REPORT DOCUMENTATION PAGE

Form Approved OMB NO. 0704-0188

1. REPORT DATE (DD-MM-YYYY)	2. REPORT TYPE	3. DATES COVERED (From - To)
31-10-2016	Final Report	16-Sep-2011 - 15-Sep-2016

4. TITLE AND SUBTITLE
Final Report: Studies in Structural, Stochastic and Statistical Reliability for Communication Networks and Engineered Systems

5a. CONTRACT NUMBER
W911NF-11-1-0428

5b. GRANT NUMBER

5c. PROGRAM ELEMENT NUMBER
611102

6. AUTHORS
Francisco J. Samaniego

5d. PROJECT NUMBER

5e. TASK NUMBER

5f. WORK UNIT NUMBER

7. PERFORMING ORGANIZATION NAMES AND ADDRESSES
University of California - Davis
Sponsored Programs
1850 Research Park Drive, Suite 300
Davis, CA 95618 -6153

8. PERFORMING ORGANIZATION REPORT NUMBER

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS (ES)
U.S. Army Research Office
P.O. Box 12211
Research Triangle Park, NC 27709-2211

10. SPONSOR/MONITOR'S ACRONYM(S)
ARO

11. SPONSOR/MONITOR'S REPORT NUMBER(S)
59131-MA.1

12. DISTRIBUTION AVAILIBILITY STATEMENT
Approved for Public Release; Distribution Unlimited

13. SUPPLEMENTARY NOTES
The views, opinions and/or findings contained in this report are those of the author(s) and should not contrued as an official Department of the Army position, policy or decision, unless so designated by other documentation.

14. ABSTRACT
The present report describes the research completed by Principal Investigator F. J. Samaniego under the support of ARO grant W911NF-11-1-0428 from 9/16/11 to 9/15/16. Research results in five topical areas within the field of Engineering Reliability are summarized. Contributions include: (1) new results are obtained on the treatment of repairable systems in which the theory of system signatures is extended to minimally repaired systems and new results are obtained for comparing pairs of used systems, (2) conditions are identified under which systems of different sizes in i.i.d components will have the same stochastic performance; such a result is essential in comparing

15. SUBJECT TERMS
reliability, systems, networks, system signatures, minimal repair, optimality, nonparametric estimation, asymptotics

16. SECURITY CLASSIFICATION OF: a. REPORT	b. ABSTRACT	c. THIS PAGE	17. LIMITATION OF ABSTRACT	15. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
UU	UU	UU	UU		Francisco Samaniego
					19b. TELEPHONE NUMBER
					916-698-7669

Standard Form 298 (Rev 8/98)
Prescribed by ANSI Std. Z39.18

Report Title

Final Report: Studies in Structural, Stochastic and Statistical Reliability for Communication Networks and Engineered Systems

ABSTRACT

The present report describes the research completed by Principal Investigator F. J. Samaniego under the support of ARO grant W911NF-11-1-0428 from 9/16/11 to 9/15/16. Research results in five topical areas within the field of Engineering Reliability are summarized. Contributions include: (1) new results are obtained on the treatment of repairable systems in which the theory of system signatures is extended to minimally repaired systems and new results are obtained for comparing pairs of used systems, (2) conditions are identified under which systems of different sizes in i.i.d components will have the same stochastic performance; such a result is essential in comparing the performance of two coherent systems of arbitrary sizes, (3) the notion of "survival signature" is applied to the comparison of the performance of systems with heterogeneous components, thereby relaxing the i.i.d assumption made on the components of coherent systems that is required for comparisons based on system signatures, (4) in comparisons among communication networks of a given size (with v vertices and n edges), new results are obtained showing that the stochastic precedence ordering (spo) can provide a total ordering of these networks with respect to the probability of total connectivity in situations where the traditional stochastic ordering (sto) fails to do so, and (5) three distinct versions of the problem of estimating component reliability from system failure-time data are treated, each resulting in consistent estimators with asymptotically normal distributions.

Enter List of papers submitted or published that acknowledge ARO support from the start of the project to the date of this printing. List the papers, including journal references, in the following categories:

(a) Papers published in peer-reviewed journals (N/A for none)

Received   Paper

TOTAL:

Number of Papers published in peer-reviewed journals:

(b) Papers published in non-peer-reviewed journals (N/A for none)

Received   Paper

TOTAL:

Number of Papers published in non peer-reviewed journals:

(c) Presentations

Number of Presentations: 0.00

Non Peer-Reviewed Conference Proceeding publications (other than abstracts):

Received Paper

TOTAL:

Number of Non Peer-Reviewed Conference Proceeding publications (other than abstracts):

Peer-Reviewed Conference Proceeding publications (other than abstracts):

Received Paper

TOTAL:

Number of Peer-Reviewed Conference Proceeding publications (other than abstracts):

(d) Manuscripts

Received Paper

TOTAL:

Number of Manuscripts:

Books

Received Book

TOTAL:

Received	Book Chapter

TOTAL:

Patents Submitted

Patents Awarded

Awards

Professor Samaniego served as the Faculty Marshall at the 2016 Graduation Ceremony of the College of Letters and Science at the University of California, Davis. This honor is traditionally bestowed upon a faculty member with a distinguished record of published research and a record of outstanding teaching and other contributions to the College and the University. In presenting Dr. Samaniego at the Ceremony, the Dean of the College pointed out his publication record, which includes three edited volumes, two research monographs, an upper-division textbook, and over 120 research articles in highly regarded Statistics journals. It was noted that he also received the Academic Senate Distinguished Teaching in 2002 and the Davis Prize of Undergraduate Teaching and Scholarly Achievement in 2004, and that he served as Theory and Methods Editor of the ASA's flagship journal, the Journal of the American Statistical Association (JASA).

Graduate Students

NAME	PERCENT_SUPPORTED	Discipline
Yin Jin	1.00	
FTE Equivalent:	**1.00**	
Total Number:	**1**	

Names of Post Doctorates

NAME	PERCENT_SUPPORTED
FTE Equivalent:	
Total Number:	

Names of Faculty Supported

NAME	PERCENT_SUPPORTED
FTE Equivalent:	
Total Number:	

Names of Under Graduate students supported

NAME	PERCENT_SUPPORTED	Discipline
Yifei Wang	0.00	BS -- Statistics -- 2016
FTE Equivalent:	**0.00**	
Total Number:	**1**	

Student Metrics

This section only applies to graduating undergraduates supported by this agreement in this reporting period

The number of undergraduates funded by this agreement who graduated during this period: 1.00

The number of undergraduates funded by this agreement who graduated during this period with a degree in science, mathematics, engineering, or technology fields:...... 1.00

The number of undergraduates funded by your agreement who graduated during this period and will continue to pursue a graduate or Ph.D. degree in science, mathematics, engineering, or technology fields:...... 1.00

Number of graduating undergraduates who achieved a 3.5 GPA to 4.0 (4.0 max scale):...... 1.00

Number of graduating undergraduates funded by a DoD funded Center of Excellence grant for Education, Research and Engineering:...... 0.00

The number of undergraduates funded by your agreement who graduated during this period and intend to work for the Department of Defense 0.00

The number of undergraduates funded by your agreement who graduated during this period and will receive scholarships or fellowships for further studies in science, mathematics, engineering or technology fields: 0.00

Names of Personnel receiving masters degrees

NAME	
Zihe Liu -- 2015 - 2016	
Total Number:	**1**

Names of personnel receiving PHDs

NAME	
Yin Jin	
Total Number:	**1**

Names of other research staff

NAME	PERCENT_SUPPORTED
FTE Equivalent:	
Total Number:	

Sub Contractors (DD882)

Inventions (DD882)

Scientific Progress

See Attachment

Technology Transfer

<u>Final Report on ARO grant W911NF-11-1-0428 (proposal # 59131-MA)</u>

September 16, 2011 - September15, 2016

Francisco J. Samaniego, Principal Investigator

Executive Summary. The present report describes the research completed by Principal Investigator F. J. Samaniego under the support of ARO grant W911NF-11-1-0428 from 9/16/11 to 9/15/16. Research results in five topical areas within the field of Engineering Reliability are summarized. Contributions include: (1) new results are obtained on the treatment of repairable systems in which the theory of system signatures is extended to minimally repaired systems and new results are obtained for comparing pairs of used systems, (2) conditions are identified under which systems of different sizes in i.i.d components will have the same stochastic performance; such a result is essential in comparing the performance of two coherent systems of arbitrary sizes, (3) the notion of "survival signature" is applied to the comparison of the performance of systems with heterogeneous components, thereby relaxing the i.i.d assumption made on the components of coherent systems that is required for comparisons based on system signatures, (4) in comparisons among communication networks of a given size (with v vertices and n edges), new results are obtained showing that the stochastic precedence ordering (spo) can provide a total ordering of these networks with respect to the probability of total connectivity in situations where the traditional stochastic ordering (sto) fails to do so, and (5) three distinct versions of the problem of estimating component reliability from system failure-time data are treated, each resulting in consistent estimators with asymptotically normal distributions.

Principle Investigator F. J. Samaniego, under the support of grant W911NF-11-1-0428 from the Army Research Office (and the grant's extension through September 15, 2016) has carried out a program of research in the areas of structural, stochastic and statistical reliability theory for systems and networks. The present document provides a summary of the research accomplished during the five-year period 9/16/11 – 9/15/16. This research includes new results on comparisons among repairable systems, equivalence among systems of varying sizes, comparisons among systems can be divided into the following five topical areas: A) extensions of system signatures to repairable systems, B) the characterization of systems of different sizes having equivalent stochastic performance under an i.i.d. assumption on component lifetimes, C) extensions of system signatures to systems with independent heterogeneous components, D) a comparative analysis of the performance of communication networks of varying design and E) three distinct versions the statistical estimation of the common reliability of system components based on system failure-time data.

1. The Signature of a System Under Minimal Repair.

The signature of a coherent system with independent and identically distributed component lifetimes is a useful tool in the study and comparison of lifetimes of systems (see Samaniego (1985, 2007)).The signature of a coherent system with n components is a vector whose k^{th} element is the probability that the k^{th} component failure is fatal to the system; it is known that it is a distribution-free measure of the design of the system. The notion of dynamic signature was introduced by Samaniego et al. (2009). In the dynamic setting, one considers a system which is inspected at some time t and found to be working, with k, say, failed components. The dynamic signature is then the signature of the system consisting of the n - k functioning components following the inspection. Lindqvist and Samaniego (2015) consider the situation when a system failure is observed; it is assumed that a minimal repair is performed and that the information available consists of the time of failure t and the number k of failed components at failure.

The PI and his collaborator develop and describe the probabilistic mechanism of the failure process following the failure and minimal repair, conditioning on the available information. The signature of the system, after repair, is referred to as the conditional dynamic signature; it is similar to the dynamic signature, and the two can be computed from the same set of residual signatures. Several examples suggest that the dynamic signature tends to dominate the conditional dynamic signature stochastically under comparable scenarios. A condition which guarantees this domination is presented. See Lindqvist and Samaniego (2015) for details.

2. On the Equivalence of Systems of Different Sizes.

The signature of a coherent system with independent and identically distributed component lifetimes has been found to be a useful tool in the study and comparison of lifetimes of engineered systems. A key result in the theory of system signatures is the representation of a system's survival distribution in terms of its signature vector (see Samaniego (1985, 2007)), which leads to several results on stochastic comparison of system lifetimes. In order to compare two coherent systems of different sizes with respect to their signatures, the smaller system needs to be represented by an equivalent system of the same size as the larger system. Here equivalence between systems means that their lifetime distributions are identical for any component distribution.

While such equivalent systems are usually represented as mixtures of coherent systems (so-called mixed systems), Lindqvist, Samaniego and Husbey (2016) demonstrate that they can be obtained in a simpler fashion by the addition of irrelevant components to the smaller system, thereby obtaining monotone systems of larger dimension but with equivalent performance to that of the original system. In addition to making the formulas for signatures of equivalent systems more transparent, the new representation lends to insights into the interpretation of mixed systems. The "opposite" problem of whether, for a given mixed system, there exist equivalent systems of smaller sizes is also considered. While there is always an equivalent mixed system of larger size, there need not be equivalent systems of smaller sizes, but efficiency and economy is well served when such systems are found to exist.

Finally, the search for equivalence of systems of different sizes is narrowed to the class of coherent systems. A sufficient condition for equivalence of coherent systems of sizes respectively n and n+1, for general n, is given, and as a special case, it follows that any k-out-of-n -system with $1 < k < n$ has an equivalent coherent system of size $n + 1$. The proof is based on first adding an irrelevant component to the smaller system, and then obtaining an equivalent coherent system by manipulating the minimal cut sets of the original system. The aforementioned paper on this work has been published in *Advances in Applied Probability*.

3. On Comparing Coherent Systems with Heterogeneous Components

A brief review of system signatures will be helpful in what follows. Samaniego (1985) introduced the signature of a coherent or mixed system as an alternative index to the structure function. While narrower in scope than the structure function, Samaniego demonstrated that a system's signature was substantially more useful. Assuming that the lifetimes of the system's components are independent and identically distributed (i.i.d.), the signature **s** of a coherent system of order n is the n-dimensional probability vector whose i^{th} element is $s_i = P(T = X_{i:n})$, where T is the system lifetime and $X_{1:n}, \ldots, X_{n:n}$ are the order statistics of the n i.i.d. component lifetimes. Under the i.i.d. assumption, the signature vector is a distribution-free function that constitutes a pure measure of the system's design. The assumption has the effect of "leveling the playing field" among systems, eliminating anomalies like the fact that a series system in good components can outperform a parallel system in poor components. The utility of signatures derives from the fact that combinatorial mathematics is applicable in their computation and the theory of order statistics for i.i.d. samples from a common continuous distribution F is applicable for identifying a system's lifetime characteristics as a function of the system's signature. Some examples of signatures: the vector **s** = (1/3, 2/3, 0) is the signature of the 3-component coherent system with minimal cut sets {1} and {2, 3}, the vector **s** = (0, 1/5, 3/5, 1/5, 0) is the signature of the widely cited 5-component bridge system and the vector **s** = $(0,..,0, 1_k , 0,.., 0) \in [0, 1]^n$ is the signature of the k-out-of-n system (which fails upon the k^{th} component failure). For details, see Samaniego (2007).

The signature vector **s** of a given n-component system, as defined above, has the very desirable and useful property of being distribution-free under the assumption that the system's components have i.i.d. lifetimes. On the other hand, when the component lifetime distributions vary, the vector **s** with i^{th} element $s_i = P(T = X_{(i)})$ for i = 1,…, n will, in general, depend on the underlying component distributions. This fact renders this particular metric inappropriate for extending the signature-based representation theorem for system reliability established by Samaniego (1985) and the preservation theorems obtained by Kochar, Mukerjee and Samaniego (1999) that identified the linkage between properties of signatures and properties of the corresponding systems under the i.i.d assumption. Indeed, it has not been clear, until very recently, that the notion of system signatures could be generalized to apply to the case of independent heterogeneous components. The paper by Coolen and Coolen-Maturi (2012) represents an important advance in the latter area. These authors defined a new metric, the system's "survival signature", which generalizes signatures to systems with heterogeneous components, is distribution-free and depends only on a system's design. This metric is defined as follows.

Definition: Consider an m-component system with components of K different types. Suppose that the system has m_k components of type k, where k = 1,…, K. Assume that the lifetimes of components of the same type are exchangeable and that the lifetimes of components of different types are independent. Then the *survival signature* of the system is a nonnegative function Φ of K variables, where $\Phi(l_1, l_2,\dots, l_K)$ for $l_k = 1,\dots, m_k$ and k = 1,…, K, represents the probability that the system works when precisely l_k components of type k are working for k = 1,…, K.

Coolen and Coolen-Maturi (2012) discussed the calculation of the survival signature under the conditions stated above, and showed that Φ does not depend on the component distributions. Further, they showed that, under the stronger assumption that, for k = 1,…, K, components of type k have i.i.d. lifetimes with common distribution F_k, the system's reliability function could be obtained via the following representation result:

$$P(T > t) = \sum_{l_1=0}^{m_1} \dots \sum_{l_K}^{m_K} \Phi(l_1, \dots, l_K) \prod_{k=1}^{K} \binom{m_k}{l_k} \left[F_k(t)\right]^{m_k - l_k} \left[1 - F_k(t)\right]^{l_k}. \qquad (1)$$

In Samaniego and Navarro (2016), the representation in (1) above is shown to be a useful tool in the comparison of systems. For example, given two systems with components of several types, results are obtained which provide conditions on the distributions of different types of components under which stochastic domination of one system over another will hold. In order to be in a position to compare two systems of arbitrary sizes, Samaniego and Navarro (2016) derived a recursive relationship between the survival signature of a given system in independent, heterogeneous components and any larger system in which the original system is embedded.

The aforementioned paper also treats the comparison of systems with heterogeneous components using an alternative, complementary tool – that of generalized distortion functions. If T is the lifetime of a coherent system (or a stochastic mixture of such – a so-called "mixed system") based on independent, heterogeneous components of K types with respective reliability functions $\overline{F}_i, i = 1, \dots, K$, then it is known that the system reliability function can be written as

$$\overline{F}_T(t) = \overline{Q}(\overline{F}_1(t), \dots, \overline{F}_K(t)), \qquad (2)$$

where $\overline{Q}$ is a continuous, decreasing function independent of { $\overline{F}_i, i = 1, \dots, K$ } and satisfying $\overline{Q}$ (0,…, 0) = 0 and $\overline{Q}$ (1,…, 1) = 1. The function $\overline{Q}$ is known in the literature as a "dual distortion function".

The representation in (2) above is utilized by Samaniego and Navarro (2016) for comparing two systems with independent heterogeneous components. In addition to results stating sufficient conditions of the component reliability for one system to dominate another in a stochastic sense, Samaniego and Navarro obtain a number a number of necessary and sufficient conditions for one system to outperform another. Examples of such results are given in the following theorem.

Theorem 3.1. Suppose that two coherent of mixed systems with lifetimes T_1 and T_2 have independent, heterogeneous components of K types with respective reliability functions $\overline{F}_i, i = 1, \dots, K$. Let $\overline{Q}_1$ and $\overline{Q}_2$ be the respective dual distortion functions satisfying equation (2). Then

(i) $T_1 \leq_{st} T_2$ holds for all $\bar{F}_1(t),\ldots,\bar{F}_K(t)$ if and only if $\bar{Q}_1 \leq \bar{Q}_2$ on the domain $(0, 1)^K$, and

(ii) $T_1 \leq_{hr} T_2$ holds for all $\bar{F}_1(t),\ldots,\bar{F}_K(t)$ if and only if $\bar{Q}_2 / \bar{Q}_1$ is decreasing on $(0, 1)^K$,

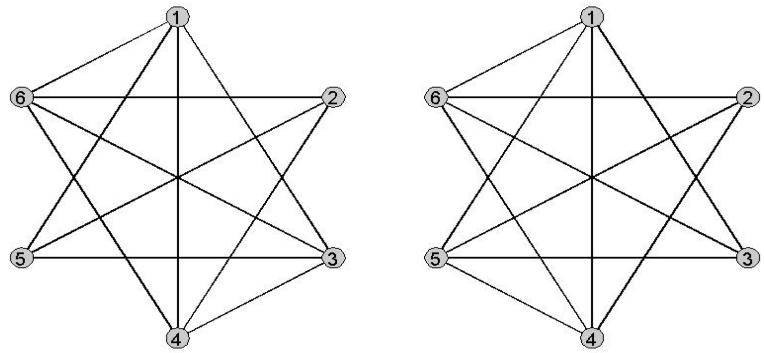
where $\leq_{st}$ stands for stochastic ordering and $\leq_{hr}$ stands for hazard rate ordering. Similar results are obtained for likelihood ratio ordering.

The two methods featured in Samaniego and Navarro (2016) provide two new, complementary tools for carrying out a comparative analysis of pairs of systems with independent, heterogeneous components. Further, using the distortion-function approach, it is shown that certain graphical procedures may be used to shed light on system comparisons when analytical results prove intractable.

4. On the Reliability of Communication Networks.

The search for Uniformly Optimal Networks (UONs) among networks G(v, n) of a given size was pioneered by Boesch, Li and Suffel (Networks, 1991). For example, they identified the unique UON among networks in the G(v, v – 1), G(v, v), G(v, v + 1) and G(v, v + 2) classes, where G(A, B) represents a communications network (i. e., an undirected graph) with A vertices and B edges. The UON in the G(v, v + 3) class was later identified by Wang (1994). Letting p = $\bar{F}(t_0)$, where F is the common lifetime distribution of the network's i.i.d. edges, these investigators identified the network G* which satisfies the inequality, for every network G in the class of interest, $P_{G^*}(T^* > t_0) = h_{G^*}(p) \geq h_G(p) = P_G(T > t_0)$ for all p $\in$ [0, 1], where T and T* represent network lifetimes and h_{G^*} is the reliability polynomial of the network C.. Equivalently, they demonstrated that $T \leq_{st} T^*$ for any lifetime T corresponding to a network in the class.

However, Myrvold, Cheung, Page and Perry (Networks, 1991) provided a collection of examples showing that for some classes of networks, e.g., the class $G\left(v, \binom{v}{2} - v/2 - 1\right)$ for any even value $v \geq 6$, a UON did not exist. They demonstrated the existence of a network in each class which dominated every other network in the class for p sufficiently large, but was inferior to an alternative network for p sufficiently small. A pair of noncomparable G(6, 11) networks, G_8 (good for small p) and G_9 (good for large p), are pictured in Figure 1 below.



G_8 Network $\qquad$ G_9 Network

Figure 1: Two G(6, 11) networks

The Myrvold et al. paper all but squelched the vigorous research that had been focusing on the identification of UON networks. The new challenges now facing researchers in this area included the problem of characterizing the collection of network families, that is (v, n) pairs, for which a universally optimal network exists and then describing a method for identifying the UON in any such family. These problems have remained open for some 20 years, and they are regarded (by most researchers) to be intractable.

Could it be that stochastic ordering is too strong a criterion to expect uniform optimality of a single member of a class G(v, n)? McAssey and Samaniego (2014) showed that, within the class of G(6,11) networks (that is, within the Myrvold et al. class with v = 6), a class which has 1365 possible network designs, but a class for which for each network has one of 9 distinct signatures. While it is known that these signatures are not totally ordered with respect to "stochastic ordering", McAssey and Samaniego (2014) showed that they are in fact totally ordered with respect to the "stochastic precedence" ordering -- defined as $X \leq_{sp} Y$ if and only if $P(X < Y) \geq P(X < Y)$. The G(6, 11) network referred to as G_9 in Figure 1 above was shown to be the uniformly optimal network in the G(6, 11) class relative to the sp ordering. In the latter ordering, the signature vector of the network G_9 dominates the signatures of all other networks of its size, and the lifetime of this network is larger than that of any other G(6, 11) network in the sp sense. The significance of this finding is that it shows that, while a uniformly optimal network relative to the stochastic ordering criterion may not exist in a the class of networks of a given size, definitive comparisons may be made relative to an alternative criterion, namely that of stochastic precedence. McAssey and Samaniego's study of the class of G(6, 11) networks effectively reopened the door to the search for uniformly optimal networks in the class G(v, n) for arbitrary v and n, demonstrating that the stochastic ordering is an unduly severe requirement when comparing the performance of communication networks, and that the less severe stochastic precedence ordering is better suited to the comparison of networks of any given size.

5. Inference about Component Reliability based on System Failure-time Data.

PI Samaniego has been working on various versions of the important problem of estimating component behavior from system failure-time data for several years. The applicability and relevance of this work stems from the fact that data on fielded systems very rarely contains information of the performance of individual components but, instead, typically consists of failure time data on the system's themselves. Solving the inverse problem of making inferences about component behavior from system failure-time data is often the only way to estimate the performance of a system'scomponents under field conditions. The material reviewed below summarizes the PI's work in this problem area as a whole. First, the basic formulation of the problem which was treated in Bhattacharya and Samaniego (2010) will be described. This work is based on data from sampled failure times from systems of a single type. The solution to the more complex problem of estimating the underlying component reliability based on failure-time data from systems of varying design is then described. The work was published in Hall, Jin and Samaniego (2015). The third version of this estimation problem deals with estimating component reliability based on failure-time data from a system of *unknown design*. This work has been completed during the present year of ARO support and is presented in Jin, Hall, Jiang and Samaniego (2016), a paper that is presently in press in the journal *Statistica Sinica*. The following paragraphs provide some details on the motivation, technical findings and envisioned applications of the three versions of the problem treated in our recent work.

The main goal of the PI's research on the estimation of component reliability from system failure-time data is to identify a asymptotically consistent estimator of the common unknown component reliability function $\bar{F}(x)$ in each of the cases above and to establish each estimator's asymptotic properties. The primary challenges in the work were mathematical in nature, as the three problems we have treated are non-standard estimation problems which seek to estimate the performance of a parameter $\bar{F}$ on which there is no direct data.

As noted above, this work applies in industrial settings in which component behavior must be inferred from the performance of fielded systems. Such settings are often encountered in the acquisitions program of the U. S. Army. The solution presented for the third scenario mentioned in the first paragraph of this Section is motivated by military considerations involving captured enemy systems of unknown design. The resulting estimator of component reliability is shown have excellent large-sample properties and, also, to provide defensible estimation with samples of moderate size.

The main results obtained for each of the three versions of the problem of estimating component reliability are summarized below.

I. Estimating $\bar{F}$ (t) from a Sample of System Failure Times.

- Given a random sample $X_1,\ldots, X_N$ of system lifetimes, the *empirical distribution* $\hat{\bar{F}}_{T,N}(t)$ based on these data is a consistent and asymptotically normal estimator of the system reliability function.
- It is well known that the component and system reliability functions R(t) and R*(t) are explicitly related via the equation R*(t) = h(R(t)), where h is the system's reliability polynomial.
- The Nonparametric Maximum Likelihood Estimator (NPMLE) of R*(t) is obtained by inverting this relationship above.
- In Bhattacharya and Samaniego (2010), the asymptotic distribution of the NPMLE of R(t) is identified in closed form. This result may be used to provide both point estimates and approximate confidence intervals for R(t) at any t > 0.

II. Estimating $\bar{F}$ (t) from the Failure Times of Systems of Varying Design.

- Given m systems whose components have i.i.d. lifetimes with reliability function $\bar{F}$ (t), f random sample of failure times is available on each system.
- Hall, Jin and Samaniego (2016) provide a consistent and asymptotically normal estimator of $\bar{F}(t)$ and show that it is asymptotically superior to all mixture estimators based on the separate NPMLEs of individual component reliability functions.

III. Estimating Component Reliability Based on Lifetime Data from a System of Unknown Design.

- In military conflicts, captured systems are usually of unknown design. In Jin, Hall, Jiang
- and Samaniego (2016), the results above are extended to such scenarios.
- It is assumed that the available data consists of pairs (T, K), where T is the failure time of a given system and K is the number of failed components at the time of system failure. The estimator of R(t) constructed in Jin, Hall, Jiang and Samaniego (2016) has been shown to be consistent and have an asymptotically normal distribution. Further, the estimator based of the data {(T, K)} is shown to be asymptotically superior to the estimator based on the known signature **s** and the failure-time data alone.

References

Bhattacharya, D. and Samaniego, F. J. (2010) On estimating component reliability based of system failure-time data, *Naval Research Logistics,* **57**, 380 – 89.

Boesch, F. T., Li, X. and Suffel, C. (1991) On the existence of uniformly optimal reliability networks, *Networks*, 21, 181 – 91.

Colbourn, C. J. (1987) *The Combinatorics of Network Reliability*, Oxford: Oxford University Press

Coolen, F.P.A and Coolen, T. (2012) On Generalizing the Signature of Systems with Multiple Types of Components, in *Complex Systems and Dependability*, eds. W. Samojski et al., Berlin: Springer, 115 – 130.

Hall, P., Jin, Y. and Samaniego, F. J. (2015) On nonparametric estimation of component reliability based on failure-time data from systems of varying design, *Statistica Sinica*, **25**, 1313 – 1335.

Jin Y., Hall, P., Jiang J. and Samaniego, F. J. (2016) Estimating Component Reliability Based on Lifetime Data from a System of Unknown Design, *Statistica Sinica*, in press.

Kochar, S., Mukerjee, H. and Samaniego, F. J. (1999) The 'signature' of a coherent system and its applications to comparisons among systems, *Naval Research Logistics*, **46**, 507 – 23.

Lindqvist and Samaniego (2015) On the signature of a system under minimal repair, *Applied Stochastic Models in Business and Industry,* **31**, 297 – 306.

Lindqvist, Samaniego and Huseby (2016) On the equivalence of systems of different sizes, *Advances in Applied Probability,* **48**, 332 – 348.

McAssey, M. and Samaniego, F. J. (2014) On Universally Optimal Networks: A Reversal of Fortune?, *Communications in Statistics –Theory and Methods*, **43**, 2452 – 2467

Myrvold, W., Cheung, K. H., Page, L. B. and Perry, J. E. (1991) Uniformly most reliable networks do not always exist, *Networks*, **21**, 417 – 19.

Navarro, J., Samaniego, F. J. and Balakrishnan, N, (2013) Mixture representations for the joint distribution of lifetimes of two coherent systems with shared components, *Advances in Applied Probability*, **45**, 1011-27.

Samaniego, F. J. (1985). On the closure of the IFR class under the formation of coherent systems, *IEEE Transactions on Reliability*, TR-34: 69–72.

Samaniego, F. J. (2007). *System Signatures and their Applications in Engineering Reliability*, New York: Springer.

Samaniego, F. J., Balakrishnan, N. and Navarro, J. (2009) Dynamic signatures and their use in the comparison of new and used systems, *Naval Research Logistics,***56**, 577 – 91.

Samaniego, F. J. and Navarro, J. (2016) On Comparing Coherent Systems with Heterogeneous Components, *Advances in Applied Probability*, Advances in Applied Probability, **48**, 88 – 111.

Wang, G. (1994) A proof of Boesch's conjecture, *Networks*, **24**, 277 - 84