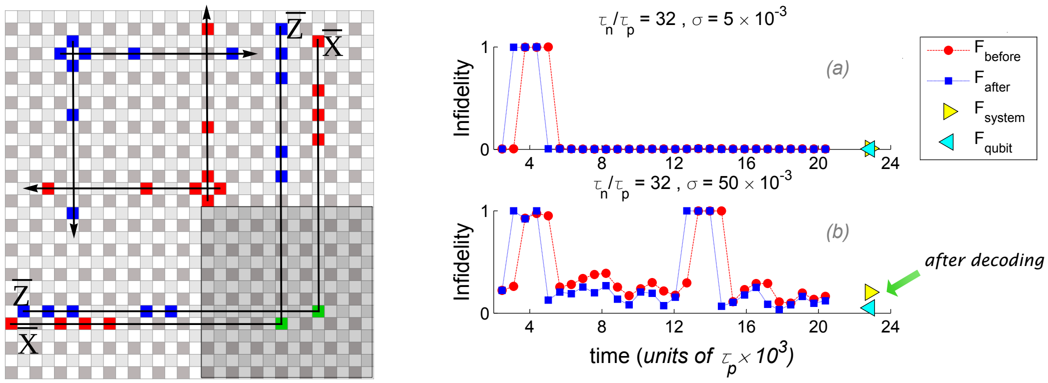

FIG. 1. **Left:** Planar layout of the hypergraph-product code[5] $[[450, 98, 5]]$. Dark and light gray squares---two sublattices of physical qubits; lines: two pairs of anticommuting logical operators (red and blue, respectively, X and Z operators, green---overlap of Z and X operators); arrows: two stabilizer generators. Other stabilizer generators are obtained by shifts over the same sublattice with periodic boundaries; all of these operators need to be measured every error-correcting cycle. Shaded region: each physical qubit uniquely corresponds to a pair of logical operators, thus $k = 98$ encoded logical qubits. Note that the toric code with the same dimensions (not shown) encodes only $k = 2$ qubits but has a larger distance $d = 15$.
Right: Simulated infidelity for a single run of repeated error correction with the $[[5, 1, 3]]$ code using the universal set of gates from Ref. 14. Time axis in units of single-pulse duration, τ_p, starts at the end of the encoding circuit. One ancilla connected via Ising couplings to each of $n = 5$ qubits in a star-like geometry was used, with correlated classical noise as a source of dephasing. Each point is taken either right before or right after one of the stabilizer measurements as indicated. Circles and squares correspond to the infidelity of the encoded state, the corresponding curves shoot up whenever an error is detected (correction is applied after the cycle of four measurements). Two infidelities at the end of the decoding correspond to full-system six-qubit fidelity and the single decoded-qubit fidelity with other qubits traced out. Note that the noise amplitude is quite large: in the absence of decoupling pulses this would require an error correction cycle of $\lesssim 5\tau_p$ for the bottom plot ($5 \times 10^2 \tau_p$ for the top plot), much shorter than $25 \times 10^2 \tau_p$ used.

The construction of new codes is based on an algebraic version[5] of the quantum hypergraph-product (QHP) ansatz introduced by Tillich and Zémor[4]. Namely, a quantum code is constructed from two binary matrices, $\mathcal{H}_1$ (dimensions $r_1 \times n_1$) and $\mathcal{H}_2$ (dimensions $r_2 \times n_2$), as a CSS code with the stabilizer [5]

$$G_x = (E_2 \otimes \mathcal{H}_1, \mathcal{H}_2 \otimes E_1), \qquad G_z = (\mathcal{H}_2^T \otimes \widetilde{E}_1, \widetilde{E}_2 \otimes \mathcal{H}_1^T). \tag{1}$$

Here each matrix is composed of two blocks constructed as Kronecker products (denoted with "$\otimes$"), and E_i and $\widetilde{E}_i$, $i = 1, 2$, are unit matrices of dimensions given by r_i and n_i, respectively. In the original construction[4], given the binary parity check matrix $\mathcal{H}_1 = \mathcal{H}_2^T$

of a classical LDPC code $[n_\mathrm{c}, k_\mathrm{c}, d_\mathrm{c}]$, the QHPC (1) is a CSS code with the parameters $[[n = n_\mathrm{c}^2 + (n_\mathrm{c} - k_\mathrm{c})^2, k = k_\mathrm{c}^2, d = d_\mathrm{c}]]$.

When redundant rows are added to the matrix $\mathcal{H}_1 = \mathcal{H}_2^T$ to make it square, $r_1 = n_1 = n_c$, the parameters of the resulting quantum code are[5] $[[n = 2n_c^2, 2k_c^2, d_c]]$. This is easiest to achieve when the classical code is cyclic, by taking $\mathcal{H}_1$ to be a square circulant matrix corresponding to the parity check polynomial. A representative example is shown in Fig. 1, Left, where the code with parameters $[[450, 98, 5]]$ is obtained from the check polynomial $h(x) = 1 + x + x^3 + x^7$ corresponding to the binary code $[15, 7, 5]$. Similar to the toric code, the qubits are mapped to the bonds of a two-dimensional square lattice, and stabilizer generators are obtained by shifting the same patterns along the lattice, with periodic boundary conditions. The *hyperbicycle* codes suggested in Ref. 6 have the same local structure except with changed lattice periodicity. This modifies the code parameters (block length n, the number of encoded qubits, and the distance d), but the upper and the lower bounds on the distance both scale as a square root of the block length, $d \propto \sqrt{n}$. This shows that the constructed codes have a finite fault-tolerant error correction threshold[7].

Compared to the original hypergraph-product construction[4], the main advantages of the new code families is the added flexibility of parameters and a somewhat higher rate. In addition, thus constructed variants of the same code could be used to construct fault-tolerant operations by code deformations.

C. Error clustering for quantum LDPC codes

The third direction of research in quantum LDPC codes was to study the consequences of the clustering of a likely error in such codes noticed in Ref. 7. In particular, we suggested in Ref. 7, that the error clustering could be used to construct a new polynomial-time algorithm for syndrome-based decoding, an extremely important problem for quantum LDPC codes[38]. We have constructed and implemented a version of cluster-based syndrome decoding algorithm which goes over all small clusters of qubits connected to non-zero syndrome bits, with subsequent minimal-weight matching using Knuth's `DLX` algorithm (this technique is an extension of the minimal-weight matching for the toric and related codes[20, 21]). The technique has the complexity scaling as $n \log n$, and is very fast for small $p \lesssim (2z)^{-1}$ where few-qubit clusters are dominant. Unfortunately, with p increasing toward the error-correction threshold, larger clusters are needed and the overall complexity rapidly increases.

Another problem related to syndrome-based decoding is that of finding the distance of a code. In the paper[10] presented at the 2013 Information Theory and Applications (ITA) workshop in San Diego, we suggested a novel classical algorithm for finding the distance of a quantum LDPC code. Again, the idea of the algorithm is to look for a possible minimum-weight codeword only among those forming connected clusters. We also review the existing algorithms for finding the minimal distance of a code and show that the algorithm constructed in Ref. 10 is exponentially faster in a certain range of parameters (all known algorithms have complexity that scales exponentially with the block length.)

This technique was further analyzed in the subsequent conference paper [11]. Here, a number of generic distance finding algorithms for binary codes have been analyzed in application to quantum LDPC codes. While the cluster-based algorithm is near-optimal with high-rate codes, the random-window (RW) algorithm has been identified as most efficient so far. While this latter algorithm has exponential complexity, it works very fast (polynomial complexity) to filter out ‘‘bad’’ codes which contain small-distance codewords. This feature may be useful for further studies searching for quantum LDPC with certain properties.

III. MULTI-QUBIT GATES BASED ON DECOUPLING PULSE SEQUENCES

Building a quantum computer with hundreds or thousands of qubits with gates concurrently operating at such an accuracy is a great physics and engineering challenge. It is pursued by a number of groups, using different physical systems for implementing qubits. However, the corresponding control algorithms need not necessarily be developed from scratch, since different physical systems may share some key properties.

In particular, qubits with always-on couplings are a natural model for several potential quantum computer (QC) architectures such as the original Kane proposal, nitrogen vacancy centers in diamond, superconducting phase qubits , and circuit QED lattices.When compared to their counterparts with tunable couplings, qubits with always-on couplings can be expected to have better parameter stability and longer coherence times. In addition there is also much to be benefited from over sixty years of development in nuclear magnetic resonance (NMR) which has resulted in an amazing degree of control available to such systems[39, 40].

Related coherent control techniques based on carefully designed pulse sequences used to selectively decouple parts of the system Hamiltonian have been further developed in application to quantum computation[41--47]. While NMR quantum computation is not easily scalable[48], it still holds several records for the number of coherently controlled qubits[40]. However, some of these records have been achieved with the help of *strongly-modulated* pulses, computer-generated single- and multi-qubit gates tailored for a particular system Hamiltonian[49--52]. While such gates can be used in other QC architectures[53], they may violate scalability.

On the other hand, NMR-inspired techniques like dynamical decoupling (DD) can also be used to control large systems with local interactions, where pulses and sequences intended for a large system can be designed to a given order in the Magnus series[54] on small qubit clusters[12]. DD is also excellent in producing accurate control for systems where not all interactions are known as one can decouple interactions with the given symmetry[55, 56]. Moreover, DD works best against errors coming from low-frequency bath degrees of freedom which tend to dominate the decoherence rates, and it does not require additional qubits. In short, DD is an excellent choice for the first level of coherence protection; it’s use could greatly reduce the required repetition rate of the QEC cycle.

This is well recognized in the research community, and applications of DD for quantum computation are actively investigated by a number of groups. However, most publications on the subject illustrate general principles using just a single qubit as an example, leaving out the issues of design and simulation of scalable approaches to multi-qubit dynamical decoupling. While the techniques for larger systems exist, they typically require longer decoupling sequences[13, 45, 55].

The goal of this part of the project is to provide a scalable benchmark implementation of a universal set of accurate gates using soft pulses for a system with always-on qubit couplings.

Specifically, we constructed[14] gates with built-in DD-protection against low-frequency phase noise for a set of qubits with bipartite Ising coupling (where qubits in partition A are only coupled to the qubits in partition B), like a chain or square lattice of qubits with the nearest-neighbor (n.n.) couplings. The constructed gates use finite-amplitude *shaped pulses* which can be implemented experimentally, are *scalable*, in the sense that the same construction works for an arbitrary bipartite graph, and they can also be executed *in parallel* for different qubits and/or qubit pairs, which is one of the necessary requirements for building a useable (large) quantum computer. The couplings need not be the same for parallel operation[15]. The gates are accurate to second order in the Magnus expansion, meaning that in the limit of very slow (classical) bath the infidelity scales as sixths or higher power of the system-bath coupling in units of inverse pulse duration.

We demonstrated the accuracy of the constructed gates by simulating the repeated Zeno effect for the $[[4, 2, 2]]$ error-detecting toric code, implemented on a five-qubit chain[14]. We have also analyzed the error operators associated with the constructed gates[15]. Namely we first analyzed the errors analytically up to a cubic order in the Magnus expansion. We further studied these errors numerically by explicitly integrating the Schrödinger equation for time evolution of clusters of up to six qubits, and gave a bound on high-order errors for qubits on a large square lattice. Using this bound, we analytically proved that with large enough toric code the present gate set can be used to implement a fault-tolerant quantum memory[15].

A. Pulse sequence design

Basic idea is that during the operation of the QC with always-on couplings, two periodic sequences are constantly executed on the two sublattices, e.g., with total period 16τ:

$$\begin{aligned} A &: \mathtt{X} - \mathtt{X} - \mathtt{X} - \mathtt{X} - -\mathtt{X} - \mathtt{X} - \mathtt{X} - \mathtt{X}, \\ B &: \ -\mathtt{X} - \mathtt{X} - \mathtt{X} - \mathtt{X}\,\mathtt{X} - \mathtt{X} - \mathtt{X} - \mathtt{X}-, \end{aligned}$$

where X represents a π pulse and $-$ a null pulse (no pulse) on the particular sublattice. When the pulses are second-order self-refocusing pulses[12], these sequences suppress the Ising couplings between the sublattices to second order in the Magnus expansion, meaning that the error in the unitary matrix scales with $(\tau J)^3$, wher τ is the pulse duration, and the corresponding infidelity scales as $(\tau J)^6$.

When we need to turn on the coupling between two neighboring qubits, the sequences on these qubits are replaced by

$$\begin{aligned} A' &: \mathtt{X} - - - \mathtt{X} - - - - - - - \mathtt{X} - - - \mathtt{X}, \\ B' &: -\mathtt{X} - - - \mathtt{X} \ - - - - \mathtt{X} \ - - - \mathtt{X} -, \end{aligned}$$

so that the Ising coupling between these qubits is only decoupled half of the time, while the coupling to other qubits continues to be removed. As a result, if the system Hamiltonian is $H = \frac{1}{2} J \sum_{\langle ij \rangle} \sigma_i^z \sigma_j^z$, the effective evolution operator becomes $U = \exp(-iJ_z \sigma_a^z \sigma_b^z t/4)$, where $t = 16\tau$ is total duration of the sequence. Repeating the sequence, we can generate an

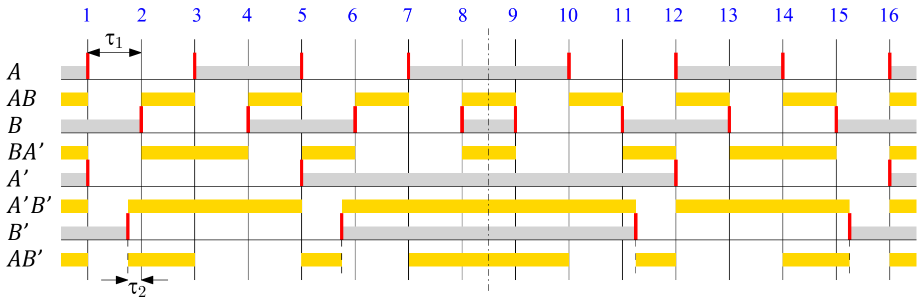

FIG. 2. (Color online) Schematic design of the ZZ-rotation gate on a bipartite Ising network using δ-pulses. Pulses are indicated with vertical red lines (all of them are π pulses around the x-axis). Sequences A and B are applied on idle qubits of the two sublattices. The regions shaded in gray correspond to time intervals where the signs of σ^z on the corresponding sublattice is not inverted, while yellow shading along the intermediate line labeled AB represents the sign of the coupling term $\sigma^z \otimes \sigma^z$. All of these occupy exactly half of the total cycle duration, indicating that the corresponding leading-order average Hamiltonians are all zero. The lines labeled A' and B' correspond to a pair of qubits to be coupled. They are decoupled both from the on-site noise and from the neighboring dual-sublattice qubits as can be seen from the shading along lines labeled A', B', AB', and BA'. On the other hand, the mutual coupling (line $A'B'$) does not average to zero, which allows for the controlled-phase gate to be constructed.

arbitrary two-qubit phase rotation. The version of these sequences which allows for unequal inter-qubit couplings is schematically shown in Fig. **??**.

With addition of the single-qubit gates, we have a universal set[57]. Indeed, e.g., the CNOT gate can be constructed using the identity [58]:

$$\texttt{CNOT} = e^{i\pi/4} Y_1 X_2 \bar{X}_1 \bar{Y}_1 \bar{Y}_2 e^{-i\pi/4\sigma_1^z\sigma_2^z} Y_2, \qquad (2)$$

where $X_i \equiv \exp\left(-i\frac{\pi}{4}\sigma_i^x\right)$, $Y_i \equiv \exp\left(-i\frac{\pi}{4}\sigma_i^y\right)$, are $\pi/2$ pulses, and $\bar{X}_i$ denotes the corresponding conjugate gates. The complete pulse sequence used to implement a CNOT gate is shown in Fig. 3.

Such sequencs can be run simultaneously on many pairs of qubits as long as qubits from different pairs are not mutually coupled.

A single-qubit rotations are implemented using the dynamically corrected gates (DCGs) invented by Khodjasteh et al.[13, 59, 60] In essence, the original DCG construction[13] is a generalization of the Eulerian path DD-sequence-generation technique[44] which allows for a construction of composite pulses accurate to leading order in the Magnus expansion. We use this technique to implement arbitrary single-qubit rotations; again, on a bipartite lattice these gates can be executed in parallel on any non-neighboring qubits.

For example, to implement the toric code[20, 23], one can use square lattice with n.n. Ising couplings between the qubits, with one sublattice used for qubits, and the other for ancillae. More generally, to implement an arbitrary LDPC code, one can use the network of couplings between the qubits and the ancillae form the corresponding Tanner graph. In particular, for hypergraph-product and related codes[4, 5] one can use the square lattice layout with additional connections, see code layout in Fig. 1, Left.

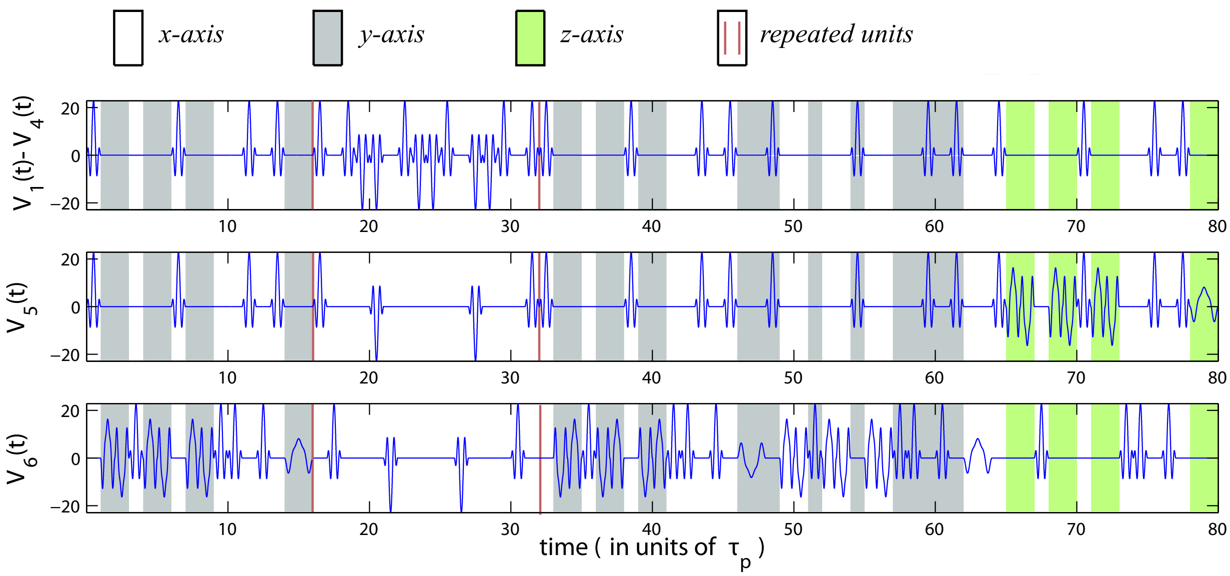

FIG. 3. (Color online) Pulse sequences used to implement the CNOT gate between qubits Q_5 and Q_6 on a star graph. It is a combination of four DCG gates and a ZZ-coupling sequence. Second-order self-refocusing pulse shapes $Q_1(\pi)$ and $Q_1(\pi/2)$ from Refs. 12 are used. Shading shows the direction of the applied pulses as indicated. The unit surrounded by vertical red lines, $16\tau_p \leq t \leq 32\tau_p$, should be repeated N_{rep} times, for the total sequence duration $16(N_{\text{rep}} + 4)\tau_p$.

B. Analysis of error operators

In Ref. 15, we have carefully analyzed the errors associated with the universal gate set based on soft-pulse dynamical decoupling sequences. When used with second-order NMR-style self-refocusing pulses, the constructed sequences eliminate the inter-qubit couplings to second order, and in addition decouple time-independent on-site Ising terms (chemical shifts) also to second order. Fluctuating Ising term (low-frequency phase noise) is decoupled to linear order; second order decoupling of such terms can also be achieved using a symmetrized version of the same construction.

The basic two-qubit gate is an arbitrary-angle ZZ-rotation; it can be viewed as a continuous family of doubled Eulerian sequences[44] which allow flexibility of the effective coupling: same average rotation rate can be achieved for qubit pairs with differing Ising couplings. These gates can also be executed in parallel on an arbitrary number of qubit pairs with the restriction that qubits from different pairs cannot be directly connected to each other. In addition to providing controlled removal of unwanted Ising couplings to quadratic order (when used with second-order NMR-style self-refocusing pulses), these sequences also decouple low-frequency phase noise to the same order.

We characterized the accuracy of the constructed gates in few-qubit systems using an extension of the analytical average-Hamiltonian expansion[12], and also numerically by integrating full quantum dynamics of clusters of up to six qubits in the presence of control pulses, coupling Hamiltonian, and additional on-site Ising terms. These simulations confirmed that the gates are working as designed, with the systematic portion of the average infidelity of a CNOT gate as small as 10^{-11} on a chain and 10^{-8} on an $n = 6$ star graph with $N_{\text{rep}} = 5$ repetitions of the basic sequence.

We also went beyond the fidelity and analyzed the weight distribution of systematic errors generated by our sequences. It turned out that single- and two-qubit errors are relatively suppressed, while errors of larger weights dominate the evolution. Such an error distribution is expected in any control scheme based on perturbation theory.

Scalable quantum computation being the primary target of the present construction, we also analyzed the error patterns that would be expected when this or similarly constructed gate sets are used in a large system. It turned out that for sequences suppressing the inter-qubit couplings to order K, when the couplings are small compared to the inverse sequence duration, dominant errors are formed by clusters involving up to $K+1$ bonds (up to $K+2$ qubits). While such clusters can sometimes merge forming larger-weight errors, we show that one can choose the parameters so that large error clusters do not form during a measurement cycle that involves several CNOT and single-qubit gates. We analyzed specifically the measurement cycle of the toric code and the corresponding planar layout of qubits and ancillae, and demonstrated that fault tolerant quantum memory can indeed be implemented using our gate set.

We also mentioned that the obtained exponential bound for the amplitude of a large error clusters is also compatible with the threshold analysis for concatenated codes with noise that involves long-range temporal and spatial correlations[61, 62]. Fault-tolerance with a concatenated code using the present gate set can be demonstrated by choosing a suitable qubit network, e.g., a linear qubit chain[63--65].

C. Network design principles

The most important parameter that governs the likelihood of a run-away large-weight error formation is the sparsity of the coupling network. It can be characterized by the maximum degree z of the corresponding graph. On a chain with $z = 2$, there are only $s + 1$ clusters with s bonds involving a given qubit; with $z > 2$, the cluster number grows exponentially with s. This growth has to be overcome by the small expansion parameter $\alpha \equiv J\tau_{\text{seq}}$: the amplitude of an error cluster involving s bonds scales as α^s.

On the other hand, when a large number of qubits are coupled to a single qubit or other quantum system like a harmonic oscillator, it would be much more difficult to control the run-away large weight error formation. We believe this applies not only to the present gate set based on decoupling sequences, but generally to any kind of control scheme where perturbation theory is used, e.g., controlled coupling schemes based on tuning qubits in and out of resonance.

IV. CONCLUSIONS

The supported research substantially advanced our understanding of the quantum LDPC codes and possibilities of their implementation. Key results include the proof of the existence of finite fault-tolerant error correction threshold for quantum LDPC codes[7], new constructions of quantum LDPC codes with simple planar qubit layouts[5, 6], an implementation of a universal gate set based on decoupling sequences[14], and the proof that these gates can be used to implement fault-tolerant quantum memory using the toric code[15].

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