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**Engineering Exotic Topological Matter and Complex Entanglement Patterns in Atomic,  
Molecular, and Optical Systems**

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MARYLAND UNIV COLLEGE PARK**

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**08/31/2018  
Final Report**

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# Final Report for “Engineering Exotic Topological Matter and Complex Entanglement Patterns in Atomic, Molecular, and Optical Systems”

Alexey V. Gorshkov

July 31, 2018

Although this is the final report for the entire award, the instructions say “please list any accomplishments that have been made since the last report submission”, so I list only the accomplishments made since the last report. The exceptions are the first five projects below, which were submitted in the previous reporting period but were published after the previous reporting period, so we repeat their discussion here for completeness.

The following projects have been completed:

1. Published the paper “**Fast State Transfer and Entanglement Renormalization Using Long-Range Interactions**” [1]. In short-range interacting systems, the speed at which entanglement can be established between two separated points is limited by a constant Lieb-Robinson velocity. Long-range interacting systems are capable of faster entanglement generation, but the degree of the speed-up possible is an open question. In this paper, we present a protocol capable of transferring a quantum state across a distance  $L$  in  $d$  dimensions using long-range interactions with strength bounded by  $1/r^\alpha$ . If  $\alpha < d$ , the state transfer time is asymptotically independent of  $L$ ; if  $\alpha = d$ , the time is logarithmic in distance  $L$ ; if  $d < \alpha < d+1$ , transfer occurs in time proportional to  $L^{\alpha-d}$ ; and if  $\alpha \geq d+1$ , it occurs in time proportional to  $L$ . We then use this protocol to upper bound the time required to create a state specified by a MERA (multiscale entanglement renormalization ansatz) tensor network, and show that, if the linear size of the MERA state is  $L$ , then it can be created in time that scales with  $L$  identically to state transfer up to multiplicative logarithmic corrections. Such MERA circuits are among the most promising approaches for quickly preparing topological states in AMO systems. However, MERA circuits for fractional Chern insulators have not yet been constructed - we are working on this. We also think that our fast state transfer and entanglement generation protocols are so promising for the case of dipoles ( $\alpha = 3$ ) in  $d = 3$  dimensions, that we applied for a patent for this protocol.
2. Published the paper “**Entanglement area laws for long-range interacting systems**” [2]. While the papers discussed in the previous two items provide protocols for fast entanglement generation in the presence of long-range interactions, it is also important to prove analytically that no faster protocols exist. This work is a step in this direction. We prove that the entanglement entropy of any state evolved under an arbitrary  $1/r^\alpha$  long-range-interacting  $d$ -dimensional lattice spin Hamiltonian cannot change faster than a rate proportional to the boundary area for any  $\alpha > d + 1$ . We also prove that for any  $\alpha > 2d + 2$ , the ground state of such a Hamiltonian satisfies the entanglement area law if it can be transformed along a gapped adiabatic path into a ground state known to satisfy the area law. These results significantly generalize their existing counterparts for short-range interacting systems, and are useful for identifying dynamical phase transitions and quantum phase transitions in the presence of long-range interactions. The area laws that we discuss are also known to be closely related to topological order [A. Kitaev and J. Preskill, Phys. Rev. Lett. 96, 110404 (2006)].
3. Published the paper “**Observation of Prethermalization in Long-Range Interacting Spin Chains**” [3]. This paper studies the effects of long-range interactions on long-time dynamics following a quantum quench. In particular, we show theoretically and, in collaboration with the group of Chris Monroe, experimentally that long-range interactions can sometimes

greatly delay the relaxation of a quantum system to thermal equilibrium by driving the system into a new type of prethermal state that retains a strong memory of the initial conditions for a long time.

4. Published the paper “**Spectrum estimation of density operators with alkaline-earth atoms**” [4]. If one treats atomic orbitals in a given trap as sites, then contact interactions between atoms in such a trap give rise to effective long-range interactions between such sites. Such long-range interactions allow one to entangle the atoms very efficiently as well as to measure their entanglement properties. In particular, in this paper, we show that Ramsey spectroscopy of fermionic alkaline-earth atoms in a square-well trap provides an efficient and accurate estimate for the eigenspectrum (equivalently entanglement spectrum) of a density matrix whose  $n$  copies are stored in the nuclear spins of  $n$  such atoms. This spectrum estimation is enabled by the high symmetry of the interaction Hamiltonian, dictated, in turn, by the decoupling of the nuclear spin from the electrons and by the shape of the square-well trap. Practical performance of this procedure and its potential applications to quantum computing, quantum simulation, and time-keeping with alkaline-earth atoms are discussed.
5. Published the paper “**Complexity of sampling as an order parameter**” [5]. While entanglement is a very popular and useful tool for studying the dynamics of quantum systems, it is interesting to consider whether other measures of the complexity of quantum states can be used. One candidate for such a measure is complexity of sampling: how difficult is it to imitate the experimental results (which are typically probabilistic in quantum mechanics and hence sample from some distribution) on a classical computer. In this paper, we consider the classical complexity of approximately simulating time evolution under spatially local quadratic bosonic Hamiltonians for time  $t$ . We obtain upper and lower bounds on the scaling of  $t$  with the number of bosons,  $n$ , for which simulation, cast as a sampling problem, is classically efficient and provably hard, respectively. We view these results in the light of classifying phases of physical systems based on parameters in the Hamiltonian and conjecture a link to dynamical phase transitions. In doing so, we combine ideas from mathematical physics and computational complexity to gain insight into the behavior of condensed matter systems. We are currently trying to understand whether complexity of sampling can be used to distinguish topological phases from trivial phases. We are also working on extending our results to long-ranged models.
6. Submitted and published the paper “**Lieb-Robinson bounds on  $n$ -partite connected correlation functions**” [6]. For pure quantum states, bi-partite connected correlations reveal the presence of bi-partite entanglement. Similarly, for pure states, multi-partite connected correlations reveal the presence of multi-partite entanglement. In this paper, we ask the question of how quickly one can generate multi-partite connected correlations in a quantum system. In particular, we prove bounds on this rate and demonstrate protocols that saturate various features of these bounds. This work is particularly timely given that current AMO experiments routinely measure such multi-partite connected correlations.
7. Submitted and published the paper “**Observation of a Many-Body Dynamical Phase Transition with a 53-Qubit Quantum Simulator**” [7]. This paper studies the effects of long-range interactions on system dynamics following a quantum quench. In particular, we demonstrate experimentally (in collaboration with the group of Chris Monroe) and explain theoretically a dynamical phase transition, i.e. a transition (as a function of system parameters) that can be observed only in dynamics.

8. Submitted the paper “**Scale-Invariant Continuous Entanglement Renormalization of a Chern Insulator**” [8]. Since AMO systems are generally isolated from the environment, they do not naturally relax to their ground states. Therefore, one needs to find ways to prepare such ground states. One of the goals of this AFOSR project was to construct an efficient quantum circuit (called a MERA) for the preparation of integer and fractional quantum Hall states. In this paper, we accomplished this goal for the case of an integer quantum Hall state. Although there have been earlier papers showing related constructions, our construction is particularly elegant in that the circuit is scale invariant and uses continuous (as opposed discrete) time evolution in continuous (as opposed to discrete) space. We also propose an experimental implementation of this circuit with alkaline-earth atoms.

## References

- [1] Eldredge, Z., Gong, Z.-X., Young, J. T., Moosavian, A. H., Foss-Feig, M., and Gorshkov, A. V. Fast quantum state transfer and entanglement renormalization using long-range interactions. *Phys. Rev. Lett.* **119**, 170503, Oct (2017).
- [2] Gong, Z.-X., Foss-Feig, M., Brandão, F. G. S. L., and Gorshkov, A. V. Entanglement area laws for long-range interacting systems. *Phys. Rev. Lett.* **119**, 050501 (2017).
- [3] Neyenhuis, B., Zhang, J., Hess, P. W., Smith, J., Lee, A. C., Richerme, P., Gong, Z.-X., Gorshkov, A. V., and Monroe, C. Observation of prethermalization in long-range interacting spin chains. *Sci. Adv.* **3**, e1700672 (2017).
- [4] Beverland, M. E., Haah, J., Alagic, G., Campbell, G. K., Rey, A. M., and Gorshkov, A. V. Spectrum estimation of density operators with alkaline-earth atoms. *Phys. Rev. Lett.* **120**, 025301 (2018).
- [5] Deshpande, A., Fefferman, B., Tran, M. C., Foss-Feig, M., and Gorshkov, A. V. Dynamical phase transitions in sampling complexity. *Phys. Rev. Lett.* **121**, 030501 (2018).
- [6] Tran, M. C., Garrison, J. R., Gong, Z.-X., and Gorshkov, A. V. Lieb-robinson bounds on  $n$ -partite connected correlation functions. *Phys. Rev. A* **96**, 052334 (2017).
- [7] Zhang, J., Pagano, G., Hess, P. W., Kyprianidis, A., Becker, P., Kaplan, H., Gorshkov, A. V., Gong, Z. X., and Monroe, C. Observation of a many-body dynamical phase transition with a 53-qubit quantum simulator. *Nature* **551**, 601 (2017).
- [8] Chu, S.-K., Zhu, G., Garrison, J. R., Eldredge, Z., Curiel, A. V., Bienias, P., Spielman, I. B., and Gorshkov, A. V. Scale-invariant continuous entanglement renormalization of a chern insulator. *submitted* (2018).

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**Abstract**

Following the instructions, I am listing only the accomplishments that have been made since the last report submission.

We pursued the project "Engineering Exotic Topological Matter and Complex Entanglement Patterns in Atomic, Molecular, and Optical Systems" along several directions. First, we have showed how long-range interactions can be used to quickly generate entanglement, including topologically relevant entanglement described by a MERA (multiscale entanglement renormalization ansatz). Second, we have proven area-law bounds on how quickly entanglement can be generated (and how much entanglement a given gapped ground state, such as a topological state, has) in the presence of long-range interactions. Third, we have studied the effect of long-range interactions on thermalization rates. Fourth, we have shown how effective long-range interactions in alkaline-earth atoms trapped in a single trap can be used to measure the entanglement spectrum of a density matrix stored in the nuclear spins of the atoms. Fifth, we have proposed to use complexity of sampling as a new order parameter signaling dynamical phase transitions,

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with potential applications to distinguishing topological states from trivial states. Sixth, we have studied how quickly multi-partite connected correlations (and hence multi-partite entanglement) can be generated. Seventh, we studied the effect of long-range interactions on dynamical phase transitions. Eighth, we constructed an elegant efficient circuit for the preparation of integer quantum Hall states.

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[1] Eldredge, Z., Gong, Z.-X., Young, J. T., Moosavian, A. H., Foss-Feig, M., and Gorshkov, A. V. Fast quantum state transfer and entanglement renormalization using long-range interactions. Phys. Rev. Lett. 119, 170503, Oct (2017).

[2] Gong, Z.-X., Foss-Feig, M., Brandao, F. G. S. L., and Gorshkov, A. V. Entanglement area laws for long-range interacting systems. Phys. Rev. Lett. 119, 050501 (2017).

[3] Neyenhuis, B., Zhang, J., Hess, P. W., Smith, J., Lee, A. C., Richerme, P., Gong, Z.-X., Gorshkov, A. V., and Monroe, C. Observation of prethermalization in long-range interacting spin chains. Sci. Adv. 3, e1700672 (2017).

[4] Beverland, M. E., Haah, J., Alagic, G., Campbell, G. K., Rey, A. M., and Gorshkov, A. V. Spectrum estimation of density operators with alkaline-earth atoms. Phys. Rev. Lett. 120, 025301 (2018).

[5] Deshpande, A., Fefferman, B., Tran, M. C., Foss-Feig, M., and Gorshkov, A. V. Dynamical phase transitions in sampling complexity. Phys. Rev. Lett. 121, 030501 (2018).

Submitted during this reporting period:

[6] Tran, M. C., Garrison, J. R., Gong, Z.-X., and Gorshkov, A. V. Lieb-Robinson bounds on n-partite connected correlation functions. Phys. Rev. A 96, 052334 (2017).

[7] Zhang, J., Pagano, G., Hess, P. W., Kyprianidis, A., Becker, P., Kaplan, H., Gorshkov, A. V., Gong, Z. X., and Monroe, C. Observation of a many-body dynamical phase transition with a 53-qubit quantum simulator. Nature 551, 601 (2017).

[8] Chu, S.-K., Zhu, G., Garrison, J. R., Eldredge, Z., Curiel, A. V., Bienias, P., Spielman, I. B., and Gorshkov, A. V. Scale-invariant continuous entanglement renormalization of a chern insulator. submitted (2018).

### **New discoveries, inventions, or patent disclosures:**

**Do you have any discoveries, inventions, or patent disclosures to report for this period?**

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**Please describe and include any notable dates**

Applied for a patent on Ref [1] above. U.S. Patent Application 15/802,146, filed Nov 2, 2017; International Patent Application PCT/US2017/063892, filed Nov 30, 2017.

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