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14. ABSTRACT
Sensitivity testing is a commonly encountered problem in defense and heavy industries. A main purpose is to estimate the critical stimulus level of an experimental object by subjecting the test specimens under a variety of stress levels. The sample size in such testing tends to be small due to high cost and testing time. Thus efficient sequential procedure for choosing stress levels adaptively is desired because it can reduce the required sample size. Due to its practical importance, this problem has received a lot of attention in the past decades. However, much less

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Report Title

Final Report: Binary and Hybrid Response Data in Sensitivity Testing: Sequential and Bayesian Optimal Designs

ABSTRACT

Sensitivity testing is a commonly encountered problem in defense and heavy industries. A main purpose is to estimate the critical stimulus level of an experimental object by subjecting the test specimens under a variety of stress levels. The sample size in such testing tends to be small due to high cost and testing time. Thus efficient sequential procedure for choosing stress levels adaptively is desired because it can reduce the required sample size. Due to its practical importance, this problem has received a lot of attention in the past decades. However, much less is known or has been done when the response takes a hybrid form, which has two parts. In addition to the binary response, if it is a response (e.g., explosion), there is a further measurement on the nature of the response (e.g., explosive energy). This measurement is on a continuous scale and forms the second part of the response. It can provide valuable information beyond the binary response. A major part of this project is to develop efficient sequential design procedures for conducting sensitivity testing with hybrid response. The second part of the project concerns fixed sample and sequential designs of sensitivity testing with binary data.

Enter List of papers submitted or published that acknowledge ARO support from the start of the project to the date of this printing. List the papers, including journal references, in the following categories:

(a) Papers published in peer-reviewed journals (N/A for none)

Received		Paper
04/11/2017	5	Heng Su, C. F. Jeff Wu. CME Analysis: a new method for unraveling aliased effects in two-level fractional factorial experiments, Journal of Quality Technology, (01 2015): 1. doi: 360,897.00
04/11/2017	6	Rahul Mukerjee, C. F. Jeff Wu, Ming-Chung Chang. Two-level minimum aberration designs under a conditional model with a pair of conditional and conditioned factors, Statistica Sinica, (06 2015): 0. doi: 360,898.00
04/11/2017	7	Simon Mak, V. Roshan Joseph. Minimax and Minimax Projection Designs Using Clustering, Journal of Computational and Graphical Statistics, (08 2015): 0. doi: 362,332.00
04/11/2017	8	Simon Mak, V. Roshan Joseph. Support Points, Annals of Statistics, ( ): . doi: 1,016,562.00
04/11/2017	9	Dianpeng Wang, Yubin Tian, C. F. Jeff Wu. A skewed version of the Robbins-Monro-Joseph procedure for binary response, Statistica Sinica, ( ): 1679. doi: 1,039,022.00
04/11/2017	10	Lulu Kang, V. Roshan Joseph. Kernel Approximation: From Regression to Interpolation, SIAM/ASA Journal on Uncertainty Quantification, ( ): 112. doi: 1,039,023.00

TOTAL: 6

Number of Papers published in peer-reviewed journals:

(b) Papers published in non-peer-reviewed journals (N/A for none)

Received Paper

TOTAL:

Number of Papers published in non peer-reviewed journals:

(c) Presentations

C. F. Jeff Wu (plenary talk), "Quality improvement: from autos and chips to nano and bio," The 1st Sino-US Research Conference on Quality, Analytics, and Innovations, July 2016, Shanghai.

C. F. Jeff Wu (plenary talk), "A fresh look at effect aliasing and interactions: some new wine in old bottles," Japan Statistical Society, September 2016, Kanazawa, Japan (as the inaugural Akaike Memorial Lecture).

Roshan Joseph V. "Simulation and Optimization using Minimum Energy Designs," JSM, Chicago, August 2016.

Roshan Joseph V. Discussion of three talks on "Optimal Experimental Design for Physical Models", JSM, Chicago, August 2016.

S. Mak "Support points: A new way to compact distributions". Poster
presentation at Georgia Statistics Day 2016. October 2016, Atlanta, U.S.A.

S. Mak "Support points: A new way to compact distributions". Best Student Paper presentation at INFORMS Annual Meeting. November 2016,
Nashville, U.S.A.

Number of Presentations: 6.00

Non Peer-Reviewed Conference Proceeding publications (other than abstracts):

Received Paper

TOTAL:

Number of Non Peer-Reviewed Conference Proceeding publications (other than abstracts):

Peer-Reviewed Conference Proceeding publications (other than abstracts):

Received Paper

TOTAL:

Number of Peer-Reviewed Conference Proceeding publications (other than abstracts):

(d) Manuscripts

Received Paper

TOTAL:

Number of Manuscripts:

Books

Received Book

TOTAL:

Received	Book Chapter

07/23/2015 4.00 Yijie D. Wang, C. F. J. Wu . Bayesian cubic spline in computer experiments, New York: Springer, (12 2015)

TOTAL: 1

Patents Submitted

Patents Awarded

Awards

C. F. Jeff Wu:

George Box Medal, ENBIS, 2017

Akaike Memorial Lecture Award (inaugural), Institute of Statistical Mathematics, Tokyo, and Japan Statistical Society, 2016

Simon Mak:

Best Student Paper Award, Quality, Statistics, and Reliability Section, INFORMS Annual Meeting, Nashville, November 2016.

Best Student Poster winner, Georgia Statistics Day, October 2016.

Graduate Students

NAME	PERCENT_SUPPORTED	DISCIPLINE
Simon Mak	100	Industrial Engineering/Statistics
FTE Equivalent:	**1.00**	
Total Number:	**1**	

Names of Post Doctorates

NAME	PERCENT_SUPPORTED
FTE Equivalent:	
Total Number:	

Names of Faculty Supported

NAME	PERCENT_SUPPORTED	National Academy Member
Chien-Fu Jeff Wu	0.08	Yes
Roshan J. Vengazhiyil	0.08	
FTE Equivalent:	**0.16**	
Total Number:	**2**	

Names of Under Graduate students supported

NAME	PERCENT_SUPPORTED
FTE Equivalent:	
Total Number:	

Student Metrics

This section only applies to graduating undergraduates supported by this agreement in this reporting period

The number of undergraduates funded by this agreement who graduated during this period: 0.00

The number of undergraduates funded by this agreement who graduated during this period with a degree in science, mathematics, engineering, or technology fields:...... 0.00

The number of undergraduates funded by your agreement who graduated during this period and will continue to pursue a graduate or Ph.D. degree in science, mathematics, engineering, or technology fields:...... 0.00

Number of graduating undergraduates who achieved a 3.5 GPA to 4.0 (4.0 max scale):...... 0.00

Number of graduating undergraduates funded by a DoD funded Center of Excellence grant for Education, Research and Engineering:...... 0.00

The number of undergraduates funded by your agreement who graduated during this period and intend to work for the Department of Defense 0.00

The number of undergraduates funded by your agreement who graduated during this period and will receive scholarships or fellowships for further studies in science, mathematics, engineering or technology fields:...... 0.00

Names of Personnel receiving masters degrees

NAME
Total Number:

Names of personnel receiving PHDs

NAME
Total Number:

Names of other research staff

NAME	PERCENT_SUPPORTED
FTE Equivalent:	
Total Number:	

Sub Contractors (DD882)

Inventions (DD882)

Scientific Progress

1. The Robbins-Monro stochastic approximation has been widely used in military sensitivity testing. A modification of the procedure for binary data by the co-PI Roshan Joseph V. (Joseph, 2004, Biometrika) is shown to speed up the convergence of the procedure in a large simulation study by the PI Jeff Wu (Wu-Tian, 2014, JSPI). This procedure is now known as the Robbins-Monro-Joseph (RMJ) procedure by its users including those in the military. However, the Wu-Tian study also found that RMJ can sometimes behave erratically. The funded work and recent paper by Wang-Tian-Wu (2014) proposed a skewed version of the RMJ to nudge the iterations away from making too many test runs that give little information for the iteration scheme. It is shown to outperform the original RMJ procedure. This method has been communicated to a statistics group at the US Army RDECOM ARDEC. For related information, see the section on technology transfer.

The CME analysis paper by Su and Wu (2017) fully develops a new data analysis strategy first proposed by Wu in his 2011 Fisher Lecture at the Joint Stat Meetings in Miami (and its written version published in Journal of American Statistical Association in 2015). This strategy allows the identification and estimation of two-factor interactions even in resolution IV or III designs by reparametrizing the interactions into conditional main effects.

2. The work for this proposed problem has gone slower than others because the PI was waiting for some nearly realistic examples from his Army contacts. For proprietary reasons, it was not coming. Some work on a closely related problem (which does not need such real world motivation) has been done but no manuscript is ready yet. The PI Wu has presented this work in the “Conference on Applied Statistics in Defense (CASD)” in Washington D. C., October, 2016.

3. The Bayesian optimal design criterion is computationally expensive to evaluate and thus the optimization is not easy. We have used ideas from computer experiments to deal with this situation. In the minimax design paper by Mak and Joseph (2015), an efficient algorithm is proposed for generating minimax distance designs using clustering methods. The other two papers propose new modeling methods for computer experiments: Kang and Joseph (2015) proposed kernel interpolation method and Wang and Wu (2015) proposed a Bayesian version of cubic splines for computer experiments. Mak and Joseph (2016) deals with finding a representative point set of an arbitrary probability density function, known as support points, which can be used to numerically evaluate the high dimensional integrals appearing in the Bayesian optimal design criterion. Support points have many other applications including uncertainty quantification, Bayesian computation, stochastic optimization, and big data analysis.

Technology Transfer

A skewed version of the Robbins-Monro-Joseph (RMJ) procedure as developed by Wang-Tian-Wu (2014) has been communicated to Doug Ray and Paul Roediger at ARDEC. Based on the information we gave them, they have written an R code for this new procedure and shared with other users in the military.

Manuscripts and R codes were sent to our contacts in Army (Dr. Doug Ray, US Army RDECOM ARDEC).

Final Report of W911NF-14-1-0024

Binary and Hybrid Response Data in Sensitivity Testing:
Sequential and Bayesian Optimal Designs

Submitted by
Chien-Fu Jeff Wu and Roshan Joseph Vengazhiyil
Georgia Institute of Technology
Atlanta, GA 30332-0205

1. Introduction

Sensitivity testing is a commonly encountered problem in defense and heavy industries. A main purpose is to estimate the critical stimulus level (or threshold) of an experimental object by subjecting the test specimens under a variety of stress levels. In pyrotechnics applications, the stimulus level may be the drop height of an explosive and the response is binary (i.e., response or non-response). See Figure 1 for an example. A threshold can be defined as the stress level that results in, say, 99% probability, of explosion. The sample size in such testing tends to be small due to high cost and testing time. Thus efficient sequential procedure for choosing stress levels adaptively is desired because it can reduce the required sample size. This problem, known as sensitivity testing, is the focus of the first part of this project.

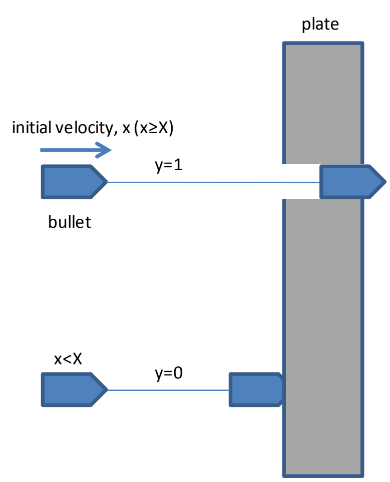

Figure 1. An example of sensitivity testing.

The second part of this project focuses on optimal experimental designs. Because the physical experiments can be costly, many of the experiments are done in computers using computer models that simulate the physical system. However, computer experiments can also be time consuming and expensive. Thus efficient experimental designs are needed to reduce the cost of experimentation. The proposed methods have applications beyond computer experiments. They can be used not only in battlefield simulations but also in a real battle on distributing and allocating the resources. They also have applications in Bayesian computation and stochastic optimization, which can help develop better sensitivity testing methods.

2. Scientific Accomplishments

2.1 Sensitivity testing and related problems

The Robbins-Monro stochastic approximation has been widely used in military sensitivity testing. A modification of the procedure for binary data by the co-PI Roshan Joseph V. (Joseph, 2004, Biometrika) is shown to speed up the convergence of the procedure in a large simulation study by the PI Jeff Wu (Wu-Tian, 2014, JSPI). This procedure is now known as the Robbins-Monro-Joseph (RMJ) procedure by its users including those in the military. However, the Wu-Tian study also found that RMJ can sometimes behave erratically. The funded work and recent paper by Wang-Tian-Wu (2014) proposed a skewed version of the RMJ to nudge the iterations away from making too many test runs that give little information for the iteration scheme. The technical ideas are as follows. In the Joseph 2004 paper, he used a quadratic loss function centered around the target (i.e., a percentile of interest, say 95% or 99% percentile) to derive his procedure. If overshooting is deemed less desirable (as in the following example) than undershooting, a skewed quadratic loss with a more severe loss above target than below target can be employed in deriving the procedure. In the WTW 2014 paper, it is shown to outperform the original RMJ procedure. Furthermore, by choosing the skewness ratio properly it can determine the percentage of experimental runs that are above target, which is very useful for potential users. This method has been communicated to a statistics group at the US Army RDECOM ARDEC. Here is a possible scenario for a military application: to test the loading capacity of a newly developed aircraft (e.g., bomber), the RMJ procedure (or other procedure used in the military like D-optimal) can be used. But overloading (above the percentile) can cause instability of the aircraft in the test flight. Thus it is highly desirable to have as many tests done below the percentile. The skewed RMJ is ideally suited for this and other military applications.

The CME analysis paper by Su and Wu (2017) fully develops a new data analysis strategy first proposed by Wu in his 2011 Fisher Lecture at the Joint Stat Meetings in Miami (and its written version published in Journal of American Statistical Association in 2015). This strategy allows the identification and estimation of two-factor interactions even in resolution IV or III designs by reparametrizing the interactions into conditional main effects (cme's). Until this work, it has been deemed impossible to estimate interactions for resolution III or IV designs because two-factor interactions are aliased with other two-factor interactions. By using cme's as the basis functions in the effect selection, it becomes possible to estimate interactions WITHOUT the need of adding more runs. So it can have a big impact in practice. SAS/JMP is looking into incorporating the method in its software. The Mukerjee-Wu-Chang 2017 paper is a theoretical extension of the Su-Wu paper by considering for the first time some extension of the widely used minimum aberration criterion for the traditional class of fractional factorial designs into the new situation where the cme's are used in lieu of the two-factor interactions for the same class of designs. The work on hybrid responses has gone slower than others because the PI was waiting for some nearly realistic examples from his Army contacts. For proprietary reasons, it was not coming. Some work on a closely related problem (which does not need such real world motivation) has been done but no manuscript is ready yet. The PI Wu has presented this work in the "Conference on Applied Statistics in Defense (CASD)" in Washington D. C., October, 2016.

2.2 Optimal design and related problems

Minimax distance designs have applications in several problems such as sensor placement, security surveillance, and uncertainty quantification. A minimax design D is obtained as

$$min_D max_{x \in \mathcal{X}} ||x - Q(x, D)||,$$

where $\mathcal{X}$ is the experimental region and $Q(x, D)$ is the closest point in D to $x \in \mathcal{X}$ under the norm $||.||$. An example of minimax design for seven points inside $[0,1]^2$ is shown in Figure 2.

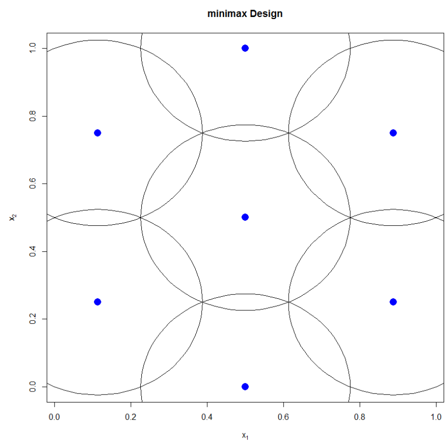

Figure 2. A 7-point minimax design for two variables inside a square.

Generating a minimax design is extremely computationally intensive because we need to search over the entire experimental region to find the worst distance. Therefore, these designs have not received much attention in the literature. Mak and Joseph (2017) proposed a novel algorithm for generating minimax designs using clustering methods, which is published in the Journal of Computational and Graphical Statistics. This is probably the first algorithm which can efficiently generate nearly optimal minimax designs for large samples and in high dimensions. The right most panel in Figure 3 shows a 20-point minimax design over the state of Georgia generated by the proposed algorithm (mMc-PSO). It is compared with a few other designs. It can be seen that the minimax design generated by the proposed algorithm has the smallest worst distance to a point in the region from a design point.

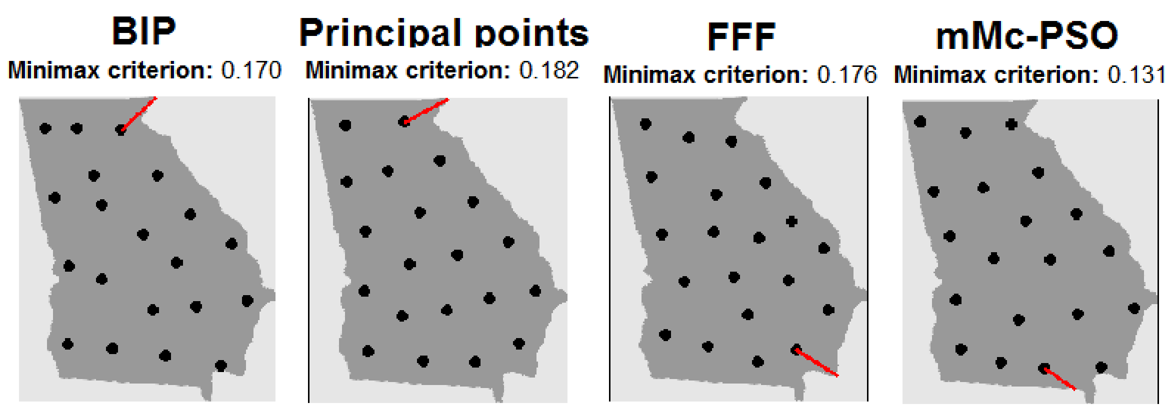

Figure 3. Four different 20-point designs for the state of Georgia. The red line on each plot connects the point in Georgia furthest from the design to its nearest design point, with its length equal to the minimax criterion of the design. Of these four designs, the new method *mMc-PSO* provides the best minimax design.

Minimax designs have many interesting military applications outside computer simulations. If we want to destroy an unknown target in a certain region with a fixed number of bombs, a minimax design gives the optimal solution. Figure 4 shows an illustration.

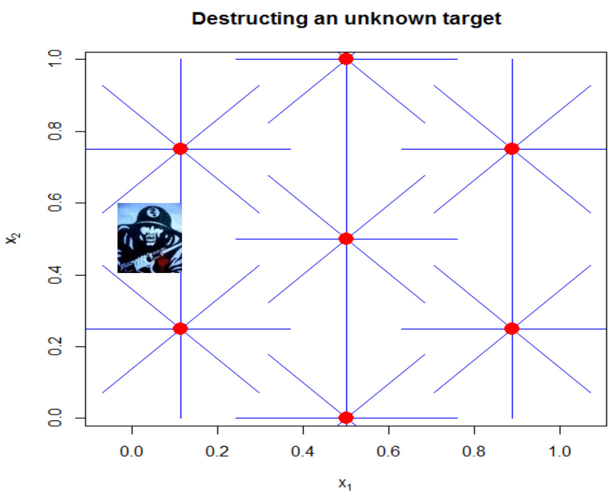

Figure 4. A minimax design solution minimizes the chance of missing the unknown target.

Another major work done as part of the project is to find representative points of an arbitrary probability distribution, known as support points (Mak and Joseph 2017). The support points $x_1, \ldots, x_n$ of a random variable Y with distribution function F is defined as

$$argmin_{x_1,\ldots,x_n} \frac{2}{n}\sum_{i=1}^{n} E||x_i - Y||_2 - \frac{1}{n^2}\sum_{i=1}^{n}\sum_{j=1}^{n}||x_i - x_j||_2,$$

where the expectation inside the first sum is taken with respect to the distribution F. This is a very difficult optimization problem, but we have developed an efficient optimization procedure by exploiting the difference-of-convex structure of the objective function.

Roughly speaking 100 support points can be as good as 10,000 Monte Carlo samples. Support points have many applications in diverse fields including uncertainty quantification, Bayesian computation, stochastic optimization, and big data analysis. An example of support points for three different distributions is shown in Figure 5. A comparison of support points with Monte Carlo and Quasi-Monte Carlo for a bivariate normal distribution is shown in Figure 6.

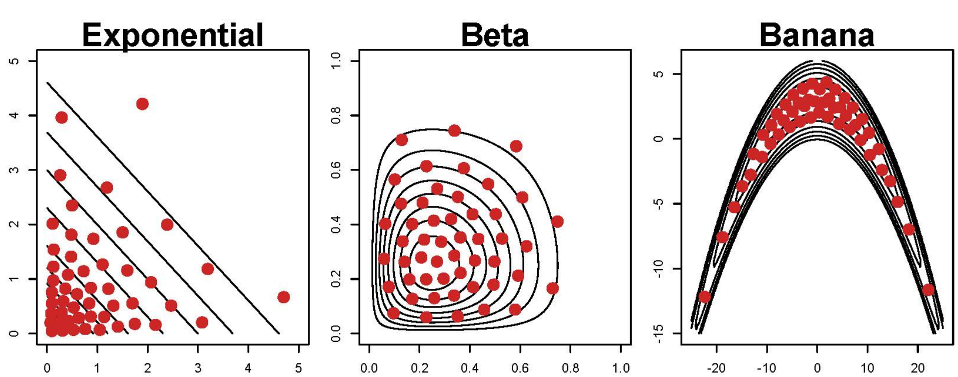

Figure 5. 50 support points for the two-dimensional Exp(1), Beta(2,4) and the banana-shaped distribution. Lines represent density contours.

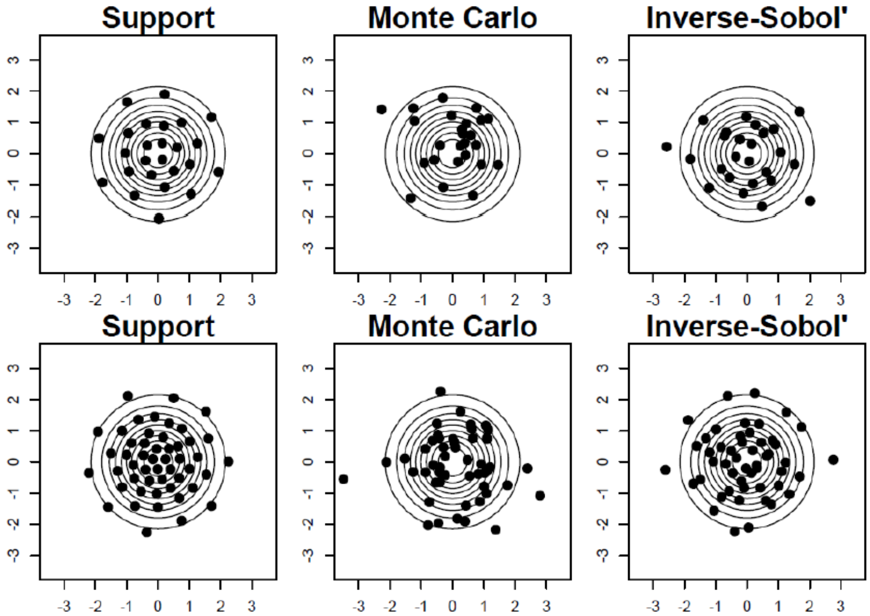

Figure 6. The top panel shows 25 points and the bottom panel shows 50 points from a bivariate normal distribution produced by three different methods. Lines represent density contours.

A major aim of the project is to develop efficient optimal and sequential designs for sensitivity experiments. The Bayesian formulation for solving this problem is computationally very expensive because of the high dimensional integrals present in the optimal design criterion. Support points can be used to efficiently compute these integrals and thus reduce the overall cost of optimization. Two other papers under this topic propose new modeling methods for computer experiments: Kang and Joseph (2015) proposed kernel interpolation method and Wang and Wu (2015) proposed a Bayesian version of cubic splines for computer experiments.

3. Interactions with Students

Three Ph.D. students (Dianpeng Wang, Li Gu, and Simon Mak) were partially supported by this grant. Based on their research, they have written journal papers joint with Professors Jeff Wu and Roshan Joseph V. and also presented the results in many conferences. Dianpeng Wang graduated in 2015 and is now a postdoctoral student at the Chinese Academy of Sciences, Beijing. Li Gu graduated in 2016 and now working as a Quantitative Associate in Well Fargo & Company, Charlotte. Simon Mak is a fourth year student and is expected to graduate in 2018.

4. Interactions with Military

- Wu gave a webcast talk on “Sensitivity testing of DoD systems” for the OA Forum on November 9, 2015. Joseph gave a webcast talk on “Space-filling designs” for the OA Forum on April 4, 2015. OA Forum is a regular web conference for statisticians in the military.
- An R package was written by Paul Roediger (US Army RDECOM ARDEC) for the skewed Robbins-Monro-Joseph procedure (Wang-Tian-Wu, 2015). It has been used in military applications.
- An R package was written by Paul Roediger (US Army RDECOM ARDEC) for the 3pod procedure (Wu-Tian, 2014). The 3pod procedure has been promoted by Doug Ray of ARDEC and widely used in the military, as an alternative to Neyer’s D-optimal procedure for sensitivity testing. D-optimal is listed as a Mil-Standard. The 3pod work was funded by a previous ARO grant.
- Joseph and Wu have provided software codes and manuscripts to Doug Ray (ARDEC), Jim Brownlow (TSS Edwards AFB), and Jim Higdon (Eglin, AFB) on computer experiments and uncertainty quantification.

5. References

1. Dianpeng Wang, Y. Tian, and C. F. J. Wu (2015). A skewed version of the Robbins-Monro-Joseph procedure for binary response. *Statistica Sinica*, 25, 1679-1689.
2. Joseph, V. R. (2004). “Efficient Robbins-Monro Procedure for Binary Data”. *Biometrika* 91, 461-470.
3. Kang, L. and Joseph, V. R. (2016). “Kernel Approximation: From Regression to Interpolation”. *Journal of Uncertainty Quantification*, 4, 112-129.
4. Mak, S. and Joseph, V. R. (2017). “Minimax and Minimax Projection Designs Using Clustering,” *Journal of Computational and Graphical Statistics*, to appear.
5. Mak, S. and Joseph, V. R. (2017+). “Support Points,” Under review in *Annals of Statistics*.
6. Mukerjee, R., C. F. J. Wu and M. C. Chang (2017). Two-level minimum aberration designs under a conditional model with a pair of conditional and conditioned factors. *Statistica Sinica*, to appear.
7. Su, Heng and C. F. J. Wu (2017). CME Analysis: a new method for unraveling aliased effects in two-level fractional factorial experiments. *J. Quality Technology*, 1-10.
8. Wu, C. F. J. and Y. Tian (2014). Three-phase optimal design of sensitivity experiments (with discussions). *J. Statistical Planning and Inference*, 149, 1-32.
9. Wu, C. F. J. (2015). Post-Fisherian experimentation: from physical to virtual. *Journal of American Statistical Association,* 110*,* 612-620.