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A Generalized Sum-Rate Optimizer for Cooperative Multiuser Massive MIMO Link Topologies

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ABSTRACT Large-scale, or massive, multiple-input multiple-output (MIMO) systems are typified by the number of antennas contributing to a communication link. This type of link can consist of single nodes with a large number of antennas or a large number of cooperating nodes—each contributing a small number of antennas. Such massive systems naturally lead to link topologies that are not often considered in studies of smaller scale cooperative MIMO scenarios. For this system to be economically practical, each node participating in the massive link likely has limited transmit power capability, and therefore properly limiting the per-node transmit power must be incorporated into the signal processing algorithm. This paper develops a generalized multiuser massive MIMO (G4M) optimization algorithm for colocated or cooperative signaling, subject to any sum-, per-antenna, or per-node power constraint, and that can also accommodate nonlinear precoding and detection and any number of antennas. Using the G4M algorithm, a number of topologies unique to cooperative massive MIMO are described, demonstrating the facility this algorithm provides in optimizing the performance of multiuser massive links with atypical topologies.

INDEX TERMS Large MIMO systems, multiuser channels, array signal processing.

I. INTRODUCTION

The defining characteristic of multiuser massive multiple-input multiple-output (MIMO) communication, including the so-called ''massive MIMO effect'' [1]–[3], is an excess of antennas compared with the number of users. The objective of cooperative massive MIMO is to combine the possibly limited capabilities of many small nodes to allow the same significant overall communication performance of massive MIMO. This concept of cooperation on a massive scale is intriguing, as adding capacity to existing infrastructure could be as simple as adding more low-cost modules that can coordinate with those already present. In this way, the idea is similar to the concept of cooperative MIMO where multiple radios work together to accomplish the MIMO signal processing [4]–[9]. The combination of massive MIMO and cooperative nodes leads to a plethora of new and interesting multiuser topologies enabled by complex interaction of antennas and users participating in a link. These topologies are the focus of the work detailed in this paper.

Limitations on transmit power handling for each node can have a significant impact on the type of signal processing that can be accommodated by this type of massive MIMO system. Specifically, when the coordinating nodes transmit to multiple users [10], if the transmit precoding is developed assuming a sum-power constraint, it is assumed that some nodes can transmit high power while others transmit reduced power (i.e. power ''sharing''). However, the limited power capability of each node will likely preclude such a possibility, and therefore either the total transmit power must be reduced or a more practical power constraint, such as a per-antenna or per-node power constraint, must be deployed [11]–[13]. While some work has appeared focusing on multi-user MIMO signaling under per-antenna power constraints [14]–[17], the computational burden of these algorithms, or emphasis on a specific multiuser topology, makes them poorly suited as the number of antennas or users becomes large. This is the case for massive MIMO systems where researchers in the field face a wide variety of channel topologies.

In addition to the required power allocations for massive antenna systems, the multi-user/antenna/node nature of the analyzed systems, whether cooperative or not, leads to both typical and atypical link topologies. Capacity limits and algorithms of common topologies of single-user, multi-antenna channels is well documented [10], [18]. The simplest multiuser channels, the broadcast channel [19]–[21] (a single user transmitting to multiple users) and multiple-access channels [22]–[24] (multiple users transmitting to a single user), can be optimized for maximum sum rate using dedicated algorithms. More complex channels such as the interference channel [25], [26] (multiple simultaneous single-user channels) or X-like channels [27], [28] (multiple simultaneous broadcast or multiple-access channels) have also received attention in the literature. Expanding onto even larger scales, multi-cell networks [29], [30] (multiple multiuser channels communicating with multiple receivers) are possible where single-cell multiuser channels have been adapted to solve an even larger problem formulation including when feedback is limited [31]. Though the work on multiuser channels is expansive, approaches are often limited to particular topologies which may be difficult to apply to the new topologies possible in massive systems.

Given the limitations of existing algorithms, new approaches to evaluating massive MIMO systems are required to provide meaningful, rather than merely theoretical, results illustrating the benefit of such networks. Research on massive MIMO links ranges from information-theoretic analyses of asymptotic antenna growth [32]–[34] to applications of well-known communications schemes such as multicarrier CDMA [35] or V-BLAST [36] to MIMO systems with up to 600 transmit and receive antennas. The work in [2] identifies various practical concerns associated with large arrays and develops transmission and reception schemes for massive MIMO that require low complexity and low power. In [37] massive MIMO spectral efficiency is addressed when using a smaller number of antennas. Despite all this prior work, efficient signaling algorithms that can accommodate a wide variety of link topologies and power constraints remain relatively immature for multiuser massive MIMO channels.

Recently, we proposed a new iterative beamforming algorithm [38] that can efficiently compute the transmit and receive beamforming weights under a per-antenna power constraint. The algorithm can accommodate nonlinear processing such as dirty-paper coding (DPC) [10], [39] or successive interference cancellation (SIC). Our prior work showed that in many cases, the communication rate achievable using this iterative solution for a MIMO broadcast channel with a per-antenna power constraint approaches that achieved using optimal iterative waterfilling (IWF) with a traditional sum-power constraint [22], [40]. Therefore, from the standpoint of performance and computational efficiency, this iterative algorithm certainly represents a potential candidate for massive systems where the number of data streams can become very large. However, this algorithm has not been developed for the complex topologies that arise when the transmitter consists of multiple nodes, each with multiple antennas and each satisfying its own per-node constraint. Furthermore, the approach has not yet been applied to study the potential performance of massive MIMO communication links with atypical topologies.

Iterative solutions are a common approach to utility maximization in wireless networks [41]–[43]. Though this work also employs an iterative approach, the novelty of this paper lies in the following:

- A generalization of the per-antenna power constraint from [44] into a per-node power constraint for any spatial topology and multiplexing. This is accomplished by introducing an antenna enumeration function into the Lagrange functions that can handle complex power allocations and thus introduces more general topological analyses.
- For any algorithm, a proof of optimality for all possible topologies would prove difficult; indeed, except in special cases, the problems are more often NP-hard. Since the number of different topologies increases drastically with massive MIMO, many new algorithms compare with existing or sub-optimal solutions. This work's comparative simulations show the proposed algorithm can approach an optimal solution in a variety of single- and multi-user topologies with known optima.
- Given confidence in solutions for common channels, the facility of the proposed algorithm and its ability to look at the topological effect of massive MIMO systems. Only a few of the numerous possibilities are examined, with other applications including the study of hardware considerations, green networks, radio cognition, and scalability.

II. ANALYSIS FRAMEWORK

Consider the cooperative massive MIMO system depicted in Fig. 1 that shows a combination of large and small antenna

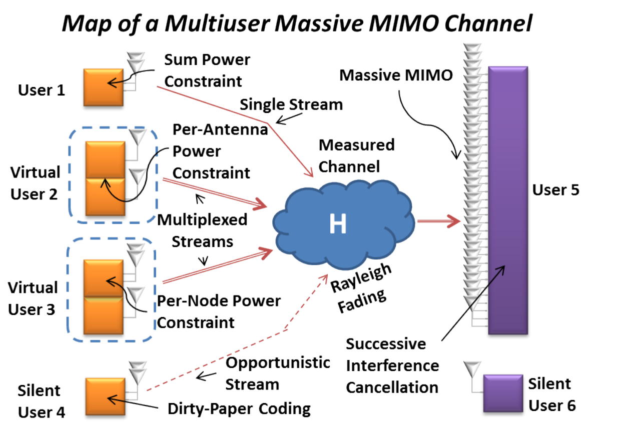

FIGURE 1. A possible topology of a next-generation cooperative massive MIMO multiuser system with heterogeneous hardware. Nodes can have any combination of: sum-, per-antenna, or node power constraints. Additionally, nodes may: be equipped with any number of antennas, use various methods of precoding or detection, or participate tangentially in the channel.

arrays consisting of modules or *nodes*, each of which may have multiple antennas. These arrays form, possibly virtual, *transmitters* (users) that communicate with multiple mobile, and possibly virtual, *receivers* (also called users). Although not explicitly shown, it is possible that the overall network consists of multiple virtual transmitters with large arrays, consisting of multiple nodes each with multiple antennas, in addition to the multiple mobile receivers. A *sublink* in this system refers to a single data stream communicated from one (possibly virtual) user to another. The sublink from the mth user to the nth user is spatially precoded with a $N_{t,m} \times 1$ transmit beamforming vector $\mathbf{b}_{(m,n)}$ and, if the nth user has multiple antennas, received with a unique receive beamforming vector $\mathbf{w}_{(m,n)}$. Each sublink experiences interference from other sublinks, the extent of which is controlled by optional application of nonlinear precoding (DPC) or detection (SIC) and spatial signal processing.

Given this general topology, the sublink from the mth to the nth user has a weighted received signal that can be written as

$$y_{(m,n)} = \mathbf{w}^{\dagger}_{(m,n)}\mathbf{H}_{(m,n)}\mathbf{b}_{(m,n)}x_{(m,n)} + \mathbf{w}^{\dagger}_{(m,n)}\left(\sum_{(i,j)}^{\mathcal{I}_{(m,n)}} \mathbf{H}_{(i,n)}\mathbf{b}_{(i,j)}x_{(i,j)} + \boldsymbol{\eta}_{(m,n)}\right), \quad (1)$$

where $\{\cdot\}^{\dagger}$ is a conjugate transpose, $\mathbf{H}_{(m,n)}$ is the $N_{r,n} \times N_{t,m}$ virtual MIMO channel transfer matrix from the mth user to the nth user, $x_{(m,n)}$ is the Gaussian symbol transmitted over the (m,n) sublink, and $\boldsymbol{\eta}_{(m,n)}$ is additive white Gaussian noise (AWGN) whose variance is σ_{η}^2. The total power per channel use is given by $P_T = \sum_{(m,n)}^{\mathcal{L}} \mathbf{b}^{\dagger}_{(m,n)}\mathbf{b}_{(m,n)}$, where the list[1] $\mathcal{L}$, of cardinality N_l, contains the duples of all existing sublinks and the notation $\sum_{(m,n)}^{\mathcal{L}}$ indicates summing over all members of the list $\mathcal{L}$.

The sublinks forming the list $\mathcal{I}_{(m,n)}$ appearing in (1) are considered spatial interference to the (m,n) data stream. Without nonlinear DPC or SIC, $\mathcal{I}_{(m,n)}$ contains all duples in $\mathcal{L}$ other than (m,n). However, if nonlinear processing is used, the interference list $\mathcal{I}_{(m,n)}$ is reduced. For example, consider the simple case of a point-to-point (P2P) channel where user 1 is transmitting with $N_{t,1} = 2$ while user 2 is receiving with $N_{r,2} = 2$ antennas. In this case it is reasonable to assume two multiplexed sublinks that interfere with each other. We then create the superlist

$$\mathcal{I} = \begin{array}{c} \left|\begin{array}{cc} 0 & 1 \\ 1 & 0 \end{array}\right| \\ (1,2)\ (1,2) \end{array} \begin{array}{c} (1,2) \\ (1,2) \\ \end{array} \quad (2)$$

where column and row labels represent the corresponding sublink interference and are included for convenience. Entries into the matrix $\mathcal{I}$ represent boolean answers to the question: "Does the (m,n) sublink (row) receive interference from the (i,j) sublink (column)?" Each diagonal element of $\mathcal{I}$ is zero since a stream does not interfere with itself.

The signal-to-interference-plus-noise ratio (SINR) $\rho_{(m,n)}$ for the received signal $y_{(m,n)}$ can be calculated assuming noise of variance σ_{η}^2 and unity-norm receiver beamforming vectors as

$$\rho_{(m,n)} = \frac{|\mathbf{w}^{\dagger}_{(m,n)}\mathbf{H}_{(m,n)}\mathbf{b}_{(m,n)}|^2}{\sigma_{\eta}^2 + \sum_{(i,j)}^{\mathcal{I}_{(m,n)}} |\mathbf{w}^{H}_{(m,n)}\mathbf{H}_{(i,n)}\mathbf{b}_{(i,j)}|^2} = \frac{\nu_{(m,n)}}{\delta_{(m,n)}}. \quad (3)$$

The total sum-rate is then

$$R = \sum_{(i,j)}^{\mathcal{L}} \log(1 + \rho_{(i,j)}) \quad (4)$$

which assumes no joint detection on the data streams.

The novelty and objective of the generalized multiuser massive MIMO maximization (G4M) algorithm is to find the beamforming vectors $\mathbf{b}_{(m,n)}$, $\mathbf{w}_{(m,n)}$, and power distributions, with a per-node power constraint, that maximize the rate in (4) via simple linear iterations for any link topology defined by $\mathcal{L}$ and $\mathcal{I}$.

To formulate this per-node power constraint, let $\mathbf{B}_m$ represent the matrix whose columns are the transmit beamforming vectors $\mathbf{b}_{(m,n)}$, and let $\hat{\mathbf{b}}_i^T$ represent the ith row of $\mathbf{B}_m$, where $\{\cdot\}^T$ indicates a transpose. With this notation, $\hat{\mathbf{b}}_i^{\dagger}\hat{\mathbf{b}}_i$ represents the power transmitted from the ith transmit antenna. To maintain notational compactness, we introduce the antenna enumeration function (AEF) $u_{k,m}(q)$ that returns the integer index of the qth transmit antenna for the kth node. For example, if the mth user has a large array consisting of three nodes,[2] where nodes 1, 2, and 3 respectively have three, one, and two antennas, then one possible form for $u_{k,m}(q)$ is

	$u_{k,m}(q)$		
k (node)	$q=1$	2	3
1	1	2	3
2	4	-	-
3	5	6	-

We further let $\hat{N}_{t,k}$ represent the number of transmit antennas for node k; thus, the total number of transmit antennas available to user m becomes

$$N_{t,m} = \sum_{k=1}^{N_{n,m}} \hat{N}_{t,k} \quad (5)$$

where $N_{n,m}$ is the number of nodes cooperating in the link for the mth user. Note that for a traditional cooperative MIMO system with a per-antenna power constraint, each node has a single antenna with AEF $u_{k,m}(1) = k$. Similarly, the traditional non-cooperative sum-power constraint is achieved by considering the user as a single node with AEF $u_1(q) = q$. These two extreme cases, per-antenna power and sum-power,

[1] It is important to use the programmatic definition of list. A set has no duplicate members where a list can; multiplexing is allowed with duplicate duples in the same sublink list.

[2] It is imperative in this discussion to distinguish "user" from "node". A user has no antennas in itself; rather, it consists of a number of cooperating nodes that are each equipped with multiple antennas.

constraints have been well studied; however, the AEF allows the G4M algorithm to analyze any per-node power constraint.

The per-node power constraint is now mathematically formulated by constraining the power associated with the rows of the matrix $\mathbf{B}_m$, or

$$\sum_{q=1}^{\hat{N}_{t,k}} \hat{\mathbf{b}}^{\dagger}_{u_{k,m}(q)} \hat{\mathbf{b}}_{u_{k,m}(q)} \leq P_k \tag{6}$$

$$\sum_{k=1}^{N_{n,m}} P_k = P_{T,m} \tag{7}$$

where $N_{n,m}$ is the total number of nodes cooperating for the mth user, $P_{T,m}$ is the total available transmit power for the mth user, and P_k is the allowable transmit power from the kth node. We use a Lagrange multiplier formulation to include this constraint into a cost function for maximizing (4). The Lagrange functions containing the power constraint become

$$\lambda_f(\cdot) = \sum_{k=1}^{N_n} \lambda_k \left[\sum_{q=1}^{\hat{N}_{t,k}} \hat{\mathbf{b}}^{\dagger}_{u_{k,m}(q)} \hat{\mathbf{b}}_{u_{k,m}(q)} - P_k \right] \tag{8}$$

$$\Lambda = \sum_{(i,j)}^{\mathcal{L}} \log\left(\nu_{(i,j)} + \delta_{(i,j)}\right) - \log\left(\delta_{(i,j)}\right) - \lambda_f(\cdot) \tag{9}$$

where λ_k is a Lagrange variable and $\nu_{(i,j)}$ and $\delta_{(i,j)}$ are the numerator and denominator of $\rho_{(i,j)}$ from (3).

Using the the Lagrange multiplier formulation, maximization of (4) subject to the per-node power constraint is obtained by finding the maximum of (9). This is achieved by first finding the gradient of Λ with respect to $\mathbf{b}^*_{(m,n)}$. Setting the gradient to zero and gathering terms leads to equations for the rate-maximizing beamformers given by

$$\mathbf{b}_{(m,n)} = \xi_{(m,n)} \mathbf{Q}^{-1}_{(m,n)} \mathbf{A}_{(m,m,n)} \mathbf{b}_{(m,n)} \tag{10}$$

$$\mathbf{Q}_{(m,n)} = \sum_{k=1}^{N_n} \lambda_k \sum_{q=1}^{\hat{N}_{t,k}} \mathbf{J}_{u_{k,m}(q)} + \sum_{(i,j)}^{\overline{\mathcal{I}}_{(m,n)}} \xi_{(i,j)} \rho_{(i,j)} \mathbf{A}_{(m,i,j)} \tag{11}$$

$$\mathbf{A}_{(m,i,j)} = \mathbf{H}^{\dagger}_{(m,j)} \mathbf{w}_{(i,j)} \mathbf{w}^{\dagger}_{(i,j)} \mathbf{H}_{(m,j)} \tag{12}$$

where $\overline{\mathcal{I}}_{(m,n)}$ represents the list of links for which $(m,n) \in \mathcal{I}_{(i,j)}$ and $\mathbf{J}_i$ is a matrix with a single 1 on the ith row and ith column and zeros elsewhere. Furthermore, using $\nu_{(i,j)}$ and $\delta_{(i,j)}$ as the numerator and denominator of $\rho_{(i,j)}$, we have $\xi_{(i,j)} = 1/[\nu_{(i,j)} + \delta_{(i,j)}]$. Similarly, the receive beamformer can be expressed as

$$\mathbf{w}_{(m,n)} = \frac{1}{\rho_{(m,n)}} \tilde{\mathbf{Q}}^{-1}_{(m,n)} \tilde{\mathbf{A}}_{(m,n,n)} \mathbf{w}_{(m,n)} \tag{13}$$

$$\tilde{\mathbf{Q}}_{(m,n)} = \mathbf{I} + \sum_{(i,j)}^{\mathcal{I}_{(m,n)}} \tilde{\mathbf{A}}_{(i,j,n)} \tag{14}$$

$$\tilde{\mathbf{A}}_{(i,j,n)} = \mathbf{H}_{(i,n)} \mathbf{b}_{(i,j)} \mathbf{b}^{\dagger}_{(i,j)} \mathbf{H}^{\dagger}_{(i,n)}. \tag{15}$$

where $\mathbf{I}$ is the identity matrix. The receive beamforming vector is equivalent to the MMSE receiver and is written as in (13) for symmetry in the iterations.

This development also provides a procedure for computing the Lagrange multipliers λ_k without a separate numerical optimizer. Consider summing over the inner-product of each transmit vector and gradient of Λ

$$\sum_{(i,j)}^{\mathcal{L}} \mathbf{b}^{\dagger}_{(i,j)} \nabla_{(m,n)} \Lambda = 0 \tag{16}$$

which, after some algebra, allows the kth Lagrange variable to be isolated and written as a function of the power constraints

$$\lambda_k = \frac{1}{P_k} \sum_{(m,n)}^{\mathcal{L}} \mathbf{b}^{\dagger}_{(m,n)} \left(\sum_{q=1}^{\hat{N}_{t,k}} \mathbf{J}_{u_{k,m}(q)} \right) \mathbf{F}_{(m,n)} \mathbf{b}_{(m,n)} \tag{17}$$

$$\mathbf{F}_{(m,n)} = \xi_{(m,n)} \mathbf{A}_{(m,m,n)} - \sum_{(i,j)}^{\overline{\mathcal{I}}_{(m,n)}} \xi_{(i,j)} \rho_{(i,j)} \mathbf{A}_{(m,i,j)}. \tag{18}$$

The preceding equations must be applied iteratively to compute the beamformers that maximize the rate. Specifically, we begin by initializing all values of $\mathbf{b}_{(m,n)}$ and $\mathbf{w}_{(m,n)}$. We can then compute the Lagrange multipliers using (17) and update the beamformers using (10) and (13). This iteration continues until convergence to, as observed numerically, a local maximum.

III. G4M APPLICATION

The parameterization seen in massive MIMO multiuser channels is an immediate indicator of its analysis complexity: number of nodes (N_n), antennas per node ($\hat{N}_{t,k}$), total transmit antennas ($N_{t,m}$), power per node (P_k), number of receiving users (N_u), antennas per receiver ($N_{r,n}$), precoding and interference (denoted through the interference lists $\mathcal{I}$), network topology ($\mathcal{L}$), achievable sum-rate (R), channel transfer functions ($H_{(m,n)}$), and so on. Any alteration of even just one of these parameters can have significant consequences on network performance.

TABLE 1. Generalized multiuser massive MIMO maximization (G4M) algorithm.

	Preliminary Steps (User Defined)
U1	Determine multiuser channel topology via sublink duple list $\mathcal{L}$
U2	Define [non]linear precoding with interference list $\mathcal{I}$
U3	Define per-node P_k power constraints
U4	Define antenna enumeration function $u_{k,m}(q)$
U5	Realize channels $\mathbf{H}_{(m,n)}$ for each sublink from step (U1)
	Iterative Steps (Algorithm Defined)
A1	Evaluate all Lagrange variables λ_k from Eq. (17)
A2	Evaluate all transmit beamformers $\mathbf{b}_{(m,n)}$ from Eq. (10)
A3	Evaluate all receive beamformers $\mathbf{w}_{(m,n)}$ from Eq. (13)
A4	Go to (A1)

The G4M algorithm accounts for each of these parameters and easily facilitates analysis of massive MIMO multiuser channels without the need for third-party numerical solvers that may become cumbersome with a massive number of antennas. Consider the steps in Table 1 which are broken up into "User Defined" (U1-U5) and "Algorithm Defined" (A1-A4). The user defined step allows one to create a particular massive MIMO topology with all the parameters listed above as suited for the scenario under examination. Once defined, the algorithmic steps are simply an iteration over the streams' weights and power allocations. Note that this iteration is completely interlaced and both beam weights and power allocations are updated each iteration.

Table 1 shows the abbreviated steps for G4M while the following provides a more detailed explanation. Simulations in this work were generated using MATLAB® [45]; however, any mathematical programming language can accomplish the same tasks since the vast majority are simply linear operations. To begin G4M the communication topology of interest is defined:

- U1: Determine the multiuser channel topology (e.g. broadcast, MIMO X, P2P channels) by defining all possible sublink duples for the list $\mathcal{L}$. For example, a simple MIMO X channel with users 1 and 2 as transmitters and users 3 and 4 as receivers might have a link list $\mathcal{L} = \{(1,3),(1,4),(2,3),(2,4)\}$. In MATLAB this is just a two-column matrix where each row is a sublink, the first column is the transmitter, and the second column is the receiver node.
- U2: Define the type of precoding (e.g. DPC or linear) and detection (e.g. SIC or linear) used for each sublink in $\mathcal{L}$. This is done by generating the interference superlist defining the spatial interference of each sublink with all other sublinks. In the previous MIMO X example where each transmitter uses DPC for its own data, but not for the competing transmitter (i.e. no transmitter cooperation), then the interference superlist becomes

$$\mathcal{I} = \begin{array}{c|cccc|l} & 0 & 1 & 1 & 1 & (1,3) \\ & 0 & 0 & 1 & 1 & (1,4) \\ & 1 & 1 & 0 & 1 & (2,3) \\ & 1 & 1 & 0 & 0 & (2,4) \\ \multicolumn{1}{c}{} & \multicolumn{1}{c}{(1,3)} & (1,4) & (2,3) & \multicolumn{1}{c}{(2,4)} & \end{array} \tag{19}$$

  which is simply an $N_l \times N_l$ matrix containing only 1's and 0's. Note that the interference superlist must also take into account the order of encoding or decoding when DPC or SIC are assumed, respectively.
- U3: Define per-node power constraints. A heterogeneous virtual transmitter might consist of nodes with vastly different power capabilities. Defining P_k, the total power per-node, allows these types of scenarios to be analyzed.
- U4: Define antenna enumeration function $u_{k,m}(q)$. Again, with heterogeneous virtual transmitters each node participating in a link might have a varying number of antennas available to participate. The AEF maps the antenna number from the individual nodes to the antenna number of the virtual node. Depending on the heterogeneity this can be written as a subfunction in MATLAB or as a simple linear relationship as is examined in Section IV.
- U5: Realize channels $\mathbf{H}_{(m,n)}$ for each sublink from step (U1); the G4M algorithm is transparent to what channel spatial effects are used including uncorrelated or correlated channels and popular channel models [46].

Once these parameters are defined no further interaction is required to optimize the sum-rate. The following iterations are performed to arrive at a, at least local, maximum. Though not optimal, random starting values of λ_k, $\mathbf{b}_{(m,n)}$, and $\mathbf{w}_{(m,n)}$ are used in this work:

- A1: Evaluate all Lagrange variables λ_k from Eq. (17). Each of these are scalar values for a total of N_l variables. Intermediate matrices including $\mathbf{F}_{(m,n)}$ (18) and $\mathbf{A}_{(m,i,j)}$ (12) need to be calculated first but are simple matrix products.
- A2: Evaluate all transmit beamformers $\mathbf{b}_{(m,n)}$ from Eq. (10) where $\mathbf{b}_{(m,n)}$ from the right side of Eq. (10) is taken from the previous iteration.
- A3: Evaluate all receive beamformers $\mathbf{w}_{(m,n)}$ from Eq. (13) where $\mathbf{w}_{(m,n)}$ from the right side of Eq. (13) is taken from the previous iteration. These have an analogous computational complexity to the transmit weights but are still simply linear algebra.
- A4: Repeat A1-A3 until some convergence criterion is reached.

The complexity of G4M is difficult to show theoretically but there are some observations common in all topologies. At each iteration all N_l weights are updated and the number of iterations is roughly proportional to the number of links. Since the weights have the form of MMSE beamformers the complexity at each iteration can be considered roughly N_l^2 times greater than a basic MMSE weight calculation of the single-stream single-user case.

IV. TOPOLOGIES

The following results provide representative behavior of such massive MIMO topologies that may arise as a function of many of the key parameters explained above; however, an exhaustive attempt to demonstrate the full potential of the G4M algorithm is not reasonable here. To keep the results tractable, certain parameters of the massive MIMO multiuser channel remain fixed for almost all simulations. All receive users are assumed real (i.e. not virtual or no receive cooperation) though the number of users can change. The statistics of the channel also remain the same, except in the case of channel correlation, although individual channel realizations will be used and the results will be averaged.

A. G4M PERFORMANCE IN COMMON TOPOLOGIES

An attempt to mathematically demonstrate a maximum convergence for *all* topologies with a single algorithm appears infeasible. These first simulations simply demonstrate the performance of the G4M algorithm as applied to various MIMO channel topologies massive or otherwise. It is important to keep in mind that the general nature of the algorithm prevents a realistic comparison across all possible algorithms and topologies. The purpose of this first result is to compare the same G4M algorithm with well-known rate maximizing algorithms in common, but relatively unrelated, channels. The core idea being that such a demonstration gives confidence when applying G4M to channels that are not well-known or do not have corresponding rate maximization algorithms; thus, G4M is shown to facilitate research into the potential of massive systems using only a single algorithm.

The stopping criteria for the following simulations was chosen under certain conditions. When the sum-rate optimal solution is known (e.g. water-filling solution) then the G4M algorithm stops once it achieves 99% of this value. When the optimal solution is not necessarily known (e.g. MIMO X channel) then the stopping criteria is when the sum-rate increases by less than 10^{-4} between iterations.

Fig. 2A simulates the massive MIMO single-user channel with $N_{r,2} = N_{t,1} = 100$ and $P_{T,1} = 10$ with the optimal water-filling solution compared with G4M. Note, both arrive at the same sum-rate solution though each approaches the optimization problem very differently. Fig. 2B shows the sum-rate growth as the number of users is increased for a fixed number of receiver antennas $N_{r,n} = 4$ and transmit antennas $N_{t,1} = 100$. Both IWF (simulated here as colocated) and G4M (simulated here as cooperative) use DPC for this simulation and arrive at the same sum-rate, but G4M uses a stream-based approach while IWF optimizes the input covariance matrices. This simulation is performed using the G4M algorithm with DPC resulting in the interference list

$$\mathcal{I} = \begin{array}{c|ccccc|l} & 0 & 1 & 1 & \dots & 1 & (1,2) \\ & 0 & 0 & 1 & \dots & 1 & (1,3) \\ & 0 & 0 & 0 & \dots & 1 & (1,4) \\ & \vdots & \vdots & \vdots & \dots & \vdots & \quad\vdots \\ & 0 & 0 & 0 & \dots & 0 & (1,N_u-1) \\ \hline & (1,2) & (1,3) & (1,4) & \dots & (1,N_u-1) & \end{array} \tag{20}$$

which is simply an upper-triangular matrix which can also be generated in MATLAB using $\mathcal{I} = \text{triu}(\text{ones}(N_l, N_l), 1)$. Additionally, since this simulation is interested in comparing a fully colocated broadcast channel (DPC and IWF) with a fully and equally distributed cooperative broadcast channel the AEF becomes

$$u_{k,m}(1) = k \tag{21}$$

which defines each cooperating node to have a single antenna. Care is taken to ensure both algorithms have equal power per channel use.

Fig. 2C is the dual multiple-access channel with a fixed number of transmitters (eight) and transmit antennas $N_{t,m} = 2$ while the number of receive antennas is swept. To keep this simulation from being redundant from the previous broadcast channel results, the G4M algorithm is setup to not use successive interference cancellation while IWF is allowed to do so. Expected results are shown; at a low number of receive antennas SIC is crucial and outperforms a link that only uses linear processing; however, as the number of antennas increases, interference cancellation is less important and G4M, with its interference list, approaches the IWF-SIC solution. Finally, Fig. 2D is the non-massive MIMO X channel with two transmitters simultaneously communicating independently with both receivers; transmitters have $N_{t,m} = 2$ antennas while receivers have $N_{r,n} = 3$. Though not a massive MIMO link, the topology was chosen to closely follow the interference alignment results from [27]. Note that the G4M performance lies between IA and the single-user MIMO (perfect cooperation) curves.

Fig. 3 shows convergence of G4M as characterized by the total number of beam updates (iterations) that must be performed before the algorithm converges. These simulations were run for a downlink channel using $N_{t,m} = 10$ transmit antennas, $N_{r,n} = 2$ receive antennas, $N_u = 10$ receivers, and capturing the current sum-rate at each iteration. G4M with

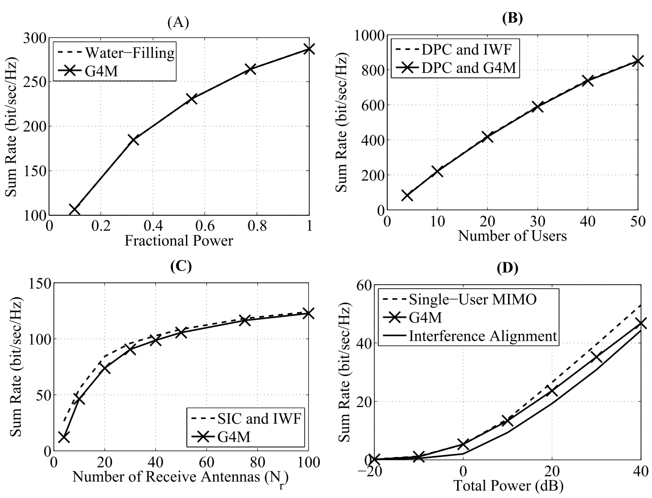

FIGURE 2. Comparison of G4M to various well-known topologies: (A) Single-user massive MIMO, (B) Downlink massive MIMO, (C) Uplink massive MIMO, and (D) MIMO X channels.

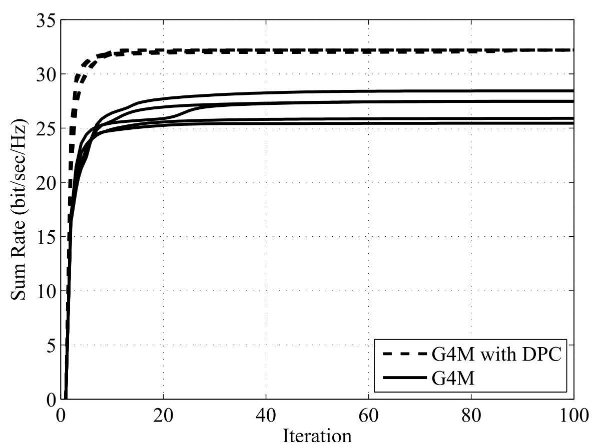

FIGURE 3. Typical convergence rate for the G4M algorithm. These simulations were run in a downlink channel using $N_{t,1} = 10$ and $N_{r,n} = 2$ antennas with the current sum-rate at each iteration shown. G4M with both nonlinear DPC and strict beamforming are shown for five different initial conditions each.

both nonlinear DPC and strict beamforming are shown. With DPC the interference superlist follows the form of (20) while with strictly linear beamforming the list becomes

$$\mathcal{I} = \begin{array}{c|l} \left|\begin{array}{ccccc} 0 & 1 & 1 & \dots & 1 \\ 1 & 0 & 1 & \dots & 1 \\ 1 & 1 & 0 & \dots & 1 \\ \vdots & \vdots & \vdots & \dots & \vdots \\ 1 & 1 & 1 & \dots & 0 \end{array}\right. & \begin{array}{c} (1,2) \\ (1,3) \\ (1,4) \\ \vdots \\ (1,11) \end{array} \\ (1,2)\ (1,3)\ (1,4)\ \dots\ (1,11) & \end{array} \tag{22}$$

which is a symmetric matrix with ones down the diagonal which can also be generated in MATLAB using $\mathcal{I} = \text{ones}(N_l, N_l)$ - eye(N_l).

The curves in Fig. 3 represent a single realization of the wireless downlink channel matrix with five different initial conditions for the G4M algorithm and different interference lists as described above. It it interesting to note the convergence behavior depending on the type of system used in the downlink channel. The nonconvexity of the optimization problem is evident with strict beamforming as demonstrated by convergence to local maxima. When DPC is used, the algorithm tends to converge quicker and is less dependent on the initial conditions. Though Fig. 3 is just a single snapshot of a particular channel, similar behavior is observed for different channel topologies.

B. HARDWARE CONSIDERATIONS

A key parameter that is often overlooked in massive antenna networks is the impact of hardware limitations. For example, with a sum-power constraint the power allocated to each antenna, by way of a power amplifier, can vary significantly as network capacity maximization is attempted via iterative water-filling. On the other hand, a per-antenna power constraint absolutely sets the power being used on each amplifier and more easily stays within its linear or non-distorting range. Fig. 4 shows the cumulative distribution functions (CDF) of the power allocation per antenna for various power constraints. Shown are CDFs of both $N_{t,1} = 10$ antennas with $N_n = 1$ (sum-power constraint) or $N_n = 5$ (two-antenna per-node power constraint) cooperating nodes and $N_{t,1} = 100$ antennas with $N_n = 1$ and $N_n = 50$ cooperating nodes. As a baseline the $N_n = N_{t,1}$ CDF is also shown. The AEF for all cases in this simulation can be written as

$$u_{k,m}(q) = (k-1) + q \tag{23}$$

where k ranges over all nodes and q ranges over the maximum number of antennas per node $\hat{N}_{t,k}$. For example, for the cooperative case, the seventh antenna on the virtual transmitter is found on the first antenna of the fourth cooperating node given that each node is contributing two antennas to the virtual link. Note that for a large number of antennas, the sum-power constraint can lead to large variation in the powers at each antenna that may not be achievable for practical amplifiers. With fewer antennas or many antennas with a per-antenna constraint, the variation in antenna power is far less drastic.

Another hardware consideration in massive networks is that all links between individual antennas may not be equivalent. For example, a single antenna may experience massive fading, occlusion due to environment, or antenna failure. In these cases the equivalent sum-rate seen between per-node and sum-power constraints will no longer hold; a single antenna cooperating node with an occluded antenna will completely lose the power contribution from that node while a node with sum-power constraint will simply reallocate power around that antenna. Fig. 5 shows the performance alterations as an increasing number of antennas fail for whatever reason. In this simulation a fully-loaded broadcast channel with $N_{t,1} = N_u = 10$ antennas and users and either $N_n = 1, 5, 10$ nodes cooperating on the transmit side. Note that cooperating nodes that are MIMO-enabled are able to compensate for antenna failure while single-antenna nodes immediately suffer from the loss; however, by reallocating power around failed antennas one places a larger burden on

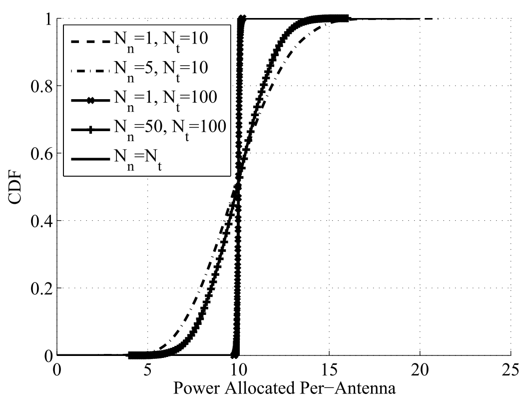

FIGURE 4. Cumulative distribution function (CDF) of powers assigned to antennas for various schemes. Shown are CDFs of both $N_{t,1} = 10$ antennas with $N_n = 1$ or $N_n = 5$ cooperating nodes and $N_{t,1} = 100$ antennas with $N_n = 1$ and $N_n = 50$ cooperating nodes. As a baseline the $N_n = N_{t,1}$ CDF is also shown; this is a step at the per-antenna power constraint.

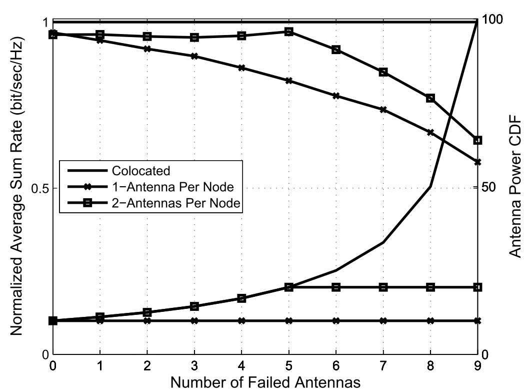

FIGURE 5. A fully-loaded broadcast channel with $N_{t,1} = N_u = 10$ antennas and either $N_n = 1, 5, 10$ nodes cooperating on the transmit side. Antenna "failures" cause the sum-rate to decrease (left y-axis) versus the colocated antenna link while the power distribution per antenna (right y-axis) increases more on colocated antennas.

the power amplifiers of antennas that are used as seen by the increased average power allocation.

C. ENERGY SAVINGS

An exciting possibility of massive MIMO is the return in energy savings due to tighter "beams" formed from the widely-separated antennas, meaning energy radiating from each antenna is steered more easily to the desired receiver thus increasing the received power and avoiding interference from/with others. This savings in power can potentially lead to more *green* networks in the future since more efficient beams lead to less required power due to less energy wasted in free-space. Fig. 6 shows the required power necessary to achieve a specific sum-rate as the number of transmit antennas is increased for G4M with and without DPC. This simulation begins with a nominal sum-rate for a $N_{t,1} = N_u = 10$ broadcast channel. Transmit antennas are then increased and the amount of power required to hit the nominal sum-rate is tabulated. Two results are worth emphasis. First, there are orders of magnitude in saving potential by moving to massive MIMO. Second, even nonlinear beamforming can drastically reduce the required power by simply exploiting the spatial freedom enabled by a massive MIMO system. Since not all networks are interested in simply maximizing throughput, the potential of massive MIMO becomes even more attractive for low-energy sensor or green networks.

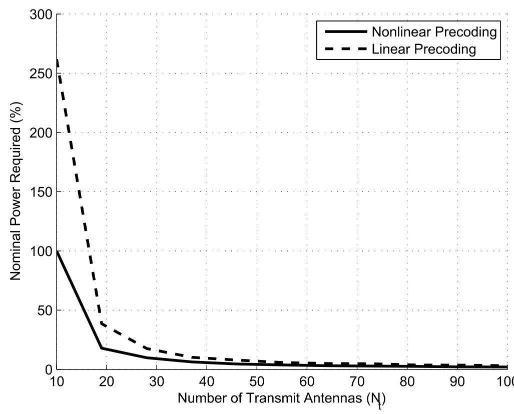

FIGURE 6. Energy savings due to massive antenna links. This simulation begins with a nominal sum-rate for a $N_{t,1} = N_u = 10$ broadcast channel. Transmit antennas are then increased and the amount of power required to hit a nominal sum-rate is tabulated. Massive networks, even with linear beamforming, can potentially result in huge energy savings once rate requirements are met.

D. SPATIAL GROWTH

The sum-rate growth as a function of the number of participating nodes is important for various reasons. In a colocated network, the network designer must understand how the number of antennas, and sum- or per-antenna power constraints affects the overall available throughput. Also, for a cooperative network, it is important to understand the benefits of adding single-antenna or MIMO nodes into the cooperation. Fig. 7 shows the spectral growth of the link when either one-,

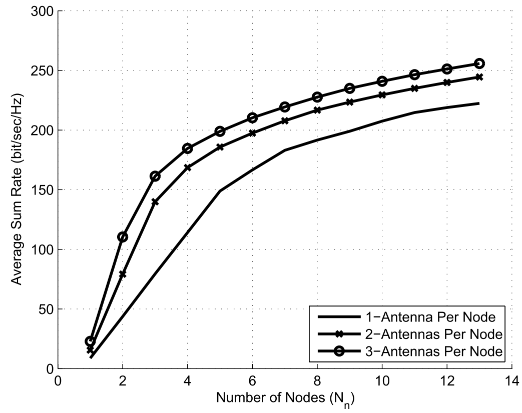

FIGURE 7. Performance improvement of cooperative links that add either one-, two-, or three-antenna nodes. The number of users is fixed at $N_u = 10$ while the total number of transmit antennas will be $N_{t,1} = N_n\hat{N}_{t,k}$.

two-, or three-antenna nodes are added in participation with the link. The number of users is fixed at $N_u = 10$ while the total number of transmit antennas will be $N_{t,1} = N_n\hat{N}_{t,k}$. There are two basic behaviors shown in this figure. For a limited number of antennas, cooperating with MIMO-enabled nodes results in a more impactful effect on the throughput. This is true until the degrees of freedom for the number of users are satisfied at which point adding nodes has a similar growth effect regardless of the number of antennas per node.

Spatial growth will help increase link throughput but the more massive a MIMO-enabled node becomes the greater the potential correlation between channels due to geographical constraints of fitting so many colocated antennas into a single hardware unit. Consider a scenario comparing a colocated antenna P2P link using any possible precoding and an uncorrelated cooperative P2P link using only linear precoding. (This scenario is more realistic since colocation allows greater flexibility in precoding methods.) Fig. 8 shows the loss in

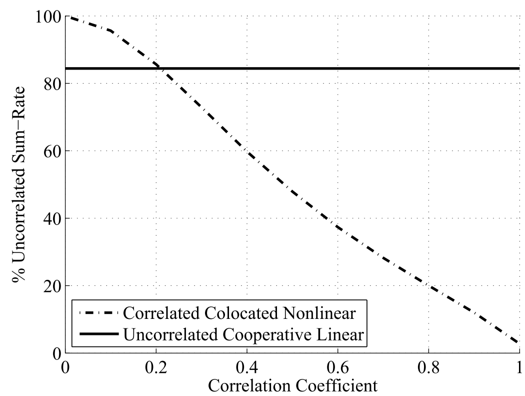

FIGURE 8. Performance loss as a large number of antennas ($N_{t,1} = N_{r,2} = 100$) become more spatially correlated. The correlated colocated curve represents the optimal (perhaps nonlinear) loss in performance as antenna correlation increases. The uncorrelated cooperative curve is for the G4M algorithm using linear processing and widely spaced, but cooperating, antennas.

performance for an $N_{t,1} = N_{r,2} = 100$ P2P link with a total power constraint of $P_T = 100$. To parameterize the channel correlation into a single value the method in [47] is used for both transmit and receive correlation. It is interesting to note that for even small values of correlation the advantages of tightly-packed colocated antennas can become quickly lost.

E. "DEAF" AND "SILENT" NETWORKS

As has been shown, one potential of massive networks is to mitigate the effects of interference caused by having many users accessing the wireless channel at the same time. Analogous to this potential is mitigating the effects of adjacent multiuser channels or networks. The behavior of the channel under consideration will be categorized into one of two types: deaf and silent. For purposes of these results, a deaf network is one that does not care about adjacent networks and simply tries to maximize its own sum-rate; this would be analogous to a "primary multiuser" in cognitive radio. A silent network is the opposite where the multiuser channel will communicate where spatially possible but cannot disrupt the adjacent network's sum-rate; this is analogous to a "secondary multiuser" in cognitive radio terminology.

The G4M framework provides a simple heuristic approach to analyzing these types of networks. For explanatory purposes, consider a simple small-scale interference list for a MIMO X or interfering broadcast channel. If entered into the G4M algorithm the maximum achievable sum-rate would be found by optimizing the beamforming vectors of both transmitters; however, for deaf and silent networks we assume that the adjoining network cannot be altered at all - their interference is as-is. Consider writing the interference list $\mathcal{I}_{(m,n)}$ as a matrix that contains the entry of 1 if the (m, n) sublink (row) receives interference from the (i, j) sublink (column). For deaf networks, we use the virtual interference superlist

$$\hat{\mathcal{I}} = \begin{array}{c} \left|\begin{array}{cccc} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right| \begin{array}{c} (1,3) \\ (1,4) \\ (2,3) \\ (2,4) \end{array} \\ \begin{array}{cccc} (1,3) & (1,4) & (2,3) & (2,4) \end{array} \end{array} \tag{24}$$

where users 1 and 2 are the transmitters and 3 and 4 are the receivers. Given this interference list the optimization will take place for a deaf network without consideration of the other multiuser channel. Similarly, for a silent network with the interference list

$$\hat{\mathcal{I}} = \begin{array}{c} \left|\begin{array}{cccc} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 \end{array}\right| \begin{array}{c} (1,3) \\ (1,4) \\ (2,3) \\ (2,4) \end{array} \\ \begin{array}{cccc} (1,3) & (1,4) & (2,3) & (2,4) \end{array} \end{array} \tag{25}$$

beams will be updated that only optimize the other channel by minimizing our own interference to that channel. The resulting sum-rate given these beamformer solutions is simply what can be achieved once the link is silent.

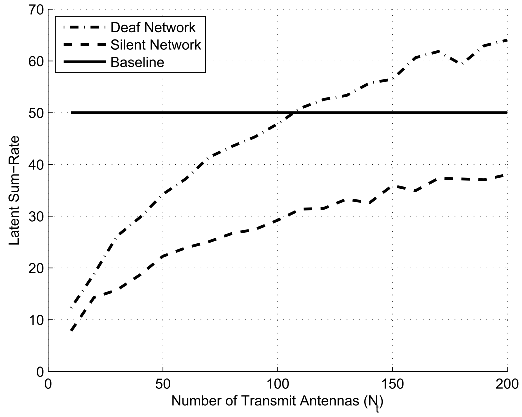

FIGURE 9. **Performance of "deaf" and "silent" networks as defined in Section IV-E. The massive broadcast channel in question with $N_u = 10$ users is adjacent to another network with total cross-channel interference. The baseline curve is the sum-rate with $N_u = N_{t,m} = 10$ and no interfering adjacent network.**

Figure 9 shows the performance of deaf and silent networks as a function of transmitting antennas. The massive broadcast channel in question with $N_u = 10$ users is adjacent to another network with total cross-channel interference. The baseline curve is the sum-rate with $N_u = N_{t,m} = 10$ and no interfering adjacent network. In both cases, with limited spatial freedom there are not many gains for either network. As the link becomes more massive the increase in available throughput is noteworthy; indeed the deaf network will outperform the baseline rate even though an adjacent network is interfering. Likewise, the silent network can achieve a significant throughput without interfering with the other network at all. Though this is just a heuristic look at such topologies the conclusions emphasize the gains of massive antenna networks and suggests future research directions.

V. CONCLUSION

A generalized multiuser massive MIMO maximization (G4M) algorithm has been extended to account for arbitrary distribution of participating nodes and number of antennas per node in a massive MIMO multiuser channel. This algorithm provides a simple framework to analyze the different parameters in such networks whether or not cooperation or other parameters are being considered. It is shown that the achievable rate of a multiuser channel with sum-power constraints is similar to that with per-node power constraints except in pathological cases such as antenna failure. Massive networks must take into account hardware considerations of nodes especially when sum-power optimizers are allowed to maximize link throughput. Massive networks also have application in both "green networks", where energy considerations are paramount and are vastly improved with the spatial antenna gain, and in cognitive-like scenarios where a multiuser link may want to be either "deaf" or "silent" depending on the network design.

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