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14. ABSTRACT
The research objective of this proposal is on analyzing long time asymptotics, control and statistical inference problems for wide family of stochastic network systems to address fundamental issues arising from data-intensive network applications. Our methodology is based on weak convergence approximations for the network systems critical loading, stochastic simulations, applying statistical inference methods stochastic differential equations to the limit approximating models. The focus of the research will be on a class of stochastic networks that exhibit the non-Markovian characteristics of data traffic from, e.g., long range dependence, self similarity and/or heavy tail

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Report Title

Final Report: Probabilistic and Statistical Analysis of Complex Stochastic Networks (Research Topic Area: Probability and Statistics)

ABSTRACT

The research objective of this proposal is on analyzing long time asymptotics, control and statistical inference problems for wide family of stochastic network systems to address fundamental issues arising from data-intensive network applications. Our methodology is based on weak convergence approximations for the network systems critical loading, stochastic simulations, applying statistical inference methods stochastic differential equations to the limit approximating models. The focus of the research will be on a class of stochastic networks that exhibit the non-Markovian characteristics of data traffic from, e.g., long range dependence, self similarity and/or heavy tail phenomena. Furthermore, in order to broaden the applicability of the problems considered past, we intend to study more general state-dependent network models.

Enter List of papers submitted or published that acknowledge ARO support from the start of the project to the date of this printing. List the papers, including journal references, in the following categories:

(a) Papers published in peer-reviewed journals (N/A for none)

Received		Paper
06/02/2017	14	Libs Sun, Chihoon Lee, Jennifer Hoeting. A penalized simulated maximum likelihood method to estimate parameters for SDEs with measurement error, Computational Statistics, ( ): . doi: 1,016,392.00
06/02/2017	20	S. Chen, S. Lenhart, J.D. Day, C. Lee, M. Dulin, and C. Lanzas. Pathogen transfer through environment-host contact: an agent-based queueing theoretic frame- work, Mathematical Medicine and Biology, ( ): . doi: 1,042,976.00
06/02/2017	19	Zhenyu Cui, Kai Wang, Chihoon Lee, Yanchu Liu. Uniqueness of Equilibrium in the Financial System under Partial Multilateral Netting, Operations Research Letters, ( ): . doi: 1,042,963.00
06/02/2017	16	Hamed Ghoddusi, Yiying Cheng, Chihoon Lee, Yaozhong Hu. The Expected Hitting Time Approach to Optimal Price Adjustment Problems, European Journal of Operational Research, ( ): . doi: 1,042,920.00
06/02/2017	18	Chihoon Lee, Amy Ward. Pricing and Capacity Sizing of a Service Facility: Customer Abandonment Effects, Production and Operations Management, ( ): . doi: 1,042,948.00
06/02/2017	17	Zheng Han, Yaozhong Hu, Chihoon Lee. On the pricing barrier control in a regime-switching regulated market, Quantitative Finance, ( ): . doi: 1,042,942.00
06/02/2017	15	Zhenyu Cui, Chihoon Lee, Yanchu Liu. Single Transform Formulas for Pricing Asian Options in a General Approximation Framework under Markov Processes, European Journal of Operational Research, ( ): . doi: 1,042,770.00
08/14/2015	1	Chihoon Lee, Anatolii A. Puhalskii. Non-Markovian State-Dependent Networks in Critical Loading, Stochastic Models, (02 2015): 0. doi: 10.1080/15326349.2014.964414 362,876.00
08/14/2015	2	Chihoon Lee, Yaozhong Hu, Myung Hee Lee, Jian Song. Parameter estimation for reflected Ornstein–Uhlenbeck processes with discrete observations, Statistical Inference for Stochastic Processes, (12 2015): 0. doi: 10.1007/s11203-014-9112-7 362,877.00
08/14/2015	3	Chihoon Lee, Jennifer A. Hoeting, Libo Sun. A penalized simulated maximum likelihood approach in parameter estimation for stochastic differential equations, Computational Statistics & Data Analysis, (04 2015): 0. doi: 10.1016/j.csda.2014.11.007 362,878.00
08/14/2015	5	Zheng Han, Yaozhong Hu, Chihoon Lee. Optimal pricing barriers in a regulated market using reflected diffusion processes, Quantitative Finance, (06 2015): 0. doi: 10.1080/14697688.2015.1034163 362,882.00
08/14/2015	6	Libo Sun, Chihoon Lee, Jennifer A. Hoeting. Parameter inference and model selection in deterministic and stochastic dynamical models via approximate Bayesian computation: modeling a wildlife epidemic, Environmetrics, (08 2015): 0. doi: 10.1002/env.2353 362,883.00
08/14/2015	9	Chihoon Lee, Amy R. Ward. Optimal pricing and capacity sizing for the GI/GI/1 queue, Operations Research Letters, (12 2014): 0. doi: 10.1016/j.orl.2014.09.005 362,886.00

08/30/2016	12	Susan Wei, Chihoon Lee, Lindsay Wichers, J. S. Marron. Direction-Projection-Permutation for High-Dimensional Hypothesis Tests, Journal of Computational and Graphical Statistics, ( ): 549. doi: 1,016,386.00

TOTAL: **14**

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Received	Paper

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(d) Manuscripts

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08/29/2016	8.00	Chihoon Lee, Jian Song. On drift parameter estimation for reflected fractional Ornstein-Uhlenbeck processes., Stochastics (09 2014)

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2015 Summer Invited Researcher, Statistical Research Institute (SRI), Seoul National University, Korea

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2014 Short-term Visiting Fellow, National Institute for Mathematical and Biological Synthesis (NIMBioS), TN

Graduate Students

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FTE Equivalent:
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FTE Equivalent:
Total Number:

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NAME	PERCENT_SUPPORTED

FTE Equivalent:
Total Number:

Names of Under Graduate students supported

NAME	PERCENT_SUPPORTED
FTE Equivalent:	
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Student Metrics

This section only applies to graduating undergraduates supported by this agreement in this reporting period

The number of undergraduates funded by this agreement who graduated during this period: 0.00

The number of undergraduates funded by this agreement who graduated during this period with a degree in science, mathematics, engineering, or technology fields:...... 0.00

The number of undergraduates funded by your agreement who graduated during this period and will continue to pursue a graduate or Ph.D. degree in science, mathematics, engineering, or technology fields:...... 0.00

Number of graduating undergraduates who achieved a 3.5 GPA to 4.0 (4.0 max scale):...... 0.00

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NAME
Total Number:

Names of personnel receiving PHDs

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Names of other research staff

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Sub Contractors (DD882)

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Scientific Progress

Technology Transfer

Final Report of "Probabilistic and Statistical Analysis of Complex Stochastic Networks" supported by the Army Research Office[1]

Period: 05/15/2014–05/14/2017

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1 Statement of the research problem studied

The main research objective is on analyzing long time asymptotics, control and statistical inference problems for a wide family of stochastic queueing network systems to address fundamental issues arising from data-intensive network applications.

More precisely, the key goals of the proposed research are:

(A) to establish weak convergence limit theorems for various performance related processes of the networks in critically loading non-Markovian networks, akin to standard diffusion approximation results available for Markovian networks;

(B) to study a statistical inference theory for constrained stochastic processes and to investigate precise mathematical results that connect the asymptotic properties of the estimators with those of underlying stochastic networks. To narrow down our focus, we study a drift parameter estimation problem for reflected *fractional* Ornstein-Uhlenbeck processes, which are relevant to a wide class of non-Markovian stochastic networks.

(C) to propose and study a penalized simulated maximum likelihood in parameter estimation for stochastic differential equations (SDEs) with or without measurement errors.

A distinguishing feature of the proposed research is the interplay between probability (e.g., stochastic analysis, weak convergence theory) and statistics (e.g., parameter estimation, sequential analysis in diffusion processes). Constrained limit stochastic processes obtained using heavy traffic approximations of stochastic networks give parsimonious models that are much more tractable for statistical inference. In general, estimation of unknown network characteristics, such as arrival and processing rates, customer abandonment rates, coefficients of variations, routing probabilities, using typical time series measurements on buffer networks and idleness, is a challenging task. For example, many important aspects of performance of a network such as traffic intensities may not be directly observable and therefore such performance measures and their related model parameters need be statistically inferred from the available, often partially observed, data.

[1] Agreement Number W911NF-14-10216 and Proposal Number 65380-MA
[2] Research performed at CSU during 05/15/2014–08/15/2015
[3] Research performed at Stevens during 08/16/2015–05/14/2017

2 Summary of the most important results

We describe our main research findings related to the research goals (A)–(C) stated above. The cited references are listed in Section 4.

2.1 Weak limit approximations of non-Markovian state-dependent networks (related to (A))

In Lee and Puhalskii (2015), we consider a stochastic network system where the arrival and service processes are time-changed “primitive” processes. The time-change is given by the integral of the queue-length (or, waiting times) dependent rate and the underlying primitive processes (e.g., arrival, service and routing processes) are not required to be Markovian. To ensure the existence and uniqueness of the solution to the equations governing the network, we impose suitable sub-linear growth condition on the total arrival rate to the network and some asymptotic bounds on the network primitives to guarantee that the sample paths of the queue-length processes are nonexplosive.

Our main results are in the form of limit theorems under critical loading regime: We consider a sequence of networks with similar structure indexed by $n \in \mathbb{N}$ and the critical loading condition is assumed to hold in the limit as $n \to \infty$. Our main result states that:

> Lee and Puhalskii (2015): If the network primitives satisfy invariance principles with continuous-path limits (not necessarily Markov processes), then the multi-dimensional queue-length processes (or other performance related processes such as the waiting times and the completed works, etc.), when suitably scaled and normalized, converge to a constrained continuous-path process on some polyhedral cones (e.g., non-negative orthant).

As a consequence we establish the following corollaries:

(i) If the scaled limits of the primitives are diffusion processes, the limit stochastic process is a reflected diffusion with a state-dependent drift and diffusion coefficients.

(ii) One can consider independent sequences of weakly dependent random variables; under suitable moment and mixing conditions which imply the invariance principle, our limit theorem results continue to hold.

(iii) Similar limit theorems will hold for a class of fluid queueing network models fed by a large number of heavy-tailed ON/OFF sources in heavy traffic, in which case a multi-dimensional reflected fractional Brownian motion process on the positive orthant, with nonzero drift and Hurst parameter $H \in (1/2, 1)$, appears as the approximating limit process.

The main contribution of this work is incorporating *general* arrival and service processes. This is achieved by applying an approach different from the one used by the papers in the past. The proofs of previous results rely heavily on the martingale weak convergence theory. They are quite involved, on the one hand, and do not seem to be easily extendable to more general arrival and service processes, on the other hand. In our approach, we, in a certain sense, return to the basics and employ ideas that have proved their worth in the setup of generalized Jackson networks. We show that continuity considerations may produce stronger conclusions at less complexity.

2.2 Statistical inference for constrained stochastic processes (related to (B))

2.2.1 Drift parameter estimation for reflected fractional Ornstein-Uhlenbeck process

In Lee and Song (2016), we consider the following one-dimensional reflected fractional Ornstein-Uhlenbeck (RFOU) process $\{W_t\}_{t\geq 0}$ on the positive real line:

$$\left.\begin{array}{c} dW_t = -\gamma W_t dt + \sigma dB_t^H + dL_t \,, \\ W_t \geq 0 \quad \text{for all } t \geq 0 \,, \end{array}\right\}$$

where $\gamma \in (0,\infty)$, $\sigma \in (0,\infty)$, $\{L_t\}_{t\geq 0}$ is a cumulative local time of the state process at zero, and $B^H = (B_t^H)_{t\geq 0}$ is a one-dimensional fractional Brownian motion with the Hurst parameter $H \in (0, \frac{1}{2}) \cup (\frac{1}{2}, 1)$.

For example, the above reflected fractional Ornstein-Uhlenbeck process arises as the key approximating process for queueing systems with reneging or balking customers (with long range dependent arrival process). Notice this step can be rigorously justified from the weak convergence results in previous Section 2.1. Then, the drift parameter γ carries the physical meaning of customers' reneging (or, balking) rate from the system.

Our main results are concerned with estimating the unknown drift parameter $\gamma \in (0,\infty)$ based on observations of the state process $\{W_t\}_{t\geq 0}$. We summarize our main findings.

> Lee and Song (2016): Assume the state process $\{W_t\}$ is observed until the observed Fisher information of the process exceeds a predetermined level of precision h, i.e., we observe $\{W_t\}$ over the random time interval $[0, \tau^H(h)]$. Then we can construct a sequential estimation scheme and prove that the sequential estimation plan has the following desirable properties:
>
> (a) it is unbiased; (b) the estimation plan is closed, i.e., the time of the observation $\tau^H(h)$ is finite with probability 1; (c) its mean squared error is a constant that does not depend on the parameter γ to be estimated.

Such results would be of ample use in applications to several areas such as engineering, financial and biological modeling where unknown parameter estimation is based on relatively shorter time observation, which commonly arises in practical situations.

Our technical approach is based on establishing uniform exponential moment estimates of the above reflected fractional Ornstein-Uhlenbeck process, which, in conjunction with certain integral representations and the fundamental martingales of fractional Brownian motions, leads to two types of fractional Girsanov formulas. Then, we obtain the standard MLE and prove its strong consistency and asymptotic normality. Furthermore, we derive the explicit expression for the sequential MLE and show that it is unbiased, uniformly normally distributed (over the entire parameter space which is the real line), and efficient in the mean square error sense.

2.2.2 Reflected Ornstein-Uhlenbeck process based on discrete observations

In Hu et al. (2015), we consider the parameter estimation problem for the reflected Ornstein-Uhlenbeck processes based on *discrete time* observations. More precisely, our main results states that:

> Hu et al. (2015), Theorem: For a fixed time interval $h > 0$, assume that the discrete time process $\{W_{kh} : 1 \leq k \leq n\}$ are observable. The following simple formula for the stationary moment of the reflected Ornstein-Uhlenbeck process ($H = 1/2$) can be obtained: For all $\alpha > -1$, $\alpha \neq 0$,
>
> $$\mathbb{E}|W(\infty)|^{\alpha} = \Gamma\left(\frac{\alpha+1}{2}\right)\frac{\sigma^{\alpha}}{\sqrt{\pi}}\gamma^{-\alpha/2},$$
>
> where $\Gamma(\cdot)$ denotes the Gamma function. Hence, a natural method of moment estimator of γ is given by
>
> $$\hat{\gamma}_{\alpha,n} := \frac{\left(\Gamma((\alpha+1)/2)\right)^{2/\alpha}\sigma^2}{\pi^{1/\alpha}}\left(\frac{1}{\frac{1}{n}\sum_{k=1}^{n}|W(kh)|^{\alpha}}\right)^{2/\alpha}$$
>
> for an arbitrary $h > 0$ and any $\alpha > -1$, $\alpha \neq 0$. *Strong consistency and asymptotic normality* of the method of moment estimator are established.

We assume that only the state process itself (not the local time process) is observable and the observations are made only at discrete time instants. Our approach is of the method of moments type and is based on the explicit form of the invariant density of the process. The method is valid irrespective of the length of the time intervals between consecutive observations. Currently, we are working on expanding the above-described results to statistical hypothesis testing problems, which can be employed to capture some particular features buried in the networks and, in particular, anomaly detections of underlying stochastic networks.

2.3 A penalized simulated maximum likelihood approach in parameter estimation for SDEs (related to (C))

2.3.1 Penalized importance sampling for parameter estimation in SDEs

From an inferential viewpoint, practitioners must contend with two major challenges when inferring parameters from the data: (i) in the multivariate state space, some state variables are completely unobserved; (ii) observed data are quite sparse over time. Bayesian approaches for parameter estimation for stochastic differential equations (SDEs) models may suffer a slow rate of convergence when the data are sparsely sampled over time and also as the dimension of SDE models increases. On the other hand, frequentist approaches generally have certain restrictions as well that may limit their application, such as assuming state-independent diffusion coefficients or that the data are observed without error. The penalized simulated maximum likelihood (PSML) approach in parameter estimation for SDE has been proposed in a paper Sun et al. (2015a) to overcome some of the aforementioned limitations.

More precisely, in Sun et al. (2015a), we consider the problem of estimating parameters of multi-dimensional SDEs with discrete-time observations that are either completely or partially observed. The transition density between two observations is generally unknown or intractable in most of practical situations.

> In Sun et al. (2015a), we propose an importance sampling approach with an auxiliary parameter (a penalty term) which improves approximation of the transition density. The penalty term we add to the log likelihood is a constraint on selecting the importance sampler. Such a penalized maximum likelihood framework is shown to produce more accurate and efficient parameter estimates. Simulation studies in three different models illustrate promising improvements of the new penalized simulated maximum likelihood method. The new procedure is designed for the challenging case when some state variables are unobserved and moreover, observed states are sparse over time.

2.3.2 Parameter inference and model selection problems

In the study of a engineering, ecological, or environmental dynamical process, the choice of underlying dynamical model (also known as the process model) is usually based upon expert knowledge or non-generalizable, ad hoc preference. Moreover, it is often the case that parameters of the model are not estimated using statistical functions of observed data. The objectives of the paper Sun et al. (2015b) are as follows: (i) to investigate a systematic statistical approach to select a process model that is consistent with the observed data and (ii) to produce parameter estimates and quantify associated uncertainties based on the observed data.

We undertake these goals under a hierarchical model framework and demonstrate our approach is effective. In many contexts, model selection is typically performed via a likelihood ratio test (LRT), the Akaike information criterion (AIC), or the Bayesian information criterion (BIC). However, such approaches are not suitable for the dynamical models that are often used in engineering and biology because the likelihood is intractable. Approximate Bayesian computation (ABC) is a methodology to estimate the model parameters when the likelihood is difficult to compute. A simulation-based procedure and a distance function between simulated data and the observed data are used instead of the likelihood in ABC. In Sun et al. (2015b), we incorporate the ABC sequential Monte Carlo algorithm into a hierarchical model framework and perform parameter estimation (with credible intervals) and dynamical model selection among a set of ODEs, SDEs, and CTMCs.

> In Sun et al. (2015b), the main novel contribution is that under a hierarchical model framework we compare three types of dynamical models: ordinary differential equation (ODE), continuous time Markov chain (CTMC), and stochastic differential equation (SDE) models. To our knowledge model selection between these types of models has not appeared previously. Since the practice of incorporating dynamical models into data models is becoming more common, the proposed approach may be very useful in a variety of applications.

2.3.3 A penalized simulated maximum likelihood in parameter estimation for SDEs with measurement error

Statistical inference for stochastic differential equation models is a challenging problem. The likelihood for the model parameters is often unknown due to discrete and partial observations. Moreover, measurement error can cause difficulties in estimating the parameters of the stochastic differential equation models.

As with many other SDE parameter estimation methods, the penalized simulated maximum likelihood (PSML) approach of Sun et al. (2015a) assumes that the data are observed without measurement error and thus known initial conditions. However, engineering data, especially sampled from complex stochastic networks, are typically contaminated by measurement error. Ignoring these errors could have substantial effects on the parameter estimates and lead to erroneous conclusions.

> Sun et al. (2017) extend the PSML framework by incorporating modified importance samplers to allow for measurement error, including unknown initial conditions, and provide a theoretical study of the asymptotic properties of the estimators obtained by the PSML approach of Sun et al. (2015a). Simulation studies for two canonical stochastic models show our method has favorable performance as compared to existing samplers. Furthermore, we establish consistency and asymptotic normality for the proposed estimators.

3 Collaborative research activities on stochastic networks with applications to biological networks

The PI spent a portion of Spring 2015 at SAMSI (Statistical and Applied Mathematical Sciences Institute) in NC as a research fellow. During the visit, the PI was involved with a working group on the topics of "Ecology of Infectious Diseases." In particular, with Dr. Cristina Lanzas at North Carolina State University and Dr. Shi Chen at UNC-Charlotte, we applied stochastic network theory to the dynamics of infectious diseases and obtained the following research outcomes.

The basic reproduction number (R_0) was estimated using a Bayesian framework. In addition, correlations between external covariates (e.g., location, ambient temperature, dietary, and probiotic usage) and prevalence/R_0 were quantified. We then couple a compartmental model to reconstruct the infection dynamics of these serotypes and quantify their risk in the population. We incorporate different sensitivity levels of detecting different serotypes and evaluate their potential influence on the estimation of basic reproduction numbers. Our modeling and analysis of this system can be readily expanded to other pathogen systems in order to estimate the pathogen and external factors that influence spread of infectious agents. A paper, Chen et al. (2016), based on this collaboration has been recently published in *Applied and Environmental Microbiology.*

Environment-hosts pathogen transfer quantification: An agent-based queueing theoretic framework

Queueing theory studies the properties of waiting queues and has been applied to investigate direct host-to-host transmitted disease dynamics. However, little has been studied for its potential usage in modeling environmentally transmitted pathogens. Currently, we explore a queueing theory modeling framework to study the in-hospital contact process between environments and hosts and potential pathogen transfer, where environments are modeled as servers, and hosts are treated as customers. Two types of servers (short and long, with the same server utilization) are investigated. We also consider various forms of transfer functions that map contact duration to the amount of pathogen transfer based on existing literatures (assuming constant pathogen amount). More precisely, we propose a case study of real in-hospital contact process and apply numerical simulations for the stochastic queues to analyze the amount of pathogen transfer under the eight different transfer functions. Next, we take pathogen decreasing during inter-arrival time into consideration, and assess pathogen transfer load in the same stochastic queues.

Our study highlights the importance of the interactions among contact process (between host and environment), transfer functions that map pathogen transfer quantity, and pathogen demography during the contact process. The modeling framework can be readily extended to more complicated queueing networks by adjusting number/type of servers and customers.

The work is expected to have impacts beyond its theoretical contributions in applied ecology, with advances likely to be influential in disease control and health policy areas. Our paper, Chen et al. (2017), is under revision for *Mathematical Medicine and Biology.*

4 Publications during tenure of ARO grant (05/15/2014– 05/14/2017)

1. L. Sun, C. Lee, and J. Hoeting (2017). A penalized simulated maximum likelihood in parameter estimation for SDEs with measurement error. Revision submitted to *Computational Statistics*.

2. S. Chen, S. Lenhart, J.D. Day, C. Lee, M. Dulin, and C. Lanzas (2017). Pathogen transfer through environment-host contact: an agent-based queueing theoretic framework. Under revision for *Mathematical Medicine and Biology.*

3. Z. Cui, C. Lee and Y. Liu (2017). On a General Framework for Pricing Asian Options Under Markov Processes. Revision submitted to *European Journal of Operations Research.*

4. C. Lee and J. Song (2016). On drift parameter estimation for reflected fractional Ornstein-Uhlenbeck processes. *Stochastics: An International Journal of Probability and Stochastic Processes (formerly Stochastics and Stochastics Reports).* Vol. 88, No. 5, pp. 751–778.

5. Z. Han, Y. Hu, and C. Lee (2016). Optimal pricing barriers in a regulated market using reflected diffusion processes, *Quantitative Finance*, Vol. 16, No. 4, pp. 639–647.

6. S. Wei, C. Lee, L. Wichers, G. Li and J. S. Marron (2016). Direction-Projection-Permutation for High Dimensional Hypothesis Tests. *Journal of Computational and Graphical Statistics*, Vol. 25, No. 2, pp. 549–569.

7. S. Chen, C. Lee, M. Sanderson, and C. Lanzas (2016). Basic reproduction number and transmission dynamics of common serogroups of enterohemorrhagic Escherichia coli. *Applied and Environmental Microbiology.* Vol. 82, No. 18, pp. 5612–20.

8. C. Lee and A. A. Puhalskii (2015). Non-Markovian state dependent networks in critical loading, *Stochastic Models*, Vol. 31, No. 1, pp. 43–66.

9. Y. Hu, C. Lee, M. Lee, and J. Song (2015). Parameter estimation for reflected Ornstein-Uhlenbeck processes with discrete observations, *Statistical Inference for Stochastic Processes*, Vol. 18, No. 3, pp. 279–291.

10. L. Sun, C. Lee, and J. Hoeting (2015a). Penalized importance sampling for parameter estimation in stochastic differential equations, *Computational Statistics and Data Analysis*, Vol. 84, pp. 54–67. (This paper has won the 2nd place prize from the American Statistical Association Section on Statistics and Environment student paper competition.)

11. L. Sun, C. Lee, and J. Hoeting (2015b). Parameter inference and model selection in deterministic and stochastic dynamical models via approximate Bayesian computation: modeling a wildlife epidemic. *Environmetrics*, Vol. 26, No. 7, pp. 451–462.

12. C. Lee and A. R. Ward (2014). Optimal pricing and capacity sizing for GI/GI/1 queue, *Operations Research Letters*, Vol. 42, No. 8, pp. 527–531.

Papers under initial review stage:

1. C. Lee and A. R. Ward (2017). Pricing and capacity sizing of a service facility: Customers abandonment effect. Under review for *Production and Operations Management*.

2. Z. Han, Y. Hu and C. Lee (2017). Optimal pricing barriers in a regulated market using Markov modulated reflected diffusion processes. Under review for *Quantitative Finance*.

3. Z. Cui, C. Lee and Y. Liu (2017). Uniqueness of equilibrium in the financial system under partial multilateral netting, Under review for *Operations Research Letters*.

4. Y. Cheng, H. Ghoddusi, Y. Hu, and C. Lee (2017). Optimal adjustment of energy prices. Under review for *European Journal of Operations Research*.